EEE 1201
Introduction to Electrical Engineering

S. M. Naimur Rahman Sayam
Lecturer
Department of CSE

The important thing about a problem is not its solution, but the strength we gain in finding the solution.

—Anonymous

BASIC CONCEPTS

Electric circuit

An electric circuit is an inter connection of electrical elements

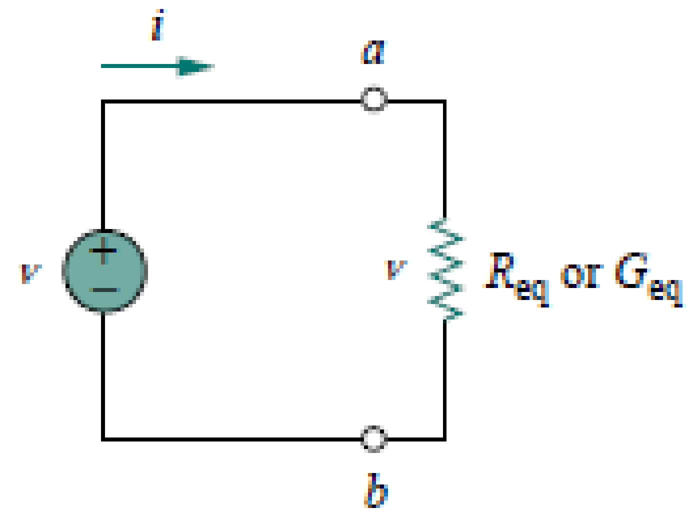


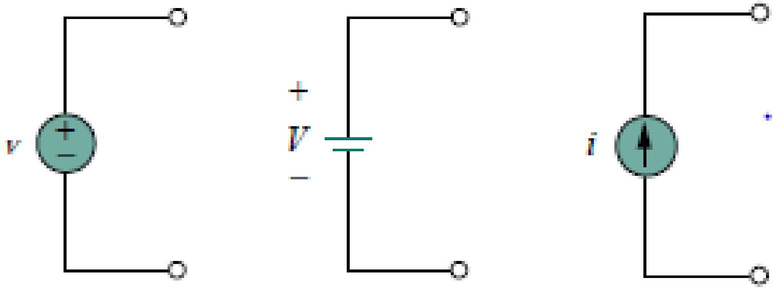
Fig: Electric circuit

CIRCUIT ELEMENTS

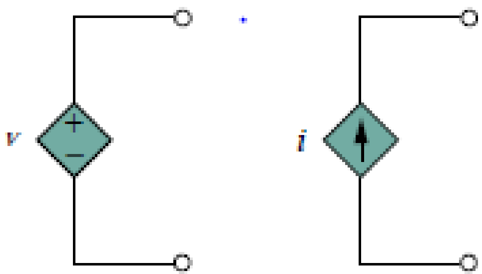
1. Passive elements - Resistors, Capacitors, Inductors
2. Active elements – Voltage source, Current source

There are two kinds of sources

1. **Independent**



2. **Dependent**



Electric current is the time rate of change of charge, measured in amperes (A)

$$\boxed{i = \frac{dq}{dt}}$$

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf). This emf is also known as voltage or potential difference.

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

$$v_{ab} = \frac{dw}{dq}$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms.

$$R = \rho \frac{\ell}{A}$$

ρ is known as the resistivity of the material in ohm-meters

Material	Resistivity ($\Omega \cdot m$)
Silver	1.64×10^{-8}
Copper	1.72×10^{-8}
Aluminum	2.8×10^{-8}
Gold	2.45×10^{-8}
Carbon	4×10^{-5}
Germanium	47×10^{-2}
Silicon	6.4×10^{2}
Paper	10^{10}
Mica	5×10^{11}
Glass	10^{12}
Teflon	3×10^{12}

Power is the time rate of expending or absorbing energy, measured in watts (W)

$$\boxed{p = \frac{dw}{dt}}$$

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

$$\boxed{p = vi}$$

BASIC LAWS

Ohm’s law

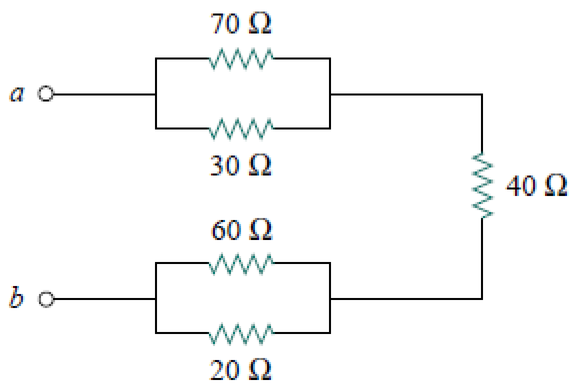
At constant temperature, the potential difference across a conductor is directly proportional to the flow of electrons through the conductor.

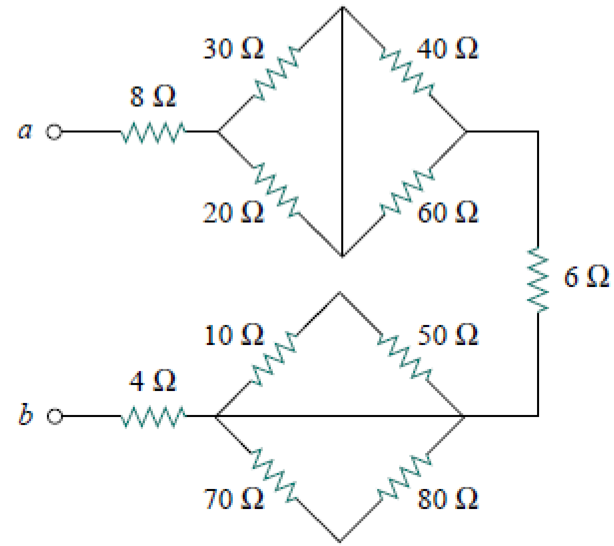
$$v \propto i$$

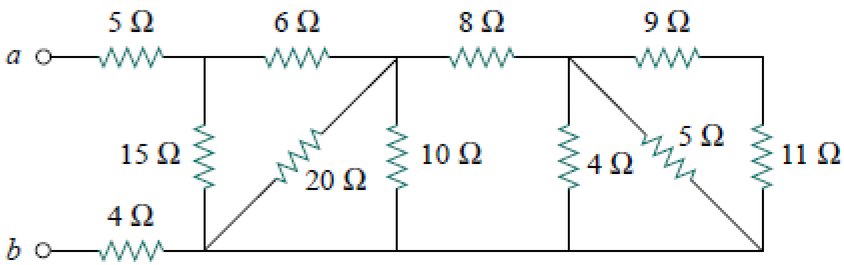
$$\boxed{v = iR}$$

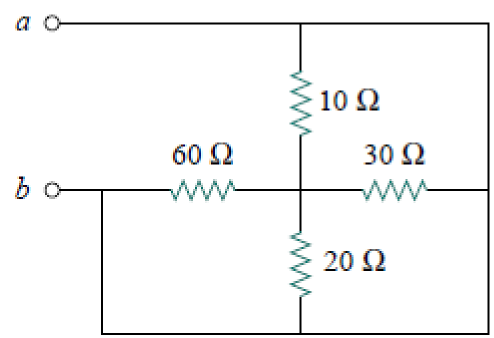
Series Parallel Circuit

Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits:

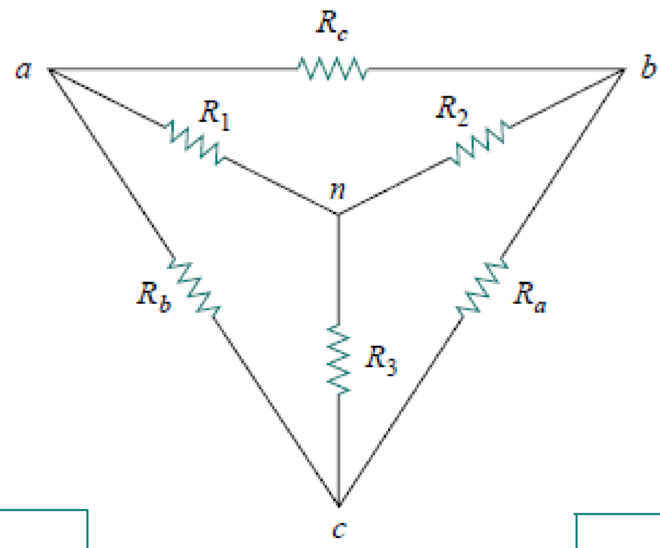








Delta to Wye Conversion



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

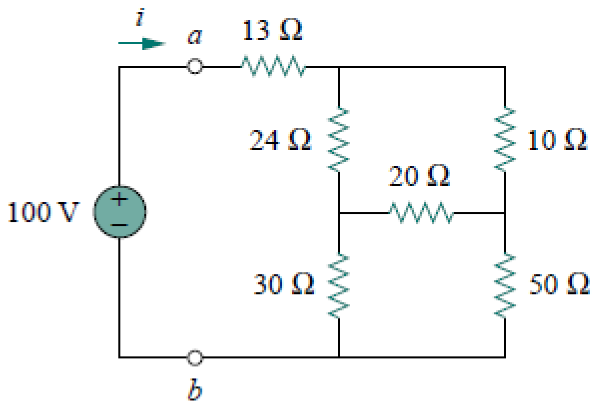
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

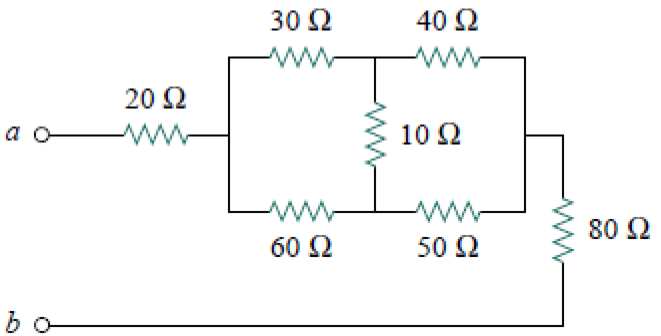
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

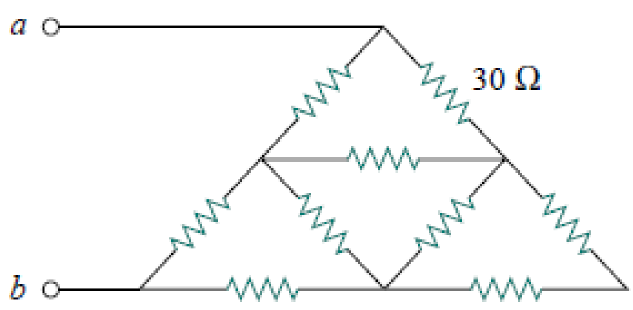
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Obtain the equivalent resistance at the terminals a-b for each of the circuits:







All resistors have a value of 30

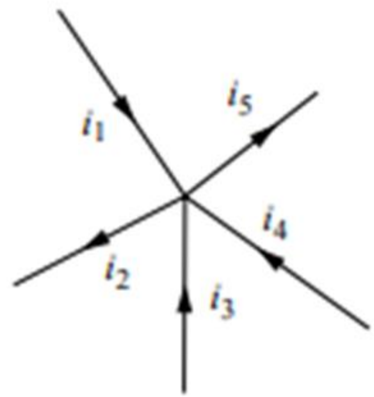
Kirchhoff’s laws

A branch represents a single element such as a voltage source or a resistor.

A node is the point of connection between two or more branches.

A loop is any closed path in a circuit.

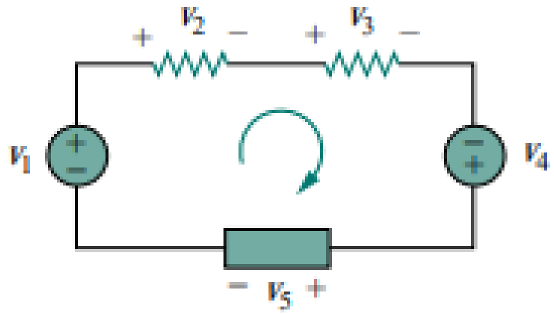
Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



$$i_1 + i_3 + i_4 = i_2 + i_5$$

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path(or loop) is zero.



v2 + v3 + v5 = v1 + v4

$$\boxed{\text{Sum of voltage drops} = \text{Sum of voltage rises}}$$

EXAMPLE 2.6

Determine v_o and i in the circuit shown in Fig. 2.23(a).

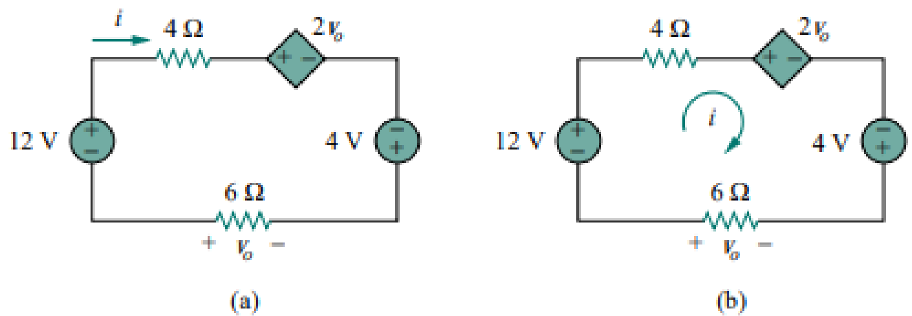


Figure 2.23 For Example 2.6.

EXAMPLE 2.7

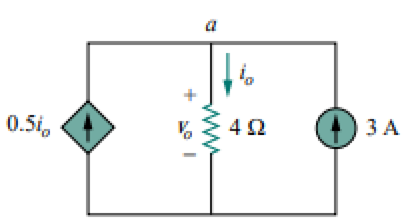


Figure 2.25 For Example 2.7.

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

Solution:

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \qquad \Longrightarrow \qquad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

PRACTICE PROBLEM 2.7

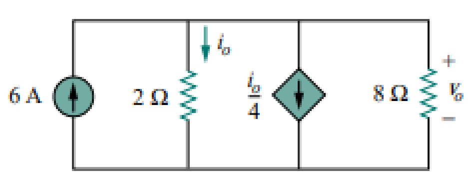


Figure 2.26 For Practice Prob. 2.7.

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 8 V, 4 A.

PRACTICE PROBLEM 2.8

Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.

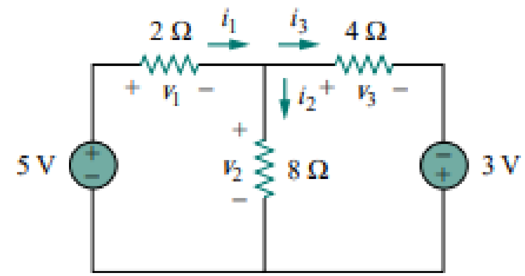


Figure 2.28 For Practice Prob. 2.8.

2.33 In the circuit of Fig. 2.97, find R if $V_o = 4V$.

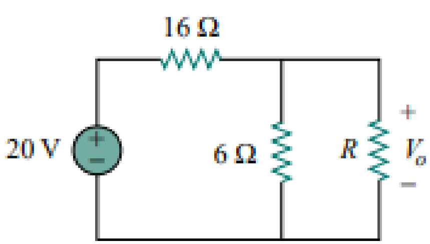


Figure 2.97 For Prob. 2.33.

2.25 Calculate v_1, i_1, v_2, and i_2 in the circuit of Fig. 2.89.

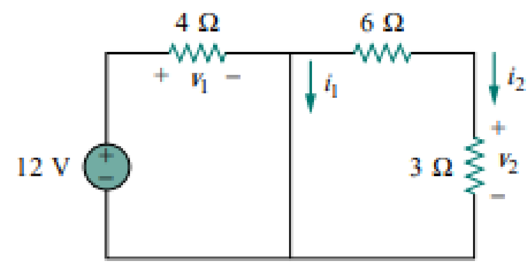
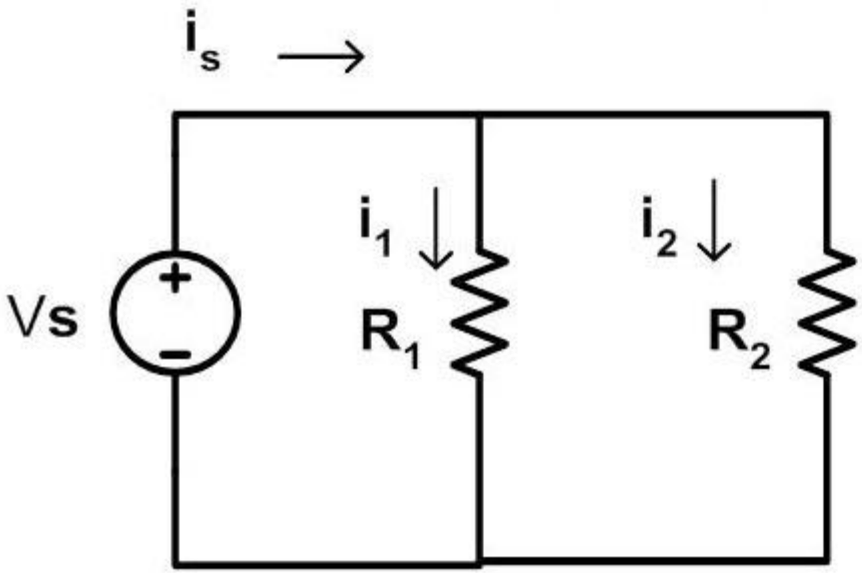


Figure 2.89 For Prob. 2.25.

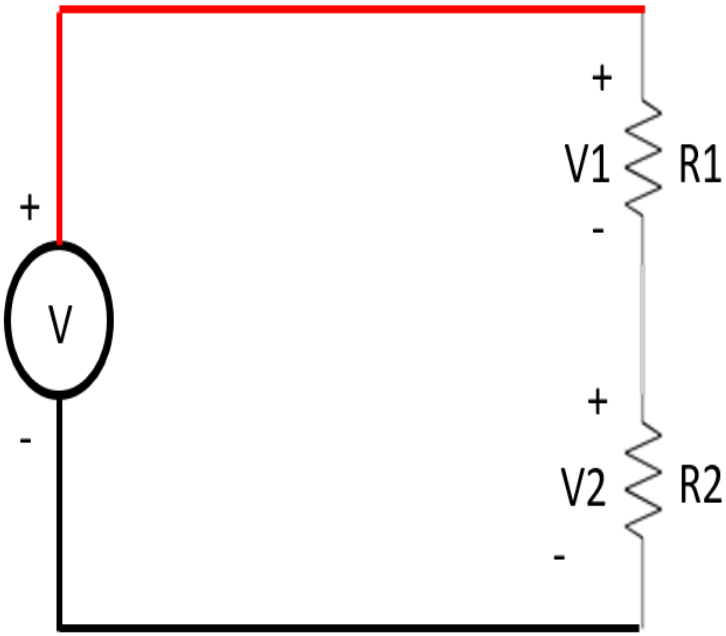
Current Divider Rule (CDR)

- Whenever current has to be divided among resistors in parallel, use current divider rule principle.



$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$



V1 is the voltage across R1:

$$V1 = \frac{R1}{R1+R2} * V$$

V2 is the voltage across R2:

$$V2 = \frac{R2}{R1+R2} * V$$

Voltage division rule

Methods of analysis

Nodal analysis

- **Current flows from a higher potential to a lower potential in a resistor**

$$\boxed{i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}}$$

PRACTICE PROBLEM 3.1

Obtain the node voltages in the circuit in Fig. 3.4.

Answer: $v_1 = -2$ V, $v_2 = -14$ V.

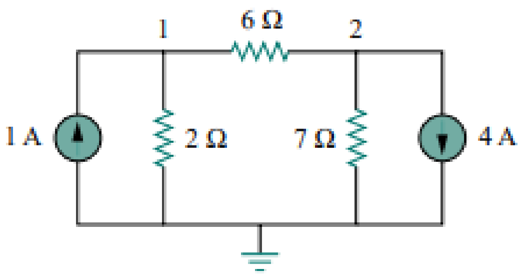


Figure 3.4 For Practice Prob. 3.1.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

PRACTICE PROBLEM 3.3

Find v and i in the circuit in Fig. 3.11.

Answer: -0.2 V, 1.4 A.

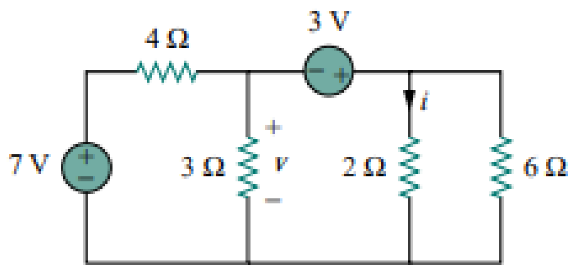


Figure 3.11 For Practice Prob. 3.3.

EXAMPLE 3.3

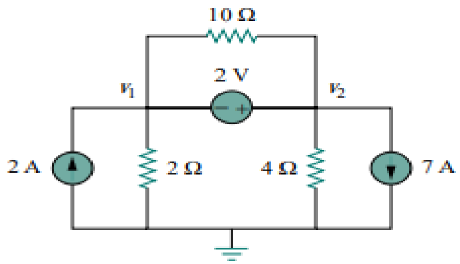


Figure 3.9 For Example 3.3.

For the circuit shown in Fig. 3.9, find the node voltages.

Solution:

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Longrightarrow \quad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \tag{3.3.1}$$

To get the relationship between v_1 and v_2, we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \quad \Longrightarrow \quad v_2 = v_1 + 2 \tag{3.3.2}$$

From Eqs. (3.3.1) and (3.3.2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Longrightarrow \quad v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

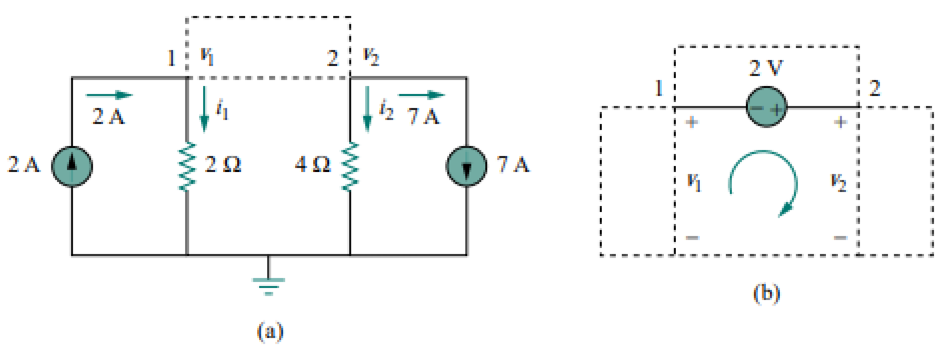


Figure 3.10 Applying: (a) KCL to the supernode, (b) KVL to the loop.

Mesh analysis

A mesh is a loop which does not contain any other loops within it.

PRACTICE PROBLEM 3.6

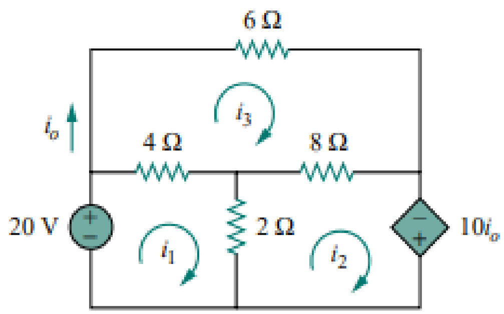


Figure 3.21 For Practice Prob. 3.6.

Using mesh analysis, find i_o in the circuit in Fig. 3.21.

Answer: -5 A.

PRACTICE PROBLEM 3.5

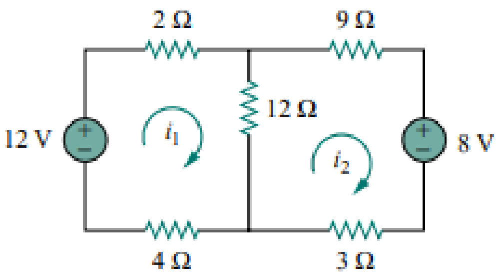


Figure 3.19 For Practice Prob. 3.5.

Calculate the mesh currents i_1 and i_2 in the circuit of Fig. 3.19.

Answer: $i_1 = \frac{2}{3}$ A, $i_2 = 0$ A.

A supermesh results when two meshes have a (dependent or independent) current source in common.

PRACTICE PROBLEM 3.7

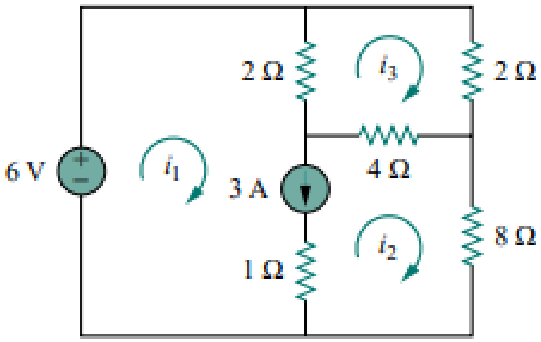


Figure 3.25 For Practice Prob. 3.7.

Use mesh analysis to determine i_1, i_2, and i_3 in Fig. 3.25.

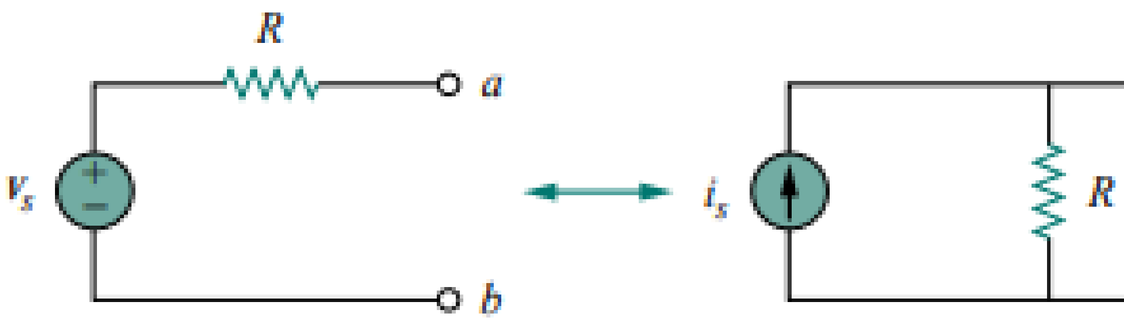
Answer: $i_1 = 3.474$ A, $i_2 = 0.4737$ A, $i_3 = 1.1052$ A.

@ Network Analysis

Circuits Theorem

1. Source transformation

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



PRACTICE PROBLEM 4.6

Find i_o in the circuit of Fig. 4.19 using source transformation.

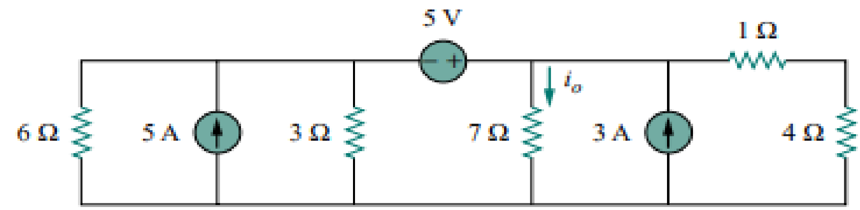


Figure 4.19 For Practice Prob. 4.6.

Answer: 1.78 A.

EXAMPLE 4.6

Use source transformation to find v_o in the circuit in Fig. 4.17.

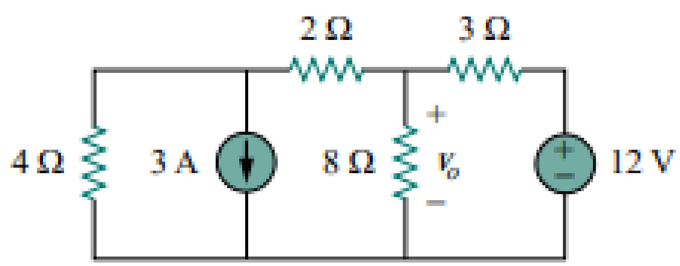


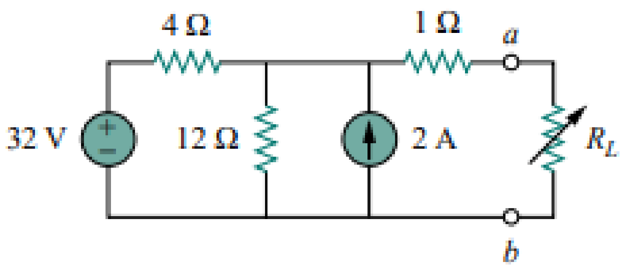
Figure 4.17 For Example 4.6.

2. Thevenin's theorem

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

EXAMPLE 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27,



PRACTICE PROBLEM 4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in Fig. 4.30. Then find i.

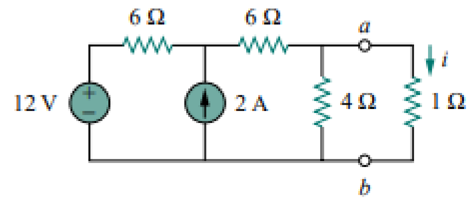


Figure 4.30 For Practice Prob. 4.8.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, $i = 1.5$ A.

PRACTICE PROBLEM 4.9

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Answer: $V_{Th} = 5.33$ V, $R_{Th} = 0.44\ \Omega$.

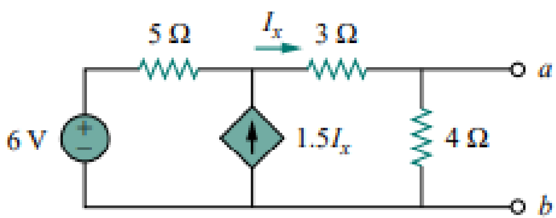


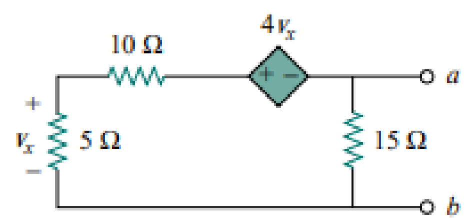
Figure 4.34 For Practice Prob. 4.9.

PRACTICE PROBLEM 4.10

Network Analysis

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.



3. Norton's theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

PRACTICE PROBLEM 4.11

Find the Norton equivalent circuit for the circuit in Fig. 4.42.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

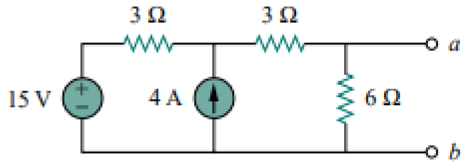
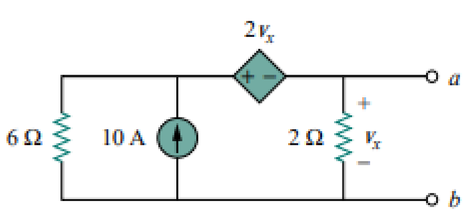


Figure 4.42 For Practice Prob. 4.11.

PRACTICE PROBLEM 4.12

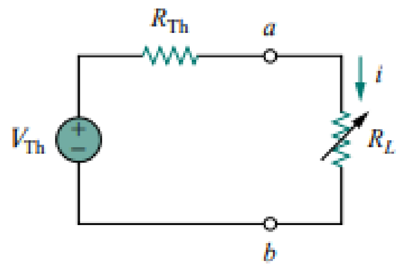


Find the Norton equivalent circuit of the circuit in Fig. 4.45.

Answer: $R_N = 1\ \Omega$, $I_N = 10$ A.

4. Maximum Power Transfer Theorem

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).



The power delivered to the load is,

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

EQUATION 1

To prove the maximum power transfer theorem, we differentiate ‘p’ with respect to RL and set the result equal to zero. We obtain,

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$$0 = (R_{\mathrm{Th}} + R_L - 2R_L) = (R_{\mathrm{Th}} - R_L)$$

which yields

$$\boxed{R_L = R_{\mathrm{Th}}}$$

showing that the maximum power transfer takes place when the load resistance RL equals the Thevenin resistance RTh.

The maximum power transferred is obtained,

$$\boxed{p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}}$$

Putting RL=RTH in equation 1

EXAMPLE 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

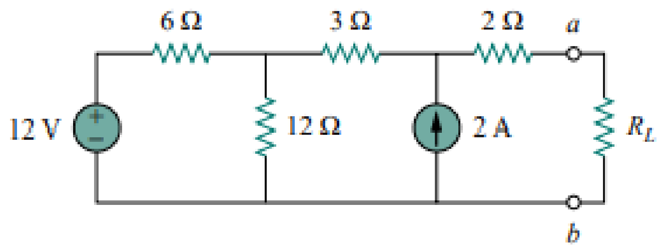
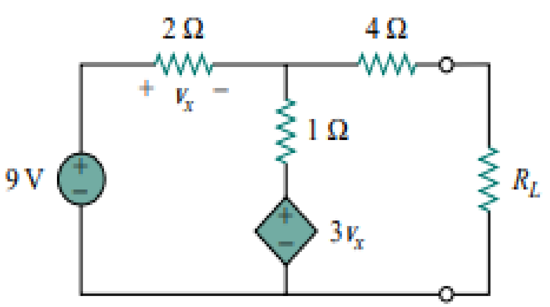


Figure 4.50 For Example 4.13.

PRACTICE PROBLEM 4.13



Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.22 Ω, 2.901 W.

5. Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

PRACTICE PROBLEM 4.3

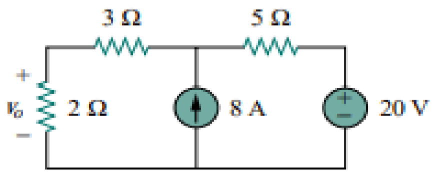


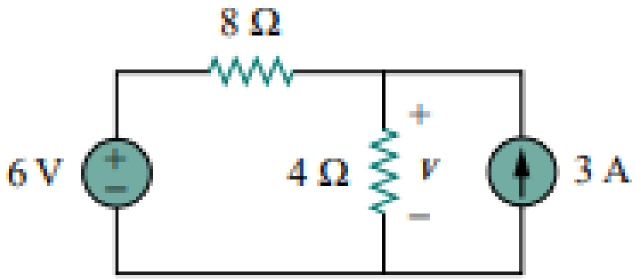
Figure 4.8 For Practice Prob. 4.3.

Using the superposition theorem, find v_o in the circuit in Fig. 4.8.

Answer: 12 V.

EXAMPLE 4.3

Use the superposition theorem to find v in the circuit



AC
Circuits

A man is like a function whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator the smaller the fraction.

—I. N. Tolstroy

Complex Numbers and Phasors

$z = x + jy \quad j = \sqrt{-1}$ RECTANGULAR FORM

$x = r\cos\theta \qquad y = r\sin\theta$

$z = r\angle\phi$ POLAR FORM

$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\dfrac{y}{x}$

$z = re^{j\phi}$ EXPONENTIAL FORM

$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\dfrac{y}{x}$

$z = x + jy = r\angle\phi = re^{j\phi}$

$e^{j\phi} = \cos\phi + j\sin\phi$ Euler's Identity

$\cos\phi = \mathrm{Re}\{e^{j\phi}\}$ Real part

$\sin\phi = \mathrm{Im}\{e^{j\phi}\}$ Imaginary part

We can convert COMPLEX numbers from one form to the other form.

Mathematical Operations of Complex Numbers

ADDITION: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

SUBTRACTION: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

MULTIPLICATION: $z_1 z_2 = r_1 \; r_2 \angle \phi_1 + \phi_2$

DIVISION: $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$

RECIPROCAL: $\dfrac{1}{z} = \dfrac{1}{r} \angle - \phi$

SQUARE ROOT: $\sqrt{z} = \sqrt{r} \angle \phi / 2$

COMPLEX CONJUGATE: $z^* = x - jy = \sqrt{r} \angle - \phi = re^{-j\phi}$

SINUSOIDS

Consider the sinusoidal voltage

$$v(t) = V_m \sin \omega t$$

where

$$V_m = \text{the } \textit{amplitude} \text{ of the sinusoid}$$
$$\omega = \text{the } \textit{angular frequency} \text{ in radians/s}$$
$$\omega t = \text{the } \textit{argument} \text{ of the sinusoid}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

T is called the period of the sinusoid

The reciprocal of the period of the sinusoid is the number of cycles per second, known as the cyclic frequency, f of the sinusoid.

$$\boxed{f = \frac{1}{T}}$$

$$\omega = 2\pi f$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

EXAMPLE 9.1

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

PRACTICE PROBLEM 9.1

Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Answer: 5, -60°, 12.57 rad/s, 0.5 s, 2 Hz.

leading and lagging

To find leading and lagging, firstly we need to convert a wave into **+ Sine** wave

EXAMPLE 9.2

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution:

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\sin(\omega t + 50^\circ - 90^\circ)$$
$$= 10\sin(\omega t - 40^\circ) = 10\sin(\omega t - 10^\circ - 30^\circ)$$

But $v_2 = 12\sin(\omega t - 10^\circ)$. Comparing the two shows that v_1 lags v_2 by 30°. This is the same as saying that v_2 leads v_1 by 30°.

PRACTICE PROBLEM 9.2

Find the phase angle between

$$i_1 = -4\sin(377t + 25^\circ) \quad \text{and} \quad i_2 = 5\cos(377t - 40^\circ)$$

Does i_1 lead or lag i_2?

Answer: 155°, i_1 leads i_2.

Phasors

- Amplitude and phase difference are two principal concerns in the study of voltage and current sinusoids.

- Phasor will be defined from the cosine function in all our proceeding study. If a voltage or current expression is in the form of a sine, it will be changed to a cosine by subtracting from the phase.

- **Example**
- Transform the following sinusoids to phasors:
  - $i = 6\cos(50t - 40^{o})$ A
  - $v = -4\sin(30t + 50_{o})$ V

Firstly we need to convert a wave into **+ cosine** wave

Solution:

a. $I = 6\angle -40^{\circ}$ A

b. Since $-\sin(A) = \cos(A+90^{o})$;

$v(t) = 4\cos\,(30t+50^{o}+90^{o}) = 4\cos(30t+140^{o})$ V

Transform to phasor => $V = 4\angle 140^{\circ}$ V

Practice Problem 9.6 Find $v = v_1 + v_2$

$$v_1 = -10\sin(\omega t + 30°) \qquad v_2 = 20\cos(\omega t - 45°)$$

Let $v = -10\sin(\omega t + 30°) + 20\cos(\omega t - 45°)$

Then, $v = 10\cos(\omega t + 30° + 90°) + 20\cos(\omega t - 45°)$

Taking the phasor of each term

$$\mathbf{V} = 10\angle 120° + 20\angle -45°$$

$$\mathbf{V} = -5 + j8.66 + 14.14 - j14.14$$

$$\mathbf{V} = 9.14 - j5.48 = 10.66\angle -30.95°$$

Converting **V** to the time domain

$$v(t) = \underline{\mathbf{10.66\cos(\omega t - 30.95°)V}}$$

PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Element	Impedance
R	$\mathbf{Z} = R$
L	$\mathbf{Z} = j\omega L$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$

$$\mathbf{Z} = R + jX$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistance* and $X = \text{Im}\,\mathbf{Z}$ is the *reactance*.

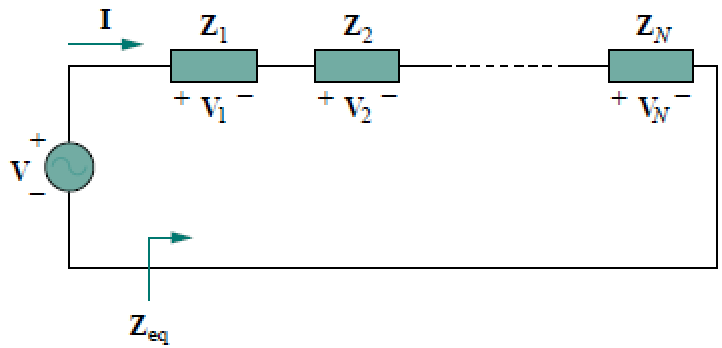
The Admitance, Y of a circuit is the reciprocal of impedance measured in Simens (S).

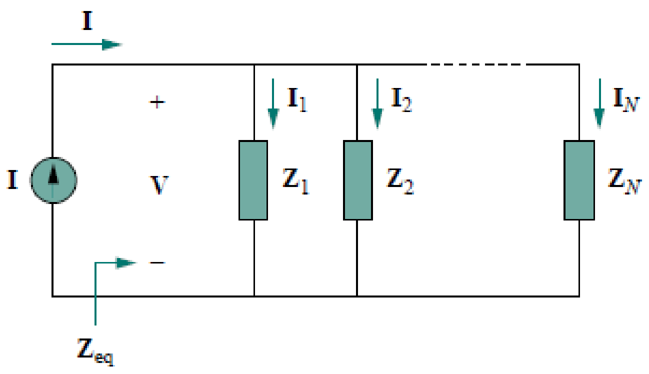
$$\boxed{\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}}$$

$$\boxed{\mathbf{Y} = G + jB}$$

where $G = \text{Re}\,\mathbf{Y}$ is called the *conductance* and $B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos).

IMPEDANC E COMBINATIONS

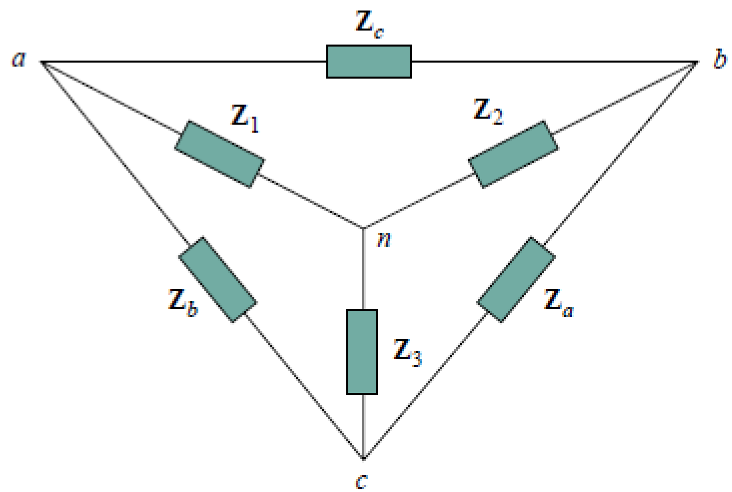




$$\boxed{\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N}$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

Delta-Y Conversion



Y-Δ *Conversion:*

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-*Y Conversion:*

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

EXAMPLE 9.10

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

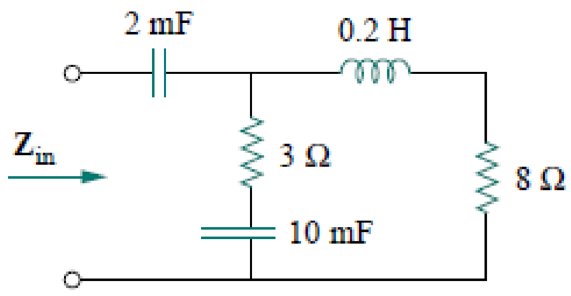


Figure 9.23

Solution:

Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the 10-mF capacitor

$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega$$

EXAMPLE 9.11

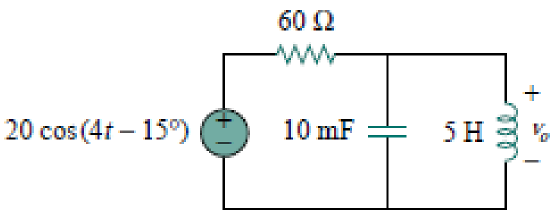


Figure 9.25 For Example 9.11.

Determine $v_o(t)$ in the circuit in Fig. 9.25.

Solution:

To do the analysis in the frequency domain, we must first transform the time-domain circuit in Fig. 9.25 to the phasor-domain equivalent in Fig. 9.26. The transformation produces

$$v_s = 20\cos(4t - 15^\circ) \quad \Longrightarrow \quad \mathbf{V}_s = 20\underline{/-15^\circ}\ \text{V}, \qquad \omega = 4$$

$$10\ \text{mF} \quad \Longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5\ \text{H} \quad \Longrightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

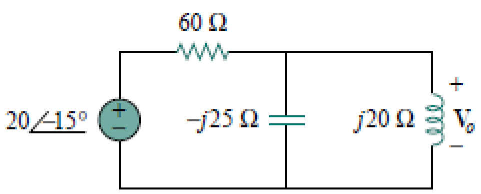


Figure 9.26 The frequency-domain equivalent of the circuit in Fig. 9.25.

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

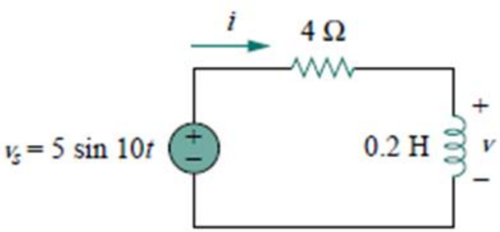
By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\underline{/-15^\circ})$$
$$= (0.8575\underline{/30.96^\circ})(20\underline{/-15^\circ}) = 17.15\underline{/15.96^\circ}\ \text{V}.$$

We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\text{V}$$

PRACTICE PROBLEM 9.9



Refer to Fig. 9.17. Determine $v(t)$ and $i(t)$.

Answer: $2.236 \sin(10t + 63.43°)$ V, $1.118 \sin(10t - 26.57°)$ A.

PRACTICE PROBLEM 9.10

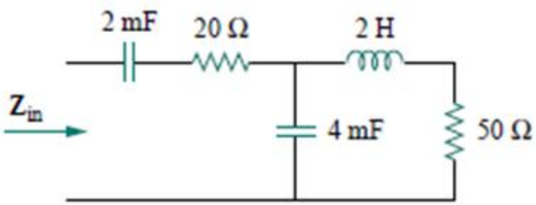


Figure 9.24 For Practice Prob. 9.10.

Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 10$ rad/s.

Answer: $32.38 - j73.76\ \Omega$.

EXAMPLE 9.12

Find current **I** in the circuit in Fig. 9.28.

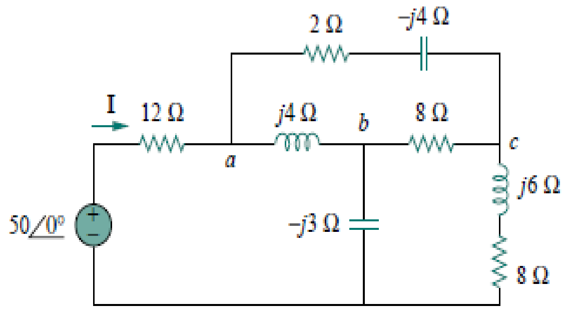


Figure 9.28 For Example 9.12.

Solution:

The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.29. We obtain the Y impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6+j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\ &= 13.6 + j0.8 + \frac{j0.2(9.6+j2.8)}{9.6+j3}\\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$

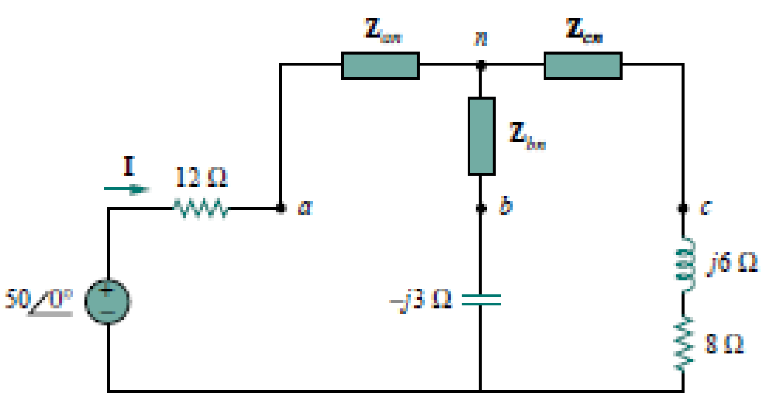


Figure 9.29 The circuit in Fig. 9.28 after delta-to-wye transformation.

EXAMPLE 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

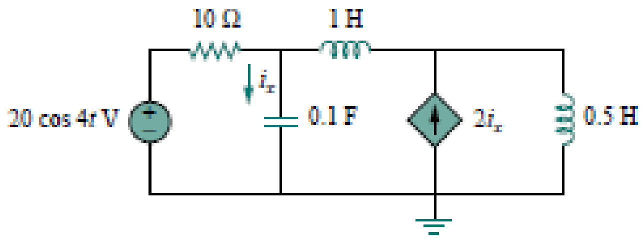


Figure 10.1 For Example 10.1.

$$20\cos 4t \implies 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$
$$1\text{ H} \implies j\omega L = j4$$
$$0.5\text{ H} \implies j\omega L = j2$$
$$0.1\text{ F} \implies \frac{1}{j\omega C} = -j2.5$$

Thus, the frequency-domain equivalent circuit is as shown in Fig. 10.2.

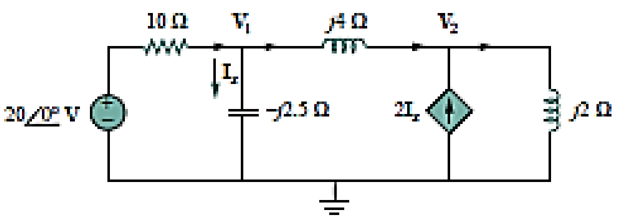


Figure 10.2 Frequency-domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \qquad (10.1.1)$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad (10.1.2)$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

EXAMPLE 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

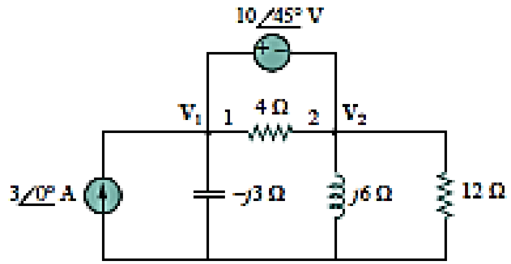


Figure 10.4 For Example 10.2.

Solution:

Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \tag{10.2.1}$$

But a voltage source is connected between nodes 1 and 2, so that

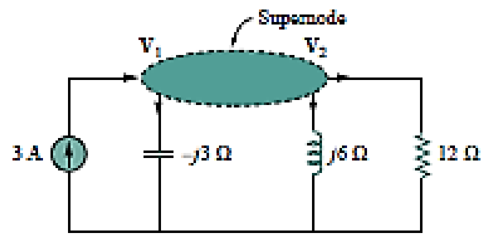


Figure 10.5 A supernode in the circuit of Fig. 10.4.

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} \tag{10.2.2}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135^\circ} = (1 + j2)\mathbf{V}_2 \quad \Longrightarrow \quad \mathbf{V}_2 = 31.41\underline{/-87.18^\circ}\text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} = 25.78\underline{/-70.48^\circ}\text{ V}$$

EXAMPLE 10.3

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

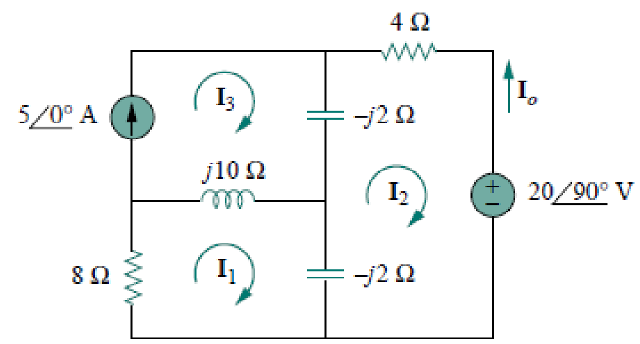


Figure 10.7 For Example 10.3.

Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad (10.3.1)$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0 \qquad (10.3.2)$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad (10.3.3)$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad (10.3.4)$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ}\text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ}\text{ A}$$

SUPERPOSITION THEOREM

The theorem becomes important, if the circuit has sources operating at different frequencies.

EXAMPLE 10.5

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit in Fig. 10.7.

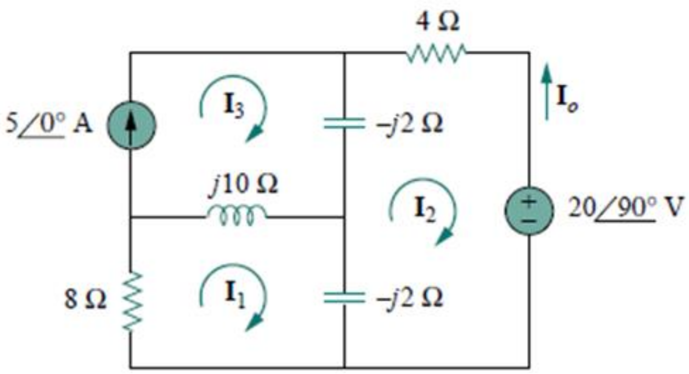


Figure 10.7 For Example 10.3.

Solution:

Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \tag{10.5.1}$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8+j10)}{-2j+8+j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{10.5.2}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{10.5.3}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{10.5.4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{10.5.5}$$

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \tag{10.5.6}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

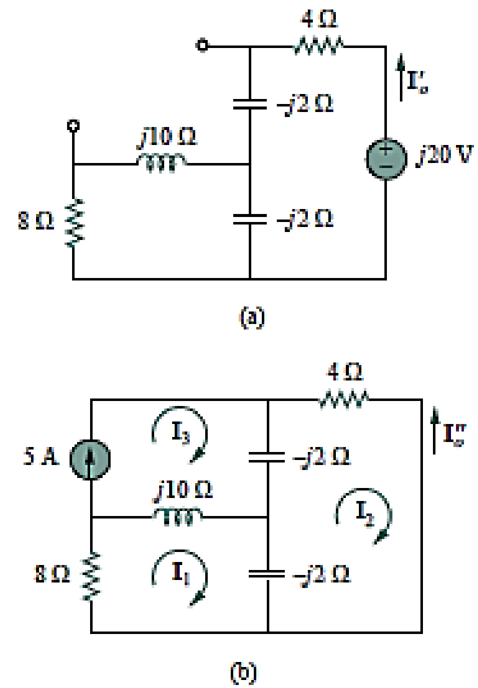


Figure 10.12 Solution of Example 10.5.

Current $\mathbf{I}''_o$ is obtained as

$$\mathbf{I}''_o = -\mathbf{I}_2 = -2.647 + j1.176 \tag{10.5.7}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529 = 6.12\underline{/144.78^\circ} \text{ A}$$

THEVENIN AND NORTON EQUIVALENT CIRCUITS

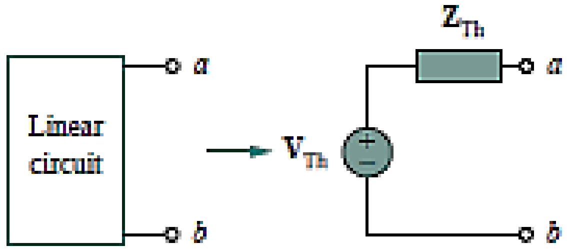


Figure 10.20 Thevenin equivalent.

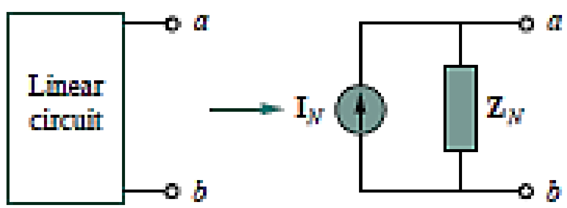


Figure 10.21 Norton equivalent.

$$\boxed{V_{Th} = Z_N I_N, \qquad Z_{Th} = Z_N}$$

PRACTICE PROBLEM 10.8

Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.24.

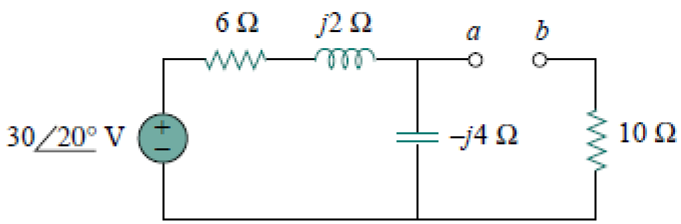


Figure 10.24 For Practice Prob. 10.8.

Answer: $\mathbf{Z}_{\mathrm{Th}} = 12.4 - j3.2\ \Omega,\ \mathbf{V}_{\mathrm{Th}} = 18.97\angle -51.57^\circ$ V.

AC POWER ANALYSIS

MAXIMUM AVERAGE POWER TRANSFER

$$R_L = R_{\text{Th}};$$

$$X_L = -X_{\text{Th}}$$

$$\boxed{\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*}$$

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.

This result is known as the *maximum average power transfer theorem* for the sinusoidal steady state. Setting $R_L = R_{\text{Th}}$ and $X_L = -X_{\text{Th}}$ in Eq. (11.15) gives us the maximum average power as

$$\boxed{P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}} \qquad (11.20)$$

EXAMPLE 11.5

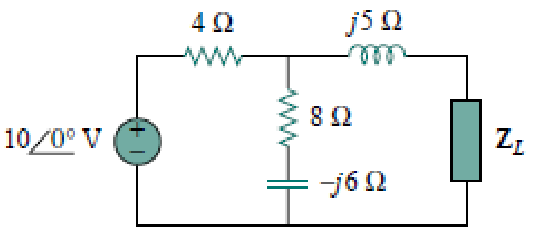


Figure 11.8 For Example 11.5.

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?

Solution:

First we obtain the Thevenin equivalent at the load terminals. To get $\mathbf{Z}_{Th}$, consider the circuit shown in Fig. 11.9(a). We find

$$\mathbf{Z}_{Th} = j5 + 4 \parallel (8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467\ \Omega$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. 11.8(b). By voltage division,

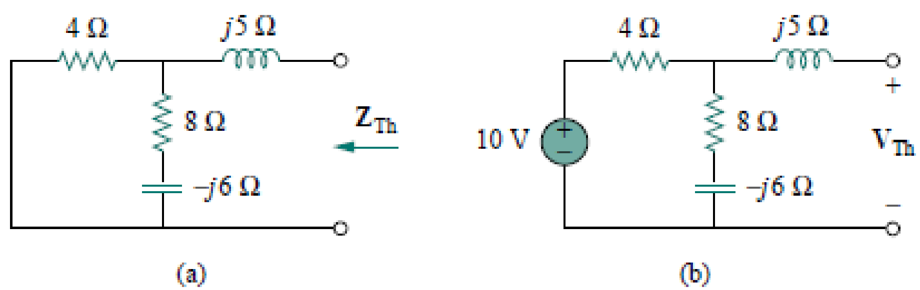
$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454\underline{/-10.3^\circ}\ \text{V}$$

The load impedance draws the maximum power from the circuit when

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 2.933 - j4.467\ \Omega$$

According to Eq. (11.20), the maximum average power is

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\ \text{W}$$



EFFECTIVE OR RMS VALUE

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

$$\boxed{X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Average power OR Real power: **(which is measured in watts, W)**

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Apparent power: **(It is measured in volt-amperes or, VA)**

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}}$$

Reactive power: **(The unit of Q is the volt-ampere reactive, VAR)**

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

Complex Power: **(It is measured in volt-amperes or, VA)**

$$\mathbf{S} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^*$$

$$\mathbf{S} = P + jQ$$

Power Factor

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

The power factor is dimensionless, since it is the ratio of the real power to the apparent power.

The power factor indicates, how much real power we consume from total power supply.

EXAMPLE 11.10

Determine the power factor of the entire circuit of Fig. 11.18 as seen by the source. Calculate the average power delivered by the source.

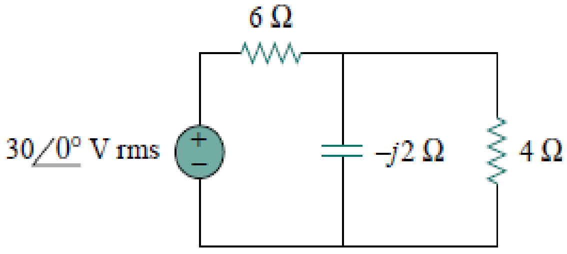


Figure 11.18 For Example 11.10.

Solution:

The total impedance is

$$\mathbf{Z} = 6 + 4 \parallel (-j2) = 6 + \frac{-j2 \times 4}{4 - j2} = 6.8 - j1.6 = 7\angle -13.24^\circ\ \Omega$$

The power factor is

$$\text{pf} = \cos(-13.24) = 0.9734 \qquad \text{(leading)}$$

since the impedance is capacitive. The rms value of the current is

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\angle 0^\circ}{7\angle -13.24^\circ} = 4.286\angle 13.24^\circ\ \text{A}$$

The average power supplied by the source is

$$P = V_{\text{rms}} I_{\text{rms}}\ \text{pf} = (30)(4.286)0.9734 = 125\ \text{W}$$

or

$$P = I_{\text{rms}}^2 R = (4.286)^2(6.8) = 125\ \text{W}$$

where R is the resistive part of $\mathbf{Z}$.

EXAMPLE 11.11

The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Solution:

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\underline{/-10^\circ}, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\underline{/+50^\circ}$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \left(\frac{60}{\sqrt{2}}\underline{/-10^\circ}\right)\left(\frac{1.5}{\sqrt{2}}\underline{/-50^\circ}\right) = 45\underline{/-60^\circ}\text{ VA}$$

The apparent power is

$$S = |\mathbf{S}| = 45\text{ VA}$$

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\underline{/-60^\circ} = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is

$$P = 22.5\text{ W}$$

while the reactive power is

$$Q = -38.97\text{ VAR}$$

(c) The power factor is

$$\text{pf} = \cos(-60^\circ) = 0.5\text{ (leading)}$$

It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\underline{/-10^\circ}}{1.5\underline{/+50^\circ}} = 40\underline{/-60^\circ}\ \Omega$$

which is a capacitive impedance.

11.38 For the entire circuit in Fig. 11.63, calculate:

(a) the power factor

(b) the average power delivered by the source

(c) the reactive power

(d) the apparent power

(e) the complex power

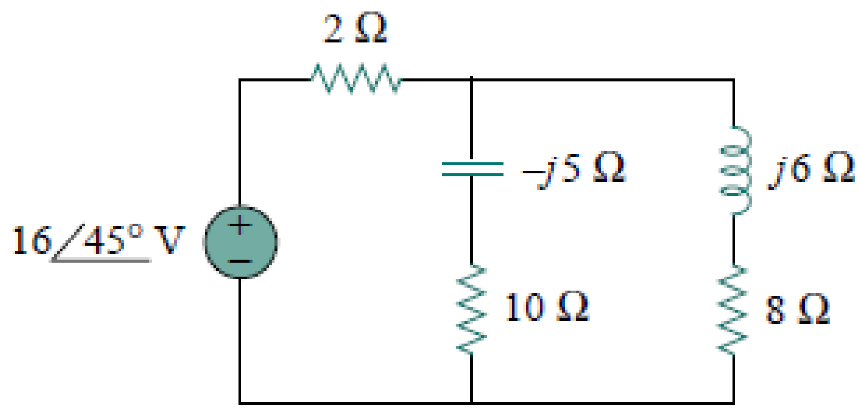


Figure 11.63 For Prob. 11.38.

Thank you

If A is success in life, then A equals X plus Y plus Z. Work is X, Y is play and Z is keeping your mouth shut.

—Albert Einstein