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Assignment 3

Exercise 3.1

$$Standard\ deviation = \sqrt{p * (1 - p)}$$

$$P = \frac{84}{100}$$

Sd = 0,367

Emax = 0.03

$$N \geq \left(\frac{1{,}96 * 0{,}366}{0{,}03} \right)^2$$

N = 573,679

We need to survey at least 574 flights to be 95% confident of an estimate within 3 percentage points of the true population.

Exercise 3.2

Hypothesis:
The claim is that first-born and second-born twins have the same mean brain volume.

Null hypothesis:
$H_0: \mu_1 = \mu_2$

Alternative hypothesis:
$H_a: \mu_1 \neq \mu_2$

Significance level:
$\alpha = 0.1$

Test statistic:
We have the sample means $\bar{x}1$: 1121.7 and $\bar{x}2$: 1130.2.

The samples are dependent and paired.
We have n = 10 pairs.
We are looking for the sample mean of differences $\bar{D}$.
$\bar{D} = (42 + 8 + 9 + -28 + -36 + -94 + 37 + 92 + -71 + -144) / 10$
$\bar{D}$: -18,5

We can now calculate the test statistic t:
The test statistic has a t-distribution with n – 1 = 9 degrees of freedom.
Observed value:
$t = (\bar{D} + (\mu_1 - \mu_2)) / (S_d / \sqrt{n})$
$t = (18{,}5 + 0) / (56{,}679 / \sqrt{10})$
$t = 1{,}032166$

Critical values:
The critical values are $-t_{9,0.05}$ and $t_{9,0.05}$, so we get -2,262 and 2,262

1,032166 < 2,262 so it is outside the critical region. We do not reject the null hypothesis.

Conclusion:
There is not enough evidence to warrant the rejection of the claim that first-born and second-born twins have the same mean brain volume.

Exercise 3.3

a)We assume the variances are not equal, since they come from different populations.

The function t.test(Alice, Bob, var.equal = FALSE) gives:

We have get confidence interval of [-0.037, 0.482]
The sample mean of Alice is 3.9616 hours, the sample mean of Bob is 3.7392 hours.
The point estimate of the μ is 3.9616 – 3.7392 = 0,2224.

The test statistic is 1.7026, in a t-distribution with 91.778 degrees of freedom.

b) We will use a two-tailed (Since the manager claims that the average hours for Alice and Bob should be the same) t-test. We assume independent samples, since the working hours do not necessarily come from the same evenings, so there is no relationship between the samples. We assume that the standard deviations are not equal.

Hypothesis:
Claim of manager: Alice and Bob work on average the same number of hours.

Null hypothesis:
$\mu_1 = \mu_2$

Alternative hypothesis:
$\mu_1 \neq \mu_2$

Significance level:
$\alpha = 0.05$

Test statistic:
Again, with function R, t.test(Alice,Bob,var.equal=FALSE), we get test statistic t = 1.7026 in a t-distribution of 92 (rounded).

P value:
0.09203 (Also obtained through t.test)

0.09203 > 0.05, so we do not reject the null hypothesis.

Conclusion:
There is not enough evidence to warrant the rejection of the claim that Alice works on average the same number of hours.

c) We will use a right-tailed t-test (Since Alice claims that she works more than Bob). Just as with b, we use independent samples, and we assume the standard deviations are not equal.

Hypothesis:
Claim of Alice: Alice works on average more than Bob.
Null hypothesis:
$\mu_1 = \mu_2$ (or $\mu_1 - \mu_2 = 0$)

Alternative hypothesis:
$\mu_1 - \mu_2 > 0$

Significance level:
$\alpha = 0.05$

Test statistic:
When using the function t.test(Alice,Bob,var.equal=FALSE, alternative = ‘greater’). we get test statistic t = 1.7026 in a t-distribution of 92 (rounded).

P value:
0.04601 (Also obtained through t-test)
0.04601 < 0.05, so we reject the null hypothesis.

Conclusion:
There is enough evidence to warrant the rejection of the claim that Alice works on average the same number of hours.

d) The ratio of the P-values is 2 : 1. This is due to the fact that for the first test, for the claim of the manager, we use a two-tailed test, and for the second test, for the claim of Alice, we use a right-tailed test.

Exercise 3.4
a) The point estimate of P (proportion of shifts worked more than 3.75 hours) is $P_{alice} - P_{bob}$
$P_{alice} - P_{bob}$ = 0.66-0.52 = 0.14

b)

Claim of Alice: "The proportion of evenings on which
Alice worked more than 3.75 hours is larger than the proportion of evenings on which Bob worked more than 3.75 hours”

Null hypothesis:
$P_{alice} = P_{bob}$

Alternative hypothesis:
$P_{alice} > P_{bob}$

Significance level:
α = 0.05

Test statistic:

$$Z_p = \frac{\hat{P}_{alice} - \hat{P}_{bob}}{\sqrt{\bar{P}(1-\bar{P})/n_{alice} + \bar{P}(1-\bar{P})/n_{bob}}}$$

$$Z_p = \frac{0.14}{\sqrt{\bar{P}(1-\bar{P})/n_{alice} + \bar{P}(1-\bar{P})/n_{bob}}}$$

$$\bar{P} = \frac{X_{alice} + X_{bob}}{n_{alice} + n_{bob}}$$

$$\bar{P} = 0.59$$

$$Z_p = \frac{0.14}{\sqrt{0.59(1-0.59)/50 + 0.59(1-0.59)/50}}$$

Z_p= 1.423

P value:
Since Z has a standard normal distribution table 2 gives us the following P-value:
P(Z>1.423) = 0.077

0.077 > 0.05, so we reject the null hypothesis.

Conclusion:
There is enough evidence to reject the Alice’s claim “The proportion of evenings on which
Alice worked more than 3.75 hours is larger than the proportion of evenings on which Bob worked more than 3.75 hours”

Appendix

3.3a/b) > t.test(Alice,Bob,mu=0)

```
     Welch Two Sample t-test

data:  Alice and Bob
t = 1.7026, df = 91.778, p-value = 0.09203
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.03703828  0.48183828
sample estimates:
mean of x mean of y
 3.9616    3.7392
```

```
3.3c)> t.test(Alice,Bob,var.equal = FALSE, alternative =
"greater")

     Welch Two Sample t-test

 data:  Alice and Bob
 t = 1.7026, df = 91.778, p-value = 0.04601
 alternative hypothesis: true difference in means is greater than
0
 95 percent confidence interval:
  0.005352246         Inf
 sample estimates:
 mean of x mean of y
    3.9616    3.7392
```

```
3.4a)
pAlice = mean(Alice>3.75)
pBob = mean(Bob>3.75)
pAlice

[1] 0.66

pBob

[1] 0.52
```

```
3.4b)
pooledSampleP = (sum(Alice>3.75) + sum(Bob>3.75))/(length(Alice)
+length(Bob))
pooledSampleP

[1] 0.59
```