As an optional project, or possible day 5 activity before wrapping up, we can give the following project ideas for students to learn more on their own.

1. CHSH game using GKP states

   CHSH game is a game involving two cooperating parties, Alice and Bob who will play against the referee Charlie. Alice and Bob can determine their strategy before the game and they are not allowed to communicate classically after the game starts (let's say by separating them far enough to preclude communication at the speed of light). At the start of the game, Charlie picks uniformly random bits. At each round, Charlie sends one bit to Alice, x, and another bit, y, to Bob. Subsequently, both Alice and Bob each needs to determine a single bit to give to Charlie, let's say a and b, respectively. Alice and Bob win this round if the following condition is satisfied:

   $$x \; AND \; y = a \; XOR \; b,$$

   which is checked and recorded by Charlie, otherwise they lose the round.

   Using a simulation based on GKP states, demonstrate that if Alice and Bob share an entangled pair, they can win with a probability higher than 75% (there exist a strategy which can get ~85% probability of winning). Without the entangled pair, Alice and Bob will only be able to win at most with 75% probability. Student can also provide a statistical analysis on how confidence can we trust the probability (since it is impossible to obtain infinitely many samples).

   An analysis of this game shows that no classical local hidden-variable theory (a theory that obeys the special theory of relativity which can provide explanations of quantum mechanical correlations through the introduction of unobservable hypothetical entities) will be able to explain the correlations that results from entanglement, indicated by a winning probability of greater than 75% in this game.

2. The quadrature of a vacuum state is inherently Gaussian distributed. This fact might be utilized to create a random number generator. Steps:
   - Sample the quadrature of a vacuum state by performing a Homodyne measurement
   - Take at least 1 million data points
   - Bin each data point into 256 bins
   - Evaluate the min-entropy of the sample
   - Calculate the extractable randomness from leftover hash lemma
   - Perform randomness extraction to get uniform random bits
   - Test the random bits on NIST statistical test suite (https://pypi.org/project/nistrng/)

3. Gaussian Boson Sampling (GBS) has been used to show quantum supremacy. In the optional GBS section, you can learn the concept and ideal simulation of small GBS circuit. In this project, you are asked to investigate how non-idealities like channel loss and thermal noise can affect the sample distribution of the GBS. You can choose one non-ideality which can be simulated in Strawberry Fields and focus on that one factor.