PS6: Derivative shortcut rules

Instructions: In this Problem Set, you'll show how to find derivatives **step-by-step**, carefully listing out all of your steps. I am being so annoying about making you take very small steps, because it's a good way to get comfortable with all the shortcut rules.
For each problem you choose:

(a) Refer to the list on the next page to identify **all** the derivative rules you're going to need to use for your problem.

(b) Make your sheet of paper into two columns.

- In the left column, do tiny derivative steps.
- In the right column, list all the rules you are using in that step.
- Look at the example on the next page to see how detailed your steps should be.

(c) Repeat with a new problem until you have checked off **all** of the derivative rules in the reference list at least once.

- Yes, all three of the rules for trigonometric functions count separately.
- I am pretty sure you will need to do at least five problems.
- If you do extra problems, I will give you a sticker.

Throughout, carefully label any derivative you find by name, using correct derivative notation.

Here are the problems you can choose from:

(1) Find $\dfrac{dy}{dx}$ if $y = (2x-1)^8 - x + e$.

(2) If $L(y) = (\tan(y))^3 + \tan(3)$, find $L'(y)$.

(3) Find $B'(x)$ if $B(x) = \cos(e^{3x})$.

(4) Differentiate $W(x) = \dfrac{x^4 - 6x + 10}{\sqrt{7x+8}}$.

(5) Differentiate $g(q) = \dfrac{q^5}{\ln(q + \frac{1}{3})}$.

(6) Differentiate $B(x) = (Ce^{3x} - D)(2x)$, where C and D are constants.

(7) Find $f'(y)$ if $f(y) = \left(\sqrt{y^4+1}\right)\left(\ln\left(\dfrac{y}{2}\right)\right)$.

(8) Find $\dfrac{dB}{dx}$ if $B = \dfrac{e^{x+\pi}}{\sin(\pi x)}$.

(9) Differentiate $h(p) = \dfrac{(2p-1)^{1/4}}{\cos(p+5)}$.

(10) Differentiate $f(q) = (\cos q)\left(\sin\left(\frac{2}{\pi}q\right)\right)$.

(11) Differentiate $G = px^5 + qx^3 - x^r$, where p, q and r are constants.

(12) Differentiate $V = \left(3x - \frac{1}{4}\right)^{-4} - (5x)^{5/4}$.

(13) Find $\dfrac{dh}{dt}$ if $h(t) = 7^t \sin(t) + t^7 \cos(t)$.

(14) Differentiate $h(t) = -3(2t^4+7)^4 + 3$.

(Example) Find $\dfrac{dp}{dt}$ if $p(t) = \dfrac{\cos(t^2-1)}{e^{-3t}}$.

I am going to need the following derivative rules:

- quotient rule
- power rule
- exponential function rule
- trig function rule (sine)
- sum rule
- constant multiple rule
- chain rule
- constant function rule

Now I will find the derivative, step by step, and explain what rules I am using in every step. I am highlighting changes from the previous line in blue and with a box around it.

Steps	**Rules**
$\dfrac{dp}{dt} = \dfrac{[e^{-3t}]\cdot\frac{d}{dt}[\cos(t^2-1)] - [\cos(t^2-1)]\cdot\frac{d}{dt}[e^{-3t}]}{[e^{-3t}]^2}$	(quotient rule)
$= \dfrac{[e^{-3t}]\cdot\frac{d}{dt}[\cos(t^2-1)] - [\cos(t^2-1)]\cdot\boxed{[e^{-3t}\cdot\frac{d}{dt}[-3t]]}}{[e^{-3t}]^2}$	(chain rule, exponential function rule)
$= \dfrac{[e^{-3t}]\frac{d}{dt}[\cos(t^2-1)] - [\cos(t^2-1)]\cdot[e^{-3t}\cdot\boxed{(-3)}]}{[e^{-3t}]^2}$	(constant multiple rule, power rule)
$= \dfrac{[e^{-3t}]\cdot\boxed{[-\sin(t^2-1)\cdot\frac{d}{dt}(t^2-1)]} - [\cos(t^2-1)]\cdot[e^{-3t}\cdot(-3)]}{[e^{-3t}]^2}$	(chain rule, trig function rule)
$= \dfrac{[e^{-3t}]\cdot[-\sin(t^2-1)\cdot\boxed{(2t-0)}] - [\cos(t^2-1)]\cdot[e^{-3t}\cdot(-3)]}{[e^{-3t}]^2}$	(sum rule, power rule, constant function rule)

And now I am done, because I have no more “please take the derivative of” signs left. Yay!

List of derivative shortcut rules

Here is the list of shortcut rules. Check them off as you use them. Keep this list for reference!

- ☐ sum rule: $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
- ☐ constant multiple rule: $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$
- ☐ constant function rule: $\frac{d}{dx}[c] = 0$
- ☐ power rule: $\frac{d}{dx}[x^n] = nx^{n-1}$
- ☐ exponential function rule: $\frac{d}{dx}[a^x] = a^x \cdot \ln(a)$

  trigonometric function rules:
  - ◯ $\frac{d}{dx}[\sin(x)] = \cos(x)$
  - ◯ $\frac{d}{dx}[\cos(x)] = -\sin(x)$
  - ◯ $\frac{d}{dx}[\tan(x)] = \sec^2(x)$
- ☐ product rule: $\frac{d}{dx}[f(x)\cdot g(x)] = \frac{d}{dx}[f(x)]\cdot g(x) + \frac{d}{dx}[g(x)]\cdot f(x)$
- ☐ quotient rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot \frac{d}{dx}[f(x)] - f(x)\cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$
- ☐ chain rule: $[f(g(x))]' = f'(g(x))\cdot g'(x)$
- ☐ logarithmic function rule: $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

Just for reference, here are a few more that come up sometimes. They get used much less and they don't show up on this problem set; I would just be sad if I didn't write them down.

- inverse trigonometric function rules:
  - $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$
  - $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$
- rules for other trigonometric functions:
  - $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$
  - $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$
  - $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$