Math 201-06

Sample Learning Target Questions for Final Review

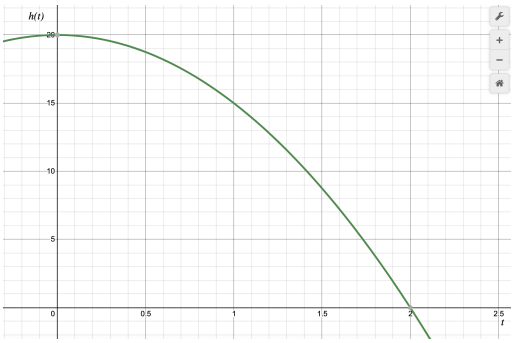
1. A ball dropped from the top of a building has height $h(t)$ (in meters) at time t (in seconds) after its dropped.

   (a) The following table provides some data for values of $h(t)$ at different times t:

t (in seconds)	0.5	0.7	0.75	0.76
$h(t)$ (in meters)	18.75	17.55	17.1875	17.112

   Use the data to determine the average velocity of the ball on the interval $[0.7, 0.76]$. Include units on your answer.

   (b) The following graph shows a plot of the function $h(t)$:



   - Use the graph to find the average velocity of the ball between $t = 1$ and $t = 1.5$ seconds, $AV_{[1,1.5]}$. Include units on your answer.
   - On the graph of $h(t)$ draw a line through $(1, h(1))$ and $(1.5, h(1.5))$; what is the slope of this line and what does that slope mean in the physical context of the function h? Write to explain, being careful and precise.

2. The following questions ask you to estimate values of the derivatives of the given functions.

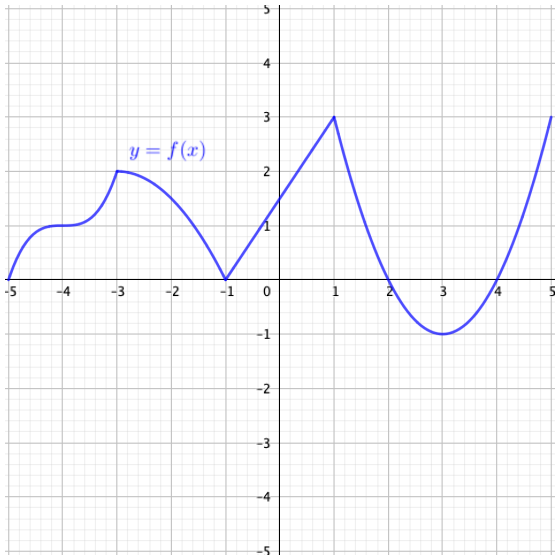
   (a) Suppose we know the following data for a function g:

t	-5	-3.5	-2	-0.5	1	2.5	4
$g(t)$	2	-6	-10	-12	-13	-13.5	-13.75

   Estimate $g'(-2)$ and $g'(2.5)$. Clearly show your work and thinking, using proper notation.

   (b) For the function f whose graph is given below, use the graph to provide accurate estimates of $f'(-4.5)$, $f'(-0.5)$, and $f'(2)$.

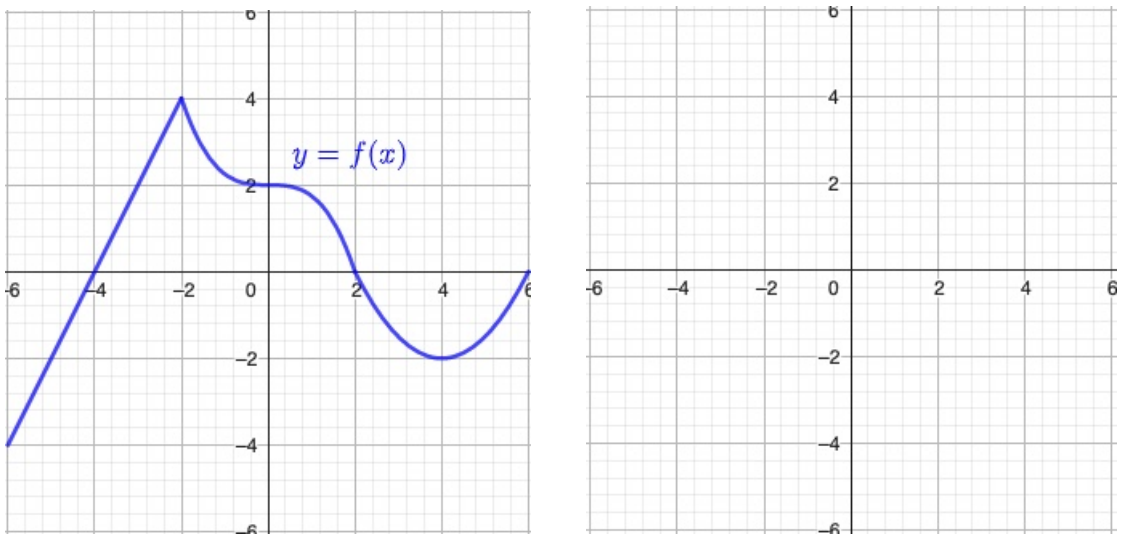
   Clearly state your results below the graph writing things like "$f'(-4.5) \approx$ ___"; write one sentence to explain your thinking.



3. Let $g(x) = 5 - \dfrac{3}{x}$.

   Use the limit definition of the derivative to determine a formula for $g'(x)$. Clearly show all of your steps using proper notation, especially proper limit notation.

4. For the function f plotted at left, sketch an accurate graph of its derivative, f', on the axes at right. Write at least one sentence below your graph to explain your thinking and process.



5. For each of the following questions, write two careful sentences that explain the meaning of the given derivative value, including correct units. As part of your response, for each piece of data you should

   - clearly identify the meaning in terms of the rate of change of a certain quantity and with relevant units.
   - explain what you expect to happen to the value of the function as the independent variable increases by one unit. For example, you could say something like "at the instant . . ., I expect that over the next ___, ..."

   (a) The value, V (in dollars), of a rare automobile is a function of the number of years, t, since the car was first manufactured, given by $V(t)$. Explain the meaning of $V'(50) = 4150$ in the context of the car's value.

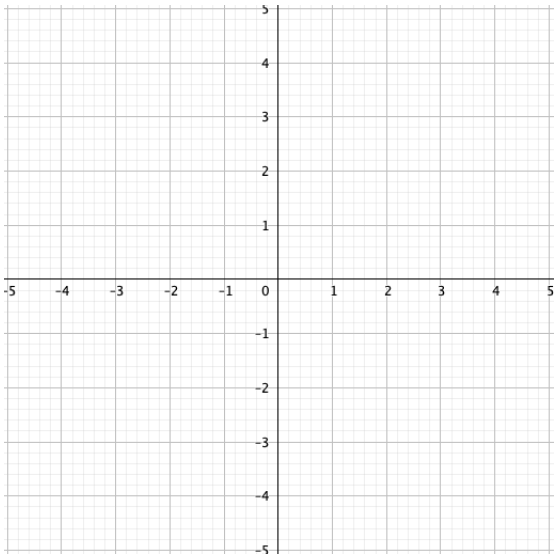
   - (meaning as a rate, with units)
   - (what I expect to happen as the input variable increases)

   (b) The cost, C (in dollars), to build a new home is a function of s (the number of square feet constructed), given by $C(s)$. Explain the meaning of $C'(2100) = 245$ in the context of building a house.

   - (meaning as a rate, with units)
   - (what I expect to happen as the input variable increases)

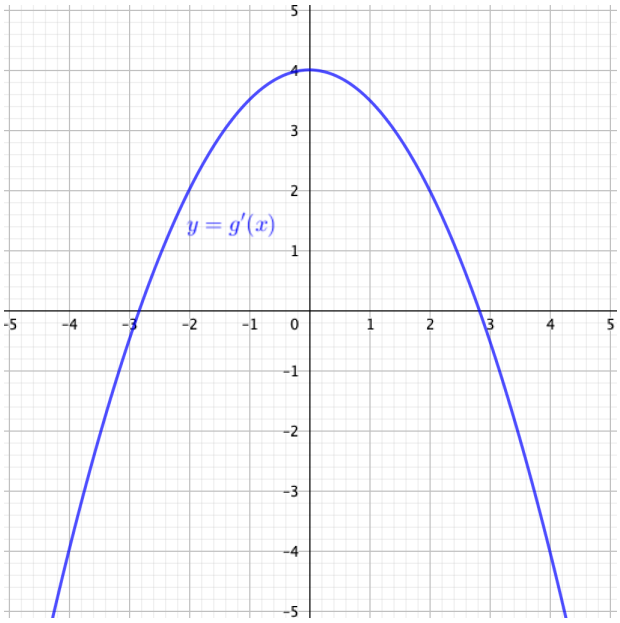
6. In both settings below, use the given information to describe the function's behavior, using not only the function's value, but also the values of the first and second derivatives.

   (a) On the axes provided, sketch a possible graph of a function $y = g(x)$ that has the following properties: $g(-1) = 1$, $g'(-1) = \frac{1}{3}$, and $g''(-1) = -\frac{1}{4}$, where your graph also satisfies having $g'(x)$ always positive and $g''(x)$ always negative. Write a careful sentence to explain why you drew your graph as you did.



   (b) The cost, C, to build a new home is a function of s, the number of square feet constructed, given by $C(s)$. Say we know that $C(1750) = 195000$, $C'(1750) = 215$, and $C''(1750) = -4$. In everyday language, explain what we know about the behavior of the cost function in the context of building a 1750 square foot house. Cite all three pieces of given information, with units.

7. For a given function $y = g(x)$, say that we know the following information: $g(-2) = -3$ and $g'(x)$ (the derivative of g) is given by the graph below.



Again, the graph here is of $g'(x)$, the derivative of $g(x)$.

Throughout the following questions, proper and correct notation is essential.

   (a) Find a formula for $L(x)$, the tangent line approximation to $y = g(x)$ at the point $(a, g(a))$, where $a = -2$.

(b) Use the tangent line approximation you found in (a) to estimate $g(-2.2)$. Clearly show your work using proper notation.

(c) Is the estimate you found in (b) too large or too small? Why?

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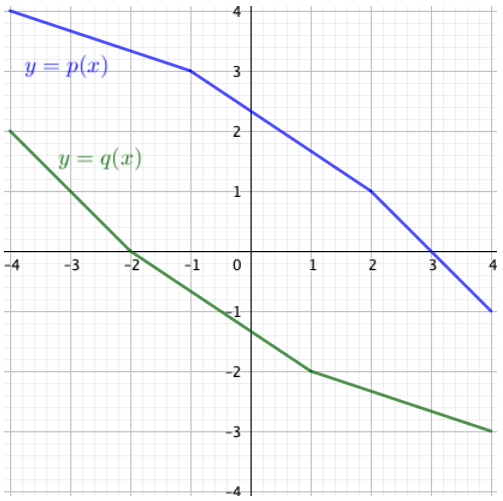
8. Consider the two piecewise linear functions p and q given by the figure below.

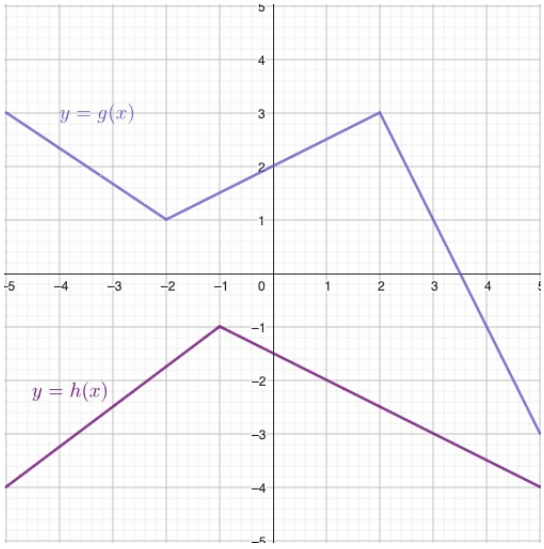


Determine the *exact* value of each of the following derivatives that involve functions that are defined in terms of p and q. Clearly show your work and thinking with proper notation, and write one sentence to explain your overall approach.

(a) Let $G(x) = p(x) \cdot q(x)$. Find $G'(3)$.

(b) Let $H(x) = \frac{q(x)}{p(x)}$. Find $H'(0)$.

9. Consider the two piecewise linear functions g and h given by the figure below.



Determine each of the following derivative values that involve functions that are defined in terms of g and h. Clearly show your work and thinking using proper notation, be sure your values are *exact* in light of the given information, and write one sentence to explain your overall approach.

(a) Let $M(x) = h(g(x))$. Determine $H'(3)$. Write your work and conclusion in the space to the right of the graph above.

(b) Let $N(x) = g(x^3)$. Determine $N'(-1)$.

(c) Let $P(x) = \ln(h(x))$. Determine $P'(1)$.

10. Consider the curve defined implicitly by the equation

$$xy^3 + 2xy = 6$$

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$ that depends on x and y. Show all of your work and thinking clearly, using correct notation.

(b) Notice that the point $(2, 1)$ lies on the curve $xy^3 + 2xy = 6$. Find the exact slope of the tangent line to curve at $(2, 1)$. Show clearly how you determined this value, and label your work with proper notation.

11. Gravel is being dumped from a conveyor belt at a rate of 10 cubic feet per minute. The gravel forms a pile in the shape of a right circular cone whose base diameter and height are always the same.

At what instantaneous rate is the height of the pile increasing when the pile is 12 ft high?

To earn a Y on this question, you must fully justify your answer using calculus, including

- *a clear, labeled diagram with variables identified*
- *appropriate equations that relate changing quantities and their rates of change*
- *proper notation involving derivatives*
- *a clear summary of your result, with correct units*

12. Suppose we want to build a cylindrical can with no top in such a way that its surface area is exactly 4 square feet. What dimensions produce the can having absolute maximum total volume? What is the absolute minimum volume that is possible?

Your task is to fully *set up* an optimization problem to be solved on an appropriate domain. *To earn a Y on this problem, you must fully justify your answer using calculus, including*

- *Draw a picture of the situation and introduce appropriate variables.*
- *Identify a constraint equation for how variables are related.*
- *State the quantity being optimized and find a formula that represents this quantity as a function of a single variable.*
- *Determine, with a sentence of justification, the domain on which the function should be optimized.*
- *Write 1-2 sentences to explain what you would do from here (*without actually doing so*) to use calculus to find the optimal box. In particular, be sure to explain how you would determine whether you had found an absolute maximum or absolute minimum.*

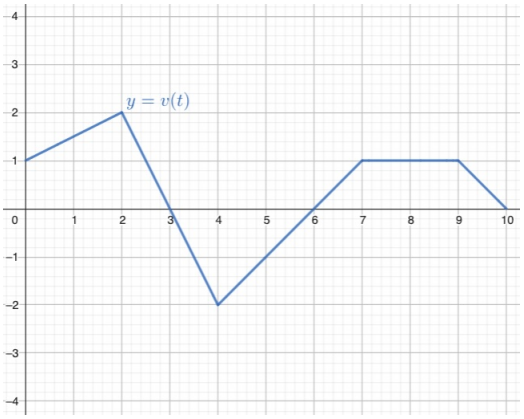


Figure 1: The graph of the velocity function, $v(t)$.

13. An object is moving along a straight path; its velocity is considered positive when moving to the right. The object's velocity $v(t)$ (in meters per second) at time t (in second) is given by the graph above.

    (a) On the time interval $[2, 6]$, determine the object's change in position, including units on your result. Clearly explain your thinking and process for determining the change in position.

    (b) On the time interval $[2, 6]$, determine the object's total distance traveled, including units on your result. Is the distance traveled on this interval the same as the object's change in position? why or why not? Clearly explain your thinking and process for determining the distance traveled.

    (c) Given an example of an interval $[a, b]$ on which the object's distance traveled is exactly 3 meters (that is, identify numerical values for a and b). Write a sentence to explain and justify your choice of interval.