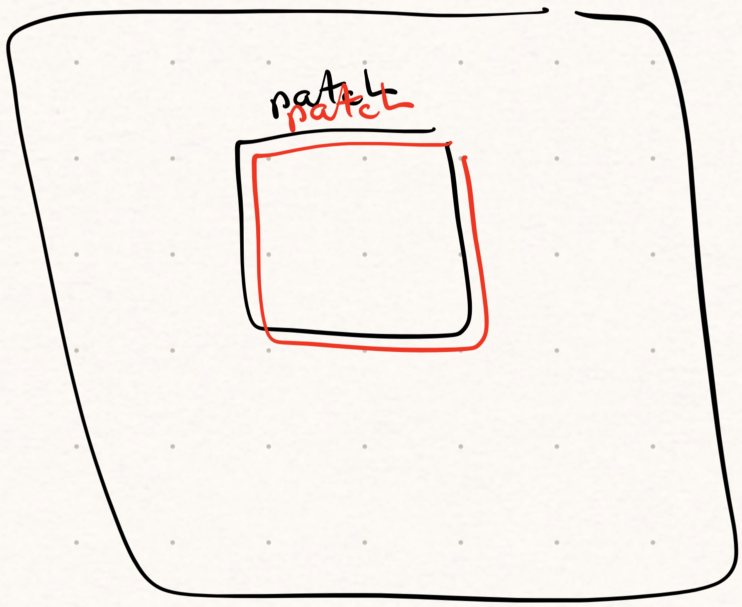


Idea: small shift

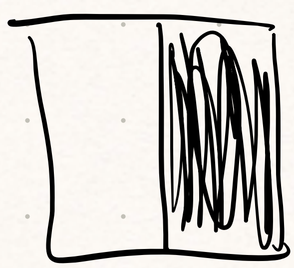
does patch look the same?

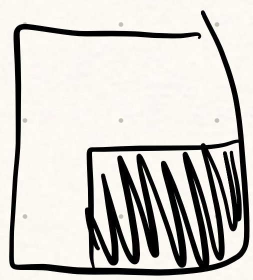
"shiftable"

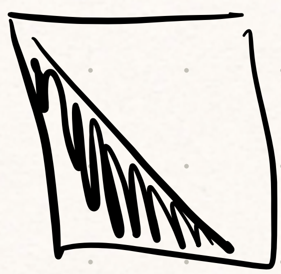
Compute "shiftability"?

Attempt 1

sobel x, y







we don't catch this case

Attempt 2

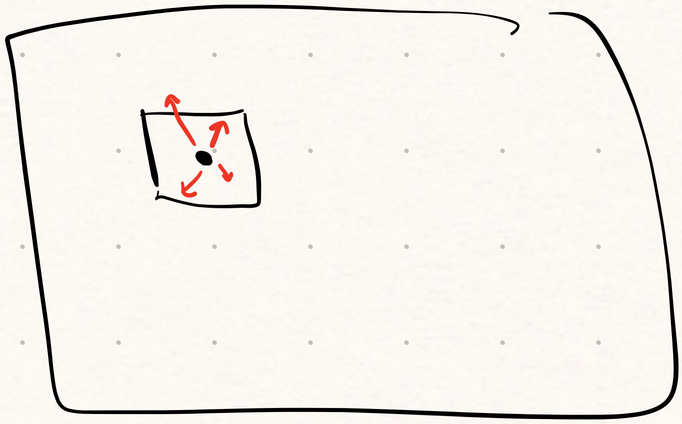
Brute force it

point of interest (x^*, y^*)

choose window W

Let (u,v) be a shift

$$E(u,v) = \sum_{(x,y)\in W} \left(I(x+u, y+v) - I(x,y) \right)^2$$



Idea: shift with <u>minimum</u> E defines shiftability

<u>for</u> each candidate point (x^*, y^*)

<u>for</u> each offset (u, v)

<u>for</u> each pixel (x, y) in patch

square / add

Attempt 3

$E(u,v)$ too expensive to compute everywhere

so approximate.

$$f(x) = f(a) + f'(a)(x-a) + \cancel{\frac{f''(a)}{2!}(x-a)^3} + \ldots$$

$$f(x) \approx f(a) + f'(a)(x-a) + ☹$$

<u>OR</u> $f(x+\Delta x) \approx f(x) + f'(x)(\Delta x)$ if we write $a \to x$, $x \to x+\Delta x$

in 2D now $f(x+\Delta x, y+\Delta y) \approx f(x,y) + f_x(x,y)\Delta x + f_y(x,y)\Delta y$

image I

image coord (x, y)

shifts (u, v)

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v$$

$$E(u,v) = \sum_{(x,y)\in W} \left(I(x+u, y+v) - I(x,y) \right)^2$$

$$= \sum_{(x,y)\in W} \left(\cancel{I(x,y)} + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v - \cancel{I(x,y)} \right)^2$$

$$= \sum \left[\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v \right]^2$$

$$I_x = \frac{\partial I}{\partial x}$$

$$I_y = \frac{\partial I}{\partial y}$$

$$E(u,v) = \sum_{(x,y)} \left[I_x u + I_y v \right]^2$$

$$= \sum_{(x,y)} \left[I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2 \right]$$

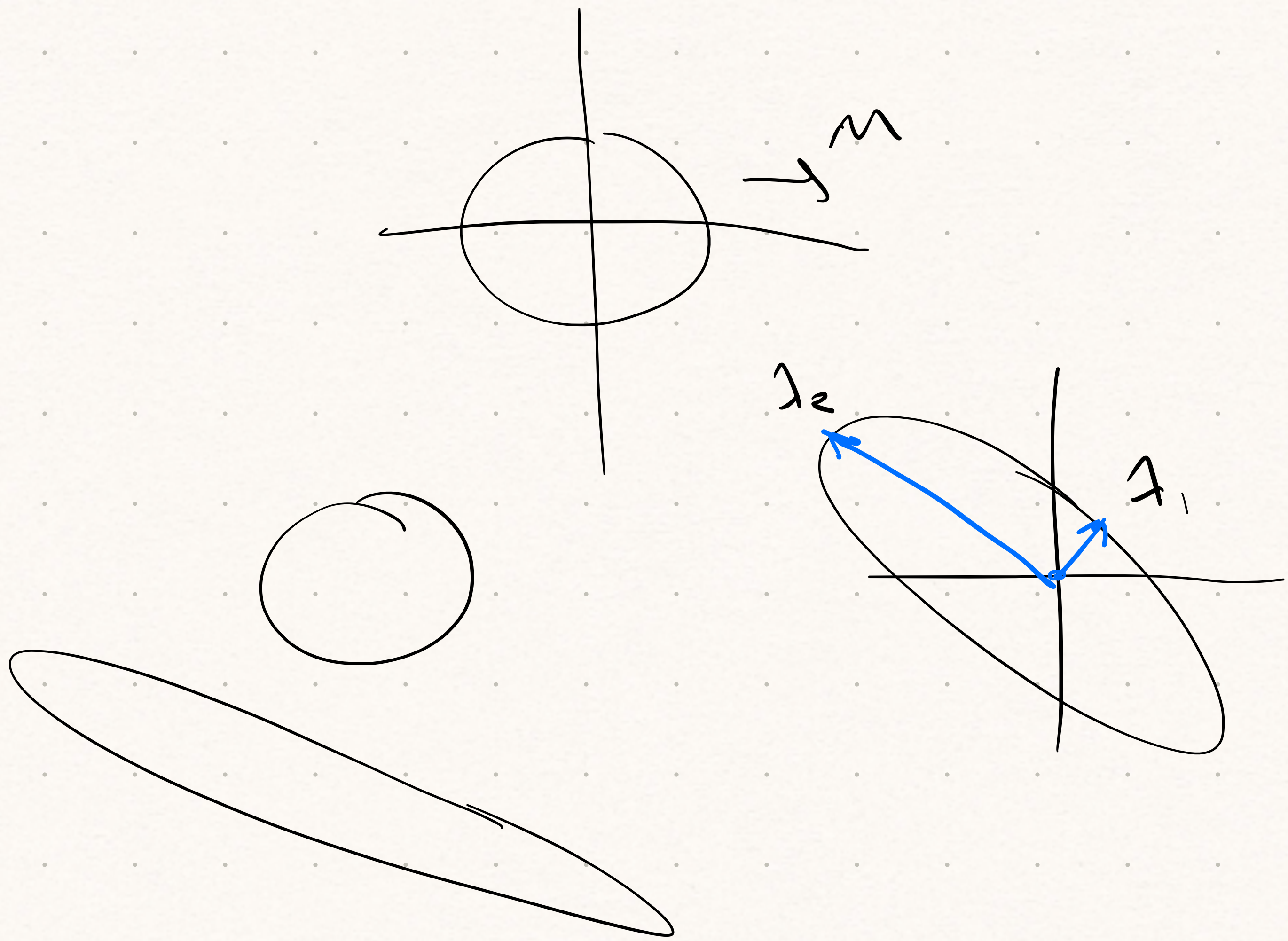
$$= \sum_{(x,y)} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$= \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \Sigma I_x^2 & \Sigma I_x I_y \\ \Sigma I_x I_y & \Sigma I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Σ: $(x,y) \in W$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \text{scalar}$$

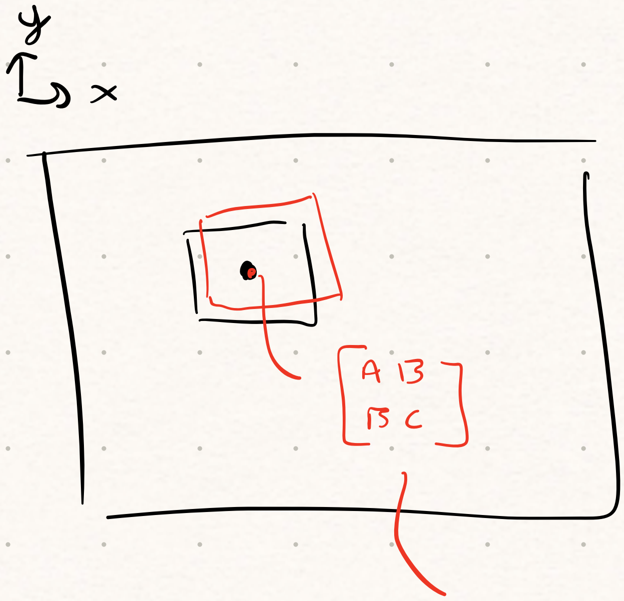
"stretch"



At each pixel (x,y)

we have A, B, C : $\begin{bmatrix} A & B \\ B & C \end{bmatrix}$

which is a "local approximation of the error function around (x,y)"



$$\begin{bmatrix} A & B \\ B & C \end{bmatrix}$$

proxy for $E(u,v)$