Physics applications of definite integrals – When is something a Δ?

Somebody asked me a good question in an exit quiz recently that I was writing a loonnggg explanation in the Canvas comments about, but then I was like, this sucks, let me write it in a document instead, lol.
The question was: When should I label some quantity with a Δ and when should I not? This is a good question that I don't think I gave a very persuasive answer to in class.
My short answer is, you should use a Δ when something is *small*, but not when something is *not small*.
For example, in the leaky bucket problem, we slice the journey of the bucket into small pieces.

- The work on each slice is *small*, so I call it ΔW.
- The distance across each slice is *small*, so I call it Δh.
- However, the force (aka the weight) is *not small*; it's some reasonable number. It does depend on h, so I'm going to call it $F(h)$ (not ΔF!).

Therefore $\Delta W = F(h) \cdot \Delta h$.
To further illustrate, here's the rest. Refer back to the original handout for the situations, and note that you may have used different letters than I'm using here.

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1. When we slice the second hand of the clock:
   - The kinetic energy of the slice is *small*, ΔK.
   - The mass of the slice is *small*, Δm.
   - The velocity of the slice is *not small* but depends on ℓ, $v(\ell)$.

   Therefore, $\Delta K = \frac{1}{2} \cdot \Delta m \cdot v(\ell)^2$.
2. When we slice the oil slick into rings:
   - The mass of each ring is *small*, Δm.
   - The density of each ring is *not small*, but depends on r, so it's called $\rho(r)$.
   - The area of each ring is *small* (thin!), but depends on r, so let's call it $\Delta A(r)$.

   Therefore, $\Delta m = \rho(r) \cdot \Delta A(r)$.
3. When we slice the satellite launch into small pieces along its journey:
   - The energy needed to push the satellite through that slice is *small*, ΔE.
   - The force the satellite feels at that slice is *not small* but depends on r, $F(r)$.
   - The distance through that slice is *small*, Δr.

   Therefore, $\Delta E = F(r) \cdot \Delta r$.

4. When we slice the magnet push into small pieces along the wire:
   - The energy needed to push the magnet through this slice is *small*: ΔE
   - The force the magnet feels at that slice is *not small* but depends on x, $F(x)$.
   - The distance through that slice is *small*, Δx.

   Therefore, $\Delta E = F(x) \cdot \Delta x$.

5. When we slice the atmosphere into small spherical shells:
   - The mass of the shell is *small*, Δm.
   - The density in that shell is *not small* but depends on h, $\rho(h)$.
   - The volume of that shell is *small* and depends on h, $\Delta V(h)$.

   Therefore, $\Delta m = \rho(h) \cdot \Delta h$.

6. When we slice the pool cue into particles:
   - The mass of the rod-particle is *small*, ΔM.
   - The distance from the rod-particle to the ball is *not small*, r.
   - The gravitational force the rod-particle exerts on the ball is *small* (at least, relative to the force the whole rod ends up exerting), ΔF.

   Therefore, $\Delta F = \dfrac{G \cdot \Delta M \cdot m}{r^2}$.
   (I think I got the letters right for the mass of the rod vs. the mass of the ball.)