PS10: Indeterminate forms; global optimization – Quick reference key

1. Show your work to decide whether each of the following limits is indeterminate or not. You don't need to compute the limits (except part H).

   (a) $\lim_{x\to 0} \dfrac{\ln(1+x)}{x} = \dfrac{0}{0}$; indeterminate

   (b) $\lim_{\Omega\to\infty} \dfrac{2^{\Omega}}{\Omega^2} = \dfrac{\infty}{\infty}$; indeterminate

   (c) $\lim_{t\to\pi} \dfrac{\cos(t)}{t} = \dfrac{-1}{\pi}$; not indeterminate

   (d) $\lim_{t\to 0} \dfrac{\cos(t)}{t} = \dfrac{1}{0}$; not indeterminate

   (e) $\lim_{z\to\infty} \dfrac{e^z+z}{2e^z+z^2} = \dfrac{\infty}{\infty}$; indeterminate

   (f) $\lim_{y\to 1} \dfrac{y^2-3y+4}{5y^2-25y+20} = \dfrac{2}{0}$; not indeterminate

   (g) $\lim_{\theta\to\frac{\pi}{2}^-} \dfrac{\tan(\theta)}{\theta-\frac{\pi}{2}} = \dfrac{\infty}{-0}$; not indeterminate

   (h) $\lim_{x\to 0} \dfrac{\cos(x)-1}{\sin(3x)-x} = \dfrac{0}{0}$; indeterminate. So:

$$\lim_{x\to 0} \frac{\cos(x)-1}{\sin(3x)-x} \overset{\text{L'H}}{=} \lim_{x\to 0} \left[\frac{-\sin(x)}{3\cos(3x)-1}\right] = \frac{0}{2} = 0.$$