INTEGRAL CALCULUS

Sample Questions

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01.Indefinite integral

i. $\int \cos^4 x \sin x \; dx$

ii. $\int \frac{1}{(\cos^2 x \sin^2 x)} dx$

iii. $\int \frac{2x^2 - 1}{(1+x)^2(x-2)} dx$

iv. $\int \frac{\sin^6 x + \cos^6 x}{(\cos^2 x \sin^2 x)} dx$

v. $\int \frac{x}{(1+x)(x+2)^2} dx$

vi. $\int \frac{\ln(\ln x)}{x} \; dx$

vii. $\int \frac{dx}{(x-a)(x-b)^2}$

viii. $\int \frac{x}{1+\cos x} dx$

ix. $\int \tan^{-1} x \; dx$

x. $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

xi. $\int e^x \frac{x^2+1}{(x+1)^2} dx$

xii. $\int \frac{e^{m \tan^{-1} x}}{(1+x^2)^2} dx$

xiii. $\int e^x \frac{1+\sin x}{1+\cos x} dx$

02.Definite integral

i. State and prove the fundamental theorem of integral calculus.

ii. $$\int_0^{\pi/2} \frac{dx}{1+\cot x}$$

iii. $$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

iv. $$\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$$

v. $$\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$$

vi. $$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

vii. $$\int_0^{\pi} \frac{x}{1+\sin x}\,dx$$

viii. $$\int_0^{\pi/2} \frac{x}{\cos x+\sin x}\,dx$$

ix. $$\int_0^{\pi} x\cos^4 x\,dx$$

x. $$\int_0^{\pi} \ln(1+\cos x)\,dx$$

xi. Prove that $$\int_0^{\pi} \ln(\sin x)\,dx = \int_0^{\pi} \ln(\cos x)\,dx = \frac{\pi}{2}\ln\frac{1}{2}$$

03.Definite integral as the limit of the sum

i. Evaluate $\int_a^b \sin x \, dx$ by first principal / definition.

ii. Evaluate $\int_a^b \cos x \, dx$ by first principal / definition.

iii. Evaluate $\int_0^{\pi/2} \cos x \, dx$ by first principal / definition.

iv. Evaluate $\int_0^{\pi/2} \sin x \, dx$ by first principal / definition.

v. Evaluate $\int_a^b x^2 \, dx$ by first principal / definition.

vi. Evaluate $\int_0^2 x^2 \, dx$ by first principal / definition.

vii. Evaluate $\int_a^b e^x \, dx$ by first principal / definition.

viii. Evaluate $\int_0^2 e^x \, dx$ by first principal / definition.

ix. Find the value of $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\cdots\cdots+\frac{1}{2n}\right]$

x. Find the value of $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+\cdots\cdots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$

xi. Find the value of $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\cdots\cdots+\frac{1}{2n}\right]$

04.Successive reduction

i. Find the reduction formula for $\int \sin^n x\, dx$

ii. Find the reduction formula for $\int \cos^n x\, dx$

iii. Find the reduction formula for $\int \sec^n x\, dx$

iv. Find the reduction formula for $\int \tan^n x\, dx$

v. Find the reduction formula for $\int \cos^m x \cos nx\, dx$

vi. Find $\int \cos^5 x \cos 2x\, dx$

vii. Find the reduction formula for $\int \sin^m x \sin nx\, dx$

viii. Find the reduction formula for $\int \cos^m x \sin nx\, dx$

ix. Find the reduction formula for $\int \sin^m x\ \cos^n x\, dx$

x. Find $\int \sin^4 x\ \cos^3 x\, dx$

xi. Find the reduction formula for $\int x^n e^{ax}\, dx$

xii. Walle's formula: Prove

$$\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \begin{cases} \dfrac{(n-1)(n-3)\cdots\cdots 5\cdot 3\cdot 1}{n(n-2)\cdots\cdots 6\cdot 4\cdot 2}\dfrac{\pi}{2} & when\, n\, is\, even \\ \dfrac{(n-1)(n-3)\cdots\cdots 6\cdot 4\cdot 2}{n(n-2)\cdots\cdots 5\cdot 3\cdot 1} & when\, n\, is\, odd \end{cases}$$

xiii. Prove

$$\int_0^{\pi/2} \sin^m x \cos^n x\, dx = \int_0^{\pi/2} \cos^m x \sin^n x\, dx$$

$$= \begin{cases} \dfrac{1\cdot 3\cdot 5\cdots\cdots (m-1)\times 1\cdot 3\cdot 5\cdots\cdots (n-1)}{2\cdot 4\cdot 6\cdots\cdots (m+n)}\dfrac{\pi}{2} & when\, m\, and\, n\, are\, even \\ \dfrac{2\cdot 4\cdot 6\cdots\cdots (m-1)}{(n+1)(n+3)\cdots\cdots (n+m)} & when\, m\, is\, odd \end{cases}$$

05.Improper Integrals

06.Differentiation under the sign of integration

07.Area of plane curve

i. Find the area bounded by the curve $y = \sin x$, the axis of x and the straight lines $x = 0$ and $x = \pi$.

ii. Show that the area between the parabola $y^2 = 4x$ and the straight line $y = 2x - 4$ is 9 square unit.

iii. Find the area of the region bounded by the parabolas $y^2 = 4b(b - x)$ and $y^2 = 4a(a + x)$.

iv. Find the area above the x-axis, included between parabola $y^2 = ax$ and the circle $x^2+y^2 = 2ax$.

v. Find the area between the curve $y^2 = \frac{(a-x)^3}{a+x}$ and the asymptote.

vi. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

vii. Find the area of the loop of the curve $xy^2 + (x + a)(x + 2a) = 0$.

viii. Find the area of the cycloid

$$x = a(\theta + \sin\theta)$$
$$y = a(1 - \cos\theta)$$

bounded by its base.

ix. Find the area bounded by the cardioid $r = a(1 - \cos\theta)$.

x. Find the area of the region enclosed by the rose curve $r = \sin 2\theta$.

xi. Find the area common to the cardioid $r = a(1 + \cos\theta)$ and the circle $r = \frac{3}{2}a$.

xii. Find the area bounded by the cardioid $r = a(1 + \sin\theta)$.

xiii. Find the area bounded by the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

xiv. Find the area bounded by the hypocycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

08.Length of plane curve

i. Find the length of the semi-cubical parabola $ay^2 = x^3$ from the vertex to the point (a, a).

ii. Find the intersected arc length of the parabola $y^2 = 4ax$ and the straight line $3y = 8x$.

iii. Find the length of the curve $8y^2 = x^2 - x^4$.

iv. Find the perimeter of the hypocycloid $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

v. Find the perimeter of the circle $x^2 + y^2 = a^2$.

vi. Find the arc length of the cycloid $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ of one cycle.

vii. Find the perimeter of the cardioid $r = a(1 + \cos\theta)$.

09.Volume and surface area of solids of revolution

i. The part of the curve $y = \sin x$ form $x = 0$ to $x = \pi$ revolves about the x-asix. Find the volume and the surface area of the solid that generated.

ii. Find the volume and area of the curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$.

iii. Find the volume and the surface area of the solid generated by revolving the cycloid
$$x = a(\theta + \sin\theta)$$
$$y = a(1 + \cos\theta)$$
about its base.

iv. Find the volume and the surface area of the solid generated by revolving the cardioid
$$r = a(1 - \cos\theta)$$
about the initial line.