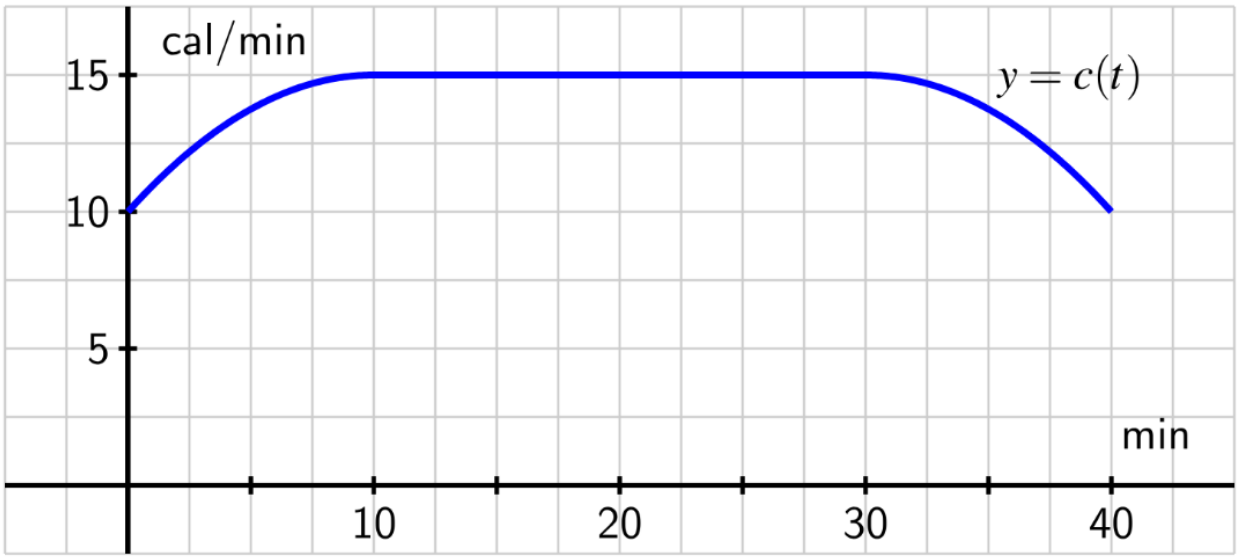
PS12: Definite integrals and the Fundamental Theorem of Calculus

During a 40-minute workout, a person riding an exercise machine burns calories at a rate of $c(t)$ calories per minute, where the function $y = c(t)$ is given by the following information:

- On the interval $0 \le t \le 10$, the formula is $c(t) = -0.05t^2 + t + 10$ (warmup);
- on the interval $10 \le t \le 30$, the formula is $c(t) = 15$ (conditioning phase);
- on the interval $30 \le t \le 40$, the formula is $c(t) = -0.05t^2 + 3t - 30$ (cooldown).

Here's a graph of $c(t)$.



1. (IN1) Shade in the area under $c(t)$ between $t = 10$ and $t = 30$. Use some simple geometry to calculate this area. Give units.
2. (INx) Write a sentence explaining what the answer to part 1 *means* in the context of the person exercising.
3. (IN2) Use a Riemann sum with 4 rectangles to approximate the area under $c(t)$ between $t = 30$ and $t = 40$:
   - How wide should each rectangle be?
   - Decide on a consistent way to choose the height of each rectangle.
     (Do you want to do the top-left corner? the top-right corner? the middle? Up to you!)
   - Sketch your four rectangles on the graph of $c(t)$.
   - Compute the four rectangle areas separately, and give units.
   - Add up the four rectangle areas to get a total area.
   - Based on your height choices, is your estimate an *over*estimate or an *under*estimate of the actual area? Why?

4. (IN3) Find a formula for an antiderivative $C(t)$ for the portion of $c(t)$ that's on the interval $0 \leq t \leq 10$.

5. (IN5) Use your antiderivative $C(t)$ to find the exact value of $\int_{t=0}^{t=10} c(t)\,dt$. Give units.

6. (IN5) Now find the *exact* value of $\int_{t=30}^{t=40} c(t)\,dt$.

   - Careful: you'll need to find a new antiderivative, because the formula for $c(t)$ is different!

7. Put it all together: Find the *exact* value of $\int_{t=0}^{t=40} c(t)\,dt$, give units, and explain what this number means in the context of the person exercising.