

Attitude determination and control systems

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1 Introduction

The cubeSat evolution has followed a historical progress similar to that of conventional spacecraft, that is, from elementary to complex. The latter has tended to become bigger and heavier. Since the late 1990s, CubeSats have increased in size from 1U to 12U and beyond to enable more complex tasks to be undertaken that require more accurate attitude control and more complex attitude determination and control systems (ADCS). However, while conventional big satellites are adapted to bigger launchers, CubeSats generally do not follow this approach, because the advantage of small satellites is the low cost of launch using light launchers and the piggy-back (ride share) launch strategy. CubeSats are subject to significant launch constraints relative to their size and mass. Consequently the auxiliary equipment including ADCS has to follow this constraint. The size and mass of ADCS are mostly defined by sensors and actuators. At the stage of feasibility study and preliminary design, the compromise between attitude requirements (accuracy and time response) and sensors and actuators has to satisfy these constraints while simultaneously achieving mission requirements. To maintain a reasonable cost for a CubeSat, the ADCS hardware that is already available in the commercial market should be considered first. This is why the chapter begins by presenting sensors and actuators. Next, the principles of attitude control are presented, and finally the mathematical techniques available to interface with the hardware chosen for the mission are reviewed, which together provide the required accuracy within the mass and size limitations. Sensors, actuators, physical principals, and algorithms suitable for CubeSats are described from a general point of view, because there are so many realizations of ADCS. It is currently impossible to fix the state of the art as the adaptation to specific missions is rapid and ongoing.

The attitude control system has to provide initial damping of the satellite angular motion after its deployment from the launcher, acquisition and maintenance of the required attitude position or attitude motion, and attitude maneuvering if the latter is required. To realize the required angular motion, the current attitude state of the satellite has to be known. The mismatch measured between the required and actual motion of the satellite is corrected by a control torque calculated via attitude control

algorithms. To undertake these two tasks, that is, determination of the current attitude state and the development of angular motion control torque, the satellite has to be equipped with sensors, actuators, an on-board computer, and a power source. The approach to the design of the attitude control system is dictated by the satellite dynamics requirements, irrespective of its size and form factor. The form factor of a satellite, particularly of a CubeSat, due to constraints and limitations of mass, size, energy capability, component redundancy, etc., forces the selection of an appropriate attitude control system design as a compromise between requirements and constraints.

The measurements of the environmental characteristics depend on the angular position and/or angular velocity of the satellite. Usually, sensors are defined as either *positioning* or *inertial*. The positioning sensor measurement samples depend on their orientation with respect to external field gradient (gravity and magnetic), aerodynamical resistance and solar radiation pressure force directions, infrared radiation of the Earth, optical measurements of stars, visible planets, limb of the atmosphere in the vicinity of the horizon, etc. This means that for the sensor's operation, an environment is compulsory, namely, no environment means no measurements. The inertial sensors do not require environmental knowledge. Their operation is based on centrifugal acceleration measurement due to the satellite's rotation. Inertial sensors directly measure the angular velocity of the satellite. Positioning sensors do not. They provide indirect measurements.

Positioning sensors become sensors, while their measurements depend on the orientation of the satellite with respect to a gradient of fields and other known directions in space, and these measurements vary as the angular position of the satellite varies. The positioning sensors are incapable of measuring the angular position of the satellite with regard to a reference frame or any direction. This means that to transform the sensor measurements to attitude knowledge, attitude determination algorithms have to be developed and incorporated. If attitude information is required to provide a real-time attitude motion control, then an onboard computer has to be used to process the attitude determination algorithms.

When the current attitude is *known*, the difference between the actual attitude and the required attitude generates control actuation commands via attitude control algorithms. These commands, via controllers, initiate actuators to develop the required control torque to reduce the difference. Control algorithms, therefore, must be implemented onboard. Within a specific attitude control system, a few or even all components listed above can be absent. The complexity of the attitude control system depends on requirements for the attitude motion and capabilities of available onboard components. For instance, passive attitude control systems do not need sensors, algorithms, a computer, or power at all.

This chapter is organized as follows: Section 2 describes the most common attitude sensors; Section 3 deals with the attitude actuators; the main types of attitude control systems are described in Section 4; attitude determination and control methods are presented, respectively, in Sections 5 and 6; concluding comments are given in Section 7.

2 Sensors

2.1 Sun sensors

There are a wide range of technologies used to measure Sun angles for a spacecraft. The Sun angle is a convenient absolute attitude measurement for a CubeSat in the inner Solar system as the Sun is easily the brightest object and very easy to detect. However, it is a point source, so it can only generate a vector (i.e., two dimensions). The actual attitude can be any rotation about that vector, so a second vector is required to generate full attitude knowledge. The second vector would typically be measured using the Earth for a LEO spacecraft, and this can be achieved either directly by observing the Earth disc or through the measurement of the Earth's magnetic field using magnetometry.

There are two basic divisions of sensing the Sun angle, usually referred to as analogue sensors and digital sensors. Analogue sensors will directly use the Sun's energy to generate current in a photodiode to be used in the attitude control computations. The maximum number of sensing elements in an analogue detector is four. Other sensors, referred to as digital Sun sensors, use multidetector arrays either dependent on linear or two-dimensional photodiode arrays. They are referred to as digital as they use some type of digital calculations or image processing to obtain the Sun angle. An analogue detection system with a local analogue to digital (AtoD) converter is still considered to be an analogue Sun sensor even if the output from the sensor head is in the digital domain.

A second classification of Sun sensors type is referred to as either *course* or *fine* Sun sensors. This is a simplistic way of describing the accuracy of the measurement of the Sun angle from different types of sensor. There is no standard definition of these terms, but sometimes the term course Sun sensor is incorrectly used to describe all analogue Sun sensors.

For detumbling and safe modes, it is appropriate to use course Sun sensors when low accuracy is acceptable and robustness when high rotation rates are more important. Once the spacecraft is in its nominal orientation and the body rates are low, a digital Sun sensor will typically give much higher performance.

Although Sun sensors will not have the absolute accuracy required for an Earth imaging mission, they are sufficient for most other aspects of a mission, such as pointing requirements for communications (even laser communications that uses internal steering mirrors). They should therefore be considered as a low-cost backup for star mapper-based ADCS solutions if the mass/volume budgets allow.

2.2 Magnetic sensors

The attitude of a spacecraft can be ascertained by measuring the vector of the Earth's magnetic field. However, the measurement needs to be processed according to two further factors. Firstly, as the field varies around the Earth's orbit, the position in orbit has to be known. Secondly, the field vector at that location in orbit has to be compared

with that measured to calculate the relative offset between the two that provides the attitude vector. The known vector at any location is created by using a mathematical model of the field, for example, using the IGRF model (the International Geomagnetic Reference Field model [1]). This predicts the field at any point and any altitude in space but is only an approximation, resulting in an inherent error.

Like Sun sensors, magnetometers are available with both analogue and digital outputs. There are a variety of ways of measuring magnetic fields with various degrees of accuracy and complexity of supporting electronics. The two most common measurement methods are fluxgate and magnetoresistive. A magnetometer produces three measurements of the perpendicular fields that are usually measured in nano-Tesla (nT) or occasionally Gauss. One Gauss is equal to 100,000 nT. The decision regarding the type and sensitivity of the magnetometers used for a CubeSat mission should be driven by the attitude knowledge requirements.

2.3 *Star mappers/trackers*

Like the ancient naval explorers, it is possible to navigate using the stars. A camera takes an image of the star field and compares it to a catalogue of star positions so that it can unambiguously recognize in which direction the camera is pointing in inertial space. Unlike the Sun, which is a point source, the star field is a two-dimensional image allowing an absolute attitude orientation to be gained from a single image.

With the use of digital cameras and the ability to measure star positions at subpixel accuracy, the star mapper provides the highest accuracy pointing knowledge, typically 0.01 degrees and better, and is therefore the main sensor used for applications needing excellent pointing such as high-resolution Earth imagery.

As the star brightness is low compared with that of the Sun, Moon, and Earth, a high accuracy star mapper will typically need a large baffle to avoid stray light entering the camera and causing noise or errors in the image. Typically, the higher the accuracy of the star mapper, the larger the baffle requirement and, for a small satellite, this can be volume prohibitive. For the CubeSat form factor, this baffle requirement limits the performance of star cameras that have to fit within the 10-cm square envelope.

2.4 *Earth sensors*

For the vast majority of communication missions, the important attitude vector to know and control is to point the antennas toward the Earth. Depending on the solar panel configuration, it may be possible to rotate around that vector without needing to have a fixed pointing orientation other than Nadir.

The best way to get an accurate knowledge of Earth's center is to look at the Earth via infrared. The Earth is a warm body in a sea of cold space. If looked at in the absorption bands of water molecules, the Earth is almost a uniform "color," which makes it very easy to identify and calculate the central point. However, working in the infrared

usually requires expensive detectors and mechanisms, which make them expensive, heavy, and power hungry, so they are not well suited to CubeSats. They are also located on the nadir face, which is often competing for real estate with the payload. As a consequence, Earth sensors are rarely found on CubeSat missions nowadays, with star mappers and gyros are used in their place.

2.5 GNSS-based attitude knowledge

Differential Global Navigation Satellite System (GNSS) measurements can be very accurate, with resolution in the range of centimeters, whereas a single GNSS measurement might be accurate to 1 m or perhaps submeter. GNSS techniques have been used on spacecraft to measure attitude by using multiple antennas on a long baseline, where the difference of a centimeter can translate into a fairly accurate attitude knowledge. Three or more antennas are needed to get an attitude fix in three dimensions.

2.6 Measuring attitude change—Gyros

All the sensors previously discussed measure an absolute attitude, so the errors in the measurement are one time and simply suffer from accuracy and noise error sources. An alternative measurement system is to use the rate of change of the satellite motion to estimate where the satellite is at the current time compared with an absolute reading of attitude sometime in the past. This technique is often used on simple CubeSats that use a Sun/magnetometer pair to give absolute attitude knowledge during the sunlight phase. When the CubeSat moves into eclipse, the primary attitude reference of the Sun is lost, so a rate sensor (gyro) is used to estimate the change in attitude since the last known absolute attitude.

There are a wide range of gyros on the market, and as would be expected, there is a trade-off between accuracy, cost, and volume. Originally, gyros were based on spinning masses, and the force created when these masses were moved off their axes was used as a measurement of rate of change. Mechanical gyros are very rarely used in space nowadays due to their inherent reliability problems.

The most accurate units are ring laser gyros (RLGs). These can measure delays in time of travel of a laser to subwavelength accuracies resulting in very high performance but at the expense of size and power. In the middle range of cost/performance are fiber-optic gyros (FOGs) followed by a wide range of microelectromechanical system (MEMS) gyros, which are the smallest and lowest cost, but do not have very good performance.

For a gyro to perform well, it must have a minimum amount of drift and noise. Drift is how accurately the rate reading reports the real measurement, and the noise is the random variation on that signal. MEMS gyros have the worst drift performance. As there is no absolute measurement, the buildup of drift on the gyro reading is critical. At each reading of the rate, the error is summed, so the error gets worse and worse over time. If the drift rate is poor, then the satellite can be pointing many

degrees off from the pointing angle that is anticipated when the satellite comes out of eclipse.

2.6.1 Stellar gyro

A stellar gyro has a built-in star camera, similar to a star mapper, but works as a rate sensor rather than an absolute measurement of attitude. It does this by correlating two sequential images of the star field to determine the rotation and translation of the image, which reflects the motion of the satellite that the camera is mounted to. The benefits are that the image does not have to be that accurate as the correlation algorithm will remove issues such as hot pixels, dead pixels, and noise.

It is understood that the stars have not moved between the two images, so this information can be used to reset the drift on the internal MEMS gyros resulting in a near drift-free rate sensor. Even if this process is implemented, the accuracy of a MEMS gyro is still dependent on the absolute measurement of attitude in sunlight by the Sun sensor and magnetometer, so it is only suitable for communication missions, not highly precise Earth observation, which requires a star mapper.

3 Actuators

3.1 Reaction wheels (flywheels)

Following Newton's third law, accelerating a mass in one direction will create an equal and opposite reaction in the other direction. A reaction wheel accelerates a relatively massive disc through the use of a high torque motor to achieve a reaction of the spacecraft in the opposite direction. Using three such wheels, in orthogonal axes, allows the spacecraft attitude to be moved in any direction required.

A reaction wheel is usually defined by two attributes. The torque, measured in newton-meters (Nm), represents the amount of force that the wheel can apply to the satellite, so the higher the torque, the more agile the spacecraft can be. The other attribute is the momentum measured in newton-meter-seconds (NmS), which defines the maximum energy that can be stored up in the inertia of the wheel.

A momentum wheel is a reaction wheel that is run at a constant high speed. It therefore generates no torque directly on the spacecraft but will produce a gyro stiffness in the axis perpendicular to the mass rotation direction. A high momentum wheel is beneficial when pointing stability is needed, as it requires significant energy (disturbance torques) to move it off its axis.

3.2 Control momentum gyro (CMG)

If a momentum wheel is mounted on a pivot, then when the pivot is driven to change the axis of the spinning wheel, a very high torque is generated on the spacecraft to resist the gyro stiffness of the wheel. This is the basic operating method of a CMG. Either two wheels with single pivots can be used to control a satellite in three axes, or a single wheel with two degrees of freedom, mounted on a dual rotating pivot,

can be used. A CMG can have a significant mass advantage over a reaction wheel with equivalent torque, but they do not scale to small sizes well, so they are less applicable to CubeSat applications. The control algorithms are complicated as there are certain positions of the CMGs where they cannot generate torque in the right direction—these are called singularities.

3.3 Fluid dynamic actuator

A relatively new technology involves the use of a heavy fluid, pumped through a tube, to achieve the same effect as a reaction wheel. The acceleration of the reaction mass will create a reactive force on the spacecraft in the opposite direction. The use of a pump relieves the bearings from carrying the full load of the mass at launch. When a conductive liquid is used, such as a liquid metal, then magnetic fields can be used to drive the fluid using the Lorentz force. This removes the need for bearings in the system, which is one of the lowest reliability components of reaction wheels; this potentially gives fluid dynamic actuator (FDA)-based systems very long lifetime.

Another major advantage of an FDA is that the fluid does not have to run around a circle; it can be any shape that encloses an area, allowing the design to be square, for example, which maximizes the inertia of the fluid by following the outline of the CubeSat frame. Much of this technology is proprietary, and its use is restricted by patents.

3.4 Magnetorquers

Magnetorquers are electromagnets. They generate a large magnetic field when energized. This interacts with the local magnetic field and generates a torque on the spacecraft when their field is offset from the local Earth's magnetic field vector, much like a compass needle will turn toward North. The unit of measure for a torque rod is magnetic moment, measured in ampere-square-meter (Am^2).

The simplest method of generating a field is a coil of copper wire. This is typically square and follows the outline of the spacecraft to maximize the surface area enclosed by the coil. However, air-cored magnetorquers require high power to generate a reasonable magnetic field and are relatively heavy as copper is a dense material. The alternative is to use a magnetorquer rod. The rod constrains and amplifies the field generated by the coil wrapped around it. The potential disadvantage of a rod is that when the coil is not energized, there will still be a small residual magnetic field, called the remnant. If the remnant is significant, it will cause a disturbance on the spacecraft attitude that has to be compensated for. The material chosen for the rod therefore has to result from a trade-off between having high amplification of the field, combined with low remnance.

The rods fabricated from the soft magnetic material can be used on their own within passive attitude control systems as a passive damper and are called hysteresis rods. Due to the magnetization reversal under angular motion of the CubeSat, the hysteresis

effect within the rods causes a transformation of the angular kinetic energy to a heat and, consequently, decreases the angular velocity.

3.5 Other methods

Spacecraft attitude can be adjusted by using a propulsion system and multiple thrusters that are switched depending on which axis needs to be corrected. Various thruster types are available and are described in Chapter 15. They are not typically used in CubeSat applications.

In low orbits the small atmospheric drag can be used to control a satellite using aerodynamics. The attitude can either be passively stabilized or actively controlled by moving the aerodynamic surfaces.

Similar to moving aerodynamic surfaces, the center of mass of a satellite can be moved through sequential movement of appendages resulting in an overall rotation of a satellite body. The appendages that are moved are typically deployed solar panels. These unconventional methods of attitude control are not typically implemented on CubeSat missions, but have been flown and are planned on upcoming missions planned, to meet specific mission requirements.

4 Attitude control classification

When trying to describe or differentiate between attitude control systems, we usually describe the physical principles under which a specific attitude control system operates. We classify the systems with regard to their attitude motion modes, passive or active types, which sensors and actuators can be implemented, what features the systems for CubeSats have, and what mathematical models are to be implemented to describe and simulate attitude motion. We need to consider the environment that can be used for attitude determination and control. This can apply to both passive and active control schemes. To generate a passive control system, no active actuators, power, computing, mathematical algorithms, or knowledge of attitude can be utilized. Active control demands all of these systems.

4.1 Gravity field and gravity-gradient ACS

Different parts of the satellite have varying distances to an external large attracting gravitational force, such as the Earth; hence the attracting forces are different too, and the sum of forces develops a *gravity-gradient torque* acting on the satellite. If the orbit of the satellite is assumed to be circular and Keplerian, then the satellite with different principal moments of inertia has 24 equilibria where principal axes coincide with the axes of the orbital reference frame (local-vertical-local-horizon frame). Four of those are stable. To increase the magnitude of the gravity-gradient torque, the satellite has to be elongated, which can be realized by booms with tip mass. To make the equilibria asymptotically stable, a *damper* has to be installed. One can use relative motion of the satellite structures with friction and elasticity in the hinge to connect

the elements, magnetic elements like hysteresis rods fabricated from soft magnetic material. Accuracy of the gravity-gradient attitude control system is on the order of a few degrees. Eccentricity of the orbit increases the amplitude of the satellite libration with regard to the local vertical. The installation of a gravity-gradient boom on a CubeSat requires a deployment mechanism. For 1U–3U CubeSats, it would occupy a dominating part of the available volume. However, a reconfiguration of the CubeSat's masses can provide a proper tensor of inertia for such type of stabilization. Bigger CubeSats usually demand higher pointing accuracy that cannot be achieved by the gravity-gradient attitude control system (GGACS). However, GGACS can be applied for a mission without high accuracy requirement (REFLECTOR with hysteresis rods, 2001; NCube-2 with magnetorquers, 2005)^a. The gravity-gradient boom provides a single-axis orientation of a satellite along the local vertical; however, a rotor spinning with a constant velocity around the pitch axis can provide a three-axis orientation.

When an axisymmetrical satellite is spun around the axis of symmetry, the axis can achieve a relative equilibrium with respect to the orbital reference frame due to compensation of the gravity-gradient torque and torque developed by the centrifugal forces due to satellite rotation together with the frame. The axis can lie in the plane perpendicular to the local horizon or in the plane perpendicular to the local vertical or coincide with the normal to the orbital plane. It depends on satellite moments of inertia and its spin velocity. These effects can be leveraged to design effective gravity gradient attitude control systems that can satisfy a range of mission requirements.

4.2 Magnetic field and magnetic ACS

Onboard measurements of the geomagnetic field are used for attitude determination using a magnetometer. It is not possible to estimate the angle of the satellite rotation about the local vector of the magnetic field and implementation of statistical methods like the least mean square (LMS) or Kalman Filter (KF) to determine three-axis orientation is required. Local methods cannot determine three-axis orientation using measurements of the geomagnetic field only. However, using another positioning sensor measuring any vector noncollinear to the local magnetic vector allows determination in three axes.

A permanent magnet, hysteresis rods, or magnetorquers interacting with the geomagnetic field develop a control torque. This torque does not have a projection onto the direction of the local vector of the geomagnetic field, and consequently, there is no control about this direction. Such a mechanical system is called “underactuated.” However, there are algorithms that enable single- and three-axis orientation of the satellite with respect to the orbital [2] or inertial reference frames. For damping the angular velocity, there exist algorithms such as the popular B-dot algorithm and the less common S-dot algorithm. The latter orients the satellite to the Sun

^aDetailed description of satellites given in parentheses a reader can find, for instance, at websites <http://www.nanosat.eu/>, Gunter's page <https://space.skyrocket.de/> and others.

using an approach similar to B-dot. For CubeSat attitude control magnetometers, Sun sensors, and magnetorquers are the most common used components (Compass-1, 2008).

A combination of a strong permanent magnet and a set of hysteresis rods can create a passive magnetic attitude control system (CubeSat XI-IV, 2003; TNS-0 #2, 2017). The system provides 10–15-degree accuracy of orientation of the permanent magnet with respect to the local geomagnetic vector. The time response of the system is strongly dependent on the initial conditions and can take a few days or even weeks to stabilize. However, the system is of low cost and high reliability if the design and fabrication stages are skillfully conducted. Roughly, the motion achieved by this type of system is a rotation of the axis with the magnet along the surface of a cone with the vertex lying in the satellite center of mass, and its half angle depends on the inclination of the satellite orbit with almost noncontrolled motion about the axis with the magnet. In polar and near-polar orbits, the axis lies almost in the orbital plane and rotates at double the orbital angular velocity of the satellite. Hysteresis rods should form a grid located at a specified distance from the magnet or lying in the plane perpendicular to the magnet and crossing the magnet in the middle. These strategies represent ways to minimize magnetization of the rods by the magnet's field and even prevent their saturation, which would cause a malfunction of the damper.

4.3 Atmospheric resistance and aerodynamical ACS

For low Earth orbit satellites with altitudes between 200 and 400 km, the resistance of the atmosphere is noticeable and, consequently, can be used to develop a restoring passive torque, while the center of pressure is shifted from the center of mass of the satellite. To shift the center of pressure, a special aerostabilizer like a sphere fixed by a rod that is connected to the satellite or “an aeroskirt” again moved from the center of mass should be installed. The problem of damping can be solved using a grid of hysteresis rods (MAK-A, 1993; PAMS, 1996; SamSat QB50, 2019) or magnetorquers. However, the lifetime of a CubeSat with an aerodynamical attitude control system can be relatively short depending on the strength (time response) of the hysteresis rods and the eventual deorbiting due to atmospheric drag.

4.4 Spinning and spin stabilization

Spinning a body with a high enough spin velocity can maintain the position of the spin axis with respect to the inertial space through gyroscopic stiffness. To decrease the angle of the spin-axis nutation after deployment of the satellite from the launcher, two types of devices can be implemented. The first one is the *passive nutation damper*. There are various designs, but they all work on a similar principle. Usually, the damper is a hollow tube filled with a viscous liquid and a moving mass, like a ball, held in the middle of the tube by two springs attached to the

opposite ends of the tube. The nutational motion of the satellite generates a centrifugal force that effects the ball. The translational motion of the ball along the tube generates a friction force in the liquid and, consequently, dissipates the energy of the nutational motion. The damper parameters to be tuned for a given spin velocity are the orientation of the tube relative to the spin axis, the mass of the ball, the viscosity of the fluid, and the spring force. Effectiveness of the damper is dramatically increased by spinning. The higher the angle of nutation, the higher the force and the higher the friction that decreases the energy of the nutational motion. The main advantage of this damper approach is that it is passive and it generates no disturbing torques. The resonant tuning of its parameters is required during development as it cannot be adjusted in space. The main disadvantage is that the damper will resist any change in the orientation of the spinning axis in space if it is required. The temperature of the liquid is a very critical parameter. Instead of a viscous liquid, eddy currents can be used. A damper based on magnetorquers offers more flexibility. They can be used as an *active nutational damper* and allow changing the spin-axis orientation and spin up the satellite. Of course the active unit requires the whole set of components of an active attitude control system (sensors, calculator, algorithm of control, and power). Both types of damper have been used on CubeSats. Spin stabilization is a popular ACS strategy for CubeSats (DICE CubeSat, 2011).

4.5 Solar radiation pressure

The use of solar radiation pressure has similarities to aerodynamical ACS because both require a shift of the center of pressure with respect to the center of mass of the satellite. The line connecting these centers has equilibrium along with the direction to the Sun. The torque generated by the solar radiation pressure plays the role of a restoring one. An active damper is required. The torque is very weak, and for it to dominate over other torques, the orbit should have a high altitude. In some missions (Nanosail-D, Cubesail, Lightsail-1, and Deorbisail) the use of the sail was intended to deorbit the satellite rather than for attitude control.

4.6 Flywheels

A flywheel (or reaction wheel) is an electrical engine with a large axisymmetrical disk fixed on its axis of rotation. Installation of three flywheels along the principal axes of inertia of the satellite allows operators to turn the satellite about these axes by varying the angular velocity of the motors due to the angular momentum conservation law, if accelerating and decelerating of the motors are fast in comparison with orbital motion of the satellite. While the satellite is affected by the disturbance torque with a nonzero average value, the corresponding flywheel speeds up permanently and within a certain time becomes saturated, that is, reaches a maximum speed for the system. For desaturation the satellite has to be retained in an angular

position, and the corresponding flywheel can be decelerated. An external torque should be applied. Magnetorquers or a thruster can be used. Of course, this approach increases the total mass and volume of the attitude control system or can decrease the lifetime of the satellite if fuel-dependent thrusters are used. Flywheel attitude control systems provide high accuracy, and with relatively small electrical power draw, the solar panels should produce enough power to maintain a long lifetime of the satellite. The system may be compact and require minimal electrical energy. CubeSats use the system widely if attitude accuracy of a fraction of degree is demanded (BeeSat-1, 2009).

A single flywheel spinning with the constant angular velocity may be fixed along a certain axis of the satellite to resist changing its attitude. This architecture is referred to as a momentum biased system. The most common architecture employs a pitch flywheel to maintain the orientation of the pitch axis of the satellite along the perpendicular to the orbital plane.

5 Attitude determination mathematical techniques

1U–3U CubeSats are usually equipped with an elementary set of sensors for attitude determination. Advanced 6U–12U CubeSats typically use a more complex set of sensors. Although such CubeSats could be classified as nanosatellites or microsatellites, we describe techniques to be used for their attitude determination also.

5.1 Local methods

The simplest approach for attitude determination is realized by a *local method* [3]. It requires the operator to measure two noncollinear vectors in the satellite reference frame and to calculate the same vectors in the reference frame one needs to determine the satellite orientation relative to. For a calculation, one should be able to simulate these vectors via mathematical models. The problem of attitude determination lies in ascertaining the orthogonal matrix \mathbf{A} called *matrix of directional cosines* or *matrix of orientation* that provides a relationship $\mathbf{A}\mathbf{V}_i = \mathbf{W}_i$. Here, $\mathbf{V}_1, \dots, \mathbf{V}_n$ is a set of the unit vectors directed to n targets, for instance, to the Earth, the Sun, or a star, along with vector of the geomagnetic field induction; $\mathbf{W}_1, \dots, \mathbf{W}_n$ are the vectors directed to the same targets but measured in the satellite-fixed frame. If measurements are subject to errors, the described task does not have a solution, generally speaking. However, there are a few ways to solve the problem like the *determined method* and the *optimum method* in the sense of a functional minimization.

The determined method called *TRIAD algorithm* allows the satellite operator to ascertain an orientation using first measurements [4]. The algorithm is rather simple, and due to this, it has been widely used since the 1970s. Let the vectors \mathbf{S} and \mathbf{H} be directed to the Sun and along the induction vector of the local geomagnetic field. They are calculated using the corresponding models. Let \mathbf{s} and \mathbf{h} be the same vectors but measured by onboard sensors. To determine the transient matrix \mathbf{A} , one has to

compose two right-hand orthonormal triads of vectors $\mathbf{G} = \left(\frac{\mathbf{H}}{|\mathbf{H}|} \frac{\mathbf{H} \times \mathbf{S}}{|\mathbf{H} \times \mathbf{S}|} \frac{\mathbf{H} \times (\mathbf{H} \times \mathbf{S})}{|\mathbf{H} \times (\mathbf{H} \times \mathbf{S})|} \right)$ and $\mathbf{g} = \left(\frac{\mathbf{h}}{|\mathbf{h}|} \frac{\mathbf{h} \times \mathbf{s}}{|\mathbf{h} \times \mathbf{s}|} \frac{\mathbf{h} \times (\mathbf{h} \times \mathbf{s})}{|\mathbf{h} \times (\mathbf{h} \times \mathbf{s})|} \right)$. The relationship between these matrices is defined via the matrix \mathbf{A} as $\mathbf{G} = \mathbf{A}\mathbf{g}$. Since the matrix \mathbf{G} is orthogonal, then $\mathbf{A} = \mathbf{g}\mathbf{G}^T$. The presented technique is simple for onboard implementation, but it does not work when the satellite moves into the eclipse part of the orbit or when the Sun and the induction vectors are collinear. The technique requires the position of the satellite in space to use models of the Sun and induction vectors. An important factor to be taken into account is the *albedo*, that is, the Sun radiation reflected by the Earth surface. In an analogue Sun sensor, the incoming radiation from the Sun and reflected by the Earth combines together with the vector \mathbf{S} to be simulated and the vector \mathbf{s} to be measured.

The TRIAD algorithm has low accuracy because only two observations are used and no measurement errors are taken into account. The *optimum algorithm* QUEST, based on the *loss function* implementation on the whole set of measurement [5], provides a better result. The loss function is $L(\mathbf{A}) = \frac{1}{2} \sum_{i=1}^n a_i |\mathbf{W}_i - \mathbf{A}\mathbf{V}_i|^2$, where n is the number of measurements and a_i is the weight of the i th measurement. The matrix \mathbf{A} is obtained by minimizing $L(\mathbf{A})$. The introduction of the cost function allows transformation of the task to quaternion statement. The quaternion corresponding to the optimum solution for any required accuracy can be determined without calculating the eigenvalues of the matrix.

5.2 Least mean squares algorithm (LMS)

Local methods do not use information about the dynamics of the satellite. Let m parameters of the satellites motion x_i , ($i = \overline{1, m}$) be measured. They can be presented as known functions of the initial conditions of motion a_1, \dots, a_6 as $\Phi_1(a_1, a_2, \dots, a_6) = b_1, \dots, \Phi_m(a_1, a_2, \dots, a_6) = b_m$ where Φ_i are known functions and b_i are measured values containing errors. The number of equations above is more than the number of variables a_1, \dots, a_6 contained there. Thus we cannot solve these equations with regular mathematical methods. Introducing the *loss function* $T = \sum_{k=1}^m (\Phi_k(a_1, a_2, \dots, a_6) - b_k)^2$ that composes the differences between the *measured* and *calculated* quantities, one can require that the loss function is minimized. To do this, one has to obtain the variables a_1, \dots, a_6 that provide minimum error in the determination of the variables b_k . The necessary conditions for minimizing T are $\frac{\partial T}{\partial a_k} = 0$, ($k = \overline{1, 6}$). The number of these equations is equal to the number of unknown variables a_1, \dots, a_6 , and they can be solved by an iterative method. This technique has the advantage that the dynamical model is used. It allows us to extrapolate the solution at a time instant. The disadvantage is that the method requires long enough intervals of measurements. This is why the method is widely used for postprocessing of CubeSat flight data.

5.3 Kalman filtering (KF)

The Kalman filter is a recursive algorithm that uses a model of dynamics and onboard sensor measurements to obtain an estimation of the state vector [6]. Sometimes the whole state vector is required to control the system. In this case the KF allows reconstruction of missing information in the presence of noise and through, generally speaking, indirect measurements. Various sensors can be used with the KF, for example, Sun sensors, magnetometers, star trackers, and positioning sensors together with an angular velocity sensor. There are algorithms that estimate orientation via Euler angles and based on vector measurements and also via quaternions. The survey in [7] gives various representations of satellite attitude. However, the most popular is the quaternion representation, due to its nonsingularity, minimum dimension, and linearity of the kinematic equations.

Nonlinear formulations of the KF exist, like the extended KF, where the nonlinearity is approximated along a nominal trajectory. Another nonlinear filter is the unscented Kalman filter (UKF), which uses a limited number of points from the state space to approximate the nonlinear dynamics. A very popular version of the UKF that uses a quaternion formulation is the USQUE [8].

Despite the wide popularity of the recursive filtering scheme of the KF, there are several problems to be solved for its implementation in real-time applications. The main problem is the tuning, that is, the choice of the matrices of measurement and motion model errors. These matrices determine the KF quality in terms of the accuracy of the state vector estimation and the convergence time of estimation. In practice the filter tuning is a heuristic process via a trial-and-error procedure. However, there are a set of automatic tunings such as the method of numerical optimization and the simplex method. Another technique uses the Monte Carlo method based on multiple runs of the KF under random settings of the simulation. A generic algorithm for KF tuning consists of randomly changing the state vector in accordance to the improvement of the estimation accuracy. The best one is chosen for the next iteration. All of the aforementioned techniques based on multiple runs of the KF simulation require big computing power and are therefore not always suitable for CubeSats. Another technique to study the accuracy of motion estimation can be used for a stationary motion. It does not require to simulate KF work and is analytical. The error matrix for a stationary motion can be obtained after convergence from the quadratic matrix equation. For single-axis motion, this equation is solved in an explicit form. In a general case, this equation can only be solved numerically. Analytical methods to tune the KF that can be used for quasi-stationary motion are developed in [9] and are based on calculation of the covariance matrix after convergence. An advantage of this technique is that it does not require the operator to simulate KF work and may be easily implemented on a CubeSat.

In one effective use case of Kalman filtering, the current output of the solar panels can be interpreted as Sun sensor's measurements, but they have to be calibrated, and the albedo is to be taken into account. It was successfully used for postattitude determination in the Munin mission (2001). However, many power systems now

use peak-power trackers, where the direct relationship between panel current and Sun orientation is no longer applicable.

6 Active attitude control approaches, techniques and algorithms

Among a wide variety of attitude control algorithms, two of them are the most relevant for CubeSat missions, namely, those based on the geomagnetic field use and the implementation of the angular momentum conservation law. Transferring this to the engineering field means the use of passive or active magnetic actuators to develop a control torque under interaction with the geomagnetic field and use of the reaction wheels, which are usually composed as a triad of flywheels. Magnetic actuators (magnetorquers) in active configuration compose a triad of mutually orthogonal rods to develop a magnetic moment in any direction. Both systems require electrical power, controller, actuators, sensors, and algorithms for attitude determination and control.

Other types of active attitude control systems are not widely used for CubeSats because they either require consumable propulsion fuel or are heavy and bulky. This chapter will therefore focus on control algorithms for magnetorquers and reaction wheels. Momentum wheel (i.e., flywheel spinning with constant velocity) sometimes is used in combination with other actuators like magnetorquers or gravity-gradient boom. It is called a *pitch flywheel* and does not require a control algorithm.

6.1 B-dot

The B-dot algorithm implemented through magnetorquers is used for initial detumbling [10]. Generally, it imitates the viscous friction of the rotated satellite with regard to the vector of the geomagnetic field induction \mathbf{B} . The magnetic dipole \mathbf{m} is calculated as $\mathbf{m} = k \, d\mathbf{B}/dt$ where k is a positive gain factor either scalar or matrix. The time derivative is calculated as a ratio of increments of the sequential measurements \mathbf{B}_{j-1} and \mathbf{B}_j of the vector \mathbf{B} and their corresponding moments of time. If the satellite is equipped with a pulse width converter, then the gain coefficient k should be chosen to adjust a time response of such a damping algorithm. Also the duty cycle of measurements should be chosen with respect to the expected angular velocity of the satellite and duration of magnetorquers' switched-on time; otherwise the magnetometer can be saturated, or, at least, measurements are disturbed, while the magnetorquers are active. This algorithm does not require measurement processing. The procedure of measurement smoothing is applied only to minimize the effect of measurement errors. In practice, magnetorquers' magnetic moments are considered either positive ($+m_0$), negative ($-m_0$), or null. Then, the instant when the control action is implemented needs to be evaluated in order to optimize the time response of the system. If a KF is already used for attitude determination, it can also be utilized to determine $d\mathbf{B}/dt$. Many studies of the satellite dynamics with B-dot algorithm have been carried out with respect to the manner of magnetic moment realization (either continuous or bang-bang style, etc.) and off-duty ratio of measurements. This algorithm can also

be used for attitude stabilization when a satellite has to rotate in inertial space with a double orbital velocity. If the magnetic torque dominates over other torques acting on the satellite, then the rotation about the satellite axis of the maximum moment of inertia is asymptotically stable. For those who begin to develop an active ACS, B-dot can be a first step.

6.2 Spin stabilization

Magnetic attitude control is also widely used to provide single-axis orientation of a satellite with respect to inertial space. It could be spin stabilization when magnetorquers maintain required spin velocity and point the spin axis to the needed direction by damping the nutation motion and turning in space. The effective way for realization of this strategy consists of three stages. At first the satellite has to be detumbled and damped, for instance, by B-dot algorithm, or to avoid damping spin velocity, the dipole moment $\mathbf{m} = k(0, 0, \mathbf{e}_3(\boldsymbol{\omega} \times \mathbf{B}))_x$ can be utilized. Here, three components in the body-fixed reference frame (indicated by the subindex x) are shown, while \mathbf{e}_3 in the scalar multiplication is the unit vector of the third axis of the reference system defining the third component of \mathbf{m} . Next, the satellite is spun up about the axis of symmetry up to the required velocity by the moment $\mathbf{m} = k(B_{2x}, -B_{1x}, 0)_x$ where B_{1x}, B_{2x} are the projections of the vector \mathbf{B} onto the first two axes of the body-fixed reference frame. In the third stage the satellite is turned to the required angular position as a gyroscope by the external control torque (in this case by the magnetic torque) with $\mathbf{m} = k(0, 0, \mathbf{e}_3(\mathbf{B} \times \Delta\mathbf{L}))_x$ where $\Delta\mathbf{L}$ is a mismatch between demanded and current angular momentum of the satellite. While the spin axis approaches the required position, the acting torque ends. The advantage of this is that there is no need to damp the axis motion at the end since as soon as the control torque ends, the angular motion of the axis stops too.

Another approach for spin stabilization is to turn the axis of symmetry to the required position and, next, to spin up the satellite. This requires an accurate turn of the axis via solving the boundary value problem.

6.3 Three-axis stabilization

Three-axis orientation of the satellite using magnetic control torque only is slightly exotic but nevertheless attractive. Exotic because the magnetic control is locally underactuated, that is, the satellite cannot be commanded to follow arbitrary trajectories in the configuration space due to the impossibility to develop a control torque along the geomagnetic induction vector. However, there are approaches to provide orientation with regard to the orbital and inertial reference frames. There are two popular techniques based on the Lyapunov control and sliding control. The first one minimizes the Lyapunov function and is close to a PD regulator (but not the same). It dictates the expression for magnetic dipole $\mathbf{m} = -k_\omega(\mathbf{B} \times \boldsymbol{\omega}) - k_a(\mathbf{B} \times \mathbf{S})$, where k_ω, k_a are positive constants to be chosen, $\mathbf{S} = (a_{23} - a_{32}, a_{31} - a_{13}, a_{12} - a_{21})$, with non-diagonal elements of matrix \mathbf{A} transforming a vector from the body fixed to the inertial reference frame, and vector $\boldsymbol{\omega}$ is the angular velocity of the satellite. The same

expression for \mathbf{m} can be utilized for the satellite stabilization with regard to the orbital reference frame relating $\boldsymbol{\omega}$ and components of the matrix \mathbf{A} to this frame. The most critical issue to the implementation of such a control is the choice of the factors k_ω , k_a , which requires skill.

The problem of the local underactuation of the magnetic control has a peculiarity by the rotation of the local vector \mathbf{B} in space due to the orbital motion of the satellite. This allows that a trajectory that leads the satellite to a needed point in the phase space can be built, as at any time the control torque has to be perpendicular to \mathbf{B} . Sliding mode control might be used for this purpose. To develop the control algorithm, a two-stage approach is used. First the surface in the phase space $x(\boldsymbol{\omega}, \mathbf{A}, t) = 0$ is to be built so that it is reachable. Second the phase point corresponding to the satellite has to move along this surface. Also the surface should allow the changing of its orientation relative to the control torque to be perpendicular to \mathbf{B} . The global controllability of three-axis magnetic control is proven in [11]. A survey of the most widely used magnetic attitude control algorithms for CubeSats was published by Ovchinnikov et al. in 2019 [12].

For the flywheel control, the Lyapunov approach can be implemented. The control torque $-k_\omega \boldsymbol{\omega}_{rel} - k_a \mathbf{S}$ is to be developed by the flywheels with angular momentum \mathbf{H} as $-\dot{\mathbf{H}} - \boldsymbol{\omega} \times \mathbf{H}$ where $\boldsymbol{\omega}_{rel}$, $\boldsymbol{\omega}$ are the angular velocities with regard to the reference (for instance, the orbital) and inertial frames, respectively. Equating these two expressions, one obtains the differential equation with respect to \mathbf{H} . Its solution is a way for control synthesis. Usually, instead of solving the differential equation, the finite difference method generated iteration formula is applied.

To use magnetorquers only to provide three-axis attitude is very attractive due to seeming simplicity. However, because of the disturbing torques affecting a required attitude of a very small CubeSat like 1U, such systems are not always feasible [13].

7 Concluding comments

Since CubeSats occupy the whole range of applications bounded on one side by missions for beginners and advanced missions on the other, their attitude requirements can range from nonstabilized to very precisely oriented. Based on the two main CubeSat developers, that is, educational or commercial one, the configuration of the attitude control system depends on the requirements of attitude motion and the ability of the designers, that is, their skills, time, and available budget. A set of attitude motion regimes is shown in Table 1. The possible combinations of sensors and actuators versus attitude required to design attitude control system starting from noncontrolled motion and up to advanced three-axis attitude motion are given. A regime purposed for the same attitude can be implemented by different configurations. For example, to avoid a chaotic angular motion without requirements of accuracy and time response can be done via implementation of B-dot algorithm with magnetometer and magnetorquers or a permanent magnet with hysteresis rods. So, Table 1 shows what sensors and actuators can be used for a given attitude motion regime realization. The subset of sensors and actuators required depends on the accuracy demanded,

Table 1 Possible combinations of sensors and actuators versus attitude required.

| Sensors and actuators required | Regime of attitude motion | | | | | | | |
|--------------------------------|---------------------------|--------------------------|------------------|-----------------------------|----------------------|------------------------------|---------------------|------------|
| | Noncontrolled motion | Chaotic-prevented motion | One axis along B | One-axis spin stabilization | One axis along Nadir | One axis along local horizon | One axis to the Sun | Three axis |
| <i>Sensors</i> | | | | | | | | |
| Magnetometer | + | + | + | + | + | + | | + |
| Sun sensor | + | | | + | + | + | + | + |
| Star tracker | | | | | + | | | + |
| Earth-sensor | | | | | + | | | |
| Gyros | | | | | + | | | + |
| GNSS | | | | | | | | |
| Fluid ring | | | + | + | + | + | + | |
| <i>Actuators</i> | | | | | | | | |
| Permanent magnet | | + | + | | | | | |
| Hysteresis rods | | + | + | | + | + | | |
| Magnetorquers | | + | + | + | + | + | | + |
| Fly wheels | | | | | + | + | + | + |
| CMG | | | | | | | | |
| Fluid dynamic actuator | | | + | | + | + | + | + |
| Propulsion | | | | + | | | | + |
| Gravity-gradient boom | | | | | + | | | + |
| Aerodynamic stabilizer | | | | | | + | | |
| Solar stabilizer | | | | | | | + | |

algorithms implemented, and budget of the designer. The latter being a key parameter for a choice of CubeSat attitude control system configuration.

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