01 Function

Definition

Function:

A relation or expression involving one or more variables. is called function.

" the function $(bx+c)$"

Domain:

The domain of a function is the set of inputs accepted by the function.

Range:

The difference between the highest and the lowest values in a set.

Inverse function:

The inverse function of a function is a function that undoes the operation of f. The inverse of f exists if and only f is bijective, and if it exists, is denoted by f^{-1}.

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[***]

Odd function: A function f is odd if the following equation holds for all x and $-x$ in the domain of f:

$-f(x) = f(-x)$ - $f(x) = f(-x)$

it's graph remains unchanged after a rotation of $180°$ about the origin.

Even function:
[***]

A function is called an even function if its graph is unchanged under reflection in the y-axis.

Suppose $f(x)$ is a function such that is said to be an even function if $f(-x)$ is equal to $f(x)$

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2.

Limit

[***]

(δ-ϵ) definition of limit:

If for each number $\epsilon > 0$, there corresponds a small positive number δ such that $|f(x) - l| < \epsilon$, when $0 < |x - a| < \delta$, then l is called the limit of the function $f(x)$.

It is written as, $\lim\limits_{x \to a} f(x) = l$

[***]

By (δ-ϵ) definition prove:

$\circledast$ $\lim\limits_{x \to 2} \dfrac{2x^2 - 8}{x - 2} = 8$ find the value of δ when $\epsilon = 1$

Soln:

Let, $f(x) = \dfrac{2x^2 - 8}{x - 2}$; $l = 8$, $a = 2$

$0 < |x - 2| < \delta$

Now, since $x \to 2$, so, $|x - 2| < \delta$ where δ is smaller than any known positive number.

[P.t.o →]

Now, $|f(x)-8|$

$= \left|\frac{2x^2-8}{x-2}-8\right|$

$= \left|\frac{2(x+2)\cancel{(x-2)}}{\cancel{x-2}}-8\right|$

$= |2x+4-8|$

$= |2x-4|$

$= 2|x-2| < 2\delta$

If we take, $2\delta = \epsilon$

then, $|f(x)-8| < \delta$

therefore, $\lim_{x\to 2} f(x) = 8$

When, $\epsilon = 1$, then $\delta = \frac{1}{2}$

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1. $\lim_{x \to 4} (2x-2) = 6$

if, $0 < |x-a| < \delta = |x-4| < \delta$

and, $|f(x) - l| < \epsilon$

Now, $|f(x) - l|$

$= |2x-2-6|$

$= |2x-8|$

$= 2|x-4| < 2\delta$

$2\delta = \epsilon$

(ii) $\lim_{x \to 3} (2x^3 - 3x^2 - 18x + 29) = 2$

if, $0 > |x - a| < \delta$

$\Rightarrow |x - 3| < \delta$

and, $|f(x) - l| < \epsilon$

Now, $|f(x) - l|$

$= |2x^3 - 3x^2 - 18x + 29 - 2|$

$= |2x^3 - 18x - 3x^2 - 27|$

$= |2x(x^2 - 9) - 3x(x^2 - 9)|$

$= |(2x - 3)(x^2 - 9)|$

$= |(2x - 3)(x + 3)(x - 3)|$

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(iii) $\lim_{x\to -2} \frac{x^2-4}{x+2} = -4$

if $0 > |x-a| < \delta$

$\Rightarrow |x-(-2)| < \delta$

$\Rightarrow |x+2| < \delta$

and, $|f(x) - l| < \epsilon$

Now, $|f(x) - l|$

$= \left|\frac{x^2-4}{x+2} + 4\right|$

$= \left|\frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} + 4\right|$

$\cancel{= |4x-2|} = |x+2|$

$\cancel{= 2}$

Definition of Left-Hand Limit and R-H Limit:

⇒ A left-hand limit means the limit of a function as it aproches from the left-hand side. On the other hand, A right-hand limit means the limit of a function as it aproches from the right hand side.

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i. $\lim_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$

$= \lim_{x \to 0} \frac{x}{\sin x} \lim_{x \to 0} x \sin \frac{1}{x}$

$= 1 \lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$

$= 1 \cdot 0 \sin\left(\frac{1}{x}\right)$

$= 0$

since, $\sin \frac{1}{x}$ is bounded, ie, $-1 \le \sin \frac{1}{x} \le 1$

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(ii) $\lim_{x\to\pi} \frac{\sin x}{\pi - x}$

$= \lim_{x\to\pi} \frac{\sin(\pi - x)}{(\pi - x)}$

$= \lim_{x\to\pi} \frac{\sin z}{z}$

$= 1$

$\sin(\pi-\theta) = \sin\theta$

Let, $\pi - x = z$

so as, $x\to\pi$, $z \to 0$

(iii) $\lim_{x\to\pi/4} (\sec 2x - \tan 2x)$

$= \lim_{x\to\pi/4} \left(\frac{1-\sin 2x}{\cos 2x}\right)$

$= \lim_{x\to\pi/4} \left\{\frac{(\cos x - \sin x)^2}{\cos^2 x - \sin^2 x}\right\}$

$= \lim_{x\to\pi/4} \frac{\cos x - \sin x}{\cos x + \sin x}$

$= 0$

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i. $\lim_{x\to 0} \dfrac{1-\cos x}{x}$

$= \lim_{x\to 0} \dfrac{2\sin^2 \frac{1}{2}x}{x}$

$= \lim_{x\to 0} \left(\dfrac{\sin \frac{1}{2}x}{\frac{1}{2}x}\right)^2 \times \frac{1}{2}x$

$= 1 \times 0 = 0$

ii. $\lim_{x\to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)$

$= \lim_{x\to 0} \left(\dfrac{1-\cos x}{\sin x}\right)$

$= \lim_{x\to 0} \dfrac{2\sin^2 \frac{1}{2}x}{2\sin \frac{1}{2}x . \cos \frac{1}{2}x}$

$= \lim_{x\to 0} \tan \frac{1}{2}x$

$= 0$

3.

4. Continuity

⊛ Definition of continuous function:

*** A function $f(x)$ is commonly said to be continuous provided its graph is a continuos curve. for the corresponding value of x.

*** ii. The function f is defined as follows:

$$f(x) = \begin{cases} -x & \text{when } x \le 0 \\ x & \text{when } 0 < x < 1 \\ 2-x & \text{when } x \ge 1 \end{cases}$$

Show that it is continuous at $x=0$ and $x=1$.

Soln:

for $x=0$

functional value $f(x) = -x = -0 = 0$

$$L.H.L. = \lim_{x \to 0^-} f(x)$$

$$= \lim_{h \to 0} f(0-h)$$

$$= \lim_{h \to 0} -0-0-0 = 0$$

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$$R.H.L. = \lim_{x\to 0^+} f(x)$$

$$= \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} 0+0+0$$

$$= 0$$

Again, at $x = 1$

functional value $f(x) = 2-x = 2-1 = 1$

$$L.H.L. = \lim_{x\to 1^+} f(x)$$

$$= \lim_{h\to 0} (1-h)$$

$$= \lim_{h\to 0} 1-0 = 1$$

$$R.H.L. = \lim_{x\to 1^+} f(x)$$

$$= \lim_{h\to 0} (1+h)$$

$$= \lim_{h\to 0} 2-1+0 = 1$$

We can see that, for both point $x=0$ and $x=1$,

$L.H.L. = R.H.L. =$ functional value

So, $f(x)$ is continuous at $x=0$ and $x=1$ (Showed.)

Q4. Differentiability

1. Define derivative of a function

⇒ The derivative of a function is defined as the instantaneous rate of change of a function at a specific point. The derivative gives the exact slope along the curve at a specific point. The derivative of the function is represented as $\frac{dy}{dx}$, which means the derivative of the function with respect to the variable.

2. Define differentiatable function:

⇒ A differentiatable function of one real variable is a function whose derivative exists at each point in its domain. In other words, the graph of a differentiable function has a non-vertical tangent line at each interior point in its domain.

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3. Define left-hand derivative and right-hand-derivative.

⇒ When we approach a point from the left of the number line, it's called left hand derivative when we approach from the right, it's a right hand derivative. These definitions are exactly the same concept as one sided limits limits from the left and limits from the right.

05. Differention

(first principle) $\boxed{***}$

(i) $\sin x$

$\Rightarrow$ By definition,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\frac{x+h+x}{2}\sin\frac{x+h-x}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cos\frac{2x+h}{2}\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \lim_{h\to 0} \cos\frac{2x+h}{2} \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{\cos 2x}{2}$$

$$= \cos x$$

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with respet to x

xii. $\frac{d}{dx}\left(\tan \ln \sin e^{x^2}\right)$ $\boxed{\star\star\star}$

$= ~~\sec^2 \tan \ln \sin~~$

$= \sec^2 \ln \sin e^{x^2} \frac{d}{dx}\left(\ln \sin e^{x^2}\right)$

$= \sec^2 \ln \sin e^{x^2} \frac{1}{\sin e^{x^2}} \frac{d}{dx}\left(\sin e^{x^2}\right)$

$= \sec^2 \ln \sin e^{x^2} \cdot \frac{1}{\sin e^{x^2}} \cdot \cos e^{x^2} \cdot \frac{d}{dx}\left(e^{x^2}\right)$

$= \sec^2 \ln \sin e^{x^2} \cdot \frac{1}{\sin e^{x^2}} \cdot \cos e^{x^2} \cdot e^{x^2} \cdot \frac{d}{dx}\left(x^2\right)$

$= \sec^2 \ln \sin e^{x^2} \frac{1}{\sin e^{x^2}} \cdot \cos e^{x^2} \cdot e^{x^2} \cdot 2x$

$= 2x \cdot e^{x^2} \cot e^{x^2} \left(\sec^2 \ln \sin e^{x^2}\right)$

Find $\dfrac{dy}{dx}$ ***

vi. $x^y y^x = 1$

$$\Rightarrow \ln(x^y y^x) = \ln 1 = 0$$

$$\Rightarrow \ln x^y + \ln y^x = 0$$

$$\Rightarrow y \ln x + x \ln y = 0$$

$$\Rightarrow y \frac{d}{dx} \ln x + \ln x \frac{d}{dx} y + x \frac{d}{dx} \ln y + \ln y \frac{d}{dx} x = 0$$

$$\Rightarrow y \frac{1}{x} + \ln x \frac{dy}{dx} + x \frac{1}{y} \cdot \frac{dy}{dx} + \ln y = 0$$

$$\Rightarrow \frac{y}{x} + \ln x \frac{dy}{dx} + \frac{x}{y} \frac{dy}{dx} + \ln y = 0$$

$$\Rightarrow \frac{dy}{dx}\left(\ln x + \frac{x}{y}\right) + \frac{y}{x} + \ln y = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\ln y - \frac{y}{x}}{\ln x + \frac{x}{y}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{-x\ln y - y}{x}}{\frac{y \ln x + x}{y}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x \ln y - y}{x} \times \frac{y}{y \ln x + x}$$

$$= \frac{-y(x \ln y + y)}{x(y \ln x + x)}$$

iv. $\frac{d^n}{dx^n} x^n$ $\boxed{***}$

$\cancel{\frac{d}{dx}(x) = 1}$

when

$n=1$; $\frac{d}{dx} x = 1$

$n=2$; $\frac{d^2}{dx^2} = \frac{d}{dx}\left(\frac{d}{dx} x^2\right)$

$= \frac{d}{dx}(2x) = 2.1$

$n=3$; $\frac{d^3}{dx^3} = \frac{d}{dx}\frac{d}{dx}\frac{d}{dx}(x^3)$

$= \frac{d}{dx}\frac{d}{dx}(3x^2) = 3.2.1$

$n=4$; $\frac{d^4}{dx^4} = 4.3.2.1$

$\therefore \frac{d^n}{dx^n} = n!$

n_x

iii. If $y = (\sin^{-1}x)^2$, show that

a. $(1-x^2)y_2 - xy_1 - 2 = 0$

b. $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$

Soln:

a. Given, $y = (\sin^{-1}x)^2$ ——— (i)

Differentiating (i) with respect to x

$$y_1 = 2\sin^{-1}x \cdot \frac{d}{dx}(\sin^{-1}x) = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1(\sqrt{1-x^2}) = 2\sin^{-1}x$$

$$\Rightarrow (1-x^2)(y_1)^2 = 4(\sin^{-1}x)^2 = 4y \text{ ——— (ii)}$$

Differentiating (ii) with respect to x

$$\Rightarrow (1-x^2)\,2y_1 y_2 + (y_1)^2(-2x) = 4y_1$$

$$\Rightarrow (1-x^2)y_2 - xy_1 = 2$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - 2 = 0 \text{ ——— (iii)}$$

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03. Pair of straight line

(i) Solution:

Let the equation be,

$$ax^2+2hxy+by^2=0 \quad \text{——— (i)}$$

Dividing (i) by bx^2 (if $b\neq 0$), we get,

$$\Rightarrow \frac{a}{b}+2\frac{h}{b}\frac{y}{x}+\left(\frac{y}{x}\right)^2=0$$

Let, m_1 and m_2 be two roots of this quadratic equation in $\left(\frac{y}{x}\right)$

Then, $\left(\frac{y}{x}-m_1\right)\left(\frac{y}{x}-m_2\right)=0$

Therefore, $\frac{y}{x}=m_1$ or, $\frac{y}{x}=m_2$

$$\Rightarrow y=m_1x \quad \text{or,} \quad y=m_2x$$

Thus equation (i) represents a pair of straight lines through the origin.

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Now form,

$$11\left(\frac{3x}{\sqrt{10}}-\frac{y}{\sqrt{10}}\right)^2+3\left(\frac{3x}{\sqrt{10}}-\frac{y}{\sqrt{10}}\right)\left(\frac{x}{\sqrt{10}}+\frac{3y}{\sqrt{10}}\right)+7\left(\frac{x}{\sqrt{10}}+\frac{3y}{\sqrt{10}}\right)^2+19$$

$$\Rightarrow 11(3x-y)^2+3(3x-y)(x+3y)+7(x+3y)^2+190=0$$

$$\Rightarrow 11(9x^2-6xy+y^2)+3(3x^2-xy+9xy-3y^2)+7(x^2+6xy+9y^2)+19$$

$$\Rightarrow 99x^2-\cancel{66xy}+11y^2+9x^2-\cancel{24xy}-9y^2+7x^2+\cancel{42xy}+63y^2+190=$$

$$\Rightarrow 115x^2+65y^2+190=0$$

$$\Rightarrow 23x^2+13y^2+38=0$$

which is my free equation.

$$\Rightarrow -8xy\cos\theta\sin\theta + 3xy\cos^2\theta - 3xy\sin^2\theta = 0$$

$$\Rightarrow 3xy\cos^2\theta - 8xy\cos\theta\sin\theta - 3xy\sin^2\theta = 0$$

$$\Rightarrow 3xy\cos^2\theta - 9xy\cos\theta\sin\theta + xy\sin\theta\cos\theta - 3xy\sin^2\theta = 0$$

$$\Rightarrow 3xy\cos\theta(\cos\theta - 3\sin\theta) + xy\sin\theta(\cos\theta - 3\sin\theta) = 0$$

$$\Rightarrow (3xy\cos\theta + xy\sin\theta)(\cos\theta - 3\sin\theta) = 0$$

$$\Rightarrow xy(3\cos\theta + \sin\theta)(\cos\theta - 3\sin\theta) = 0$$

$$\Rightarrow (3\cos\theta + \sin\theta)(\cos\theta - 3\sin\theta) = 0$$

$\Rightarrow 3\cos\theta + \sin\theta = 0$ or, $\cos\theta - 3\sin\theta = 0$

$\Rightarrow \sin\theta = -3\cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = -3$

$\Rightarrow \tan\theta = -3$

$\Rightarrow \theta = \tan^{-1}(-3)$

$\Rightarrow \cos\theta = 3\sin\theta$

$\Rightarrow \dfrac{\cancel{\cos\theta}\ \sin\theta}{\cancel{\sin\theta}\ \cos\theta} = \dfrac{1}{3}$

$\Rightarrow \tan\theta = \dfrac{1}{3}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{3}\right)$

$\tan\theta = \dfrac{1}{3}$

$\cos\theta = \dfrac{3}{\sqrt{10}}$, $\sin\theta = \dfrac{1}{\sqrt{10}}$

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01. Transformation of Co-ordinates

(iv) Given,

$$11x^2+3xy+7y^2+19=0$$

Let us replace x by $(x\cos\theta - y\sin\theta)$ and y by $(x\sin\theta + y\cos\theta)$ become,

$11(x\cos\theta - y\sin\theta)^2 + 3(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) +$
$7(x\sin\theta + y\cos\theta)^2 + 19 = 0$

$\Rightarrow 11(x^2\cos^2\theta - 2x\cos\theta y\sin\theta + y^2\sin^2\theta) + (3x\cos\theta - 3y\sin\theta)$
$(x\sin\theta + y\cos\theta) + 7(x^2\sin^2\theta + 2x\sin\theta y\cos\theta + y^2\cos^2\theta)$
$+19=0$

~~$\Rightarrow 11x^2\cos^2\theta - 22xy\cos\theta\sin\theta + 11y^2\sin^2\theta + (3x\cos\theta - 3y\sin\theta)$~~
~~$(x\sin\theta + y\cos\theta)$~~

$\Rightarrow 11x^2\cos^2\theta - 22xy\cos\theta\sin\theta + 11y^2\sin^2\theta + 3x^2\sin\theta\cos\theta$
$+3xy\cos^2\theta - 3y^2\cos^2\theta\sin\theta - 3xy\sin^2\theta + 7x^2\sin^2\theta + 14xy\sin\theta$
$\cos\theta + 7y^2\cos^2\theta + 19 = 0$

$\Rightarrow$ The cofficeint of xy in this expression:

$\Rightarrow -22xy\cos\theta\sin\theta + 3xy\cos^2\theta - 3xy\sin^2\theta \cancel{-3xy} = 0$

10. Tangent and Normal

(iv)

$$x = a\cos\theta$$

$$y = b\sin\theta$$

Now,

$$\frac{dx}{d\theta} = -a\sin\theta$$

$$\frac{dy}{d\theta} = b\cos\theta$$

$$\therefore \frac{dy}{dx} = -\frac{b\cos\theta}{a\sin\theta}$$

The equation of tangent is:

$$(Y-y) = \frac{dy}{dx}(X-x)$$

$$\Rightarrow Y - b\sin\theta = -\frac{b\cos\theta}{a\sin\theta}(X - a\cos\theta)$$

$$\Rightarrow aY\sin\theta - ab\sin^2\theta = -bX\cos\theta - ab\cos^2\theta$$

$$\Rightarrow aY\sin\theta + bX\cos\theta - ab = 0$$

$\Rightarrow ay\sin\theta + bx\cos\theta - ab = 0$ [Replacing X by x and Y by y]

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09. Partial differentiation

(iii) Verify Euler's theorem for the function $u(x,y) = ax^2 + 2hxy + by^2$.

Solution:

$$u = ax^2 + 2hxy + by^2 = x^2\left\{a + 2h\frac{y}{x} + b\left(\frac{y}{x}\right)^2\right\}$$

$$= x^2 f\left(\frac{y}{x}\right).$$

Hence, the given function is a homogenous function of degree 2.

Now, $xu_x + yu_y = x(2ax + 2hy) + y(2hx + 2by) = 2u$

and this shows that Euler's theorem is satisfied in this case.

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08. Maxima and Minima

ii. Given, $f(x) = x^3 - 9x^2 + 24x - 12$

Now,

$$f'(x) = 3x^2 - 18x + 24 \text{ ——— (i)}$$

$$f''(x) = 6x - 18 \text{ ——— (ii)}$$

For extrime point, $f'(x) = 0$

$$\Rightarrow 3x^2 - 18x + 24 = 0$$

$$\Rightarrow 3(x^2 - 6x + 8) = 0$$

$$\Rightarrow x^2 - 6x + 8 = 0$$

$$\Rightarrow x^2 - 4x - 2x + 8 = 0$$

$$\Rightarrow x(x-4) - 2(x-4) = 0$$

$$\Rightarrow (x-4)(x-2) = 0$$

$$\therefore x = 4, 2$$

$f''(x)$ when $x = 4$

$6x - 18 = 6 \cdot 4 - 18 = 24 - 18 = 6 > 0 \rightarrow$ minimum

$f''(x)$ when $x = 2$

$6x - 18 = 6 \cdot 2 - 18 = 12 - 18 = -6 < 0 \longrightarrow$ maximum

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(viii) $e^{xy} - 4xy = 2$, find $\frac{dy}{dx}$.

Given, $e^{xy} - 4xy = 2$

Differentiating w.r.t. x

$$\frac{d}{dx}\left(e^{xy} - 4xy\right) = \frac{d}{dx}(2)$$

$$\Rightarrow \frac{d}{dx} e^{xy} - 4\frac{d}{dx}(xy) = 0$$

$$\Rightarrow e^{xy}\frac{d}{dx}(xy) - 4\left(y\frac{d}{dx}x + x\frac{d}{dx}y\right) = 0$$

$$\Rightarrow e^{xy}\left(y\frac{d}{dx}x + x\frac{d}{dx}y\right) - 4\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow e^{xy}\left(y + x\frac{dy}{dx}\right) - 4\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow e^{xy}y + e^{xy}x\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$$

$$\Rightarrow x\frac{dy}{dx}\left(e^{xy} - 4\right) + y\left(e^{xy} - 4\right) = 0$$

$$\Rightarrow x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \frac{dy}{dx} = -y/x$$

Limits

(d) $\boxed{\cancel{\times\times\times}}$

given,

$$\lim_{x\to 0} \frac{x(\cos x+\cos 2x)}{\sin x}$$

$$= \lim_{x\to 0} (\cos x+\cos 2x)\frac{x}{\sin x}$$

$$= \lim_{x\to 0} (\cos x+\cos 2x)\frac{1}{\frac{x}{\sin x}}$$

$$= \lim_{x\to 0} (\cos x+\cos 2x) \times \frac{1}{\lim_{x\to 0} \frac{x}{\sin x}}$$

$$= (\cos 0+\cos 2.0) \times 1$$

$$= (1+1) \times 1$$

$$= 2 \quad \text{Ans.}$$

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Defferentiating (iii) n times with help of Lebnit'z theorem,

$$(1-x^2)y_{n+2} + {}^nC_1(-2x)y_{n+1} + {}^nC_2(-2)y_n + 0 \ldots\ldots + 0 -$$

$$\left[xy_{n+1} + {}^nC_1\, 1 \cdot y_n + 0 \ldots\ldots + 0\right] - 0 = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - n\,2xy_{n+1} - 2\frac{n(n-1)}{2}y_n - xy_{n+1} - ny_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n-1+1)n\,y_n = 0$$

$$\Rightarrow (1-x^2)_{n+2} - (2n+1)xy_{n+1} - (n-1+1)n\,y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

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