PS12: Definite integrals and the Fundamental Theorem of Calculus - QR

During a 40-minute workout, a person riding an exercise machine burns calories at a rate of $c(t)$ calories per minute, where the function $y = c(t)$ is given by the following information:

- On the interval $0 \le t \le 10$, the formula is $c(t) = -0.05t^2 + t + 10$ (warmup);

- on the interval $10 \le t \le 30$, the formula is $c(t) = 15$ (conditioning phase);

- on the interval $30 \le t \le 40$, the formula is $c(t) = -0.05t^2 + 3t - 30$ (cooldown).

1. (IN1) Shade in the area under $c(t)$ between $t = 10$ and $t = 30$. Use some simple geometry to calculate this area. Give units.

   Rectangle with height 15 cal/min and width 20 mins. Area = 300 cal.

2. (INx) Write a sentence explaining what the answer to part 1 *means* in the context of the person exercising.

   Between 10 minutes and 30 minutes, the person burned 300 total calories.

3. (IN2) Use a Riemann sum with 4 rectangles to approximate the area under $c(t)$ between $t = 30$ and $t = 40$.

   Left sum overestimate comes out to 139.0625 calories.

   Right sum underestimate comes out to 126.5625 calories.

   Midpoint sum comes out to 133.59375 calories.

4. (IN3) Find a formula for an antiderivative $C(t)$ for the portion of $c(t)$ that's on the interval $0 \le t \le 10$.

$$C(t) = \frac{-0.05}{3} t^3 + \frac{1}{2} t^2 + 10\, t$$

5. (IN5) Use your antiderivative $C(t)$ to find the *exact* value of $\int_{t=0}^{t=10} c(t)\, dt$. Give units.

$$\int_{t=0}^{t=10} c(t)\, dt = \left[\frac{400}{3}\right] - \left[0\right] \approx 133.3 \text{ calories}$$

6. (IN5) Now find the *exact* value of $\int_{t=30}^{t=40} c(t)\, dt$. Same number.

7. Put it all together: Find the *exact* value of $\int_{t=0}^{t=40} c(t)\, dt$, give units, and explain what this number means in the context of the person exercising.

$$\int_{t=0}^{t=40} c(t)\, dt = \frac{400}{3} + 300 + \frac{400}{3} = \frac{1700}{3} \approx 566.67 \text{ calories}$$