

### Improvements to Euler's method

In class we explored the idea that original flavor Euler's method is analogous to a left Riemann sum. (In general it is more complicated, since the recipe for  $dy/dt$  may involve both  $y$  and  $t$ , whereas for a Riemann sum, the recipe for heights definitely only involves  $t$ .) We then thought about possible improvements to Euler's method, inspired by improvements to left Riemann sums. I'm going to show you what each of the tables might look like for each of these methods; your job would be to figure out how to calculate things like "right slope" or "middle slope." I'm including some extra columns to give some hints about those new columns.

(Do you ever write some word like "slope" so many times in a row that it starts looking like it shouldn't be a word? lol)

#### Original flavor Euler's method

$t$	$y$	slope	$\Delta y = \text{slope} \cdot \Delta t$
0			
0.2			
0.4			
0.6			
0.8			
1.0			

#### "Improved Euler's method" aka "Heun's method" aka "explicit trapezoidal rule"

$t$	$y$	left slope	$\Delta y$	Euler $y$	next $t$	right slope	avg slope	$\Delta y = (\text{avg slope}) \cdot \Delta t$
0								
0.2								
0.4								
0.6								
0.8								
1.0								

#### Midpoint method

$t$	$y$	slope	half $\Delta y$	middle $y$	middle $t$	middle slope	$\Delta y = (\text{middle slope}) \cdot \Delta t$
0							
0.2							
0.4							
0.6							
0.8							
1.0							