Discrate mathematics : Disenete mathematic is the study of mathematical structures that are countable or otherwise distinct and separable.

Example of the structures that are discrete are combination, graphs, and logical statements.

Proposition : proposition is a statement that is either true or false. But not ~~bought~~ both at the same time.

Example : i. Dhaka is the capital of Bangladesh.

ii. What time is it?

iii. $x+1 = 10$

Example (i) is a proposition.

Example (ii) is not a proposition cause that is not true or false.

Example (iii) is not a proposition cause that is not true or false.

Negation of proposition: Let p be a proposition the statement "It is not the case that" is another proposition, called nagation of proposition p.

Example: To day is saturday.

Solution: $p \rightarrow$ To day is saturday.

Negation: It is not the case that To day is Saturday.

(i) Conjunction : Let p and q be propositions. the proposition p and q denoted $p \wedge q$ is the proposition that is true when both p and q is true and otherwise false. The proposition $p \wedge q$ is the conjunction of p and q.

Example: p → "Today is friday."

q → "It is raining today."

pattern: $p \wedge q$ / p and q

Today is friday and it is raining today.

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Truth table

(ii) Disjunction: Let p and q be propositions. The proposition p or q denoted $p \vee q$ is the proposition that is false when p and q are false and true otherwise. The proposition $p \vee q$ is the disjunction of p and q.

Example: p → "Today is friday."

q → "It is raining today."

partten: $p \vee q$ / p or q

Today is friday or it is raining today.

p	q	p ∨ q
F	F	F
F	T	T
T	F	T
T	T	T

Truth table

(iv) Implication: Let p and q be proposition. The implication $p \to q$ (p implies q) is the proposition that is False when p is true and q is false and it is true otherwise.

Example: $p \to$ "Today is friday."

$q \to$ "It is raining today."

rule: If p, then q

If today is friday, then it is raining today.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Truth table

- **Necessary Rules for expressing Implication:**

(i) If p, then q
(ii) If p, q
(iii) p is sufficient for q
(iv) q when p
(v) q if p
(vi) a necessary condition for p is q.
(vii) p implise q
(viii) p only if q
(ix) q is necessary for p
(x) q followes from p
(xi) q wheven p
(xii) a sufficient condition for q is p.

□ Convers; $q \rightarrow p$ (read as q implies p) is called the Convers q implies p.

Ex: $p \rightarrow$ "To day is friday."
$q \rightarrow$ " It is raing today."

pantten : $q \rightarrow p$

Rules: If p, then q.

$\Rightarrow$ If it is raining to day, then today is friday.

■ Inverse: $p' \rightarrow q$ (read as not p implies not q) is called the invers $p \rightarrow q$.

Ex: $p \rightarrow$ "Today is friday."

$q \rightarrow$ "It is raining today."

partten: $p' \rightarrow q'$

$p' \rightarrow$ "Today is not friday"

$q' \rightarrow$ "It is not raining today."

Rules: If p, then q

If today is not friday, then it is not raining today.

■ Contrapositive: The proposition $q' \rightarrow p'$ (read as not q implies not p) is called the contra-positive of $p \rightarrow q$.

Example: $p \rightarrow$ "Today is friday."

$q \rightarrow$ "It is raining today."

partten: $q'' \rightarrow p''$

$q' \rightarrow$ "It is not raing today."

$p' \rightarrow$ "Today in not friday."

Rules: If q', the p'

⇒ If it is not raining today, then today is friday.

Tautology: Tautology is a stetment that is always true.

Ex1: $p \vee p'$

p	p'	$p \vee p'$
F	T	T
T	F	T

Ex2: * Man is mortal
* The sun rises in the east.

Ex3: $A' \wedge B \rightarrow (A \vee B)'$ is a tautology or not.

A	B	A'	$A' \wedge B$	$A \vee B$	$(A \vee B)'$	$A' \wedge B \rightarrow (A \vee B)'$
F	F	T	F	F	T	T
F	T	T	T	T	F	F
T	F	F	F	T	F	T
T	T	F	F	T	F	T

Ex3 is not a tautology.

Contradiction: Contradiction is a stetment that is always false.

Ex1 : $p \wedge p'$

p	p'	$p \wedge p'$
F	T	F
T	F	F

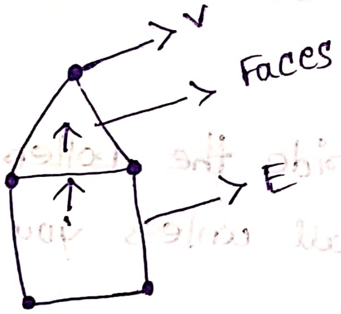
Eular Formula :-

$$F + V - E = 2$$

F = Faces

V = Vertices

E = Edges



$V = 5$

$E = 6$

$F = 3$

$$\begin{aligned} L.H.S &= F + V - E \\ &= 3 + 5 - 6 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

1. How can this english sentence be translated into a logical expression?

"you can access the internet from campus only if you are a computer science major or you are not a freshmen.

Solution:

p → "you can acess the internet from campus."

q → "you are a computer science major"

r → "you are a freshmen."

~~$p \to q \wedge \neg r$~~ $p \to q \vee \neg r$

2. How can this english sentence be translated into a logical expression?

"you can not ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Solution:

q → "you can ride the roller coaster"

p → "you are under 4 feet tall"

r → "you are older than 16 years old."

$\therefore p \wedge \neg r \to \neg q$

3. How can this english sentence be translated into a logical expression?

"The automated reply cannot be sent when the file system is full."

Solution: $q \rightarrow$ "The automated reply can be sent."

$p \rightarrow$ "The file system is full."

$p \rightarrow q'$

De-morgan's law:

① $(p \wedge q)' = p' \vee q'$

p	q	$p \wedge q$	$(p \wedge q)'$	p'	q'	$p' \vee q'$
F	F	F	T	T	T	T
F	T	F	T	T	F	T
T	F	F	T	F	T	T
T	T	T	F	F	F	F

LHS RHS

② $(p \vee q)' = p' \wedge q'$

p	q	$p \vee q$	$(p \vee q)'$	p'	q'	$p' \wedge q'$
F	F	F	T	T	T	T
F	T	F	T	T	F	T
T	F	F	F	F	T	T
T	T	T	F	F	F	F

LHS RHS

4. How can this english sentence be translated into a logical exprasion?

(i) "The diagnostic massage is stored in the buffer or it is retransmitted."

(ii) "The diagnostic message is not stored in the buffer."

Solution:

(i) p → "The diagnostic massage is stored in the buffer."

q → "it is retransmitted."

~~$p \wedge q$~~ $p \vee q$

(ii) p → The diagnostic massage is stored in the buffer.

$\therefore \neg p$

Halfman Algorithm part:

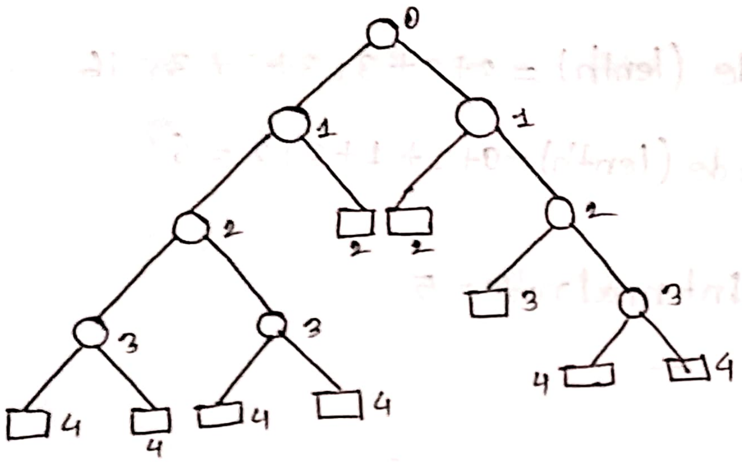
□ → External node

○ → Internal node

cann't bron child.

Q. prove that $L_E = L_I + 2n_I$

Solution:



L_E = External node (lenth) $= 2+2+3+4+4+4+4+4+4 = 31$

L_I = Internal node (lenth) $= 0+1+1+2+2+3+3+3 = 15$

n_I = number of Internal node = 8

$\therefore$ L.H.S $= L_E = 31$

$\therefore$ RHS $= L_I + 2n_I$

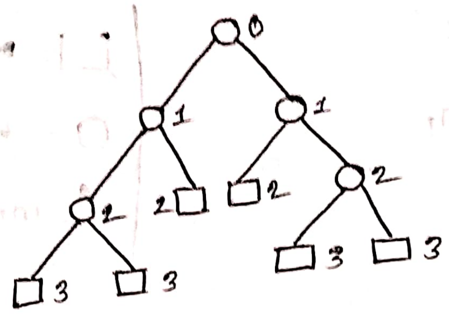
$= 15 + (2\times 8)$

$= 31.$

$\therefore$ LHS = RHS.

proved

Q:



Solution:

L_E = External node (lenth) $= 2+2+3+3+3+3 = 16$

L_I = Internal node (lenth) $= 0+1+1+2+2 = 6$

n_I = Number of Internal node $= 5$

$\therefore LHS = L_E$
$= 16$

$\therefore RHS = L_I + 2n_I$
$= 6 + (2 \times 5)$
$= 16$

$\therefore LHS = RHS$.

proved

Bicoditional: let p and q be proposition. The biconditional $p \leftrightarrow q$ (read as p implies implies q) is the proposition that is true when both p and q have the same truth value. and it's false otherwise.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Truth table

The neeessary rules for bicondition:

i. p if and only if q.

ii. p is neeessary and safieiant for q.

iii. if p then q

iv. if q then p.

v. p if q

Example:

p → "Today is friday".

q → "It is raining today."

Rules: p if and only if q.

Today is friday if and only if it is raining today.

Q: Let $P(x)$ dinote ~~that~~ the stetment "$x>3$". what are the truth values of $P(4)$ and $P(2)$?

Solution: we obtain the statment $P(4)$ by setting $x=4$ in the statment $x>3$

Hence $P(4)$ which is the statment "$4>3$" is true.

we obtain the statment $P(2)$ by setting $x=2$ in the statment $x>3$

Hence, $P(2)$ which is the statment "$2>3$" is false.

Q: Let $Q(x,y)$ dinote the statment "$x=y+3$". what are the truth values of $Q(1,2)$ and $Q(3,0)$?

Solution: we obtain the statment $Q(1,2)$ by setting $x=1, y=2$ in the statment $x=y+3$.

Hence, $Q(1,2)$ which is the statment

$1=2+3$

$1=5$ is false.

We obtain the statment $Q(3,0)$ by setting $x=3, y=0$ in the statment $x=y+3$.

Hence, ~~P(x)~~ $Q(3,0)$ which is the statment

$$3=0+3$$

$=3=3$ is true.

(i) $(p \vee (p' \wedge q))'$ and $(p' \wedge q')$

p	q	p'	q'	$p' \wedge q$	$(p \vee (p' \wedge q))$	$(p \vee (p' \wedge q))'$	$p' \wedge q'$
F	F	T	T	F	F	T	T
F	T	T	F	T	T	F	F
T	F	F	T	F	T	F	F
T	T	F	F	F	T	F	F

(ii) $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \wedge q) \wedge (p \vee r)$
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	T	T	T	T	T	T	T

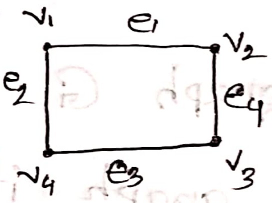
$\therefore$ $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ logicaly eqevalance.

Graph: A graph ~~long~~ nolinear data structure. A graph G consists of two things

i. A set v of elements called nodes.

ii. A set E of adges.

Example:

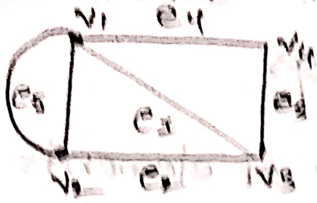


Adjacent nodes: Suppose, $e = [u, v]$, then the nodes u and v are called the ~~end~~ point of e and u and v set tobe adjacent node.

Ex:

Degree of nodes: The degree of a node u, ritten as deg(u) is the number of edges containing u.

Ex:

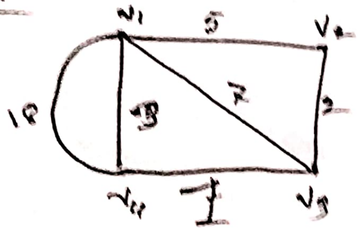


deg(v_1) = 4   deg(v_3) = 3

deg(v_2) = 3   deg(v_4) = 2

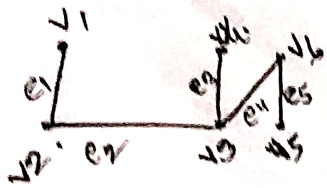
Weighted graph: A graph G is set to be weighted graph if each edge e in G is assingned a non-negative numerical value.

Ex:

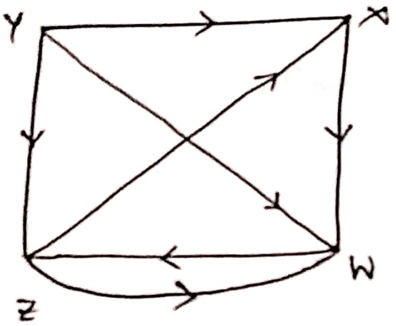


Tree graph: A connect graph T without any cycles is call Tree graph or free graph.

Ex:



* Drieeted graph : Find the adjaeeney matrix from the following graph:



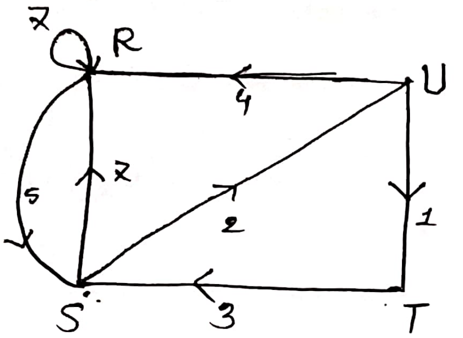
Solve:

↗	X	Y	Z	W
X	0	0	0	1
Y	1	0	1	1
Z	1	0	0	1
W	0	0	1	0

$$\therefore A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Ans:

* Find out the weight matrix from the following graph,



Sol:

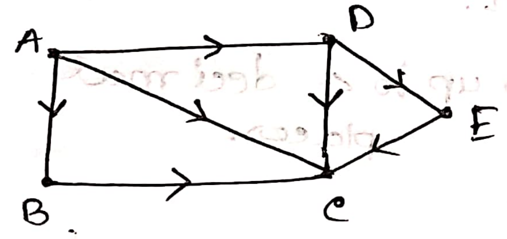
	R	S	T	U
R	7	5	0	0
S	~~7~~	0	0	2
T	0	3	0	0
U	4	0	1	0

$$\therefore A = \begin{bmatrix} 7 & 5 & 0 & 0 \\ 7 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{bmatrix}$$

Ans:

Linked representation: Linked representation is also known as adjacency structure.

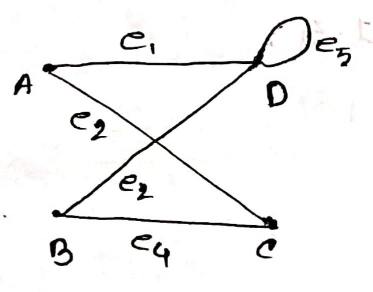
Ex:



Solve:

	Adjacency List
A	B, C, D
B	C
C	
D	C, E
E	C

Loops: An edge e is called a loop if it has identical end points that is $e = [u, u]$



Here $e_5 = [D, D]$

$\therefore e_5$ has identical end points.

* Iteration Method :

$\int_a^b f(x)dx$

$f(a) < 0$

$f(b) > 0$

$x_0 = \frac{a+b}{b}$

* find the first approximate root of the equation,

$2x^3 - 2x - 5 = 0$ up to 4 decimal places.

sol:

$\therefore f(x) = 2x^3 - 2x - 5 = 0$

$\therefore f(0) = 2 \times (0)^3 - 2 \times 0 - 5$
$= -5$

$\therefore f(1) = 2 \times 1^3 - 2 \times 1 - 5$
$= -5$

$\therefore f(2) = 2 \times 2^3 - 2 \times 2 - 5$
$= 16 - 4 - 5$
$= 7$

$a = 1, b = 2$

$x_0 = \frac{a+b}{b} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$2x^3 - 2x - 5 = 0$

or, $2x^3 = 2x + 5$

or, $x^3 = \frac{2x+5}{2}$

$\therefore x = \left(\frac{2x+5}{2}\right)^{1/3}$

$\therefore g(x) = \left(\frac{2x+5}{2}\right)^{1/3}$

$\cancel{2x^3 - 2x - 5 = 0}$

$\cancel{\Rightarrow 2x^3 -}$

$\therefore$ Iteration Method, $x_n = g(x_{n-1})$ for $n = 1, 2, 3, 4, 5, \ldots \ldots$

for $n = 1$

$$\therefore x_n = g(x_{n-1})$$

$$\therefore x_1 = g(x_{1-1})$$

$$= g(x_0)$$

$$= \left(\frac{2x_0 + 5}{2}\right)^{1/3}$$

$$= \left(\frac{2 \times 1.5 + 5}{2}\right)^{1/3}$$

$$= 1.5874$$

for $n = 2$

$$\therefore x_n = g(x_{n-1})$$

$$\therefore x_2 = g(x_{2-1})$$

$$= g(x_1)$$

$$= \left(\frac{2x_1 + 5}{2}\right)^{1/3}$$

$$= \left(\frac{2 \times 1.5874 + 5}{2}\right)^{1/3}$$

$$= 1.5989$$

for $n=3$

$\therefore x_n = g(x_{n-1})$

$\therefore x_4 = g(x_{3-1})$

$= g(x_2)$

$= g\left(\frac{2x_2+5}{2}\right)^{1/3}$

$= g\left(\frac{2\times1.5989+5}{2}\right)^{1/3}$

$= 1.60037$

for $n=4$

$\therefore x_n = g(x_{n-1})$

$\therefore x_4 = g(x_{4-1})$

$= g(x_3)$

$= g\left(\frac{2x_3+5}{2}\right)^{1/3}$

$= g\left(\frac{2\times1.60037+5}{2}\right)^{1/3}$

$= 1.60057$

for $n=5$

$\therefore x_n = g(x_{n-1})$

$\therefore x_5 = g(x_{5-1})$

$= g(x_4)$

$= g\left(\frac{2x_4+5}{2}\right)^{1/3}$

$= g\left(\frac{2\times 1.60057+5}{2}\right)^{1/3}$

$= 1.60059$

for $n = 6$

$\therefore x_n = g(x_{n-1})$

$\therefore x_6 = g(x_{6-1})$

$= g(x_5)$

$= g\left(\frac{2x_5+5}{2}\right)^{1/3}$

$= g\left(\frac{2\times 1.60059+5}{2}\right)^{1/3}$

$= 1.60059$

Ans: 1.6005

Taylor's series: Determine the taylor's series at $x=0$ for $f(x)=e^x$.

Sol:

given that,

$f(x) = e^x$

$\therefore f(0) = e^0 = 1$

Note:

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4 + \cdots$

$f'(x) = e^x$

$\therefore f'(0) = e^0 = 1$

$f''(x) = e^x$

$\therefore f''(0) = e^0 = 1$

$f'''(x) = e^x$

$\therefore f'''(0) = e^0 = 1$

$f^{iv}(x) = e^x$

$\therefore f^{iv}(0) = e^0 = 1$

$\therefore f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4 + \cdots$

or, $e^x = 1 + 1 \times x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$

or, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(Ans.)

Q2: Determine the taylor's series at, $x=0$ for $f(x) = \cos x$,

Sol: given that,

$f(x) = \cos x$

$\therefore f(0) = \cos 0 = 1$

$f'(x) = -\sin x$

$\therefore f'(0) = -\sin 0^\circ$

$= 0$

$f''(x) = -\cos x$

$\therefore f''(0) = -\cos 0$

$= -1$

$f'''(x) = \sin x$

$\therefore f'''(0) = \sin 0$

$= 0$

$\therefore f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$

or, $\cos x = 1 + 0 + \frac{(-)1}{2!}x^2 + \frac{0}{3!}x^3 + \cdots$

or, $\cos x = 1 - \frac{1}{2!}x^2 + 0 + \cdots$

Ans.