Design and Optimization of Air Core Magnetorquers for Attitude Control of LEO Nanosatellites

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***Abstract*—Magnetorquers are widely used electromagnetic actuators for attitude control of low-Earth orbit nanosatellites. Due to the limited power budget, strict volume and mass constraints of standard nanosatellite formats, the magnetorquers need to be carefully designed. An optimal magnetorquer exhibits the required magnetic dipole moment while minimizing the mass and power consumption. Here, a multiobjective optimization based on the genetic algorithm is adopted to design two air core magnetorquers. The presented approach allows a designer to set acceptable range of the design parameters, which makes it time-efficient, powerful, and versatile as it can be easily applied to different design cases. The preliminary resistance measurements of the two prototyped magnetorquers show excellent agreement with the calculations within 3%.**

***Keywords*—Magnetorquer; Optimization; Genetic algorithm; Nanosatellite; ADCS**

I. Introduction

Magnetorquers or magnetic torquers (MTQs) are electromagnetic actuators commonly used for attitude control, detumbling, and stabilization of nanosatellites in low-Earth orbit (LEO). They consist of wire coils attached to the body of a spacecraft [1]. As a current flows through the coils, a magnetic dipole moment is created, which interacts with the Earth's magnetic field exhibiting a torque $\boldsymbol{\tau}$ perpendicular to both the net magnetic moment $\boldsymbol{m}$ and the Earth's magnetic field $\boldsymbol{B}$.

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B} \tag{1}$$

The magnitude of the torque τ depends on the magnitude of the local Earth's magnetic field B, the magnitude of the magnetic dipole moment m, and the angle θ between them:

$$\tau = mB\sin(\theta). \tag{2}$$

Due to the very weak Earth's magnetic field, the magnitude of the torque that can be generated by MTQs is also very small, so MTQs have limited control authority [1]. Moreover, the torque vanishes entirely if the Earth's magnetic field and magnetic dipole moment are collinear. Thus, it is a substantial engineering challenge to design and optimize a MTQ that produces sufficient magnetic dipole moment. In this paper we describe the design procedure of two air core MTQs for Attitude and Control System (ADCS) demonstrator with one degree of rotational freedom around vertical axis. The MTQs must fit within the volume of $10 \times 10 \times 10$ cm^3, i.e. within a one-unit (1U) CubeSat format. In the following section we present how the desired magnetic dipole moment can be achieved while minimizing mass and power consumption.

II. Magnetorquer's Parameters and Equations

The magnitude of a magnetic dipole moment is given with the following equation:

$$m = NIA. \tag{3}$$

Here, N represents the number of wire turns, I the current through the wire, and A the area of the coil. The higher N, I or A, the higher the magnetic dipole moment. However, these parameters cannot be chosen arbitrarily. For example, the area A cannot exceed 10×10 cm^2. This limit is set by the 1U CubeSat format. Thus, the sides of square MTQs are limited to 10 cm. Moreover, increasing the number of turns N increases the overall mass of the MTQ and parasitic resistance of the wire, which leads to higher power consumption for the same driving current. Similarly, increasing the current for the fixed number of turns also increases the power consumption due to the parasitic resistance of wire. To systematically approach to the optimization of MTQs, a list of all design parameters with their units and descriptions are given in Table I.

The relations between the design parameters in Table I are set by the following equations:

$$P = UI, \tag{4}$$
$$U = RI, \tag{5}$$
$$R = \rho l/S, \tag{6}$$
$$S = r^2\pi, \tag{7}$$
$$O = 2(a+b), \tag{8}$$
$$l = NO, \tag{9}$$
$$A = ab, \tag{10}$$
$$D = M/V, \tag{11}$$
$$V = Sl. \tag{12}$$

Together with (3), equations (4)–(12) govern MTQ's magnetic dipole moment, power consumption, physical dimensions and mass. Although the equations are quite simple *per se*, choosing

TABLE I. MAGNETORQUER DESIGN PARAMETERS

Designation	Unit	Description
P	W	Power
U	V	Voltage
I	A	Current
R	Ω	Resistance
m	Am^2	Magnetic moment
a	m	MTQ length
b	m	MTQ width
O	m^2	MTQ perimeter
A	m^2	MTQ area
N	-	Number of turns
l	m	Wire length
r	m	Wire radius
S	m^2	Wire cross section
V	m^3	Wire volume
M	kg	Wire mass
D	kg/m^3	Wire density
ρ	Ωm	Wire resistivity

an optimal set of parametric values from Table I is a challenging task. Here, an optimal MTQ is considered to be the one that exhibits the required magnetic dipole moment while keeping the power consumption and mass at minimum. As explained above, these are opposing requirements, thus a trade-off needs to be made. To simplify the optimization process, we set the mechanical constraints first.

III. MECHANICAL CONSTRAINTS

Two MTQ frames ensure mechanical rigidity and provide support for the wire. In the prototyping phase, two frames are designed to be easily 3D-printed, i.e. the angle of the frame overhang does not exceed $45°$. This leads to the trapezoidal profile for te wire support with the overall area of 42 mm^2, as shown in Fig 1. One turn of the wire with radius r occupies the area of $4r^2$ (see Fig. 1). Thus, the ratio of the profile area to the area occupied by one turn sets the limit to the number of turns that fit within the profile. The smaller the radius r, the more turns can fit within the profile. If the wire with $r = 0.15$ mm is used (the thickest wire allowed in the optimization process), the maximum number of turns is 466. This number of turns is, however, possible only if the wire is winded perfectly. To account for the winding imperfections, the maximal number of turns is limited to 400 during the optimization process.

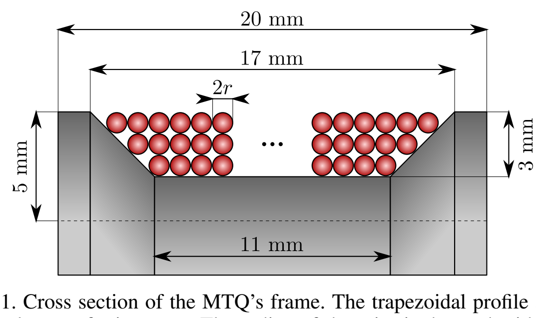

Figure 1. Cross section of the MTQ's frame. The trapezoidal profile supports multiple layers of wire turns. The radius of the wire is denoted with r.

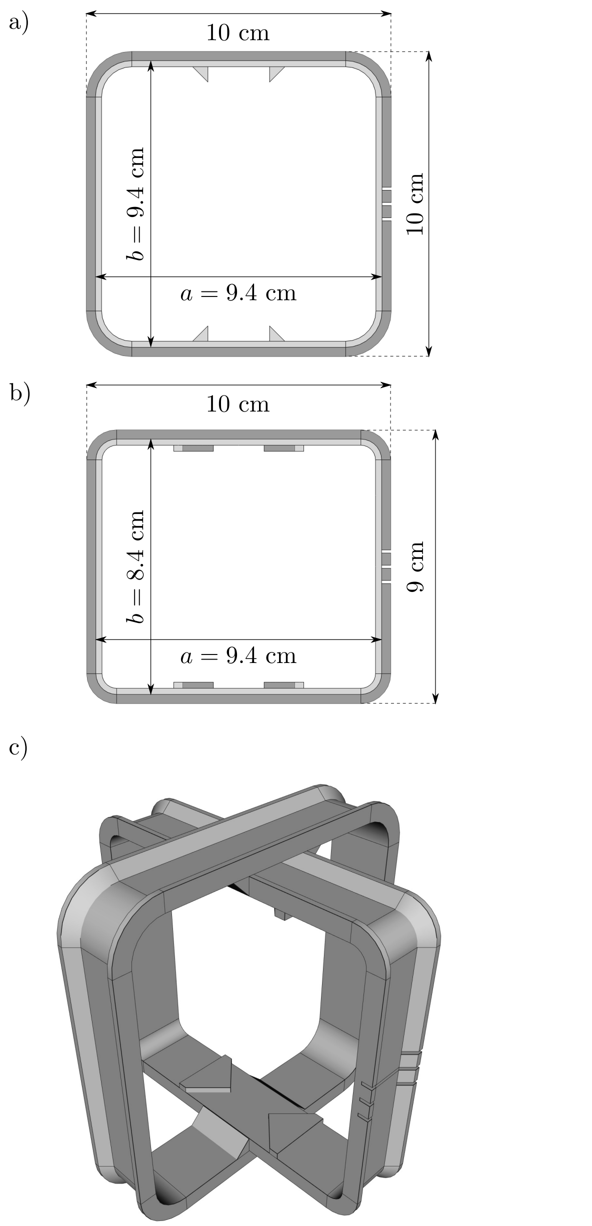

Figure 2. 2D drawing of the outer (a) and inner (b) MTQ frame. (c) 3D drawing of the assembled MTQ frames.

The constraints on the MTQ's physical size are set primarily by the 1U CubeSat format. Therefore, a side of a rectangular air core MTQ that fits within a 1U Cubesat is limited to 10 cm. The MTQ frames are designed as shown in Fig. 2. They are placed orthogonally, providing an arbitrary magnetic dipole moment in any direction within the horizontal plane. Since the frames must fit perfectly one within another, their dimensions cannot be equal. The length of the outer sides of the outer square frame (Fig. 2a) is equal to 10 cm. The inner frame is rectangular, with the outer sides of 9 cm and 10 cm (Fig. 2b). On the inner side, both frames have additional structural elements for the central alignment. Such a design leaves the volume of $9 \times 9 \times 7.6$ cm^3 within a 1U CubeSat for the rest of the satellite subsystems. The coils of the MTQs are also considered rectangular, with the sides $a = b = 9.4$ cm for the outer MTQ, and $a = 9.4$ cm, $b = 8.4$ cm for the inner MTQ, as shown in Fig. 2. It is assumed that a and b do not change significantly with each layer of wire turns, i.e. the outer layers contribute equally to the magnetic dipole moment as the inner layers. Consequently, parameters O and A (see Table I) become constant, which simplifies the optimization process. Likewise, it is assumed that the resistance and mass of the wire also do not change significantly with each layer. Furthermore, to minimize losses, a copper wire is used, which defines parameters $\rho = 1.68 \cdot 10^{-8}$ Ωm and $D = 8960$ kg/m^3.

IV. Multiobjective Optimization of Magnetorquer Design Parameters

Although fixing the values of some parameters from Table I simplifies the optimization process, as explained in the previous section, additional constraints are required for a multiobjective optimization of the MTQs. MathWorks Optimization Toolbox for Matlab environment [2], used for the MTQ optimization, distinguishes several types of constraints such as bounds, nonlinear equalities, and nonlinear inequalities. Furthermore, the objective functions that need to be satisfied during the optimization process are defined.

A. Bounds and Nonlinear Constraints

The bounds define the acceptable range of values for a certain parameter. They are set for the parameters r, N, and I, as shown in Table II.

TABLE II. Parametric Bounds

Parameter	Unit	Lower bound	Upper bound
r	mm	0.05	0.15
N	-	1	400
I	mA	1	200

The lower and upper bounds of r are chosen with respect to in-house fabrication capabilities. The wire with r smaller than the lower bound is potentially too fragile. On the other hand, if r is greater than the upper bound, the wire is too stiff. The upper bound of $r = 0.15$ mm sets the upper bound of N, as explained in the previous section. I is not mechanically constrained, however it directly effects the power consumption of the MTQ and sets the requirements on the current driver circuitry. Some driver topologies can be found in [3], [4].

Nonlinear constraints allow us to restrict the solution of the optimization to any region that can be described in terms of smooth functions. The MathWorks Optimization Toolbox offers two types of nonlinear constraints: nonlinear inequalities and nonlinear equalities. Nonlinear inequality constraints have the form $c(x) \leq 0$, where c is a vector of constraints, one component for each constraint. Similarly, nonlinear equality constraints have the form $ceq(x) = 0$ [2]. Here, m is limited by a nonlinear equality, while M and P are limited by nonlinear inequalities. Substituting the equations (3)–(12) yields the following expressions for the nonlinear constraints:

$$m = INab - m_{\text{req}}, \tag{13}$$

$$P = I^2 \rho \frac{2N(a+b)}{r^2 \pi} - P_{\max}, \tag{14}$$

$$M = 2Dr^2 \pi N(a+b) - M_{\max}. \tag{15}$$

Here, m_{req} represents the required magnetic moment. It is set to 0.3 Am^2, which is similar to the commercially available MTQs [5]. $P_{\max}$ and $M_{\max}$, representing the maximum power consumption and mass, respectively, are set to 0.5 W and 0.15 kg. If required, the parameters m_{req}, $P_{\max}$, and $M_{\max}$ can be set to other values, making this approach highly versatile.

B. Objective Functions and Optimization Procedure

In the next step, the objective functions need to be defined. These are the functions to be minimized by the optimization algorithm. For this particular optimization problem, they are quite similar to the nonlinear constraints, since already constrained power consumption and mass of the MTQs are being minimized. Therefore, the objective functions are:

$$P(r, N, I) = I^2 \rho \frac{2N(a+b)}{r^2 \pi}, \tag{16}$$

$$M(r, N) = 2Dr^2 \pi N(a+b). \tag{17}$$

The goal of the optimization process is to find the values of the parameters N, r and I that minimize the defined objective functions respecting the bounds and nonlinear constraints described above. The objective functions are optimized using the function `gamultiobj()`, which finds Pareto front of multiple fitness functions using the genetic algorithm [2].

Since the magnetic moment is directly proportional to the area of a MTQ, it is more challenging to optimize the inner, smaller MTQ for the given set of bounds and constraints. Thus, it is optimized first. The function `gamultiobj()`

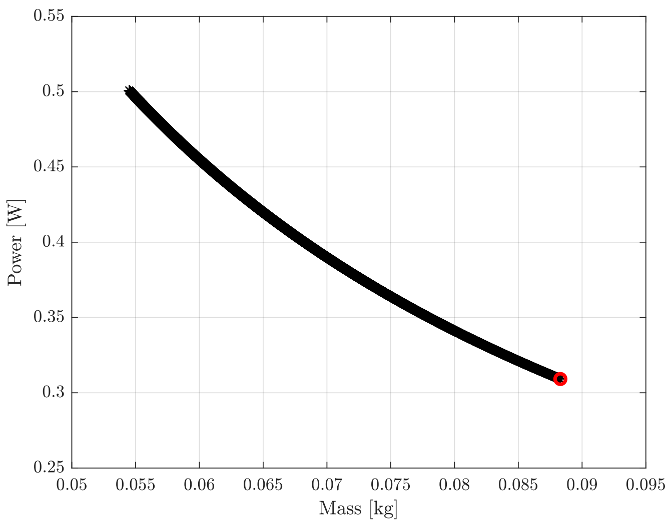

Figure 3. The Pareto front of the inner MTQ optimization with the chosen optimal point marked with the red circle.

yields the Pareto front [6] shown in Fig. 3. All the points in Fig. 3 satisfy all the bounds and constraints, thus, they are all Pareto optimal. However, one point (M, P) has to be chosen from the front as the final solution. The chosen point is the one closest to the origin, i.e. the one with the smallest distance $d_{\text{min}} = \sqrt{M^2 + P^2}$. It is marked with the red circle in Fig. 3. Finally, the initial optimization of the inner MTQ yields the point $(M = 88.3\ \text{g}, P = 309\ \text{mW})$ for $N = 393$, $r = 0.15$ mm, and $I = 96.5$ mA. Based on these results, the values of r and I are fixed for both the inner and the outer MTQ to 0.15 mm and 100 mA, respectively. Recall that $r = 0.15$ mm represents the upper bound of the wire radius (see Table II). Moreover, fixing the current simplifies the design of the future current driver circuitry that can be used for both MTQs. By defining the values of r and I, the optimization problem is reduced to analytical calculation of N from (3). Finally, using all the parameter values and the equations (4), (5), (16), and (17), one can fully characterize both the inner and the outer MTQ, as shown in Table III. Please notice that the values of all the design parameters fall within the required ranges.

TABLE III. CHARACTERISTICS OF THE OPTIMIZED MTQS

Designation	Unit	Inner MTQ	Outer MTQ
m	Am^2	0.3	0.3
a	cm	9.4	9.4
b	cm	8.4	9.4
r	mm	0.15	0.15
I	mA	100	100
N	-	380	340
M	g	85.5	80.7
P	mW	321.5	303.4
R	Ω	32.1	30.3
U	V	3.21	3.03

V. Prototyping and Preliminary Results

In the prototyping phase, both the inner and the outer MTQ frames are 3D printed based on the models designed in FreeCAD parametric modeler (see Fig. 2). The wire is winded onto the frames using a custom made coil winder based on DC electric motor with a 1:48 two-stage speed reductor. The motor speed control is implemented using the pulse-width modulation (PWM). The winder features an optical encoder and a display for keeping track of the number of turns. The winded and assembled MTQs placed on the ADCS verification setup based on a spherical air bearing [7] are shown in Fig. 4. The preliminary measurements show the resistance of 31.4 Ω and 29.4 Ω for the inner and the outer MTQ, respectively. These results are within 3% margin with respect to the calculated resistance shown in Table III. Although the preliminary measurements show promissing results, to fully benefit from the optimally designed MTQs, a high efficiency digitally controlled current driver needs to be designed, which will be the subject of our future research efforts.

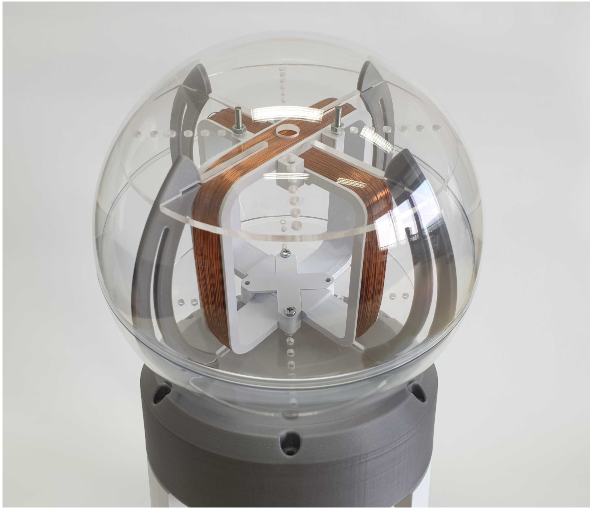

Figure 4. Assembled MTQs placed on the 3D printed spherical air bearing for ADCS verification [7].

VI. Conclusion

Due to the limited power budget, strict volume and mass constraints of standard nanosatellite formats, the optimization of MTQs for attitude control represents a substantial engineering challenge. In the presented optimization process the mass and power consumption of the two air core MTQs that fit within the 1U CubeSat format is minimized, while ensuring the required magnetic dipole moment and respecting the defined bounds of the design parameters. Here, every step of the multiobjective optimization based on the genetic algorithm is explained and justified. The preliminary resistance measurements of the two MTQ prototypes show excellent agreement with the calculations within 3%.

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