

Let's say that $D_4 = \langle r, f \rangle$, where r is the 90° ccw rotation and f is the flip over a vertical axis. Draw arrows indicating what each element of D_4 does to this list of “binary squares.”

e :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

r :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

f :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

r^2 :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

r^3 :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

rf :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

r^2f :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

r^3f :

0 0	0 1	1 0	1 1	0 1	0 0	1 0
0 0	1 0	0 1	0 0	0 1	1 1	1 0

What if $D_4 = \langle s, t \rangle$, where s is the flip over a vertical axis and t is the flip over the up-right diagonal?

e :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

s :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

t :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

st :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

sts :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

$stst$:

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

ts :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0

tst :

0	0	0	1	1	0	1	1	0	1	0	0	1	0
0	0	1	0	0	1	0	0	0	1	1	1	1	0