Math Modeling: Coordinate Transforms and Movement Calculations

Tello drones localize relative to specific mission pads. To create a larger space where drones can remain within global bounds requires knowing the location between mission pads, as a predefined map.

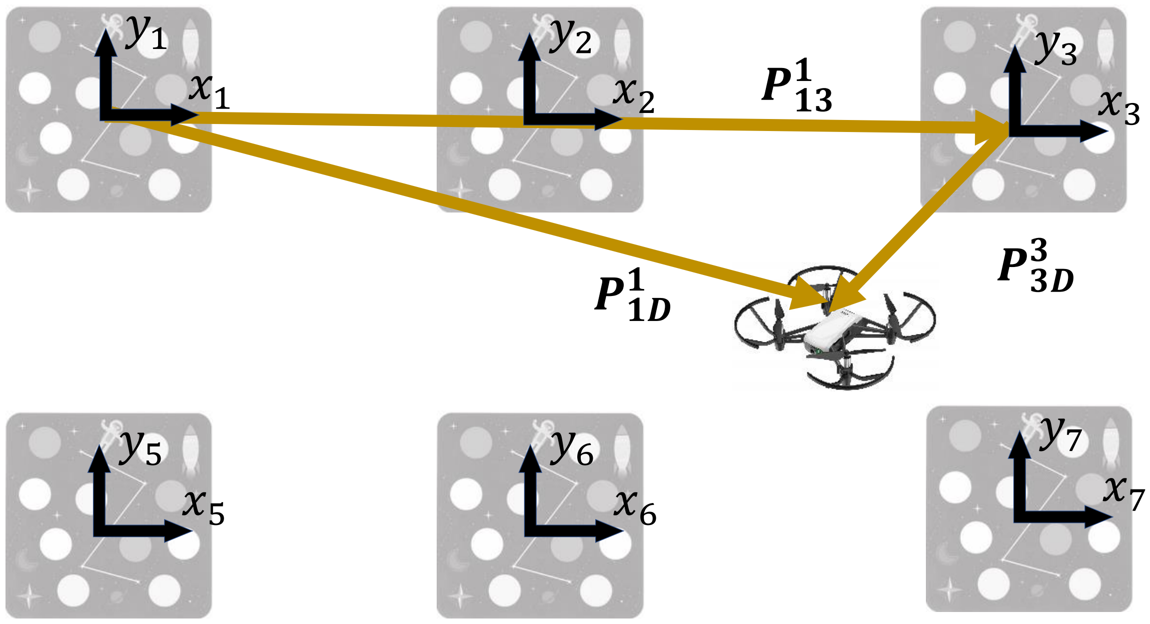

Figure 1: Mission pad

We can define these vectors as the following:

$\boldsymbol{P_{1D}^{1}}$***: Position in global coordinate frame***

$\boldsymbol{P_{3D}^{3}}$***: Position of drone in local mission pad frame***

$\boldsymbol{P_{13}^{1}}$***: Pre-determined location of mission pad***

Here we can see that if we want to find the position of the drone in the global coordinate frame, we know $\boldsymbol{P_{3D}^{3}}$ from the drone, and we predetermined $\boldsymbol{P_{13}^{1}}$**.** This should be simple vector addition, however they are technically in different coordinate frames and require a rotation matrix, resulting in:

$$P_{1D}^{1} = R_{3}^{1} * P_{3D}^{3} + P_{13}^{1}$$

However, by aligning all mission pads in the same direction, R_3^1 simplifies to the identity, resulting in:

$$P_{3D}^1 = P_{3D}^3 \rightarrow P_{1D}^1 = P_{3D}^3 + P_{13}^1$$

Now knowing the global coordinate, we can now apply proportional control whenever the drone begins to fly out of the approved airspace.

For example, if the drone crosses a boundary defined along the positive X axis of the global coordinate system, we can define an error as:

$$\boldsymbol{e} = \begin{matrix} x_{boundary} \\ 0 \\ 0 \end{matrix} - \begin{matrix} x_D \\ 0 \\ 0 \end{matrix}$$

Then, a restoring velocity, $\boldsymbol{v}$ would be based on some gain k_p.

$$\boldsymbol{v} = k_p \cdot \boldsymbol{e}$$

However, this restoring velocity is in the global frame, so let us denote it as $\boldsymbol{v^1}$. In order to put this velocity into the drone's frame, we need to multiply it back by a rotation matrix, based upon the yaw of the drone.

$$\boldsymbol{v^D} = \boldsymbol{R_1^D} \cdot \boldsymbol{v^1}$$

Where:

$$\boldsymbol{R_1^D} = \begin{matrix} \cos(yaw) & -\sin(yaw) & 0 \\ \sin(yaw) & \cos(yaw) & 0 \\ 0 & 0 & 1 \end{matrix}$$