

Applied optimization recipe

A lot of calculus students find applied optimization (and related rates) particularly tricky, and here's why: *there's no formula for doing them.* Every situation is a bit different, and what works in one case doesn't necessarily work in another.

There is, however, a *recipe* (if I was feeling very fancy I would call it a *heuristic* – google it) that you can follow to think your way through optimization problems.

1. Draw three pictures.
 - If you don't draw three, it's harder to see what's different between the pictures.
 - Draw your pictures pretty big, or you'll have a hard time labeling.
2. Determine what quantities are *different* (ie., variables) and what ones are the same.
 - If something is different between your pictures, label it with a letter.
 - If something is the same in all your pictures, label it with what number it is.
3. Write down a *constraint equation*.
 - This has to do with the quantity that's the same in your three pictures.
 - What keeps you from just using whatever numbers you want?
4. Write down an *objective function*.
 - This is the thing you want to be the biggest or smallest possible.
 - It must be a function of only one variable; use your constraint equation.
 - Creativity! Geometry! Trigonometry! Other math you remember!
5. Figure out any *endpoints*.
 - Can your one variable be any number you want? Does it have to at least be positive? Is there some kind of maximum or minimum value it could possibly be?
6. Do the *finding extrema recipe*:
 - (a) Find all the critical *numbers* of the objective function.
 - Figure out where the derivative is either zero or undefined.
 - (b) Find all the critical *values* of the objective function.
 - Plug back into the original objective function.
 - (c) Find all the endpoint values of the objective function.
 - (d) Decide who is the smallest and the largest.