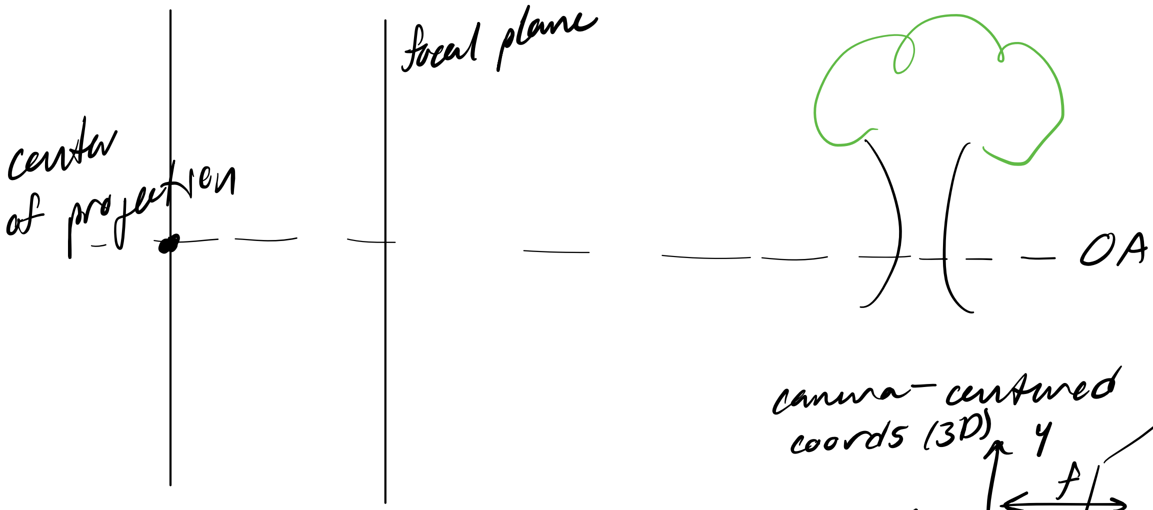
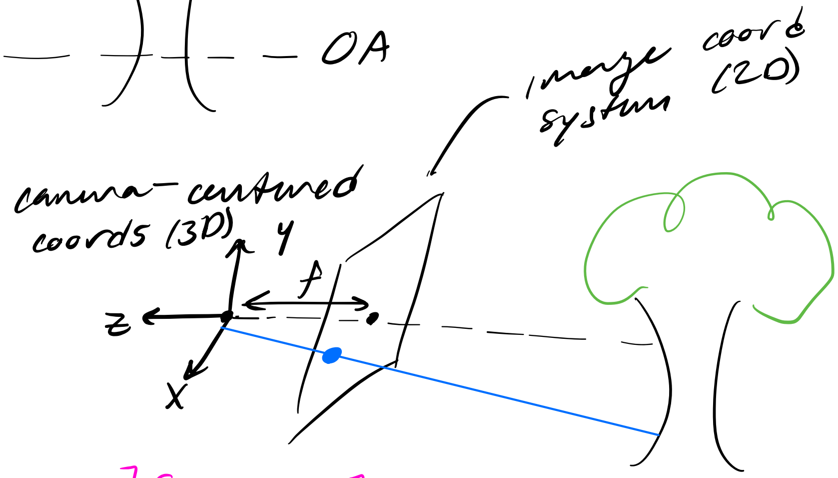
Camera Model





BIG PICTURE

$$X_{img} = [\text{intrinsic}][\text{projection}][\text{extrinsics}]\, X_{world}$$

homogeneous

Extrinsics

pose: camera's position in the world
- location
- orientation

3D rotations:
- φ, θ, ρ ☹
- quaternions
- rotation matrix!!!

3×3 matrix

↳ SO(3) → right-handed coord. systems

↳ orthogonal
- rows are unit vectors
- columns are unit vectors
- rows are orthogonal to each other

- inversion of rotation matrix gets you back to where you started

$$\underbrace{X_c}_{\text{point in camera coords.}} = \overbrace{R}^{3\times 3} \underbrace{X_w}_{\text{point in world coords.}} + \overbrace{t}^{3\times 1}$$

Question: Where is the camera center in world coords?

$$0 = R X_w + t$$

$$-t = R X_w$$

$$R^T(-t) = \underbrace{R^T R}_{I} X_w$$

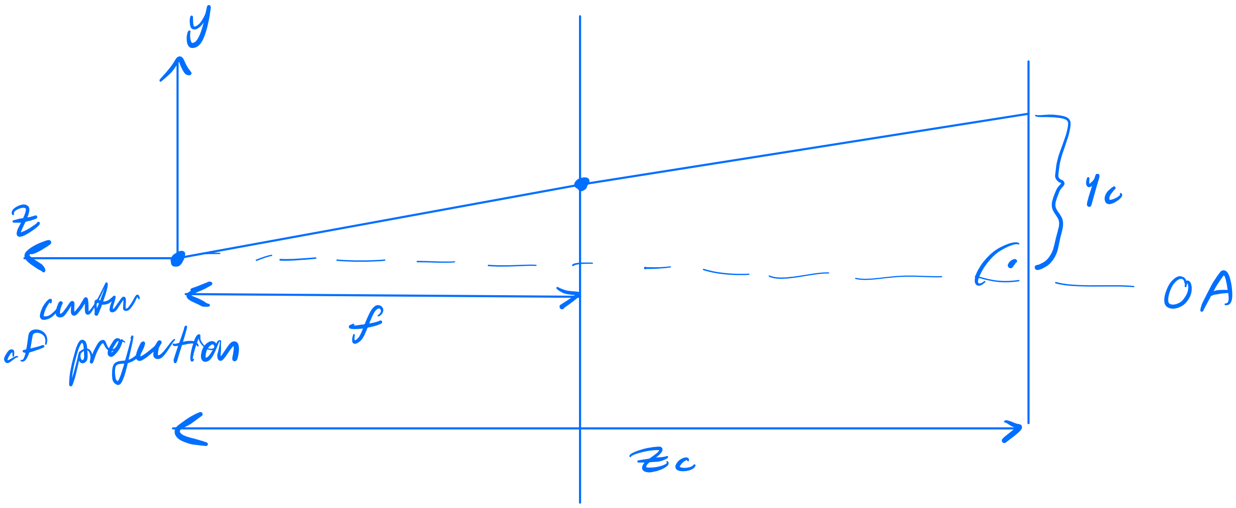
$$-R^T t = X_w$$

Question: Where is the world origin in camera coords?

$$X_c = R \cdot 0 + t$$

$$\underset{4\times 1}{\begin{bmatrix} X_c \\ 1 \end{bmatrix}} = \underset{4\times 4}{\left[\begin{array}{c|c} R_{3\times 3} & t_{3\times 1} \\ \hline 0_{1\times 3} & 1_{1\times 1} \end{array}\right]} \underset{4\times 1}{\begin{bmatrix} X_w \\ 1 \end{bmatrix}}$$

E: extrinsics: $w \rightarrow c$



lowercase for 2D coords uppercase for 3D coords

$$\frac{y_I}{-f} = \frac{y_c}{z_c}$$

$$y_I = -f\,y_c / z_c$$

$$x_I = -f\,x_c / z_c$$

$$\begin{bmatrix} x_I \\ y_I \\ 1 \end{bmatrix} \approx \begin{bmatrix} z_c x_I \\ z_c y_I \\ z_c \end{bmatrix} = \begin{bmatrix} z_c\left(-\frac{f x_c}{z_c}\right) \\ z_c\left(\frac{-f y_c}{z_c}\right) \\ z_c \end{bmatrix} = \begin{bmatrix} f x_c \\ f y_c \\ -z_c \end{bmatrix}$$

How to incorporate optical aberration?

$$\begin{bmatrix} x_I \\ y_I \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

camera projection!

$$\begin{bmatrix} x_I \\ y_I \\ 1 \end{bmatrix} \approx \underbrace{\left[\begin{array}{ccc|c} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]}_{\text{intrinsics}\ 3\times4} \underbrace{\left[\begin{array}{c|c} R & t \\ \hline 0 & 1 \end{array}\right]}_{\text{extrinsics}\ 4\times4} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}_{4\times1}$$

"radial distortion"

$$\begin{bmatrix} x_I \\ y_I \end{bmatrix} \rho\left(\left\| \begin{matrix} x_I \\ y_I \end{matrix} \right\|_2 \right)$$

radial distortion coefficient

$$\rho(x) = 1 + a_1 x + a_2 x^2$$

$$\rho(x) = 1 + \frac{a}{x+1}$$ stretching stretching factor

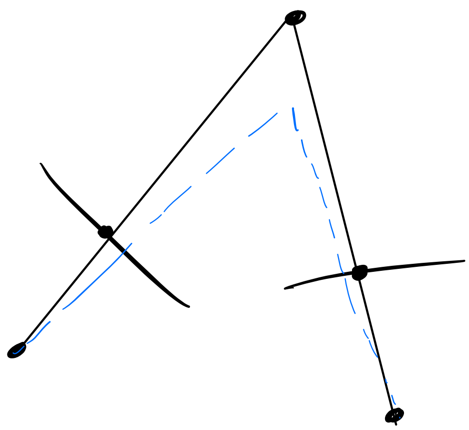
what is focal length?

get it from metadata from an image

photogrammetry?

So what can we do with all this?

Triangulation



known: projection matrices P_1, P_2

3D image coords

$\begin{bmatrix} x_{I_1} \\ y_{I_1} \end{bmatrix}, \begin{bmatrix} x_{I_2} \\ y_{I_2} \end{bmatrix}$

Compute world point X_W

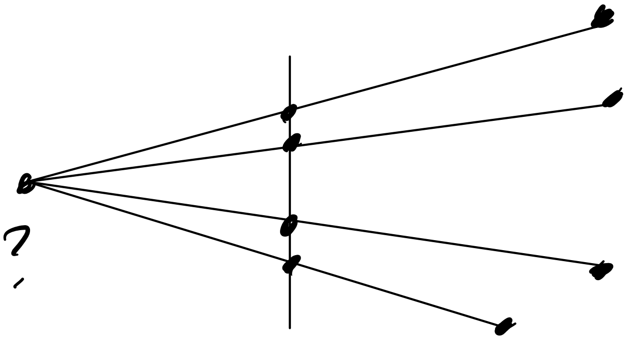
solve for $P_1 X_W \approx X_{I1}$

$P_2 X_W \approx X_{I2}$

$$\min_{X_W} \sum_{j \ldots} \|proj(X_W, P_1) - X_{I_1}\|^2 + \|proj(X_W, P_2) - X_{I2}\|^2$$

Resectioning

known: f: focal length
3D points
2D image points] correspondence



cost function:

$$\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

$$\min_{P} \sum_{j} \| \underbrace{proj(X_{W_j}, P) - X_{I_j}}_{\text{residuals}} \|^2$$

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known: 2D-2D correspondences
f_1, f_2

Compute: cam 1 @ origin
where is cam 2 in cam 1's camera coord. system?

R_{12}

t_{12}