# Inverse Transformations

T: 3×3 matrix

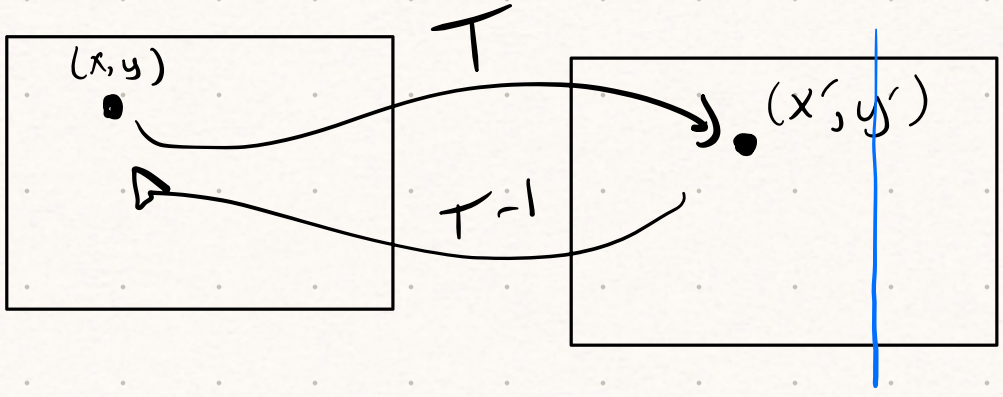
$T^{-1}$: 3×3 matrix

[ ] matrix = grid of #s

go deeper

[ ] $x \rightarrow x'$

matrix = transformation
operator
function




$T T^{-1} = I$

$T^{-1} T = I$

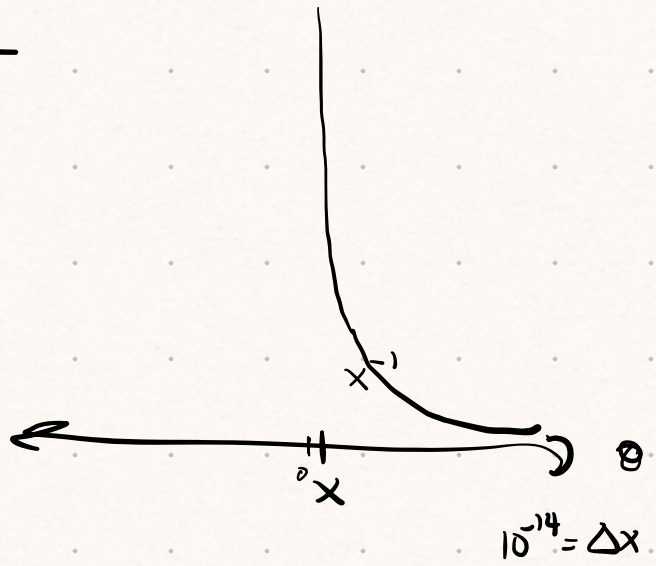
$5^{-1}$ ok

$6^{-1}$ not ok

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1}$

some matrices are 1-1 transformation

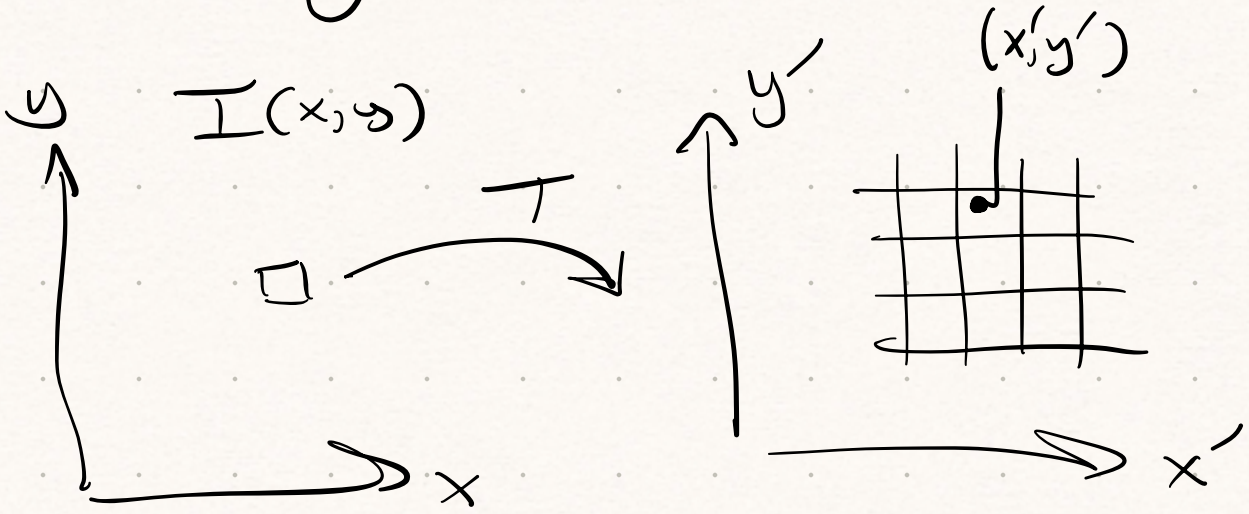
some squish 2D into a line or a point

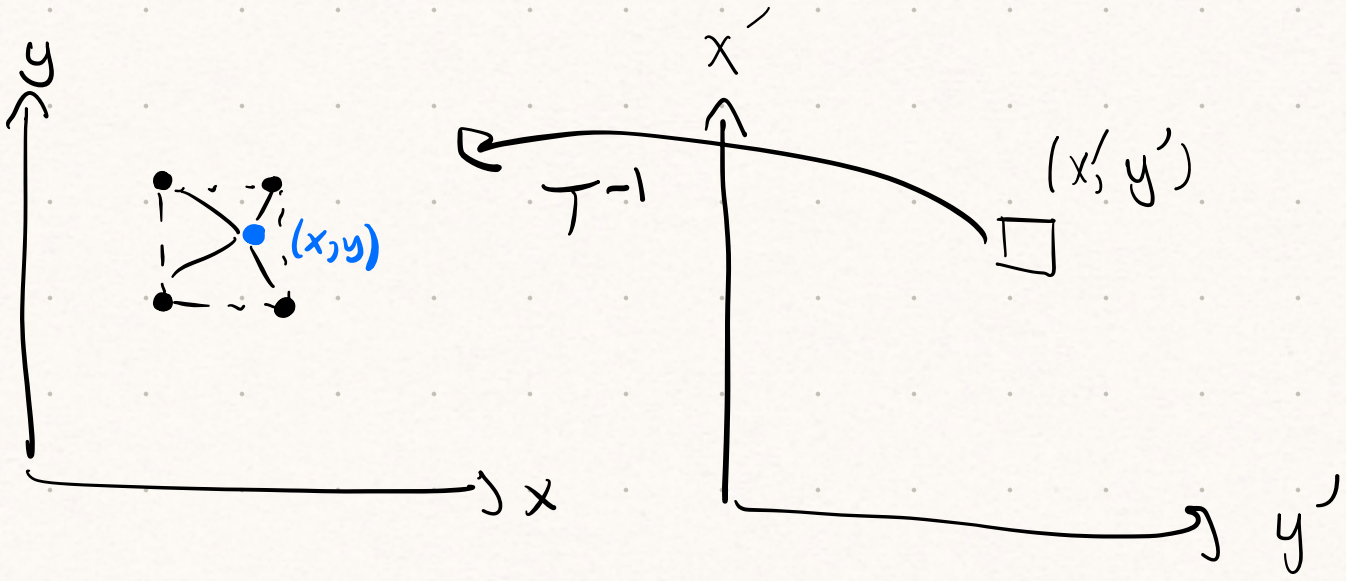



How to invert matrix?

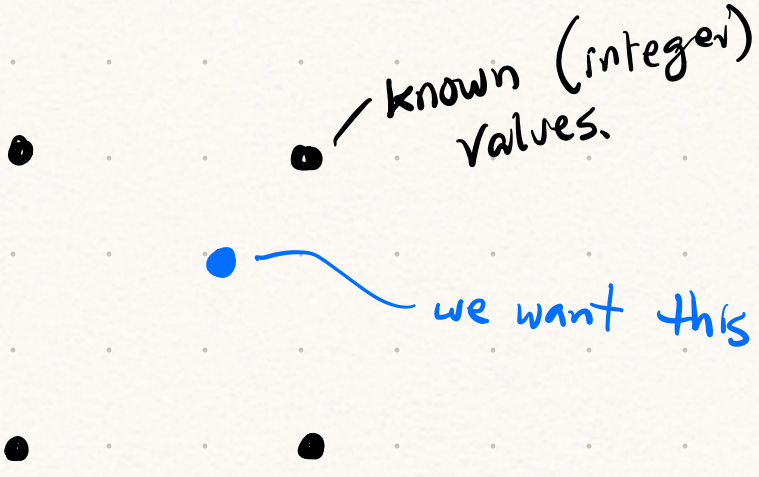
- library call
- generally, never invert a matrix
- but just 3x3? kinda OK

# Warping

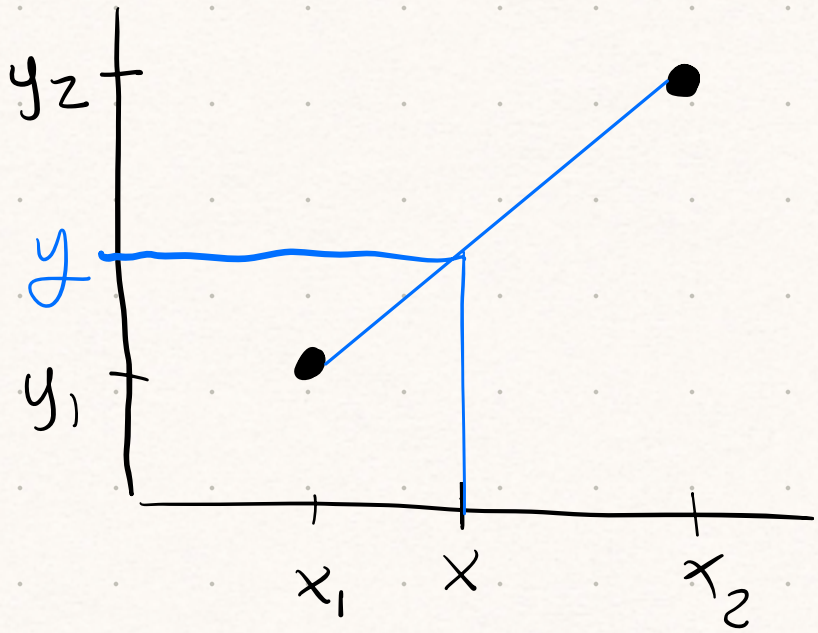



How to pick value?

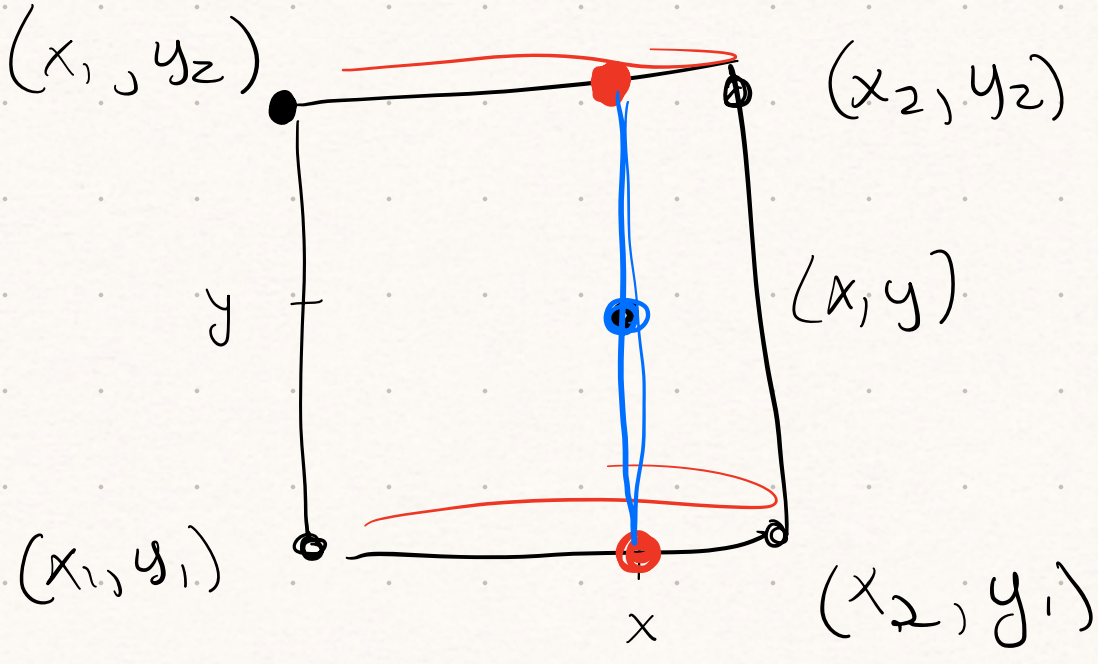


# 1D Linear Interpolation




RGB

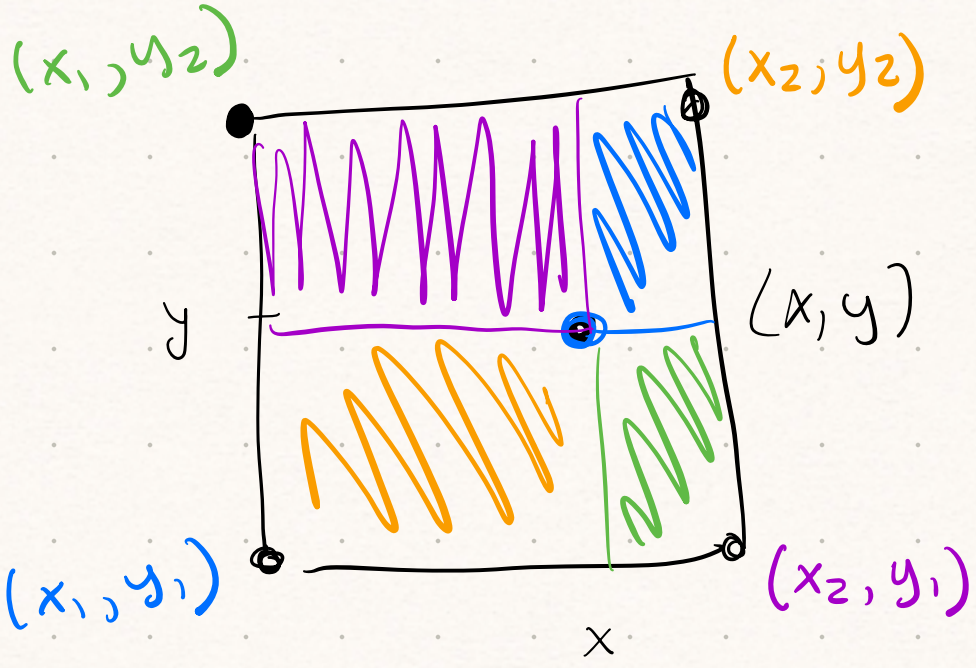
$$y = \frac{x - x_1}{x_2 - x_1} \cdot (y_2 - y_1) + y_1$$




step 1

step 2

# Bilinear Interpolation




$$I[x,y] = |x_1 - x| \cdot |y_1 - y| \, I[x_2, y_2]$$

$$+ |x_2 - x| \cdot |y_2 - y| \, I[x_1, y_1]$$

$$+ (\ldots)$$

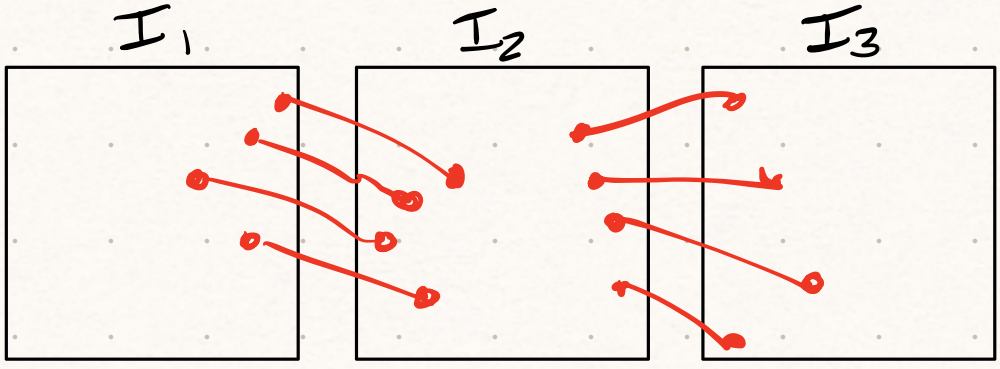
$$+ (\ldots)$$

# Panorama Stitching

CAPTURE

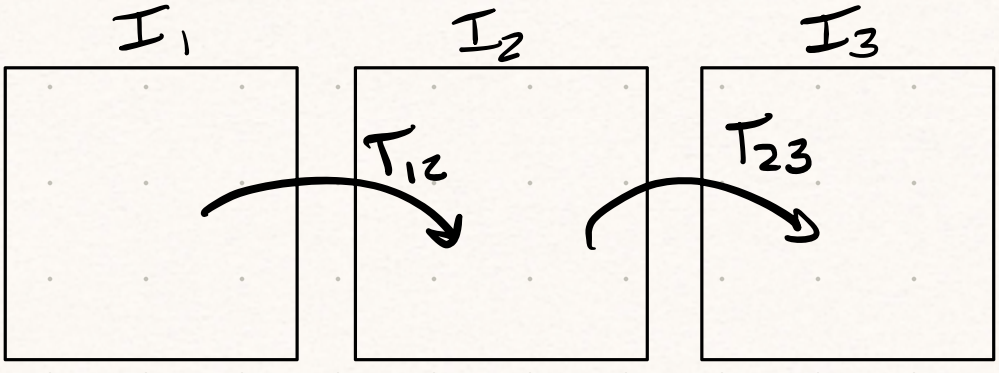
1. Capture images with some overlap
2. Feature detectron, descriptor

MATCH



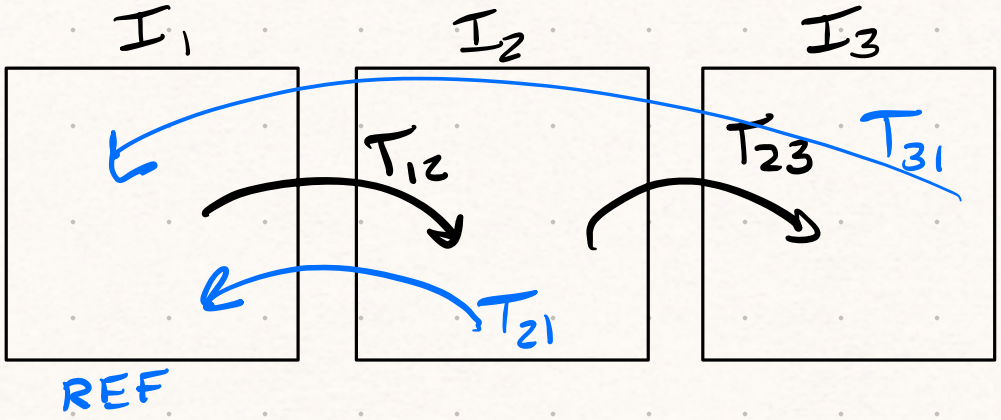

3. Fit a transformation to align neighboring pairs of images.

ALIGN PAIRWISE




4a. pick reference frame

4b. compute $T_{ir}$ for each $i$

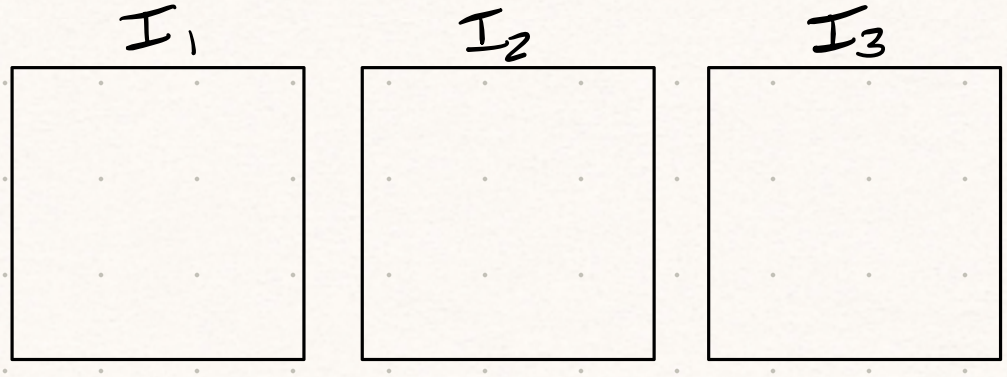


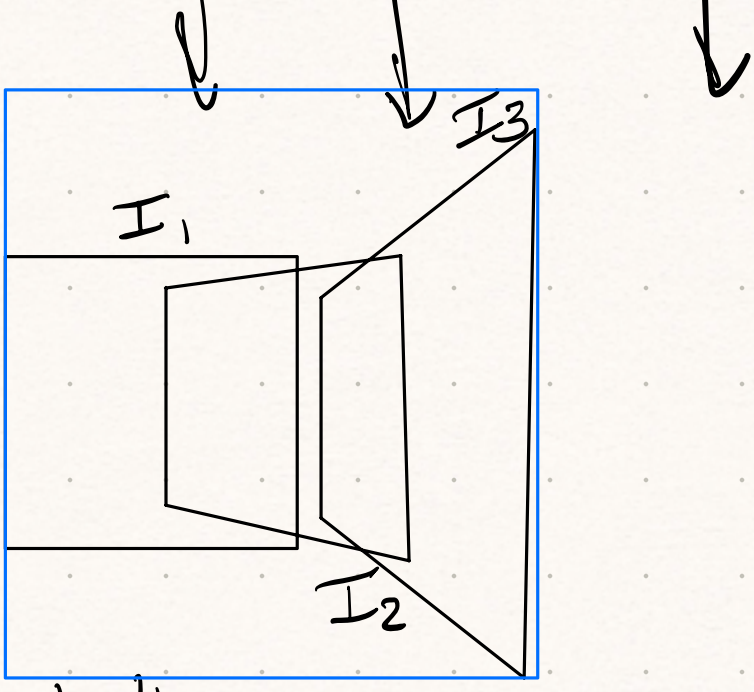

$$T_{21} = T_{12}^{-1}$$

$$T_{31} = (T_{23} T_{12})^{-1} = T_{12}^{-1} T_{23}^{-1}$$

$$T_{11} = I$$

$$T_{12}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

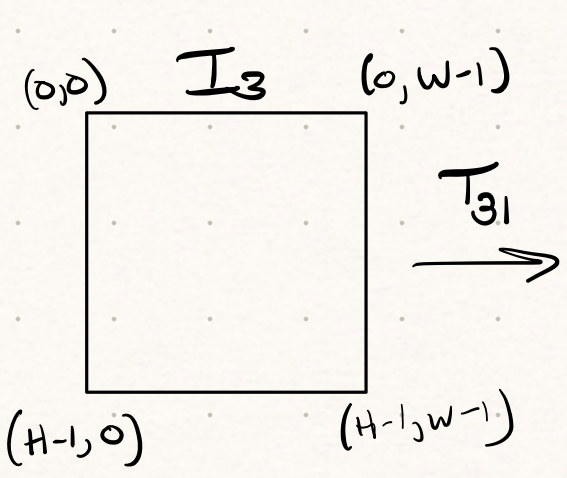


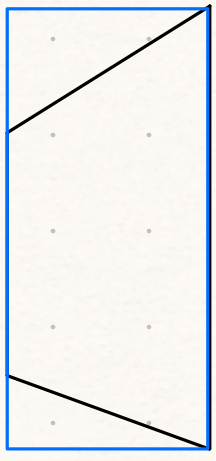




all three in $I_1$ coords

5. Create an accumulator to store output image

5a. Compute bounding box of each warped image.

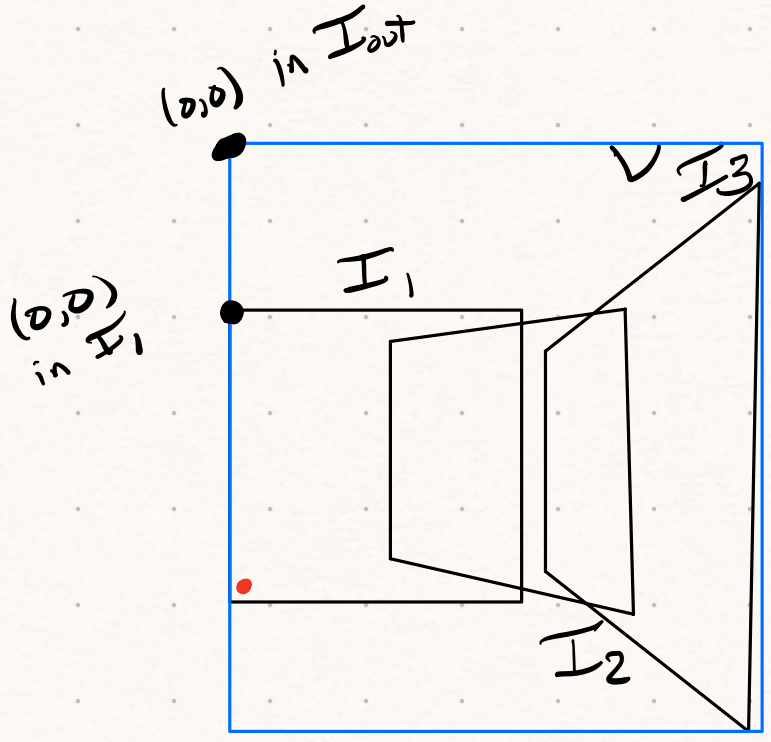


bounding box

↻ REPEAT FOR EACH IMAGE

- mins
- maxs




$T_{1,out}$ just shifts (0,0) with translation

So we have

- an empty $I_{out}$
- transformations $T_{i,out}$ for each $i$

WARP 6. Reverse warp each image into $I_{out}$

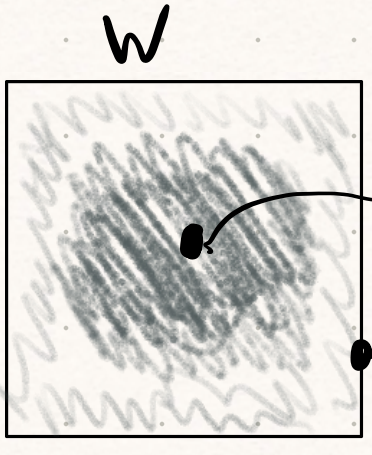
6a. Accumulate weighted pixel contributions. Store weight in a 4th channel.

6b. Normalize: divide first 3 channels by 4th channel. Discard 4th channel

CROP 7. Do this in a photo editor

(IMPROVEMENT)

6a' blend with feathering




accumulation: $I_{out}[:,:,0:4] \mathrel{+}= I * W$

$I_{out}[:,:,4] \mathrel{+}= W$

Initialize $I_{out} = 0$

For each source image $I$

grab first 3 channels

$$I_{out}[:,:,0:3] \mathrel{+}= warp(I_i * W_i, T_{i,out})$$

$$I_{out}[:,:,3] \mathrel{+}= warp(W_i, T_{i,out})$$

grab 4th channel

Normalize: $I_{final} = I_{out}[:,:,0:3] / I_{out}[:,:,3]$