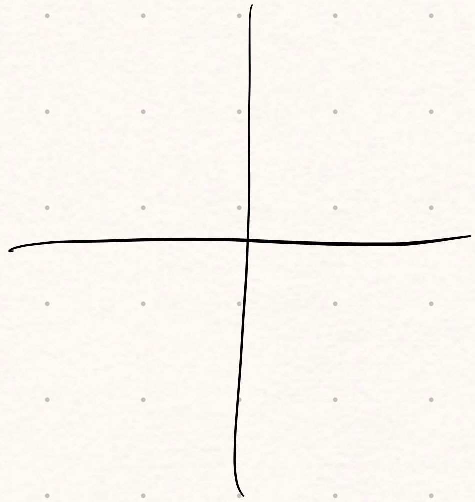
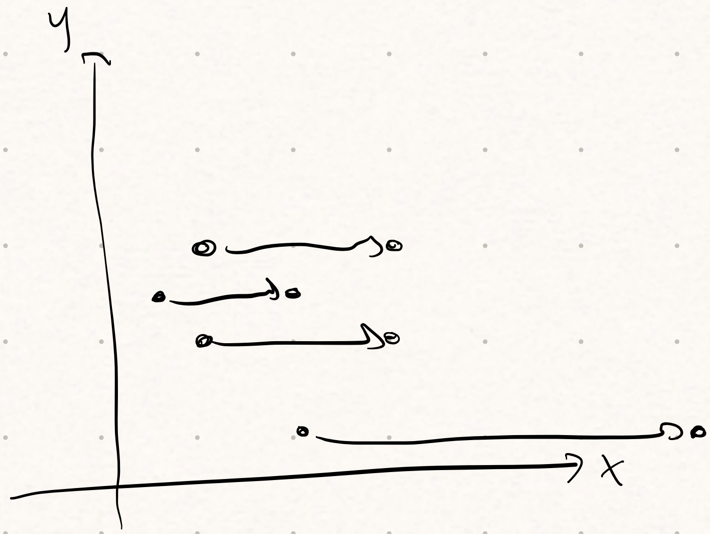
$$\underset{\mathbb{R}^2}{\begin{bmatrix} x' \\ y' \end{bmatrix}} = \underset{\text{transformation}}{\begin{bmatrix} M_{2\times 2} \end{bmatrix}} \underset{\mathbb{R}^2}{\begin{bmatrix} x \\ y \end{bmatrix}}$$



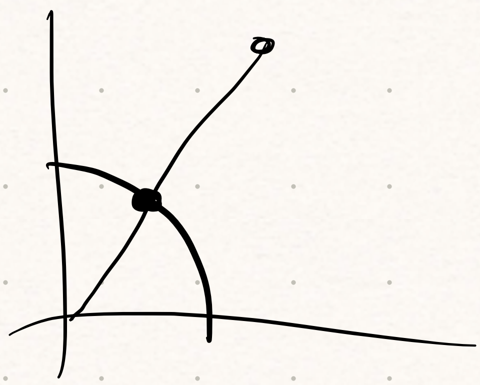
$$\begin{bmatrix} 2 & 6 \\ 0 & 1 \end{bmatrix}$$



$M\vec{v}$

$M(a\vec{v})$

$a\left(M\vec{v}\right)$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\left[\begin{array}{cc|c} \overbrace{s_x \quad h_1}^{M} & & a \\ h_2 \quad s_y & & b \end{array}\right]}_{2\times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x + h_1 y + a \\ s_y y + h_2 x + b \end{bmatrix}$$

$$= \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

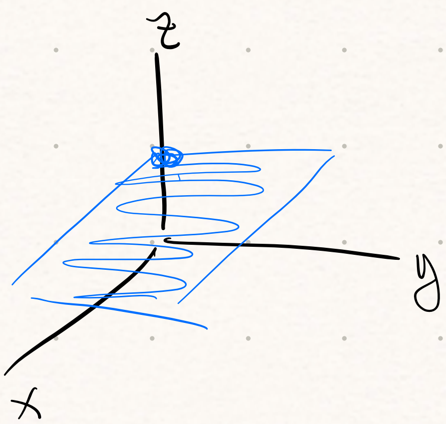
$$\downarrow$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \left[\begin{array}{c|c} M & \begin{matrix} a \\ b \end{matrix} \end{array}\right] \left[\begin{array}{c} x \\ y \\ \hline 1 \end{array}\right]$$

homogeneous coordinates

$$Ax + b \iff Mx$$

affine $\qquad\qquad$ linear



$$\begin{bmatrix} x \\ y \end{bmatrix} \longleftrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \longleftrightarrow \begin{bmatrix} x\lambda \\ y\lambda \\ \lambda \end{bmatrix}$$

$\mathbb{R}^2 \qquad \mathbb{RP}(3)$

Conclusion:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\left[\begin{array}{c:c} M & \begin{matrix} a \\ b \end{matrix} \end{array}\right]}_{\substack{2\times 3 \\ \text{matrix}}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

can represent combinations of

- scale
- translate
- rotate
- shear

one problem: <u>composition</u>

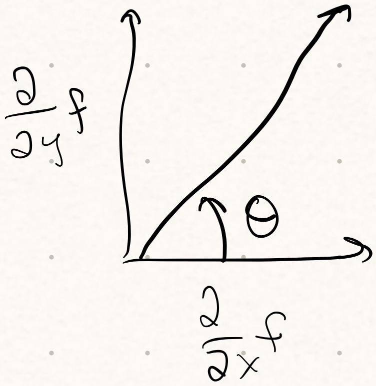
better idea:

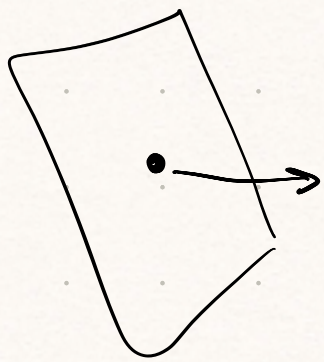
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\left[\begin{array}{cc:c} \multicolumn{2}{c:}{M} & \begin{matrix} a \\ b \end{matrix} \\ \hdashline 0 & 0 & 1 \end{array}\right]}_{3\times 3 \text{ matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

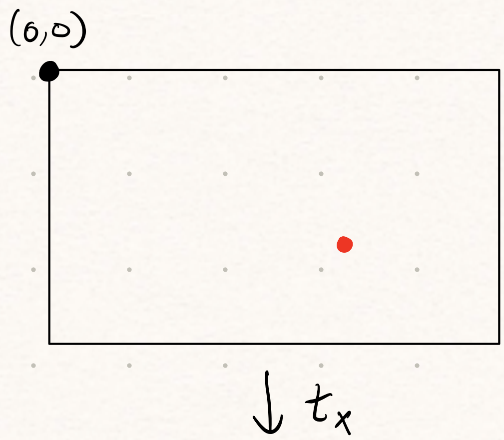
now: $T_{combo} = T_{scale} * T_{rot} * T_{translate} * \dots$ works!

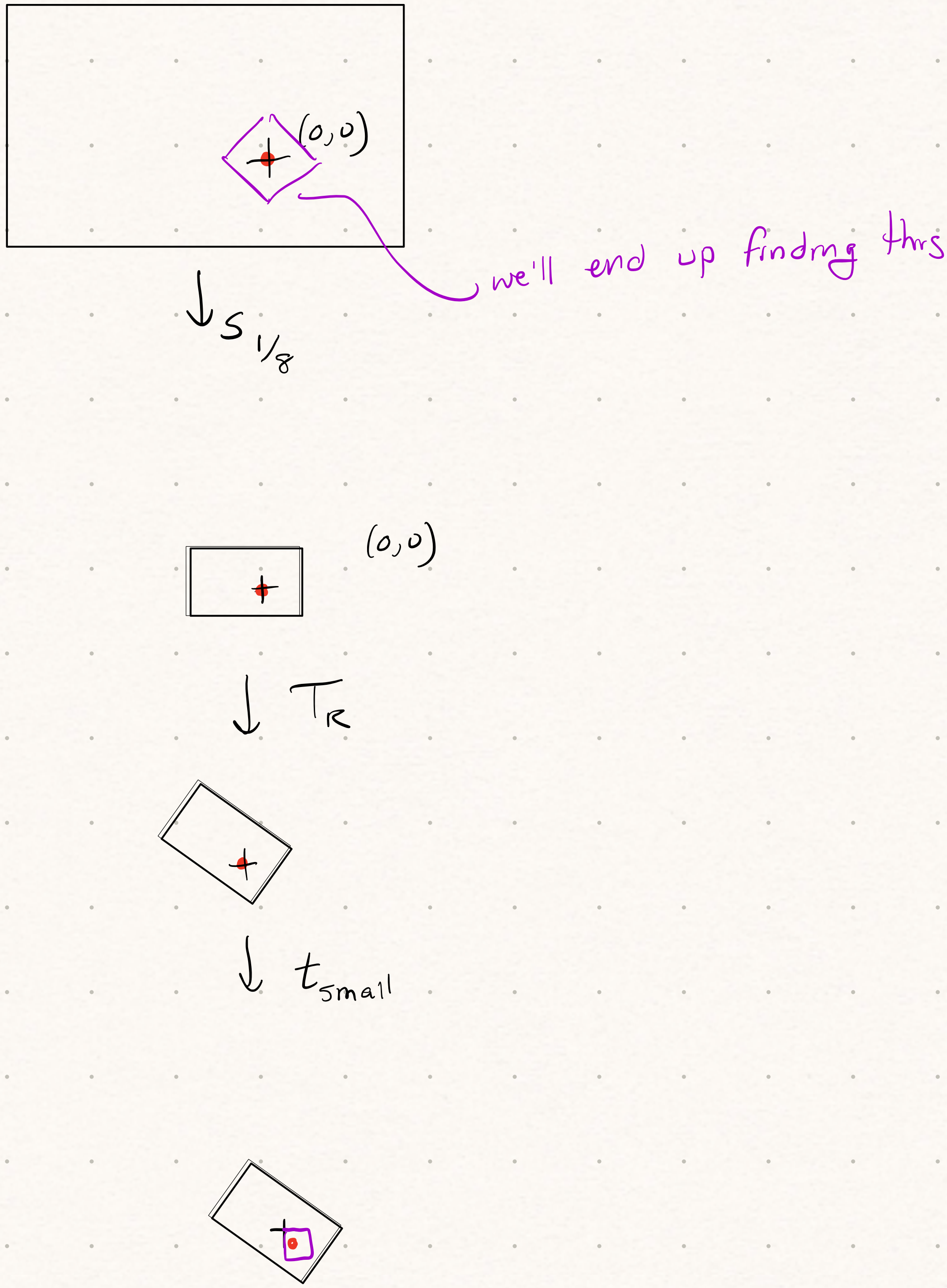
$$\left(T_{shift} * \left(T_{rot} * \left(T_{-shift} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right)\right)\right)$$

$$\nabla f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$









(0,0)
we'll end up finding this
↓ S 1/8
(0,0)
↓ T R
↓ t small