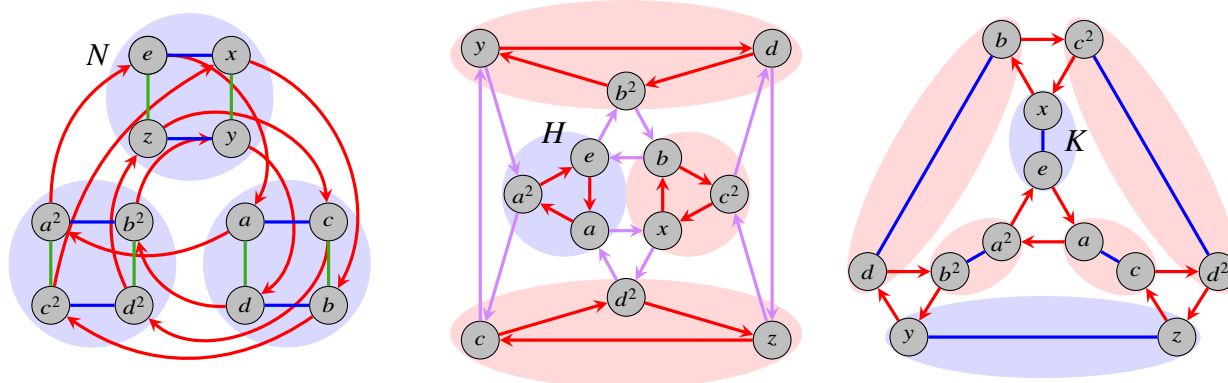


### Problem 9 from Homework #5

**Problem.** Here's an extended problem where you can explore the relationship between left cosets, right cosets, conjugate subgroups, and normalizers.

Below are three Cayley diagrams of  $A_4$ , each highlighting the left cosets of a different subgroup. These are the subgroups  $N$ ,  $H$ , and  $K$  from slide 17 of the normal-subgroups slides from class on Wednesday. To make the notation suck less and the Cayley diagrams more readable, we can take  $a = (123)$ ,  $b = (134)$ ,  $x = (12)(34)$ , and  $z = (13)(24)$ ; arrows in the Cayley diagrams are color-coded appropriately. Then:

$$N = \langle x, z \rangle; \quad H = \langle a \rangle; \quad K = \langle x \rangle.$$

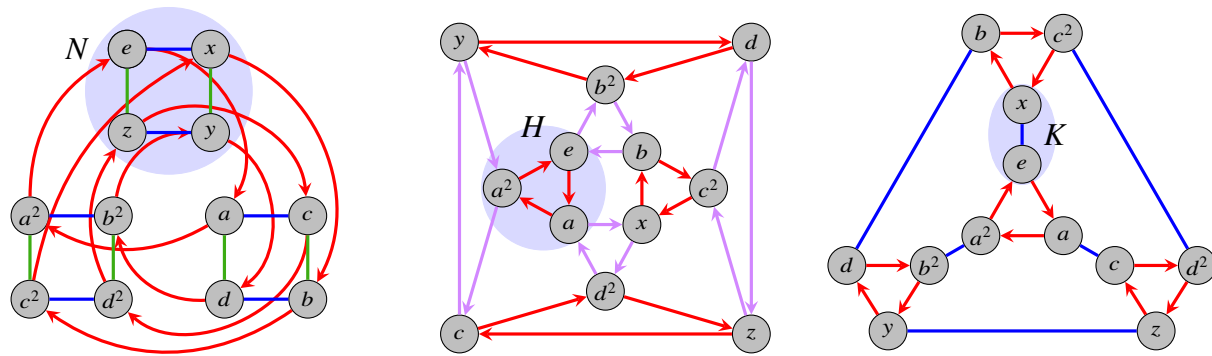


- Label each of the “coset bubbles” in each diagram above with which left coset it is. For instance,  $\{a, c, b, d\}$  is certainly  $aN$ .
- For each subgroup shown above, partition  $A_4$  into its right cosets. (Work smarter not harder: think about which elements you actually need to bother shifting by!) Write the right cosets as subsets of  $A_4$ , consisting of permutations in cycle notation. Also, highlight them by colors on a fresh copy of the Cayley diagrams – see the next page.
- Conjecture as to why I made some of the bubbles blue and some of them red. Relatedly, find  $N_{A_4}(N)$ ,  $N_{A_4}(H)$ , and  $N_{A_4}(K)$ .
- For each (non-identity) left coset  $gH$ , illustrate the construction of the conjugate subgroup  $gHg^{-1}$  on a fresh copy of the Cayley diagram – see next page. Repeat this for  $N$  and  $K$ .

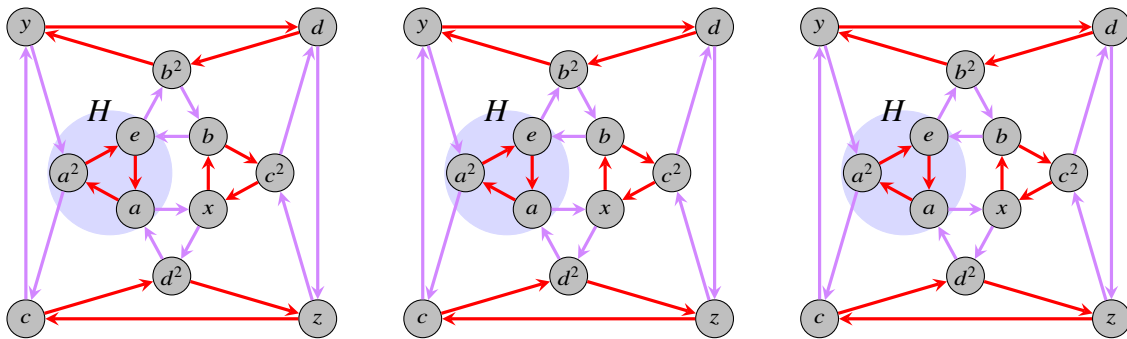
### Fresh Cayley diagrams for Problem

Please please *please* print this out and draw your coset bubbles by hand (or by marking up a pdf on a tablet). I promise that it would suck *so much* to do this in tikz.

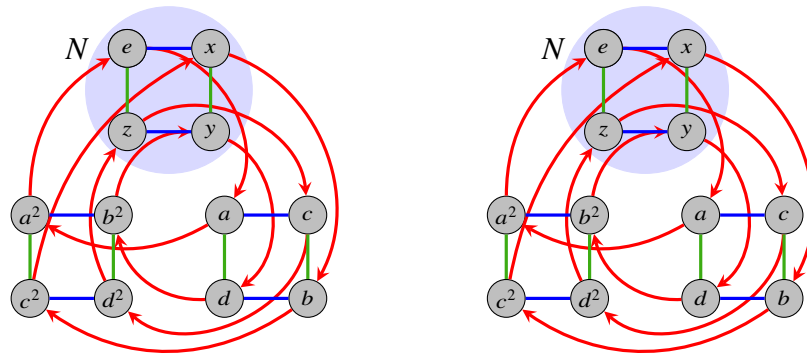
For part (b):



For part (d), subgroup  $H$  (3 copies bc  $[A_4 : H] = 4$  and I don't care about one of 'em):



For part (d), subgroup  $N$  (note  $[A_4 : N] = 3$ ):



Ran outta room, see next page

