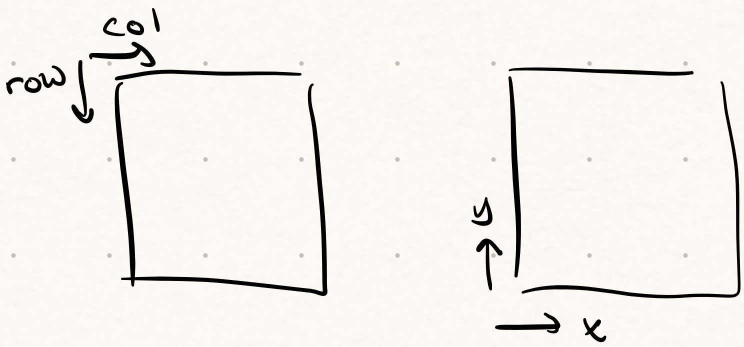
Image $W \times H \times$ channel

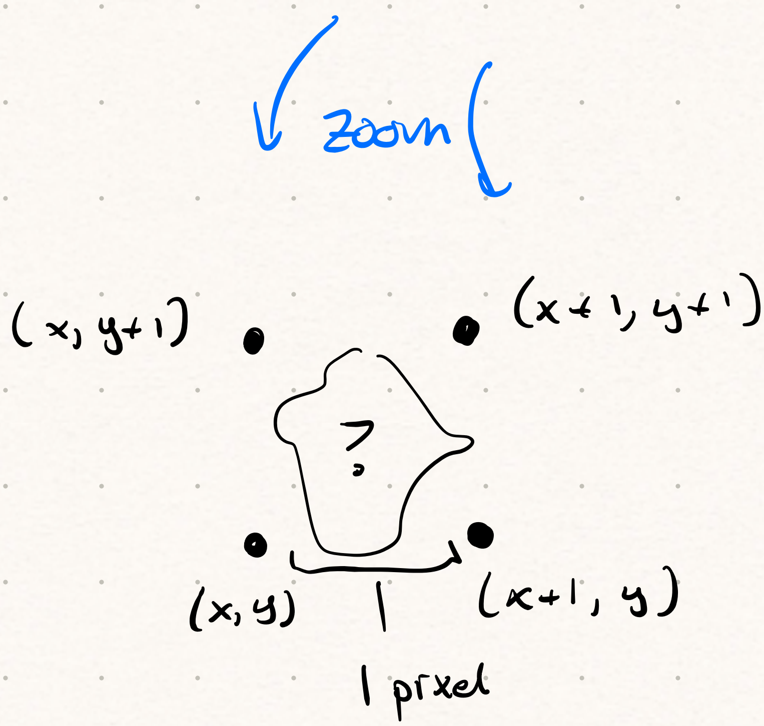
↳ no derivatives here

↳ one channel at a time

Image $W \times H$

function from coordinates to intensities





$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\Delta f = \frac{f(x+\overset{1\text{ pix}}{\Delta x}) - f(x)}{\Delta x}$$

$$\underbrace{\frac{\Delta f}{\Delta x}}_{\text{slope}} = f(x+1) - f(x)$$

$= \text{change in intensity}$

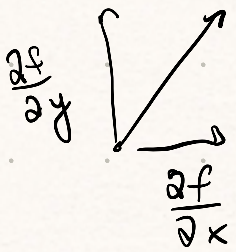
$$\frac{\Delta f}{\Delta x} = \text{slope} = \frac{f(x+1) - f(x-1)}{2}$$

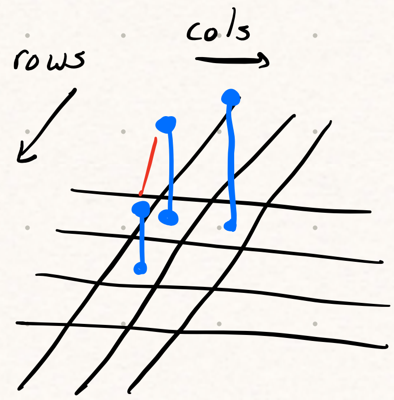
$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad \text{cross-corr}$$

$$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad \text{conv}$$

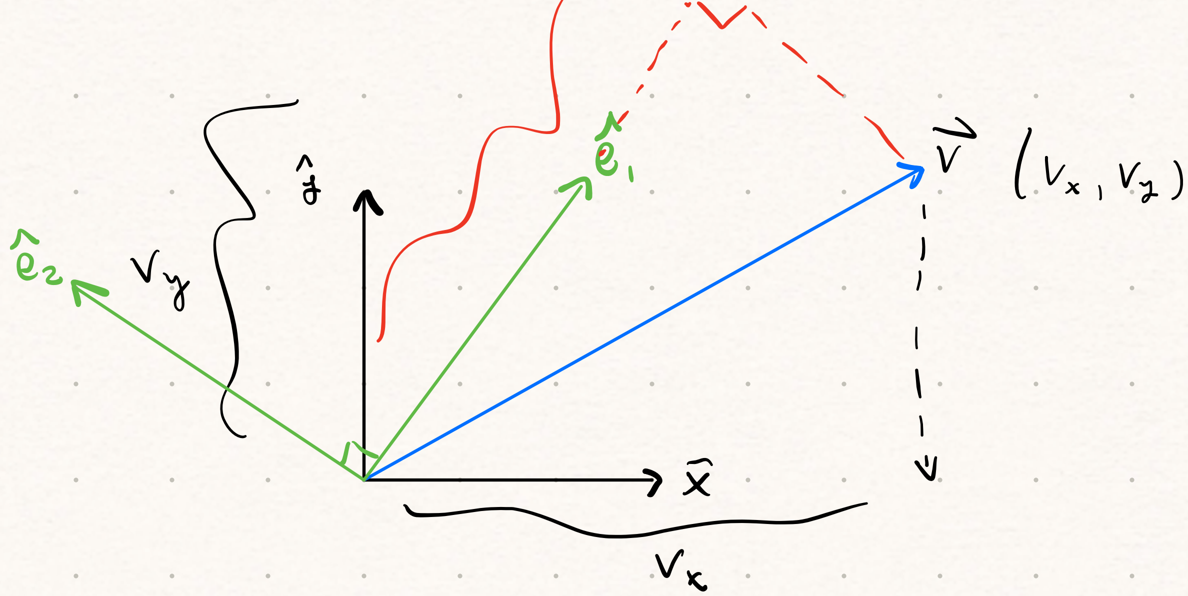
What about edges that aren't x, y?

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$





gradient mag. $\|\nabla f\|^2$?



Rules for axes:

$\mathbb{R}^n$ need n unit vectors
$\downarrow$
length 1

all orthogonal $(\hat{e}_i \cdot \hat{e}_j = 0)$

Images $\mathbb{R}^{W \times H}$: matrices with W colums an H rows

$$\hat{e}_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ & \vdots & \\ 0 & 0 & 0 \end{bmatrix}$$

$\{\hat{e}_{ij}\}$: coord system for space of images

<u>Reqs</u>: $\left.\begin{array}{l} W \times H \text{ vectors} \\ \|\vec{v}\| = 1 \\ \|\vec{v}_i \cdot \vec{v}_j\| = 0 \end{array}\right\}$ basis for $\mathbb{R}^{W \times H}$ coordinate system

$\mathbb{R}$ - real #s
$\mathbb{R}^2$ - x,y plane

$\mathbb{R}^{3\times 4}$ - matrices with real values w/ 3 rows and 4 cols.

Idea: sin/cos

$$\hat{e}_{f_x, f_y} = \left[\cos\left(\frac{i_x}{W} \cdot f_x \cdot \pi\right) \cdot \cos\left(\frac{i_y}{H} \cdot f_y \cdot \pi\right) \right]_{i_x, i_y}$$

$$\hat{e}_{f_x, f_y} \cdot \hat{e}_{f_x', f_y'} = 0 \quad \text{if} \quad f_x \neq f_x' \text{ or } f_y \neq f_y'$$

$\hat{e}_{f_x, f_y} \cdot I$ = component of the image on the (f_x, f_y) unit vector.

Discrete Cosine Transform