Lucas Faijdherbe(2594812) & Ruben van der Ham(2592271) – CS40
Assignment 4

Exercise 4.1
We are testing to see if an r = 0.876 based on n = 40 pairs are significantly different from 0, so, if the r value indicates that the Weight Watcher program is effective in reducing weight.

Hypothesis:

Null hypothesis:
H_0: $\rho = 0$

Alternative hypothesis:
H_a: $\rho \neq 0$

Significance level:
$\alpha = 0.01$

Data:
Data yields r = 0.876, n = 40.

Test statistic:
T_ρ has a t-distribution with n – 2 = 40 – 2 = 38 degrees of freedom.

Observed value $t_\rho = r \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}} = \dfrac{0.876}{\sqrt{\dfrac{1-0.876^2}{38}}} \approx 11.196$

Critical values:
We have a two-tailed test with $\alpha = 0.01$ and n = 40, so we get the critical values $-t_{38,0.01}$ and $t_{38,0.01}$, which gives: -2.712 and 2.712.

Since $t_\rho = 11.196 > 2.712$, we reject H_0.

Conclusion:
There is enough evidence to reject the claim that there is no linear correlation between the before weight and the after weight. So, one could argue that the value of r indicates that the Weight Watcher program is effective in reducing weight.

Exercise 4.2

Hypothesis:

Claimed Distribution:
For each category month, the expected value of baseball players is the same.
Therefore, the P_i of every category is also equal to all other P_i
Null hypothesis:
H_0: The above claimed distribution is agreed by the frequency counts.

Alternative hypothesis:
H_a: The above claimed distribution is not agreed by the frequency counts.

Significance level:
$\alpha = 0.05$

Data:
Data yields n = 387+329+366+344+336+313+313+503+421+434+398+371 = 4515.
P_i= 4515/12 = 376.25

Test statistic:

$$\chi^2 = \sum_{1}^{12} \frac{(E_i - O_i)^2}{O_i}$$

X^2 has a chi-squared distribution with n – 1 = 4515 – 1 = 4514 degrees of freedom.
$X^2 \approx 93.072$

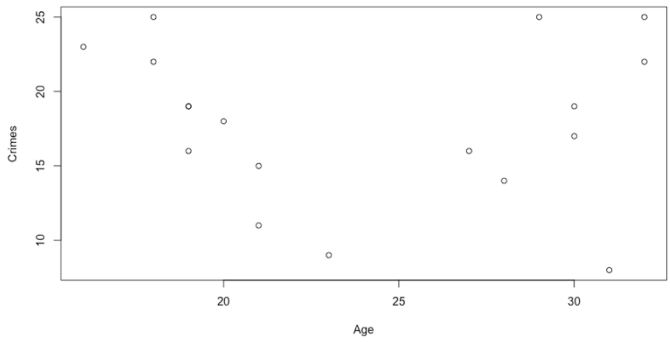
Critical values:
We have a right-tailed test with $\alpha = 0.05$ and n = 4515, so we get the critical values
$X^2_{4514,0.05}$, The closest amount of degree of freedom found in table 4 is 100, which gives: 140.169

Since $X^2 = 93.072 < 140.169$, we fail to reject H_0.

Conclusion:
There is not enough evidence to reject the claim that the frequency counts agree the claimed distribution (American-born major league baseball players are born in different months with the same frequency). Therefore we can reject the claim that “there is sufficient evidence to warrant the rejection of the claim that American-born major league baseball players are born in different months with the same frequency”.

Exercise 4.3

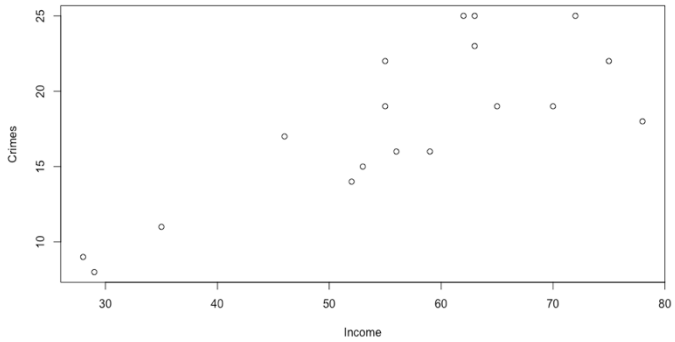
a)



Scatterplot of variables *age* and *crimes*. The sample linear coefficient is -0.071 (rounded).

If we take a look at the scatterplot, it does not seem like there is any relationship between the two variables. The sample linear coefficient is -0.071 - very close to 0, which also suggest that there is no linear relationship between the two.

b)



Scatterplot of variables *income* and *crimes*. The sample linear coefficient is 0.792 (rounded).

If we take a look at the scatterplot, we do see signs of a positive linear relationship. Since the sample coefficient is 0.792, which is very close to 1. This suggests that there is indeed a positive linear relationship.

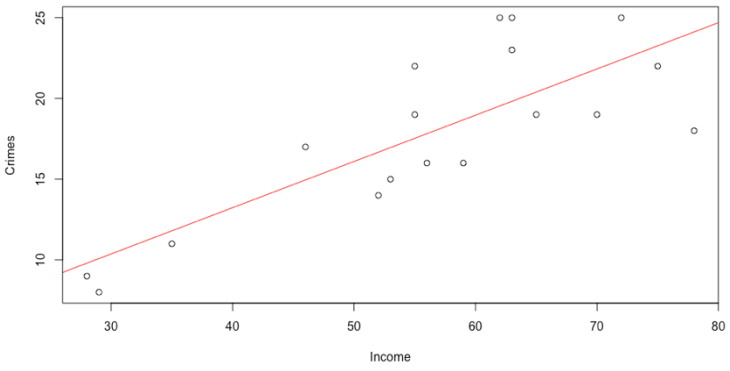
c) The fitted regression equation is given by $\hat{y} = b_0 + b_1 x$

The R function `lm(crime$crimes ~ crime$income, data = crime)` yields

Intercept: $b_0 = 1.781$

Slope: $b_1 = 0.286$

So, the fitted regression equation becomes = $\hat{y} = 1.781 + 0.286\, x$



Scatterplot of variables *income* and *crimes*, with $\hat{y}$ also plotted.

d) Claim: There is no linear relationship between the variables *income* and *crimes*.

Hypothesis:

Null hypothesis:
$\beta_1 = 0$

Alternative hypothesis:
$\beta_1 \neq 0$

Significance level:
$\alpha = 0.05$

Data:
Data yields $b_1 = 0.286$ and $S_{b_1} = 0.055$

Test statistic:
The test statistic $T_\beta = b_1 / S_{b_1}$ has a t-distribution with n-2 = 18 – 2 = 16 degrees of freedom under H_0. The observed value: $t_\beta = 0.286 / 0.055 = 5.2$

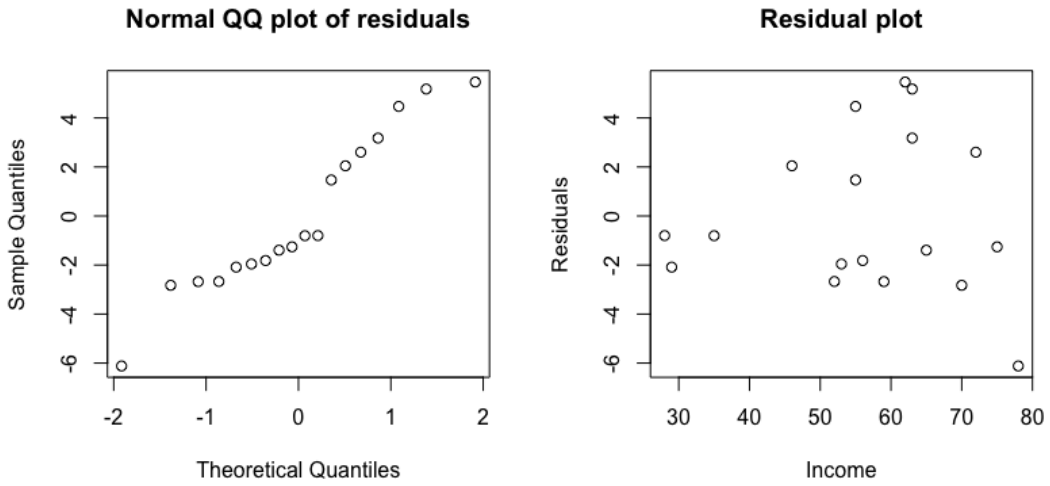
Critical values:
We have a two-tailed test with $\alpha = 0.05$ and n = 18, so we get the critical values $-t_{16,0.05}$ and $t_{16,0.05}$ which gives: -2.120 and 2.120.

Since $t_\beta = 5.2 > 2.120$, we reject H_0.

Conclusion:
The is sufficient evidence to warrant the rejection of the claim that there is no linear relationship between the variables *income* and *crimes*.

e) The requirements that have to be met are:
Errors are should be:
- Independent, something we can assume to be true
- Normally distributed
- Have a fixed standard deviation

With R, using qqnorm and plot for the residuals, we get the following plots:



From the QQ plot, we see that the plot follows approximately a straight line, so this probably comes from a normal distribution.
For the residual plot, there is no obvious pattern in the residuals. This has to be the case, otherwise there is something wrong: because our residuals look randomly placed, we can say that we have a fixed standard deviation. So we can positively say that the requirements for testing linearity are met.

Exercise 4.4

a) n = 8+12+15=35
P_{win} = 0.4
E_{win} = 0.4*35 = 14

b)
Claimed Distribution:
$P_{win} = 0.4$
$P_{draw} = 0.3$
$P_{defeat} = 0.3$

Null hypothesis:
H_0: The above claimed distribution is agreed by the frequency counts.

Alternative hypothesis:
H_a: The above claimed distribution is not agreed by the frequency counts.

Significance level:
α = 0.1

Data:
Data yields n = 35
E_{win} = 14
E_{draw} = 10.5
E_{defeat} =10.5

Test statistic:

$$\chi^2 = \sum_{win}^{defeat} \frac{(E_i - O_i)^2}{O_i}$$

X^2 has a chi-squared distribution with n – 1 = 35 – 1 = 34 degrees of freedom.
$X^2 \approx 4.714$

Critical values:
We have a right-tailed test with α = 0.1 and n = 35, so we get the critical values
$X^2_{34,0.1}$, The closest amount of degree of freedom found in table 4 is 30, which gives 43.773 of area to the right.

Since $X^2 = 4.714 < 43.773$, we fail to reject H_0.

Conclusion:
There is not enough evidence to reject the claim that the frequency counts agree the claimed distribution of Dennis.

c)
We should use a test of homogeneity. Dennis claims that one team should have the same performance of the other team, which is a different population. We are not interested in the independence of 2 variables (eg. winning and the womens soccer players. We are interested if both teams have the same proportions of winning, playing draw, and losing.

H_0:The mens and womens German soccer team have both the same chance of winning (and losing).
H_a:The mens and womens German soccer team do NOT have the same chance of winning (and losing).

d)
Null hypothesis:
H_0:The mens and womens German soccer team have both the same chance of winning (and losing).

Alternative hypothesis:
H_a:The mens and womens German soccer team do NOT have the same chance of winning (and losing).

Significance level:
$\alpha = 0.05$

Data:

	Won	Draw	Lost	**Total**
Men	8	12	15	**35**
Women	15	8	4	**27**
Total	**23**	**20**	**19**	**62**

Expected values:

	Won	Draw	Lost
Men	35*(23/62) = 12.984	35*(20/62)=11.290	35*(19/62)=10.726
Women	27*(23/62)=10.016	27*(20/62)=8.709	27*(19/62)=8.274

Test statistic:
Requirements: All E_{ij} are larger than 5, requirements met.

$$\chi^2 = \sum_{win}^{defeat} \frac{\left(E_i - O_i\right)^2}{O_i}$$

X^2 has a chi-squared distribution with (3-1)(2-1) = 2 degrees of freedom.
$X^2 \approx 8.406$

Critical values:
We have a right-tailed test with $\alpha = 0.1$, $r = 2$ and $c = 3$, so we get the critical values $X^2_{2,0.1}$, which gives 5.991 of area to the right.

Since $X^2 = 8.406 > 5.991$, we succeed to reject H_0.

Conclusion:
There is enough evidence to reject the claim that both populations (men and womens German soccer team) have equal proportions of winning (and losing).

e) $E_{men,won} = 12.984$

f)
Null hypothesis:
H_0: "Men have the same chances to win a soccer math against Italy as women have."

Alternative hypothesis:
H_a : "Men have worse chances to win a soccer match against Italy than women."

Significance level:
$\alpha = 0.01$

Data:

	Won	Draw & Defeat	**Total**
Men	8	27	**35**
Women	15	12	**27**
Total	**23**	**39**	**62**

Fisher's test

When testing we would like to reject H_0 when the frequency count in (1,1) is too low, so H_a becomes true.
We test fisher in R with: "fisher.test(matrixDouble,alt="less")"

The result of Fisher's test is:

```
Fisher's Exact Test for Count Data

data:  matrixDouble
p-value = 0.008582
#Other output
```

The P-value is 0.008582 which is below the significance level of 0.01, therefore we reject H_0.

Conclusion
There is enough evidence to reject the claim that both populations (men and womens German soccer team) have equal proportions of winning. We now know H_a is true and the men's team has worse chance of winning a match against Italy then the womens team has.

Appendix

4.3 a)

```
> crime = read.table("/Users/lucasfaijdherbe/Library/Mobile
Documents/com~apple~CloudDocs/Computer Science/Statistical
Methods/Assignments/Assignment 4/Excersises/crimemale.txt", header
= T)
> plot(crime$age, crime$crimes, xlab = 'Age', ylab = 'Crimes')
> cor(crime)
               age     income      crimes
age     1.00000000 -0.4145025 -0.07095301
income -0.41450249  1.0000000  0.79155727
crimes -0.07095301  0.7915573  1.00000000
```

b)

```
> plot(crime$income, crime$crimes, xlab = 'Income', ylab =
'Crimes')
> cor(crime)
               age     income      crimes
age     1.00000000 -0.4145025 -0.07095301
income -0.41450249  1.0000000  0.79155727
crimes -0.07095301  0.7915573  1.00000000
```

c and d)

```
> lmsim = lm(crime$crimes ~ crime$income, data = crime)
> summary(lmsim)

Call:
lm(formula = crime$crimes ~ crime$income, data = crime)

Residuals:
   Min     1Q Median     3Q    Max
-6.117 -2.054 -1.031  2.462  5.465

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.78111    3.21597   0.554    0.587
crime$income  0.28636    0.05527   5.181  9.1e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.315 on 16 degrees of freedom
Multiple R-squared:  0.6266,  Adjusted R-squared:  0.6032
F-statistic: 26.85 on 1 and 16 DF,  p-value: 9.097e-05
```

e)

```
> lmsim = lm(crime$crimes ~ crime$income, data = crime)
> par(mfrow=c(1,2))
> qqnorm(lmsim$residuals, main="Normal QQ plot of residuals")
> plot(crime$income, lmsim$residuals, main = "Residual plot", ylab
= "Residuals", xlab = "Income")
```

4.4b)

```
> observed = c(8,12,15)
> expected = c(14,10.5,10.5)
>
> x = 0
> for (i in 1:length(observed)){
+   x = x+((observed[[i]]-expected[[i]])^2/expected[[i]])
+ }
> print(paste("result = ",x))
[1] "result =  4.71428571428571"
```

d)

```
> Omen = c(8,12,15)
> Owomen = c(15,8,4)
> totalO = rbind(Omen,Owomen)
> results = matrix(totalO,ncol = 3,byrow=F)
>
>
> Emen = c(12.984,11.290,10.726)
> Ewomen = c(10.016,8.709,8.274)
> totalE = rbind(Emen,Ewomen)
> Expected = matrix(totalE,ncol = 3,byrow=F)
>
> x = 0
> for (j in 1:3){
+   for (i in 1:2) {
+     x = x+((results[i,j]-Expected[i,j])^2/Expected[i,j])
+   }
+
+ }
> print(paste("result = ",x))
[1] "result =  8.40640442413929"
```

f)

```
> matrixDouble = matrix(c(8,27,15,12),nrow = 2,byrow = F)
> fisher.test(matrixDouble,alt="less")

        Fisher's Exact Test for Count Data

data:  matrixDouble
p-value = 0.008582
alternative hypothesis: true odds ratio is less than 1
95 percent confidence interval:
 0.0000000 0.6804577
sample estimates:
odds ratio
 0.2431269
```