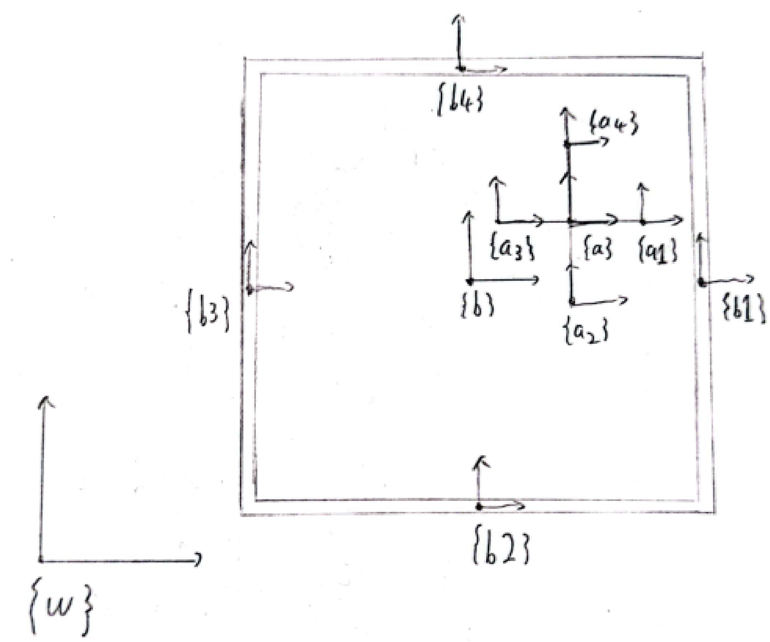
1. I chose the default project as my final project. In this project, a jack moves inside a box and impacts the box walls. There are external forces holding the box and shaking it.

2. Drawing
   (All arrow pointing right is X axis, and All arrow pointing up is Y axis)



g_wa:

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0.0 & \mathrm{x}_1(t) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0.0 & \mathrm{y}_1(t) \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_wb:

$$\begin{bmatrix} \cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0.0 & \mathrm{x}_2(t) \\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0.0 & \mathrm{y}_2(t) \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_a_a1:

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_a_a2:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_a_a3:

$$\begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_a_a4:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g_b_b1:

$$\begin{bmatrix} 1 & 0 & 0.0 & 3.0 \\ 0 & 1 & 0.0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_b_b2:

$$\begin{bmatrix} 1 & 0 & 0.0 & 0 \\ 0 & 1 & 0.0 & -3.0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_b_b3:

$$\begin{bmatrix} 1 & 0 & 0.0 & -3.0 \\ 0 & 1 & 0.0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_b_b4:

$$\begin{bmatrix} 1 & 0 & 0.0 & 0 \\ 0 & 1 & 0.0 & 3.0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0 & 1.0 \end{bmatrix}$$

g_w_a1:

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x_1(t) + 0.5\cos(\theta_1(t)) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y_1(t) + 0.5\sin(\theta_1(t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

g_w_a2:

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x_1(t) + 0.5\sin(\theta_1(t)) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y_1(t) - 0.5\cos(\theta_1(t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

g_w_a3:

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x_1(t) - 0.5\cos(\theta_1(t)) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y_1(t) - 0.5\sin(\theta_1(t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

g_w_a4:

$$\begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x_1(t) - 0.5\sin(\theta_1(t)) \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y_1(t) + 0.5\cos(\theta_1(t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

g_w_b1:

$$\begin{bmatrix}\cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & 1.0\,x_2(t) + 3.0\cos(\theta_2(t))\\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & 1.0\,y_2(t) + 3.0\sin(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_w_b22:

$$\begin{bmatrix}\cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & 1.0\,x_2(t) + 3.0\sin(\theta_2(t))\\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & 1.0\,y_2(t) - 3.0\cos(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_w_b3:

$$\begin{bmatrix}\cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & 1.0\,x_2(t) - 3.0\cos(\theta_2(t))\\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & 1.0\,y_2(t) - 3.0\sin(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_w_b4:

$$\begin{bmatrix}\cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & 1.0\,x_2(t) - 3.0\sin(\theta_2(t))\\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & 1.0\,y_2(t) + 3.0\cos(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_b1_a1:

$$\begin{bmatrix}\cos(\theta_1(t) - \theta_2(t)) & -\sin(\theta_1(t) - \theta_2(t)) & 0 & 1.0\,x_1(t)\cos(\theta_2(t)) - 1.0\,x_2(t)\cos(\theta_2(t)) + 1.0\,y_1(t)\sin(\theta_2(t)) - 1.0\,y_2(t)\sin(\theta_2(t)) + 0.5\cos(\theta_1(t) - \theta_2(t)) - 3.0\\ \sin(\theta_1(t) - \theta_2(t)) & \cos(\theta_1(t) - \theta_2(t)) & 0 & -1.0\,x_1(t)\sin(\theta_2(t)) + 1.0\,x_2(t)\sin(\theta_2(t)) + 1.0\,y_1(t)\cos(\theta_2(t)) - 1.0\,y_2(t)\cos(\theta_2(t)) + 0.5\sin(\theta_1(t) - \theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_b1_a2:

$$\begin{bmatrix}\cos(\theta_1(t) - \theta_2(t)) & -\sin(\theta_1(t) - \theta_2(t)) & 0 & 1.0\,x_1(t)\cos(\theta_2(t)) - 1.0\,x_2(t)\cos(\theta_2(t)) + 1.0\,y_1(t)\sin(\theta_2(t)) - 1.0\,y_2(t)\sin(\theta_2(t)) + 0.5\sin(\theta_1(t) - \theta_2(t)) - 3.0\\ \sin(\theta_1(t) - \theta_2(t)) & \cos(\theta_1(t) - \theta_2(t)) & 0 & -1.0\,x_1(t)\sin(\theta_2(t)) + 1.0\,x_2(t)\sin(\theta_2(t)) + 1.0\,y_1(t)\cos(\theta_2(t)) - 1.0\,y_2(t)\cos(\theta_2(t)) - 0.5\cos(\theta_1(t) - \theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_b1_a3:

$$\begin{bmatrix}\cos(\theta_1(t) - \theta_2(t)) & -\sin(\theta_1(t) - \theta_2(t)) & 0 & 1.0\,x_1(t)\cos(\theta_2(t)) - 1.0\,x_2(t)\cos(\theta_2(t)) + 1.0\,y_1(t)\sin(\theta_2(t)) - 1.0\,y_2(t)\sin(\theta_2(t)) - 0.5\cos(\theta_1(t) - \theta_2(t)) - 3.0\\ \sin(\theta_1(t) - \theta_2(t)) & \cos(\theta_1(t) - \theta_2(t)) & 0 & -1.0\,x_1(t)\sin(\theta_2(t)) + 1.0\,x_2(t)\sin(\theta_2(t)) + 1.0\,y_1(t)\cos(\theta_2(t)) - 1.0\,y_2(t)\cos(\theta_2(t)) - 0.5\sin(\theta_1(t) - \theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1.0\end{bmatrix}$$

g_b1_a4:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} - 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b2_a1:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b2_a2:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b2_a3:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b2_a4:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b3_a1:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b3_a2:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b3_a2:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b3_a3:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b3_a4:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} + 3.0\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b4_a1:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} - 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b4_a2:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} - 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b4_a3:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} - 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

g_b4_a4:

$$\left[\begin{matrix}\cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & - \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & 1.0 x_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 x_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \sin{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \sin{\left(\theta_{2}(t) \right)} - 0.5 \sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)}\\\sin{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} & 0 & - 1.0 x_{1}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 x_{2}(t) \sin{\left(\theta_{2}(t) \right)} + 1.0 y_{1}(t) \cos{\left(\theta_{2}(t) \right)} - 1.0 y_{2}(t) \cos{\left(\theta_{2}(t) \right)} + 0.5 \cos{\left(\theta_{1}(t) - \theta_{2}(t) \right)} - 3.0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1.0\end{matrix}\right]$$

3. Functions

To construct the Euler-Lagrange equations, we need to know Lagrange equations and External forces in the system. To simplify Euler-Lagrange equations, all frames will be expressed in the world frame. The Lagrange equation is composed of two components: kinetic energy and potential energy. The kinetic energy can be expressed as `0.5*Vb.transpose()*M*Vb (In python)` for each mass in the system. The Vb is the frame velocity and M is the Mass matrix. The potential energy can be expressed as m*g*y for each frame, in which m is the mass, g is the gravitational acceleration, and y is the position in the y direction in the world frame. We can get the Lagrange equation for our system after we defined kinetic energy and potential energy. We know the q includes xa, ya, theta_a, xb, yb, theta_b, and this can be used to calculate the dLdqdot and dLdq. Now the left side of the Euler-Lagrange equation can be constructed. The right side is just the force that we defined. By combining the left and right sides of the equation, the Euler-Lagrange equation is constructed.

The constraints can be determined by checking the position of frame a1, frame a2, frame a3, and frame a4 in frame b1, frame b2, frame b3, and frame b4. Because the walls on the box have width, the limitations in b1, b2, b3, and b4 are different. The position can be found by frame transformation. For example, we can use the g_b1a1[1][3] to determine the y position of a1 in frame b1. If the position of a1, a2, a3, and a4 reached the limitation, the impact function will be initialized to update the impact.

To update impacts, we need following equations to construct our impact equation for each impact condition:

$$\left.\frac{\partial L}{\partial \dot{q}}\right|_{\tau^-}^{\tau^+} = \lambda \frac{\partial \phi}{\partial q}$$

$$\left[\frac{\partial L}{\partial \dot{q}} \cdot \dot{q} - L(q, \dot{q})\right]_{\tau^-}^{\tau^+} = 0.$$

In this equation, dLdqdot, L, and qdot are constructed previously. Lambda is an unknown variable. Phi is our constraint. Combining all the functions together by substituting variables with the correct sign, we can get the impact update equations for each impact condition. When impact happened, we substitute the current pose and velocity into the function and solve them numerically for the updated velocity.

4. In the simulation, I set the length of the links on the jack to 1 and the length of the square box edge to 6. The jack center starts at point (0,2) and the box center starts at (0,0). The

force that reacted on the box can cancel the sum of the box's gravitational force and 95% of the jack's gravitational force. This should cause the box to drop slowly during the simulation. And another torque reacted on the box can generate the “shaking” behavior. In the simulation video, it is clear to see that the jack is bouncing around in the box. The movement after impact makes sense in the simulation because the y velocity of the jack relative to the contact surface filled the sign and the x velocity of the jack relative to the contact surface is almost the same as the velocity before the impact. The magnitude of the jack’s velocity may be different after impact because some energy is transferred to the angular velocity which could make the linear velocity smaller than expected. Also, because there are torques reacting on the box, some energy from external force can be transferred to the jack to make it move faster than before. This can match what I see in the video. The box in the animation moves down gradually because the impact forces reacting on the box cannot be canceled out by the force reacting in the y direction. In addition, the box is rotating back and forth by the external force that reacted on it. This is what we expected to see in the simulation. The moment of inertia is set to 1 to simplify the calculations.