Example: 9.6

$i_1(t) = 4\cos(\omega t + 30°)$, $i_2(t) = 5\sin(\omega t - 20°)$. find the sum.

Solution:

$$i_1(t) = 4\cos(\omega t + 30°)$$
$$= 4\angle 30° \; i$$

$$i_2(t) = 5\sin(\omega t - 20°)$$
$$= 5\cos(\omega t - 20° - 90°)$$
$$= 5\cos(\omega t - 110°)$$
$$= 5\angle -110° \; i$$

Now, the sum is,

$$i_1(t) + i_2(t)$$
$$= (4\angle 30°) + (5\angle -110°)$$
$$= 3.2184\angle -56.9°.$$

Note: Using calculator

CMPLX MODE

~~(4 → SHIFT → 30°) + (5~~

$(4 \rightarrow \underset{(\angle)}{\text{SHIFT}} \rightarrow 30°) + (5 \rightarrow \underset{(\angle)}{\text{SHIFT}} \rightarrow -110)$

$= \rightarrow \underset{\text{P}\angle\theta}{\underset{(+)}{\text{SHIFT}}} \rightarrow = \rightarrow 3.2184 \rightarrow =$

$\angle -56.9°$

Practice problem: 9.6

$v_1 = -10 \sin(\omega t - 30^\circ)$,

$v_2 = 20 \cos(\omega t + 45^\circ)$. find the sum.

Solution:

$$v_1 = -10 \sin(\omega t - 30^\circ)$$
$$= 10 \cos(\omega t - 30^\circ + 90^\circ)$$
$$= 10 \cos(\omega t + 60^\circ)$$
$$= 10 \angle 60^\circ$$

$$v_2 = 20 \cos(\omega t + 45^\circ)$$
$$= 20 \angle 45^\circ$$

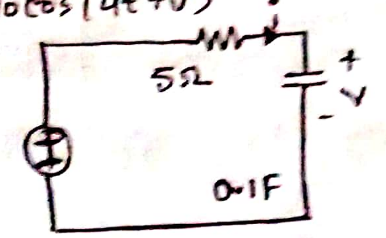
Now, the sum is,

$$v_1 + v_2$$
$$= (10\angle 60^\circ) + (20\angle 45^\circ)$$
$$= 19.14 \angle 22.80^\circ$$

using calculator

Example: 9.9

$v_s = 10 \cos(4t + 0^\circ)$



find $i(t)$ and $v(t)$.

Note:

$C \rightarrow X_C = \frac{1}{j2\pi fC} = \frac{1}{j\omega C}$

$L \rightarrow X_L = j2\pi fL = j\omega L$

Solution:

$V_s = 10 \cos(4t + 0^\circ)$

$= 10 \angle 0^\circ$

$\therefore i(t) = \frac{V}{Z}$

$= \frac{10\angle 0^\circ}{5 - J2.5}$

$= 1.788 \angle 26.56$ A

$\therefore v(t) = I X_c$

$= (1.78\angle 26.56) \times (-J2.5)$

$= 4.47\angle -116.63)$ v.

$Ac = I = \frac{V}{Z}$

$\therefore X_c = \frac{1}{J2\pi f c}$

$= \frac{1}{J\omega c}$

$= \frac{1}{J4 \times 0.1}$

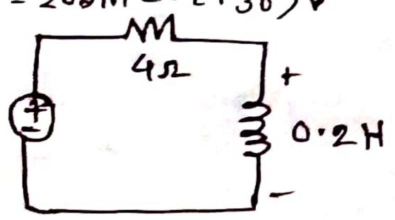
$= -J\,2.5\Omega$

$\therefore Z = 5 - J2.5$

Note: Using calculator

Practice problem: 9.9

$V_s = 20 \sin(10t + 30^\circ)$ v



find the i(t) and v(t).

Solution:

$V_s = 20 \sin(10t + 30^\circ)$

$= 20 \cos(10t + 30^\circ - 90^\circ)$

$= 20 \cos(10t - 60^\circ)$

$= 20 \cos(10t - 60^\circ)$

$= 20 \angle -60^\circ$.

$\therefore i(t) = \frac{V}{Z}$

$= \frac{20\angle -45^\circ}{4+2j}$

$= 4.47\angle -71.56$

$X_L = j\omega L$

$= j10 \times 0.2$

$= 2j$

$Z = R + X_L$

$= 4+2j$

$\therefore v(t) = IX_L$

$= (4.47\angle -71.56) \times 2j$

$= 8.94\angle 18.44$

Chapter-11

AC power Analysis

Example 11-1

Given that $v(t) = 120\cos(377t + 45^\circ)$ V and $i(t) = 10\cos(377t - 10^\circ)$ find the instaneous power and the average power absorbed by the passive linear network.

Solution:

$v(t) = 120\cos(377t + 45^\circ)$ V

$i(t) = 10\cos(377t - 10^\circ)$ A

$V_m = 120$

$I_m = 10$

$$P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{120 \times 10}{2} \cos(45^\circ + 10^\circ) + \frac{120 \times 10}{2} \cos(2 \times 377t + 45^\circ - 10^\circ)$$

$$= 600 \cos 55^\circ + 600 \cos(754t + 35^\circ)$$

$$= 344.14 + 600 \cos(754t + 35^\circ)$$

Practice problem: 11.1

$v(t) = 330 \cos(10t + 20^\circ) V$

$i(t) = 33 \sin(10t + 60^\circ) A$

find $p(t)$.

Solution:

$v(t) = 330 \cos(10t + 20^\circ) V$

$i(t) = 33 \sin(10t + 60^\circ) A$

$= 33 \cos(10t - 30^\circ) A$

$V_m = 330$

$I_m = 33$

$\theta_v = 20^\circ$

$\theta_i = 30^\circ$

$$P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{330 \times 33}{2} \cos(20^\circ + 30^\circ) + \frac{330 \times 33}{2} \cos(2 \times 10t + 20^\circ - 30^\circ)$$

$$= 5445 \cos 50^\circ + 5445 \cos(20t - 10^\circ).$$

Q- Find i_1, i_2, i_3 in the circuit fig 1.1 (a).

Fig-1.1

Solution:

loop 1,

$$30i_1 + 10i_1 - 10i_2 + 120 = 0$$

$$\Rightarrow 40i_1 - 10i_2 = -120 \text{ ——— (i)}$$

loop 2,

$$30i_2 + 10i_2 - 10i_3 + 10i_2 - 10i_1 = 0$$

$$\Rightarrow 10i_1 + 50i_2 - 10i_3 = 0 \text{ ——— (ii)}$$

loop 3,

$$30i_3 + 10i_3 - 10i_2 - 120 = 0$$

$$\Rightarrow 40i_3 - 10i_2 = 120 \text{ ——— (iii)}$$

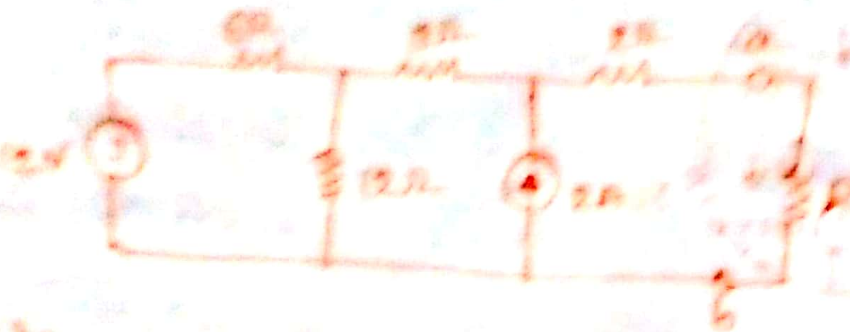
Using calculator to solve, i, ii, iii —

$i_1 = -3A$

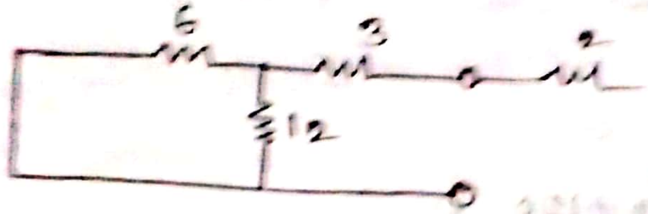
$i_2 = 0$

$i_3 = 3A$.

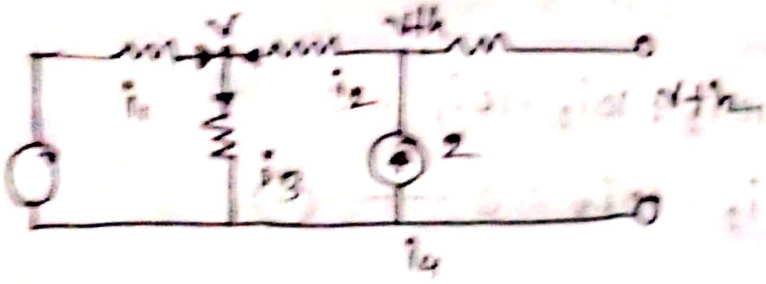
Q5: Find the value of R_L for maximum power transfer in the circuit 5.1



Solution:



$$R_{th} = (6 \| 12) + 3 + 2$$
$$= 9\Omega$$



Appling kcl node 1

$$v_{th} \Rightarrow i_1 + i_2 = i_3$$

$$\frac{12-v}{6} + \frac{v_{th}-v}{3} = \frac{v-0}{12}$$

$$\Rightarrow \frac{12-v+2v_{th}-2v}{6} = v/12$$

$$\Rightarrow 24 - 6v + 4v_{th} = v$$
$$\Rightarrow 4v_{th} - 2v = -24 \quad (1)$$

node 2.

$$i_4 = i_2$$

$$2 = \frac{V_{th} - V}{3}$$

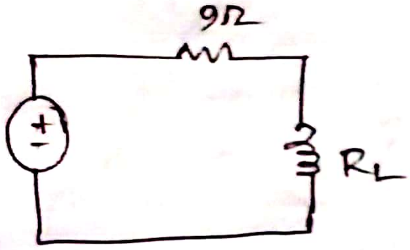
$$\Rightarrow V_{th} - V = 6 \quad \text{---} \ (ii)$$

(i) and (ii) solving we get :

$$V_{th} = 22\,V$$

$$V = 16\,V$$

equivalent circuit :



for maximum power $R_L = 9\Omega$ Ans.

and

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$= \frac{(22)^2}{4 \times 9}$$

$$= 13.44 \text{ watt}$$ Ans.

Theory:

* KCL
* KVL
* Nodale Analysis
* Mess Analysis
* Thebinin Theorem
* ~~Sinosoid~~ * Time period.
* phasors

Practice problem: 9.2

Find the phase angle between, $i_1 = -4\sin(377t + 55^\circ)$ and $i_2 = 5\cos(377t - 65^\circ)$. Does i_1 lead or lag i_2?

Solution:

$i_1 = -4\sin(377t + 55^\circ)$

$= 4\cos(377t + 55^\circ + 90^\circ)$

$= 4\cos(377t + 145^\circ)$

$= 4\angle 145^\circ$ i

$i_2 = 5\cos(377t - 65^\circ)$

$= 5\angle -65^\circ$ i

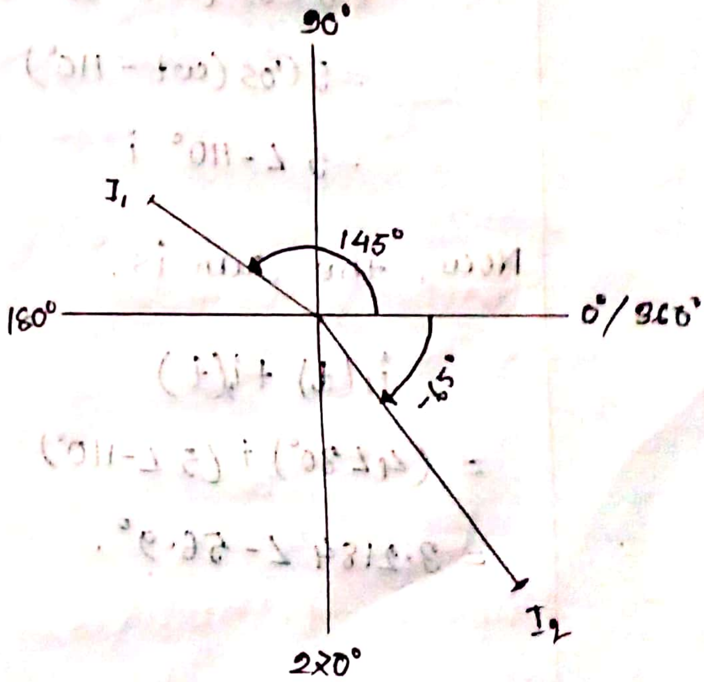
$\therefore$ phase angle $= 145^\circ - (-65^\circ)$

$= 210^\circ$

or,

$\therefore$ phase angle $= 360^\circ - 210^\circ$

$= 150^\circ$



$\therefore i_1$ leads i_2 by 210°.

Example 9.4

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution

$v_1 = -10\cos(\omega t + 50^\circ)$

$= 10\cos(\omega t + 50^\circ + 180^\circ)$

$= 10\cos(\omega t + 230^\circ)$

$= 10\angle 230^\circ$ V

$v_2 = 12\sin(\omega t - 10^\circ)$

$= 12\cos(\omega t - 10^\circ - 90^\circ)$

$= 12\cos(\omega t - 100^\circ)$

$= 12\cos(\omega t - 100^\circ)$

$= 12\angle -100^\circ$ V

$\therefore$ phase angle $= 230^\circ - (-100^\circ)$

$= 330^\circ$

OR

$\therefore$ phase angle $= 360^\circ - 330^\circ$

$= 30^\circ$

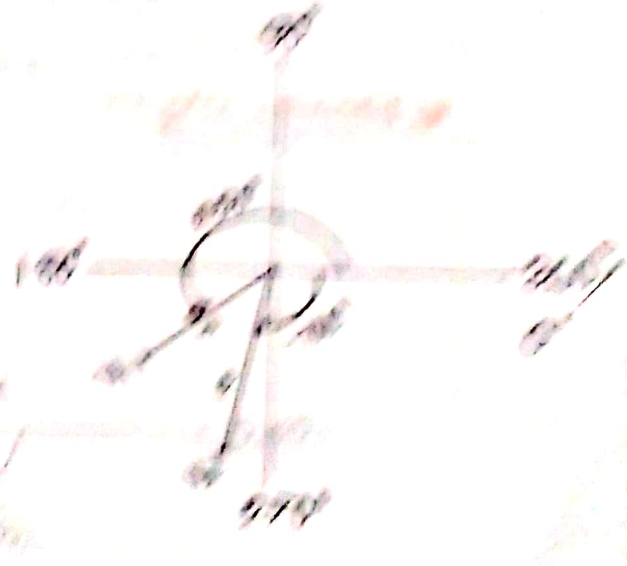


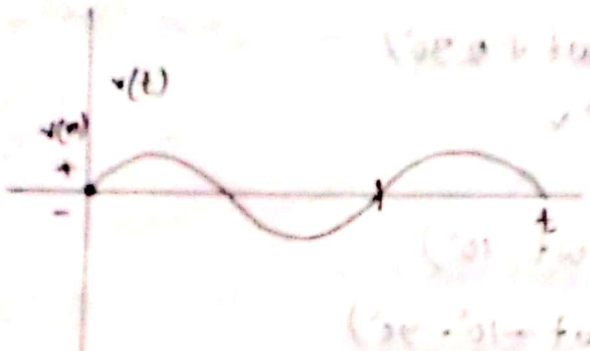
Fig 9.9

$\therefore v_2$ leads v_1 by 30°

chapter: 9

Sinusoids and phasors

Sinusoids: Sinusoids is a signal that has the form of the sine or cosine function.



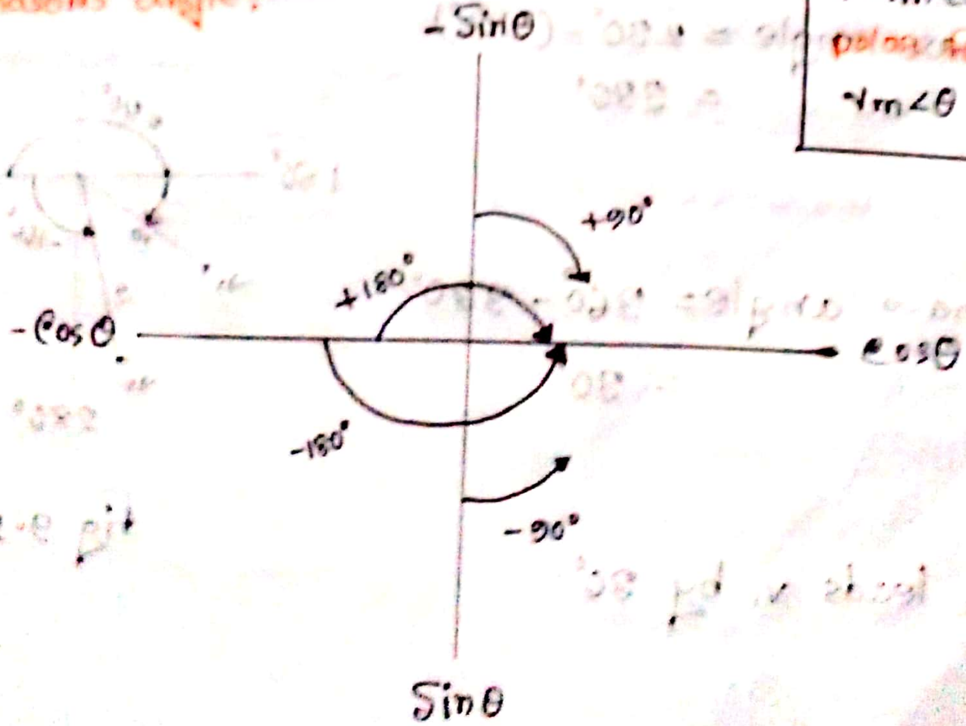
$\boxed{V_m = \text{magnitude}}$

$\therefore v(t) = V_m \sin \omega t$

$\Rightarrow v(t) = V_m \cos(\omega t + \theta)$

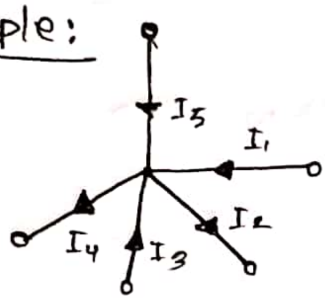
Sinusoidal form:
$v = V_m \cos \omega t + \theta$
polar form:
$V_m \angle \theta$

phasors angle:



* **KCL:** The KCL (KIRCHHOFF'S CURRENT LAW) staets that in a closed circuit, the entering current at node is equal to the current leaving at the node.

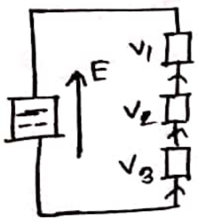
Example:



$\therefore I_1 + I_3 + I_5 = I_2 + I_4$

* **KVL:** The KVL (kirchhoff's voltage law) staets that algebraic sum of the voltage at node in a closed circuit is equal to zero.

Example:



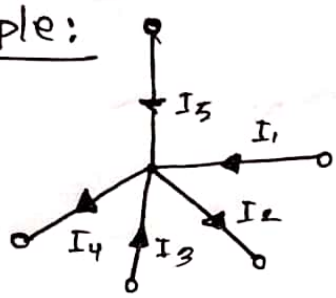
$\therefore E - V_1 - V_2 - V_3 = 0$

* **phasors:** A phasor is a complex number that represents the amplitude and phase a sinusoid.

* **Time period:** In Ac current, the "time period" is the time it takes for the current to complete one full cycle of its back-and-forth movement.

* **KCL:** The KCL (KIRCHHOFF'S CURRENT LAW) staets that in a closed circuit, the entering current at node is equal to the current leaving at the node.

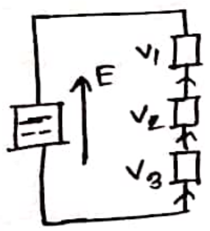
Example:



$\therefore I_1 + I_3 + I_5 = I_2 + I_4$

* **KVL:** The KVL (kirchhoff's voltage law) staets that algebraic sum of the voltage at node in a closed circuit is equal to zero.

Example:



$\therefore E - V_1 - V_2 - V_3 = 0$

$V_R = IR$

$= (50.5964 \angle 26.56) \times 3$

$= 151.789 \angle 26.56$

$\therefore$ Average power absorved by the Resistor,

$$P_{avg} = \frac{1}{2} V_R I_m \cos(\theta_v - \theta_I)$$

$$\cancel{= \frac{1}{2} \times 15}$$

$$= \frac{1}{2} \times 151.789 \times 50.5964 \times \cos(26.56 - 26.56)$$

$$= 3839.98 \text{ watt}$$

$\therefore$ Inductor absorb zero Average power.

$\therefore$ Average power supply by the source,

$$\therefore P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

$$= \frac{1}{2} \times 160 \times 50.5964 \times \cos(45 - 26.56)$$

$$= 3839.88 \text{ watt}$$

$$= 3.839 \text{ kw}.$$

Average power absorved by the Resistor,

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

$$= \frac{1}{2} \times 4.472 \times 1.118 \times \cos(56.56 - 56.56)$$

$$= 2.49 \text{ Watt}$$

Average power supplied by the source,

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

$$= \frac{1}{2} \times 5 \times 1.118 \times \cos(30^\circ - 56.56^\circ)$$

$$= 2.5 \text{ Watt.}$$

Practic problem: 11.3

In the circuit calculate the average power absorbed by resistor and inductor. find the average power supplied the voltage source.

Solution:

from circuit, $Z = 3 + J1$

$$I = \frac{V}{Z}$$

$$= \frac{160\angle 45^\circ}{3 + J1}$$

$$= 50.5964\angle 26.56$$

We know,

Avarage power,

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

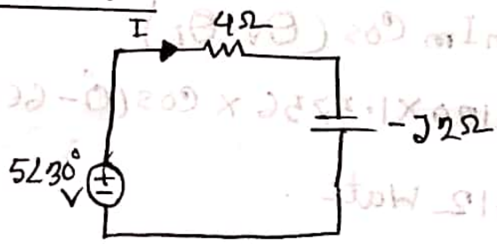
$$= \frac{1}{2} \times 800 \times 20 \cos(10^\circ - 30^\circ)$$

$$= 7.4174 \text{ KW}$$

Example: 11.3

for the circuit shown in fig 11.3 find the average power supplied by the source and the avearage power absorbed by the resistor.

Solution:



from circuit, $Z = 4 - J2\Omega$

$$I = \frac{V}{Z}$$

$$\Rightarrow \frac{5\angle 30^\circ}{4 - J2\Omega}$$

$$\Rightarrow 1.118\angle 56.56$$

$$V_R = IR$$

$$= (1.118\angle 56.56) \times 4$$

$$= 4.472\angle 56.56$$

Example: 11.2

Calculate the average absorbed by an inpedance $z = 30 - j70\Omega$ when a voltage $v = 120\angle 0°$ is applied aerossit.

Solution:

$$v = 120\angle 0°$$

$$z = 30 - j70\Omega$$

Now,

$$I = \frac{v}{z} = \frac{120\angle 0°}{30 - j70}$$

$$= 1.5756\angle 66.80$$

We know,

$$\text{Avarage power, } P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 120 \times 1.5756 \times \cos(0° - 66.88°)$$

$$= 37.12 \text{ Watt}.$$

Practice problem: 11.2

A current $I = 20\angle 30°$ A flows through an impedance $z = 40\angle -22°\Omega$ find the average power delivered to the impedance.

Solution:

$$I = 20\angle 30°$$

$$z = 40\angle -22°\Omega$$

Now,

$$v = Iz$$

$$= (20\angle 30°) \times (40\angle -22°\Omega)$$

$$\cancel{= 800\angle 10}$$

$$= 800\angle 8.$$