Assignment 3 - Econometrics 2

Department of Economics - University of Copenhagen

Sunday, 15.11.2020, 23:00

1 Introduction

Main Question For our assignment we have been given monthly consumer price index data for Mozambique (MZM) and South Africa (ZAR) as well as data for the bilateral exchange-rate. We are asked to consider the purchasing power parity (PPP) relation between these two countries.

Motivation According to the PPP theory the exchange rate between two currencies should be such that an identical good has the same price in different locations in terms of a common currency. It is unlikely that PPP holds in a strict sense, as transportation costs and trade barriers are likely to distort a solid equilibrium. The consensus view about the PPP theory is that it does not hold in the short-run and may hold in the long-run PPP[1].

Econometric model and conclusion Using co-integrated vector autoregression and vector error-correction we are able to test if the PPP relation holds in the case of Mozambique and South Africa. Our model includes the first three lags of each variable and has a co-integration of rank one. We can confirm that the PPP is constant but we can reject the strict version. We find that the variable that error corrects is most likely to be the exchange rate.

2 Description of Data

The data we are considering is the respective consumer price index of Mozambique (CPI_MZM) and South Africa (CPI_ZAR) and the exchange rate of the respective currencies Metical and Rand (MZM_ZAR, metical per rand). To analyse if economic theory holds at this example one should take geographic and economic properties of those countries into account. Mozambique is one of the poorest and least developed countries in the world, scoring rank 180 out of 189 countries in the United Nations Human Development Index of 2019[2] and rank 137 out of 141 in the Global Competitiveness Report of the World Economic Forum of the same year[3]. South Africa on the other hand is one of the most developed countries in Africa and member of the G20. In the Global Competitiveness Index it is ranked 60, above e.g. Turkey (61), Croatia (63) or India (68). Its GDP of 351.4 billion USD[4] is a multiple of Mozambique's 14.9 billion USD GDP. These insights suggest, that if in either the goods market or the currency market one economy adjusts its prices to maintain the equilibrium, which we suspect to be Mozambique. According to Corbae and Ouliaris[5], it is namely the smaller economy that adapts.

These insights also explain some of the behaviour of the CPI's of both countries. The CPI of SA is very stable and seems to follow a trend while the CPI of of MZM is much more unstable. Based on graphical inspection CPI_MZM looks like a random walk and CPI_ZAR looks like a trend-stationary variable. The exchange rate of metical to rand is also quite unstable, ranging from roughly 0.9 to 1.7. All the data is not seasonally adjusted.

3 Economic Theory

3.1 Co-integration of Univariates

A unit-root non-stationanry time series x_t (also called I(1)) is consisting of a random walk component τ, a stationary impulse response S_t and a inital value A_t. In Equation 1, we are using the co-integration vector β' (in which β_1 is standardized to 1) to obtain a linear combination $\beta' X_t$ in the bottom row,

[1] shorturl.at/rwJQW (Taylor and Taylor, 2004)
[2] http://hdr.undp.org/en/content/2019-human-development-index-ranking
[3] http://www3.weforum.org/docs/WEF_TheGlobalCompetitivenessReport2019.pdf
[4] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD
[5] D. Corbae, S. Ouliaris 1988

in which u_t is the deviation between the random walk of variable x_1 and the linear combination of x_2. The co-integration vector can also be reformulated to $\tilde{\beta}' = (-\tilde{\beta}_1 \;\; 1)$ if needed. If the two variables co-integrate, this deviation should be a stationary process with mean zero. In this case any deviation from the difference between x_1 and x_2 is only temporary and will revert to the mean over time. If u_t remains an I(1) variable, the two variables are not co-integrated.

$$
\begin{gathered}
x_{1t} = \tau_1 + S_{1t} + A_1 \qquad x_{2t} = \tau_2 + S_{2t} + A_2 \qquad \beta' = \begin{pmatrix} 1 & -\beta_2 \end{pmatrix} \\
\beta' X_t = \begin{pmatrix} 1 & -\beta_2 \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} \implies x_{1t} = \mu + \beta_2 x_{2t} + u_t
\end{gathered}
\tag{1}
$$

Using the properties shown in Equation 1, a co-integration between the variables in question of this assignment could be shown with either an Engle-Granger (two-step) co-integration analysis, a cointegrated Autoregressive Distributed Lag (ADL) model / Error Correction Model (ECM) or a co-integrated VAR model. Given the following advantages, we decide to use a co-integrated VAR because of the following advantages:

- (+) As in the simple VAR-model, there is no assumption on the causality between the variables. This leaves more leeway in the interpretation of the results. Especially, this is not the case when using Engle-Granger or ADL/ECM.
- (+) Using the vector setup of the model as well as the trace statistic, it is very handy to test for the co-integration of various different variables at once, instead of each variable individually for unit roots and their equilibrium equations.

3.2 Vector Error Correction Model

We start by considering a vector autoregressive model conditional on (X_0, X_1), with k lags, p variables, $t = 1, 2, ..., T$ observations, and with $\epsilon_t | X_{t-1}, X_{t-2} \stackrel{d}{=} N(0, \Omega)$. The VAR($k$) model (in this case with $k = 2$) can be written as shown in the top row of Equation 2. Using the lag-operator (L) we can reformulate it to:

$$
\begin{aligned}
X_t &= \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \mu + \epsilon_t \\
X_t - \Pi_1 X_{t-1} + \Pi_2 X_{t-2} &= \mu + \epsilon_t \\
X_t - \Pi_1 L X_t + \Pi_2 L^2 X_t &= \mu + \epsilon_t \\
(I_p - \Pi_1 L + \Pi_2 L^2) X_t &= \mu + \epsilon_t \\
\Theta(L) X_t &= \mu + \epsilon_t
\end{aligned}
\tag{2}
$$

$\Theta(z)_{p \times p}$ is a quadratic matrix with p dimensions of polynomials, which also gives the characteristic equation $|\Theta(z)| = 0$. The VAR model has a unit root if $\Theta(1) = I_p - \Pi_1 - \Pi_2$ such that $|\Theta(1)| = 0$. Similarly to the univariate co-integration, the VAR can be rewritten to a vector error correction model (VECM):

$$
\begin{aligned}
X_t &= \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \mu + \epsilon_t \\
X_t - X_{t-1} &= \underbrace{(\Pi_1 + \Pi_2 - I_p)}_{\Pi} X_{t-1} + \underbrace{\Pi_2}_{-\Gamma_1} (X_{t-2} + X_{t-1}) + \mu + \epsilon_t \\
\Delta X_t &= \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \mu + \epsilon_t
\end{aligned}
\tag{3}
$$

The parameter matrix Π can be reformulated to $\Pi = \Pi_1 + \Pi_2 - I_p = -\Theta(1)$. For example in Equation 4 Π has a reduced rank, so $rank(\Pi) = 1 = r < p$ with p being the number of variables. Π can therefore be decomposed into the vectors α and β (where β is again normalized in β_1).

$$\Pi_{p\times p} = \alpha_{p\times r}\beta'_{r\times p} = \begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}\begin{pmatrix}1 & -\beta_2 & -\beta_3\end{pmatrix} \tag{4}$$
$$\Delta X_t = \alpha\beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \mu + \epsilon_t$$

Given the rank of matrix Π we can derive three different cases:

- Stationarity: In case X_t is stationary, no unit roots observed, so $\Pi = -\Theta(1)$ has a full rank.
- Unit-root & cointegration: Π has reduced rank, but is not 0, so $\Pi = \alpha\beta'$. Therefore $\beta' X_t$ would be stationary, with α stating the error-correction.
- Only unit-roots: Π has reduced rank and Equation 4 is only balanced if $\Pi = 0$, so the VAR is constituted only by first differences.

3.3 Estimator

To formulate a VECM, we use the CATS module in OxMetrics. This module uses a reduced rank regression to estimate the parameters of Equation 4. As we did not further cover reduced rank regression in our course, we also cannot show its assumptions and composition.

3.4 Normalization

As shown in Equation 4, Π can be decomposed into vectors. In case of $r < p$ the reduced rank regression will yield parameter estimates for α and β which maximizes the likelihood. Beside these estimate vectors, there are also countless other combinations of α and β which yield the exact same result for Π. This process is called normalization and identification (setting a different $\beta_i = 1$). This is particularly of interest as we try to find an normalization of the parameters, which matches the economic theory.

Inference co-integration and error-correction vector $\hat{\beta}$ is considered super-consistent and $T(\hat{\beta} - \beta)$ converges in mean to zero with a mixed Gaussian distribution in which the variance is random. For the case of $\beta = (1 \quad -\beta_2)'$ this would result in Equation 5 where V_{β_2} is random. However we can estimate V_{β_2} from the data and then form the t-test statistic which will have a standard N(0,1) distribution.

$$T(\hat{\beta}_2 - \beta) \xrightarrow{d} N(0, V_{\beta_2}) \qquad t_{\beta_2=b} = \frac{\hat{\beta}_2 - b}{se(\hat{\beta}_2)} = \frac{\hat{\beta}_2 - b}{\sqrt{T^{-1}\hat{V}_{\beta_2}}} \xrightarrow{d} N(0,1) \tag{5}$$

3.5 Miss specification tests

Tests Wald tests or likelihood ratio tests for hypotheses on the parameters have limiting χ^2-distributions. The likelihood ratio statistics can be calculated as twice the difference in log-likelihoods (see trace test). Regarding the misspecification tests, they are the same as for a univariate autoregression or also a VAR case.

- Residual autocorrelation (x periods): Breusch-Godfrey LM test, H_0: No AC, $\xi_{AR} = T \cdot R^2 \xrightarrow{d} \chi^2(x)$

- Residual normality: Jarque-Bera test, H_0: Error terms are normally distributed, $\xi_{JB} = \xi_S + \xi_K \xrightarrow{d} \chi^2(2)$
- Heteroskedasticity (k regressors): Standart LM-test, H_0: Errors are homoscedastic, $\xi_{HET} = T \cdot R^2 \xrightarrow{d} \chi^2(2k)$

Trace test Given the VECM in Equation 3 and depending on the rank of Π we can estimate three different, nested models (H):

$$\begin{array}{lll} H_0: & \Delta X_t = & \Gamma_{1t-1} + \mu + \epsilon \\ H_1: & \Delta X_t = \alpha\beta' X_{t-1} + & \Gamma_{1t-1} + \mu + \epsilon \\ H_2: & \Delta X_t = \Pi X_{t-1} + & \Gamma_{1t-1} + \mu + \epsilon \end{array} \tag{6}$$

$$H_0 \subset H_1 \subset H_2 \subset ... \subset ...H_p$$

Applying a LR test statistic we can use the maximized log-likelihood values of the models to test for the co-integration rank of each. We start by comparing the model with the largest rank to the one with zero rank and descend from there, each model having one rank less.

$$LR(H_0|H_p) \rightarrow ... \rightarrow LR(H_0|H_2) \rightarrow LR(H_0|H_1)$$

$$LR(H_0|H_2) = -2(logL(H_0) - logL(H_2)) \xrightarrow{d} DF$$

This LR test statistic is also known as the trace test statistic which has a Dickey-Fuller type distribution (with $T \rightarrow \infty$), depending on the number of estimated parameters and p and r.
Given the vector-setup of the VECM, we do not have to previously check the variables for the existence of a unit root. The test stops at the smallest model not rejected. For this selected model we continue to characterize the equilibrium relationships or co-integration vector β', as well as the speed of adjustment vector α.

4 Empirical analysis

To investigate the relationship between exchange rate and price levels we will formulate a VAR model and a VECM. The advantage of a co-integrated VAR is that we allow all variables to error correct. Although we have argued that Mozambican parameters are more likely to change in an equilibrium relationship, we are on the safer side with a less restrictive model.

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \Pi_3 X_{t-3} + \Lambda \times CS + \epsilon_t \tag{7}$$

As a well specified unrestricted VAR we find that a VAR(3) satisfies all conditions on consistent estimation, shown in Table 6. We include centred seasonal dummies. CS is the matrix including those dummies and Λ contains their coefficients. p denotes the log-price-level in Mozambique, $p*$ in South Africa and e the logarithmised exchange rate.

$$X_t = (p_t, p*_t, e_t)' \tag{8}$$

Considering Table 6 the hypothesis "no auto-correlation" can not be rejected. We can account for heteroscedasticity with a robust standard error estimator. Non-normal residuals are also non-problematic because of the usage of the QMLE. We reformulate the consistently estimated model into a VECM for co-integration. We perform a trace test on the Π-matrix of our VECM, shown in Table 6. It reveals that Π has a rank of one. This means that all three variables are unit root processes that co-integrate to one non-stationary process. The reduced rank allows us to decompose the matrix into two vectors, the error correction vector α and the co-integration vector β, with

$$\alpha = (0.0161, -0.0099, -0.0553)' \text{ and } \beta = (-2.87, 3.64, 1)'. \tag{9}$$

We normalised β_3 to unity, as the logarithmised exchange rate error corrects with highest magnitude. This suggests that the exchange rate is the variable that adjusts to maintain the equilibrium. We come up with a stationary equilibrium relation:

$$\begin{aligned} e_t &= 2.87p_t - 3.64p*_1 + u_t \\ \Leftrightarrow log(\frac{MZM}{ZAR}) &= log(\frac{2.87 \times CPIMZM}{3.64 \times CPIZAR}) + u_t \end{aligned} \tag{10}$$

This equilibrium relationship allows us to answer the original research questions. Our findings indicate that the PPP is indeed constant, as we find the exchange rate to be a stationary process that error corrects deviations of prices.

For the strict PPP to hold we need both CPI's to have the same influence on the exchange rate. This implies that both coefficients, β_1 and β_2 are equal with opposite signs. Although those coefficients are non-normal we can still test this using a t-test.

$$\begin{aligned} &H_0 : \beta_1 = -\beta_2 \Leftrightarrow \beta_1 + \beta_2 = 0 \text{ vs.} \\ &H_1 : \beta_1 \neq -\beta_2 \end{aligned} \tag{11}$$

with

$$\begin{aligned} t &= \frac{\hat{\beta}_1 + \hat{\beta}_2}{se(\hat{\beta}_1 + \hat{\beta}_2)} \overset{d}{=} N(0,1) \\ t &= \frac{-2.87 + 3.64}{0.00665974} = 115.62 \end{aligned} \tag{12}$$

We can reject the null on any level of significance. The strict PPP does not hold. The impulse response functions (Figure 6) show the reactions of the variables to shocks of other variables. Unlike our original hypothesis, that South African measures are rather independent from Mozambican, South African prices react more sensitive to shocks in Mozambican prizes than the other way around. This is quite surprising and raises the question: Why would the huge economy be more sensitive?

The impulse response functions also show how the exchange rate keeps the equilibrium when price levels change. The exchange rate is more sensitive to price changes in South Africa in a sense that the shock is equaled out faster. One can see that shocks in p are have an effect for 6 months and shocks in $p*$ for about 4 months. This was already implied by the error correction vector α in Table 6. This is evidence for the PPP to hold in the long run, but not in the short run. The exchange rate needs months to account for changes in price levels. Despite paying attention to a correct computation of the impulse response functions (order in OxMetrcis) we miss contemporaneous effects of price levels on the exchange rate.

5 Discussion and Conclusion

In our empirical analysis we found statistical evidence that the PPP between Mozambique and South Africa is constant in the long run. The co-integration relationship we found suggests that the exchange rate adjusts to maintain a stable equilibrium. It needs between four and six months to adjust when price levels change. Therefore we can say that the PPP is not constant in the short run.

In section two we argued that Mozambican measures adjust rather than South African. Combining this thesis with our findings, it is the value of Mozambican metical that adjusts. This should be further investigated by looking into the exchange rate of metical and other currencies. An argument against this is that we found that South African prices react more sensitive to changes In Mozambican prices than the other way around.

The gap between the two economies is also shown in the coefficient estimates of the equilibrium. We reject the hypothesis of a strict PPP because the effect on the exchange rate of South African prices is significantly higher than the effect of Mozambican prices. Other authors[6][7][8] use perfect commodity arbitrage as an argument for the theoretical existence of the strict PPP. In the case of South Africa and Mozambique the underdeveloped infrastructure of Mozambique[9] might hinder the use of perfect commodity arbitrage.

[6] Corbae D., Ouliaris S. (1988)
[7] Kim, Y. (1990)
[8] Taylor, M.P. (1988)
[9] https://borgenproject.org/infrastructure-in-mozambique/

6 Appendix

Trace test VECM(3)						
p-r	r	Trace	Trace	crit. 5% value	p-value	pvalue
3	0	54.32	50.70	29.80	[0.0]**	[0.0]**
2	1	9.16	8.24	15.41	[0.357]	[0.448]
1	2	0.70	0.67	3.85	[0.403]	[0.413]

Table 1: Trace test for VECM(3) model with the variables p, p^* and e.

Misspecification tests VAR(3)		
	t-statistics	p-value
AR 1-7 test:	1.2850	[0.0796]
Normality test:	42.097	[0.0000]**
Hetero test:	1.3711	[0.0023]**
Sample start / end	2004(1)	2020(9)

Table 2: The table shows t-statistics and p-values of standard misspecification tests for our unrestricted VAR(3) model.

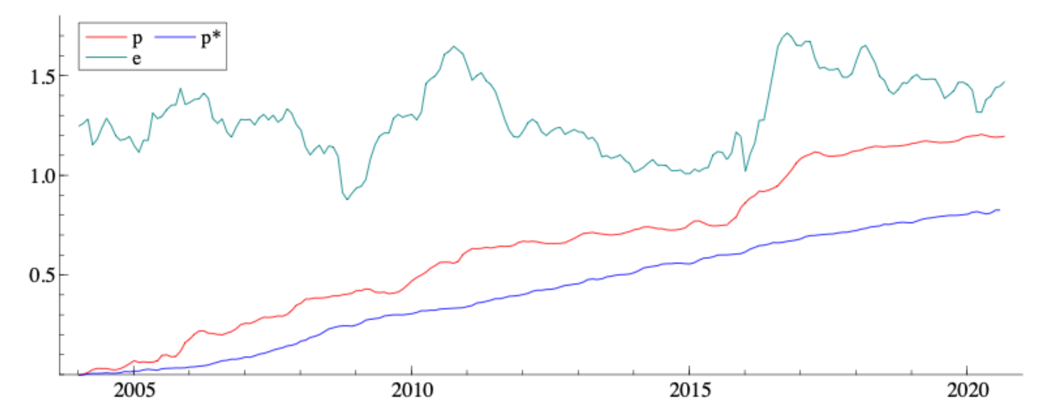

Figure 1: Time series of p, p^* and e

Table 3: Output of the CATS module for the VECM(3)

β-parameters			
	β_p	β_{p^*}	β_e
β:	-2.87 (-10.7)	3.64 (9.9)	1

α-parameters			
	α_p	α_{p^*}	α_e
α:	0.0161 (3.8)	-0.0099 (-5.2)	-0.0553 (-1.8)

Π-parameters			
	p	p^*	e
p	-0.046 (-3.6)	0.0584 (3.6)	0.0161 (3.8)
p^*	0.0284 (4.7)	-0.036 (-4.6)	-0.0099 (-5.2)
e	0.159 (1.8)	-0.201 (-1.8)	-0.0553 (-1.8)

residuals correlation and standard errors			
	p	p^*	e
p	1		
p^*	0.13	1	
e	-0.0002	0.00963	1
SE	0.00593	0.00267	0.0435

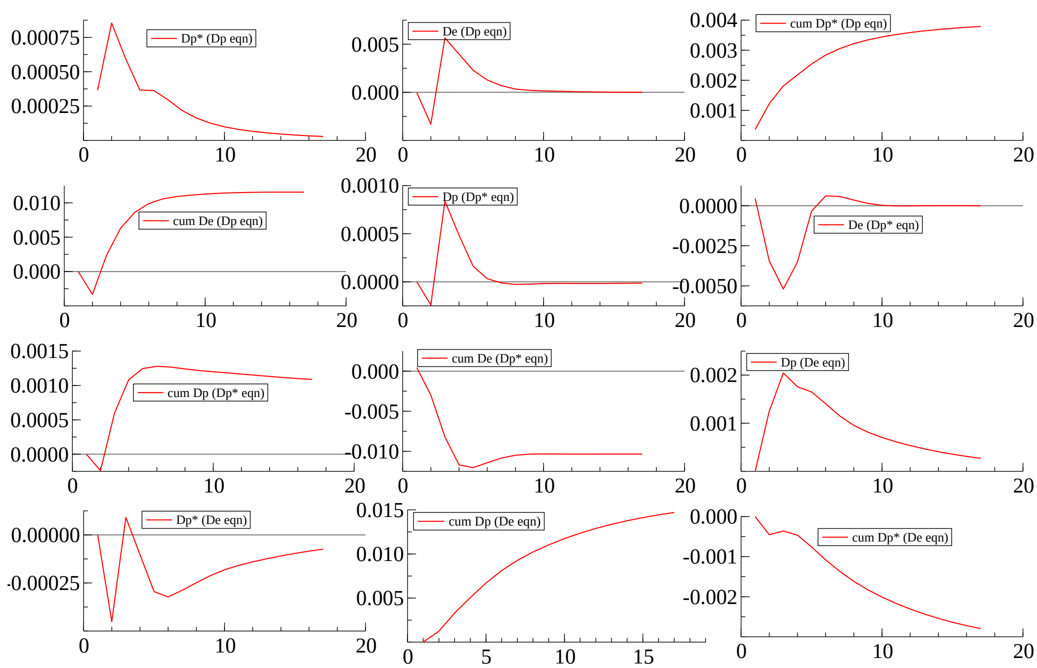

Figure 2: Impulse response functions