Homework #1

Here's your homework for this week, due Sunday 1/26 by pdf upload to Canvas. I'm also posting the .tex source on my new MATH 312 github repo, with lots of comments, so that you can see how it is made and borrow some tricks.

1. In class on Wednesday we discussed an attendance and participation policy. I think we settled on a coherent vision of a respectful and involved classroom community, and in particular, we talked about when it's okay and not okay to be absent. Please write an appropriate amount of words about what you understand the attendance and participation policy to be.

2. Get set up with a mathematical document typesetting system. Refer to this typesetting resources document for some pointers.

3. In class on Wednesday, we explored the group of symmetries of a rectangle, which I shall call **Rect**. Explore **Sq**, the group of symmetries of a square.

    - We decided that **Rect** contains 4 distinct symmetries (and we say that **Rect** has *order* 4 and write $|\,\mathbf{Rect}\,| = 4$):
        - e, the "identity" symmetry where you do nothing
        - f, a (vertical) flip
        - s, a (horizontal) spin
        - r, a half-turn rotation (so it doesn't matter whether it's cw or ccw)

      Certainly $|\,\mathbf{Sq}\,| > |\,\mathbf{Rect}\,|$ – in particular, a quarter-turn rotation is a symmetry of the square that isn't a symmetry of a rectangle. How many *distinct* symmetries are in **Sq**? Give them good letter names.

    - Here's the Cayley diagram for **Rect**:

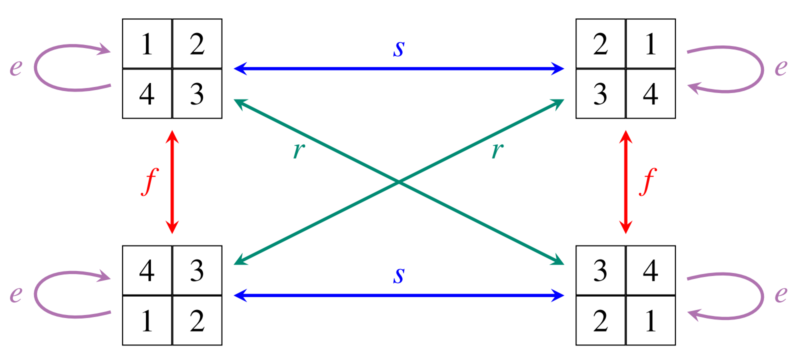


Figure 1: Cayley diagram for the group of symmetries of a rectangle.

      Draw a Cayley diagram for **Sq**. Notes:
        - It would be reasonable to omit the identity loops to reduce visual clutter.
        - Will all of your arrows be double-headed this time?
        - I've generated this diagram within LaTeX using TikZ code. Feel free to modify my source to draw your own, but also feel free to hand-draw your diagram.

- We decided that **Rect** is *generated by* any two non-trivial symmetries and we wrote several different *presentations*, such as **Rect** $= \langle f, s \mid f^2 = s^2 = e, fs = sf \rangle$.
  - See if you can determine a minimal generating set for **Sq**. (There are 12 possibilities.)
  - See if you can write a presentation for **Sq**.
- What's similar and what's different between **Rect** and **Sq**?
  - In **Rect**, every pair of symmetries *commuted* – for instance, $fr = rf$. (We say that **Rect** is *abelian*.) Is this true for **Sq**?
  - In **Rect**, every symmetry was its own *inverse* – for instance, ss (aka s^2) $= e$. (We say that every element of **Rect** is *idempotent*.) Is this true for **Sq**?

Here are some extension problems. You should try "some" of them. How many is "some"? Idk.

4. Count the symmetries of every upper-case letter in the English alphabet. Assume they're the most boring, non-decorated, sans-serif versions possible; for instance, "U" should really look like "$\bigcup$". Notice that every letter has at least one symmetry (the identity).

5. Explore **Tri**, the group of symmetries of an equilateral triangle.

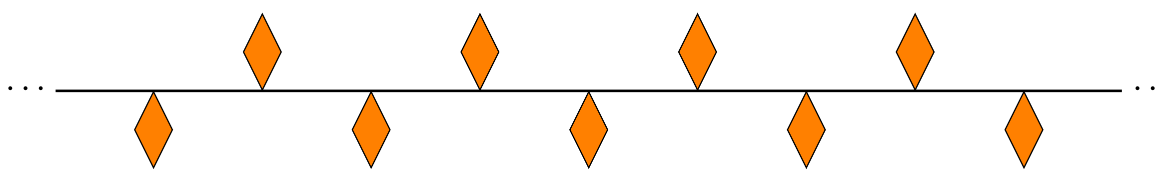
6. Write out the multiplication table for **Rect**. Convention: the box in the s row and the r column is sr (not rs).

	e	s	r	f
e				
s				
r				
f				

7. Write out the multiplication table for **Sq**.

8. Write out the multiplication table for **Tri**.

9. Here is a thing called a *frieze*. It goes on infinitely in both directions.



   Explore the group of symmetries of this figure. (Is it finite or infinte?)

10. Not every group comes from symmetries of a geometric figure (they're just nice examples to play with). Consider two light switches on a wall side by side, and think about all the possible actions that you can do to the two light switches. For example, one action is to toggle the left light switch while leaving the right one alone. Let's call this group of actions **Light**$_2$.
    (a) How many distinct actions does **Light**$_2$ have? Give these actions good letter names.
    (b) Draw a Cayley diagram for **Light**$_2$.
    (c) Find a minimal generating set for **Light**$_2$ and write a presentation.
    (d) Seem familiar?