Distributed Computation of Persistent Homology using the Blowup Complex

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September 3, 2014

1 Introduction

2 Background

Persistent homology. [Simplicial complex. Filtration.] [Homology. Persistent homology.]

Algorithms. [ELZ reduction algorithm, expressed as $D = RU$ and $R = DV$, where R is reduced and U and V are invertible and upper-triangular.] [Uniqueness of lowest-ones in the $D = RU$ decomposition [?].]

[Subtract rows to turn R into an almost-permutation matrix P. I.e., change basis to get $R = SP$, and, equivalently, $P = S^{-1}R$, where P is almost-permutation and S is invertible and upper-triangular.] [~~Explain the trick we used to construct S^{-1} in the column form without having to do row operations.~~ There doesn't seem to be a trick: to construct S^{-1} in column form, we also need row access to its elements, which is a lot of hassle. Instead we can keep $D_{>0}$ in row form, apply the row operations directly (and efficiently), and then convert it to column form on de-serialization.]

Blowup complex. [Cover C. Blowup K^C.] [Filtration by the base space filtration with ties broken by the nerve dimension.]

[Theorem: persistence of the above filtration on the blowup produces the same pairing as the filtration on the base space.]

3 Algorithm

Parallel setup. [$p+1$ processors, with one processor per cover set. One extra processor to put things together.]

Distributed computation. [Version 2 of the algorithm.]

[Optimization: only send the lowest ones in the rows that have non-zero entries in $Q^i_{>0}$.]

4 Cascade

[Brute-force repair (straight-up reduction) takes $\mathrm{O}(n^2m)$ time and up to $\mathrm{O}(n^2)$ space. (Double-check.) Use cascade repair instead.]

[Row version of the cascade algorithm. Output matrix T.]

Theorem 1. *Lowest ones of the reduced T give the correct pairing.*

Theorem 2. *$|T| = (n - m) + nm$. T can be reduced using the cascade algorithm in time $\mathrm{O}(nm^2)$, while keeping its size $\mathrm{O}(nm)$.*

5 Experiments

A Notation