**Homework #9 (due Mar 30)**

## Finish HW#8

I don't know if it was spring break brain or what, but only one person has so far turned in HW#8. So, do that. I've changed everyone's due date for HW#8 to be Mar 30.

## Automorphism groups

In class we figured out $\text{Aut}(\mathbb{Z}_n, +)$ for $n = 2 \ldots 12$. (They end up being isomorphic to $(U_n, \cdot)$, the group of integers relatively prime to $n$ with the operation multiplication mod $n$.) Let's explore $\text{Aut}(G)$ for some non-cyclic groups $G$.

**Problem 1.** Find $\text{Aut}(V_4)$. Hints:

- Say $V_4 = \langle a, b \rangle$. Any automorphism of $V_4$ is determined by what it does to those two generators. How many choices do you have for where $a$ goes? How many choices do you then have for where $b$ goes? So how many possible automorphisms do you have in total?
- Do your automorphisms commute, or does the order of composition matter?

**Problem 2.** Find $\text{Aut}(D_3)$. Hints:

- Again, automorphisms are determined by what they do to the generators.
- Also, automorphisms preserve order. Where can $r$ go? Where can $f$ go?
- Do your automorphisms commute?

## Semidirect products

**Problem 3.** The semidirect product $\mathbb{Z}_3 \rtimes \mathbb{Z}_2$ is a group of order 6 $(= 2 \times 3)$. Draw it:

- Inflate $\mathbb{Z}_2$ and insert a copy of $\mathbb{Z}_3$ in each inflated node.
- Rewire the copy of $\mathbb{Z}_3$ in the 1-node. (There is only one interesting way to do this.)
- Pop the nodes of $\mathbb{Z}_2$ and reconnect the $\mathbb{Z}_2$ arrows "straight up" – 0 to 0, 1 to 1, and 2 to 2.

**Problem 4.** I will tell you for free that $\mathbb{Z}_3 \rtimes \mathbb{Z}_2$ is isomorphic to one of the groups of order 6 that we already know. Construct an **explicit** isomorphism $\phi : \mathbb{Z}_3 \rtimes \mathbb{Z}_2 \to G$ for the correct group $G$:

(a) write a recipe for sending elements of $\mathbb{Z}_3 \rtimes \mathbb{Z}_2$ to $G$,

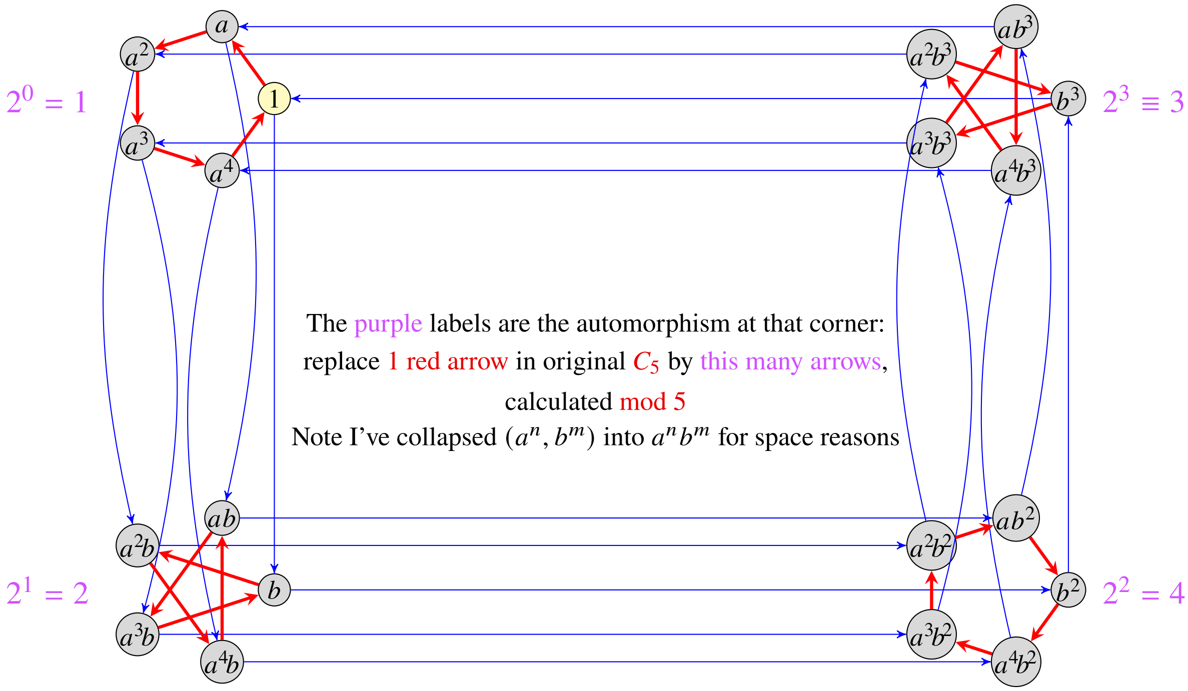
(b) show that your recipe is a homomorphism,

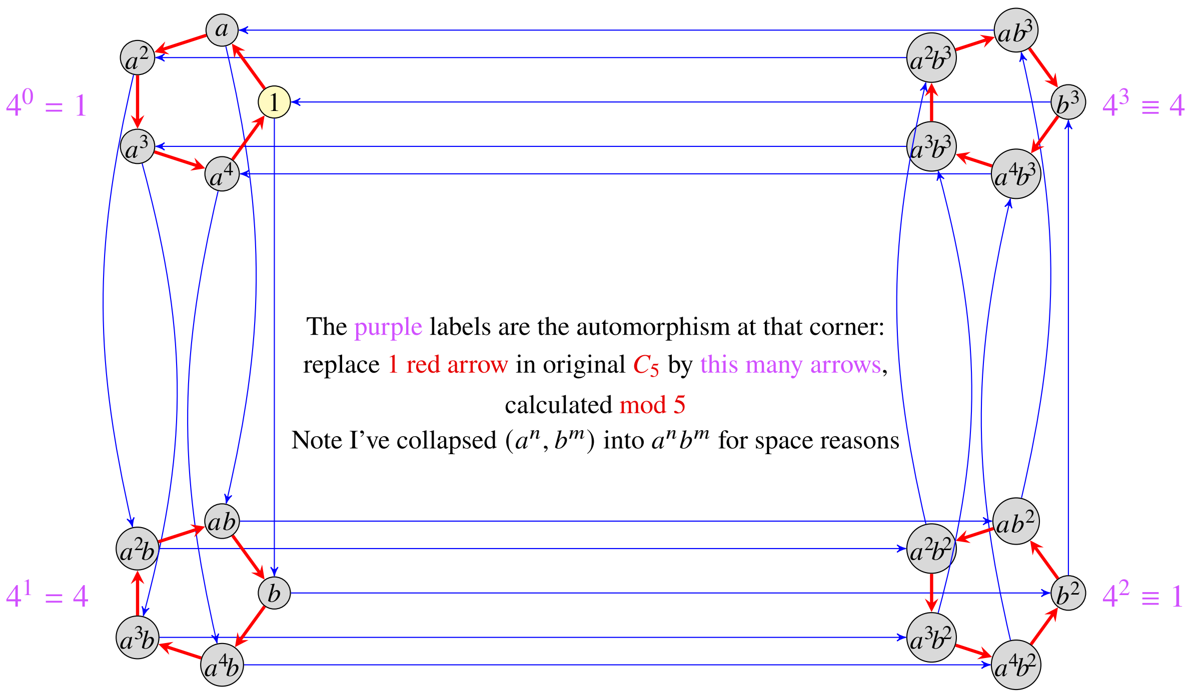
(c) show that your homomorphism is injective and surjective.

(Hint: This will be easier if you figure out a minimal set of generators for both $\mathbb{Z}_3 \rtimes \mathbb{Z}_2$ and $G$, and then figure out how to map the generators over correctly.)

**Problem 5.** In class we constructed three different versions of $C_5 \rtimes C_4$; two are drawn below. $C_5$ and $C_4$ were "compatible" because there were automorphisms of $C_5$ that had order dividing $|C_4|$. Choose two other cyclic groups that are "compatible" and construct their semidirect product.

(Look at the code here and conclude that you should **not** try to draw these in tikz, lol.)




**Problem 6.** (Bonus!) Try constructing $V_4 \rtimes C_3$, and also $C_3 \rtimes V_4$.
(I promise both of them are "compatible" in the sense above.)