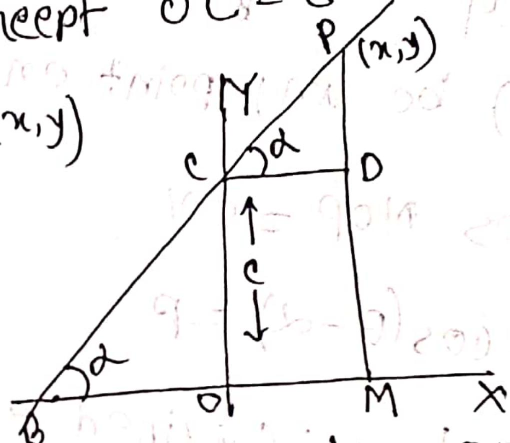
~~(vi) Let AB be the given line whose equation in the normal form is,~~

$$\cancel{x\cos\alpha + y\sin\alpha = p} \quad \text{——(i)}$$

① Let AB be the straight line whose inclination is $\alpha\,(\neq 90^\circ)$ and slope m. Let AB cut off intercept $OC = c$ from the axis of y. Let $P(x,y)$



be any point on AB. Draw the ordinate PM and CD parallel to OX to meet PM at D.

$$\text{then, } \tan\alpha = \frac{PD}{CD} = \frac{PM - DM}{OM}$$

$$= \frac{PM - OC}{OM}$$

$$\text{or, } m = \frac{y-c}{x}$$

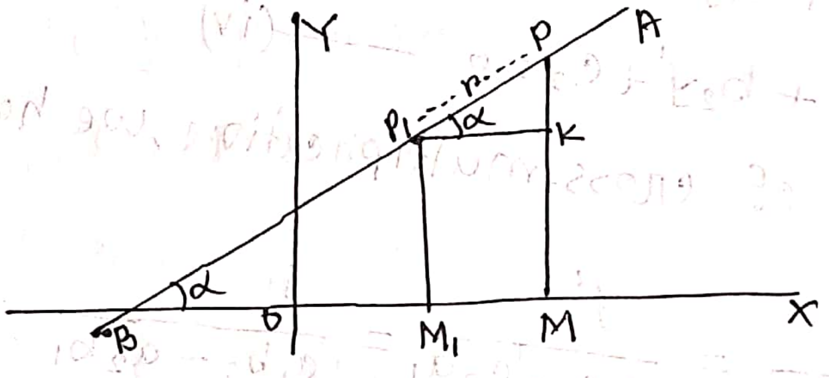
$$\therefore y = mx + c$$

(ii) Let AB be the straight line passing through the fixed point $P_1(x_1, y_1)$. Let its inclination be α. Let $P(x, y)$ be any point on AB, where $P_1P = r$

Draw P_1M_1, PM perpendiculars to OX and P_1K perpendicular to PM. The $\angle PP_1K = \alpha$,

$$PK = PM - KM = PM - P_1M_1 = y - y_1$$

$$P_1K = M_1M = OM - OM_1 = x - x_1$$



Now $P_1K = P_1P \cos\alpha$, $PK = P_1P \sin\alpha$

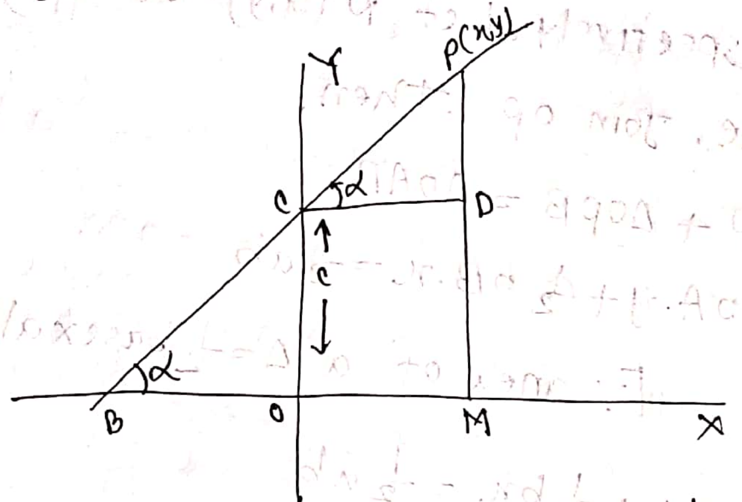
$$\therefore \frac{P_1K}{\cos\alpha} = \frac{PK}{\sin\alpha} = PP_1$$

$$\text{or} \quad \frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} \quad \text{——(ii)}$$

which is the required equation.

?.

(iii) Let the straight line passing through the two given point (x_1, y_1) and (x_2, y_2) have slope m.



the equation of straight line passing through (x_1, y_1) and having slope m is

$$y - y_1 = m(x - x_1) \quad \text{——— (i)}$$

Since it also passes through (x_2, y_2), therefore

$$y_2 - y_1 = m(x_2 - x_1)$$

$$\text{or, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

substituting this value of m in (i), we get

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } \frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

which is the required equation.

(iv) Let the straight line AB cut off intercepts $OA = a$ and $OB = b$ from the axes of x and y respectively. Let $P(x, y)$ be any point on the line. Join OP. Then,

$$\Delta OAP + \Delta OPB = \Delta OAB$$

$$\text{or, } \frac{1}{2} OA \cdot y + \frac{1}{2} OB \cdot x = \frac{1}{2} ab$$

[$\because$ area of a $\Delta = \frac{1}{2}$ base $\times$ altitude]

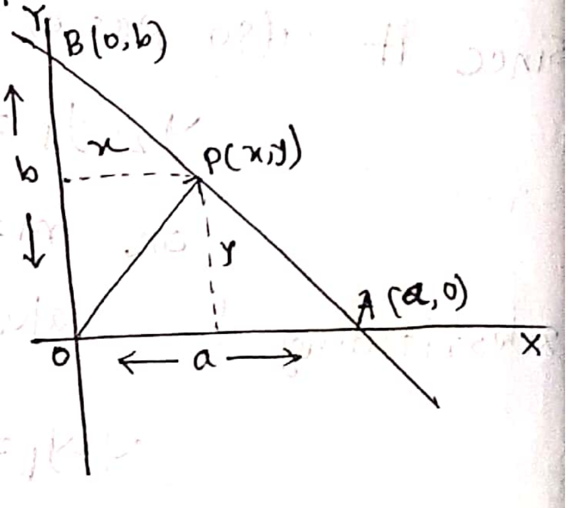
$$\text{or, } \frac{1}{2} ay + \frac{1}{2} bx = \frac{1}{2} ab$$

Dividing by $\frac{1}{2} ab$, we get

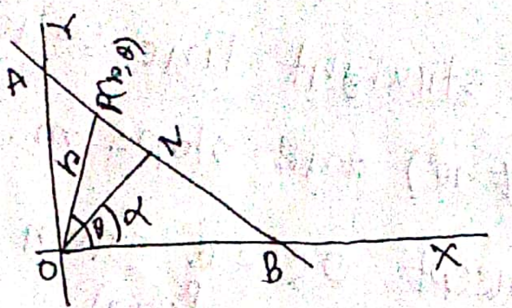
$$\frac{x}{a} + \frac{y}{b} = 1$$

which is therefore the required equation,

?



(v) Let AB ~~be~~ be the straight line. Draw ON perpendicular to AB. It is given that $ON = p$ and $\angle XON = \alpha$. We are to find its equation.



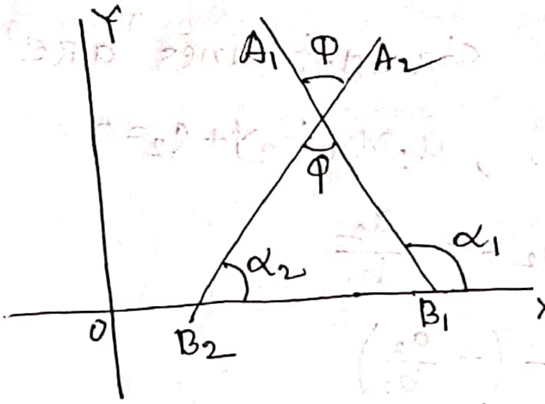
Let $P(r, \theta)$ be any point on AB. Then $\angle NOP = \theta - \alpha$.

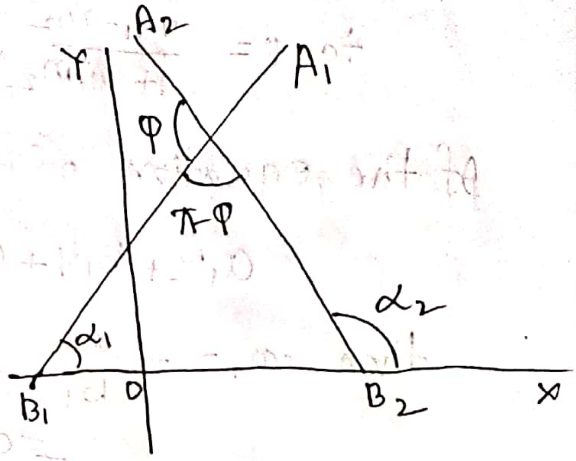
Now $OP \cos NOP = ON$

or, $r \cos(\theta - \alpha) = p$

which is the required equation.

vii) Let A_1B_1 $(y=m_1x+c_1)$ and A_2B_2 $(y=m_2x+c_2)$ be the two given lines.





Let α_1 and α_2 be the inclinations of A_1B_1 and A_2B_2 respectively. Let φ be the angle through which A_2B_2 must be rotated in the anticlockwise sense in order to coincide with A_1B_1. Clearly φ is the angle between the lines measured in the sense just described.

In Fig a, $\alpha_1 = \alpha_2 + \varphi$ or, $\varphi = \alpha_1 - \alpha_2$ ——(i)

and Fig b, $\alpha_2 = \alpha_1 + (\pi - \varphi)$, or, $\varphi = \pi + (\alpha_1 - \alpha_2)$ ——(ii)

Using the fact that $\tan(\pi+\theta) = \tan\theta$, we have from both (i) and (ii),

$$\tan\varphi = \tan(\alpha_1 - \alpha_2)$$

$$= \frac{\tan\alpha_1 - \tan\alpha_2}{1+\tan\alpha_1 \tan\alpha_2}$$

Since $\tan\alpha_1 = m_1$ and $\tan\alpha_2 = m_2$, we obtain

$$\tan\varphi = \frac{m_1 - m_2}{1 + m_1 m_2} \text{ ——— (iii)}$$

If the equation of the straight lines are

$$a_1x + b_1y + c_1 = 0 \;,\; a_2x + b_2y + c_2 = 0$$

then $m_1 = -\frac{a_1}{b_1}$, $m_2 = -\frac{a_2}{b_2}$

$$\tan\varphi = \frac{-\frac{a_1}{b_1} - \left(-\frac{a_2}{b_2}\right)}{1 + \left(-\frac{a_1}{b_1}\right)\left(-\frac{a_2}{b_2}\right)}$$

$$\Rightarrow \tan\varphi = \frac{a_2b_1 - a_1b_2}{a_1b_2 + b_1b_2}$$

$$\Rightarrow \varphi = \tan^{-1}\left(\frac{a_2b_1 - a_1b_2}{a_1b_2 + b_1b_2}\right)$$

i. If the lines are parallel, $\varphi = 0$ and hence $m_1 = m_2$.

ii. If the lines are perpendicular to each other, $\varphi = 90°$ and hence $m_1m_2 + 1 = 0$ or $m_1m_2 = -1$ or $a_1a_2 + b_1b_2 = 0$

(viii) Let $a_1x + b_1y + c_1 = 0$ ——— (i)

$a_2x + b_2y + c_2 = 0$ ——— (ii)

be the two given straight lines.

The point of intersection of the straight lines is common to both. Therefore, its coordinates, say, (x', y') will satisfy both the equation (i) and (ii)

$\therefore$ $a_1x' + b_1y' + c_1 = 0$ ——— (iii)

$a_2x' + b_2y' + c_2 = 0$ ——— (iv)

$\therefore$ by the rule of cross-multiplication, we have

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y'}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$\therefore$ the coordinates of the point of intersection of the lines are

$$x' = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y' = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

?

(ix) The equation of line parallel to $3x+5y+8=0$ is

$3x+5y+k=0$ —— (i)

The required line passes through $(-1,2)$

so, $3(-1)+5(2)+k=0$

$\Rightarrow -3+10+k=0$

$\Rightarrow 7+k=0$

$\Rightarrow k=-7$

~~so the required line is~~

Putting the value of k into equation (i), we get

$3x+5y-7=0$

(x) Given the equation

$7x-6y+6=0$ —— (i)

$2x+9y-5=0$ —— (ii)

Multiplying (i) by 2 and (ii) by 7, and subtract them,

we get $14x-12y+12-14x-63y+35=0$

$\Rightarrow -75y+47=0$

$\Rightarrow -75y=-47$

$\Rightarrow y=\frac{47}{75}$

Replacing the value of y into equation (ii), we get

$$2x+9\,\frac{47}{75}-5=0$$

$$\Rightarrow 2x+\frac{3\times47}{25}-5=0$$

$$\Rightarrow 2x+\frac{141-125}{25}=0$$

$$\Rightarrow 2x+\frac{16}{25}=0$$

$$\Rightarrow 2x=-\frac{16}{25}$$

$$\Rightarrow x=\frac{-8}{25}$$

The intersection point of (i) and (ii) is $\left(\frac{-8}{25}, \frac{47}{75}\right)$

The equation of the line perpendicular to the line $x-3y+19=0$ is $3x+y+k=0$ ——— (iii)

Since the line (iii) is passing through the point $\left(\frac{-8}{25}, \frac{47}{75}\right)$

$$3\times\left(\frac{-8}{25}\right)+\frac{47}{75}+k=0$$

$$\Rightarrow \frac{3\times3(-8)+47}{75}+k=0$$

$$\Rightarrow \frac{-72+47}{75}+k=0$$

$$\Rightarrow \frac{-25}{75}+k=0$$

$$\Rightarrow k=\frac{1}{3}$$

Putting the value of k into equation (iii), we get

$$3x+y+\frac{1}{3}=0$$

$$\Rightarrow 9x+3y+1=0$$