Main assumptions

(i) TRUTH-CONDITIONALITY: Meaning is truth-conditional.
Intuition: To understand the meaning of a sentence we need to understand the conditions under which it is true.

(ii) COMPOSITIONALITY: Meaning is compositional.
Intuition: To understand the meaning of a sentence we need to understand the meaning of its parts which can combine; additionally, we need to know how they combine.

Basic set theory

UNITS

(i) Elements: atomic units — denote with lower-case letters $a, b, c, \ldots$

(ii) Ordered pairs: Pairs of elements where the order matters — denote a pair as $\langle a, b\rangle$

(iii) Sets: groups of elements or ordered pairs — denote with upper-case letters $A, B, C, \ldots$

OPERATIONS/RELATIONSHIPS

(i) Membership: If a is in a set A, we represent it with $a \in A$; if a set B is empty, we use the empty set symbol $\varnothing = B$

(ii) Operations on and relationships between sets:

A intersects B:	$A \cap B$	the set of a such that $a \in A$ **and** $a \in B$
A unions B:	$A \cup B$	the set of a such that $a \in A$ **or** $a \in B$
A is a subset of B:	$A \subseteq B$	if $a \in A$, then $a \in B$
A is a superset of B:	$A \supseteq B$	if $a \in B$, then $a \in A$

Set theory exercises

1. True or false

(a) $1 \in \{1, 2, 3, 4, 5, 6\}$ — TRUE

(b) $\{2, 3, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}$ — TRUE

(c) $1 \subseteq \{1, 2, 3, 4, 5, 6\}$ — FALSE, 1 is not a set

(d) $\langle 1, 2\rangle \in \{1, 2, 3, 4, 5, 6\}$ — FALSE, no ordered pairs on right

(e) $\{\langle 1, 2\rangle, \langle 3, 4\rangle\} \cap \{1, 2, 3, 4, 5, 6\} = \{\langle 1, 2\rangle, \langle 3, 4\rangle\}$ — FALSE, no ordered pairs in set on right of $\cap$

(f) $\{7, 8, 9\} \cup \{1, 2, 3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ — TRUE

(g) $\{7, 8, 9\} \cap \{1, 2, 3, 4, 5, 6\} = \varnothing$ — TRUE

(h) $\{\langle 1, 2\rangle, \langle 3, 4\rangle\} \subseteq \{a, c, x, \langle 1, 2\rangle, 1, 6, s, t, \langle 3, 4\rangle, 10\}$ — TRUE

(i) If $A \subseteq B$, then there always exists a $b \in B$ such that $b \notin A$. — FALSE, B could equal A

(j) $A \cap \varnothing = A$ — FALSE

(k) $A \cup \varnothing = A$ — TRUE

Basic linguistic mappings

(i) names, definite NPs (DPs) — elements/individuals $a, b, c, \ldots$

$[\![\text{Ted}]\!] = t$

(ii) nouns, adjectives, intransitive verbs — sets of individuals $A, B, C, \ldots$

$[\![\text{dance}]\!] = \{x\colon x \text{ dances}\}$

(iii) transitive verbs — sets of ordered pairs $\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$

$[\![\text{like}]\!] = \{\langle x, y\rangle\colon y \text{ likes } x\}$

Basic compositions

(i) simple predication — S $\rightarrow$ DP VP; VP $\rightarrow$ V or VP = *is* + AdjP

S is true iff $[\![\text{DP}]\!] \in [\![\text{VP}]\!]$

$[\![\text{Ted dances}]\!]$ is true iff $t = [\![\text{Ted}]\!] \in [\![\text{dance}]\!] = \{x\colon x \text{ dances}\}$

(ii) transitive VP — S $\rightarrow$ DP_1 VP; VP $\rightarrow$ V DP_2

S is true iff $\langle [\![\text{DP}_2]\!], [\![\text{DP}_1]\!]\rangle \in [\![\text{VP}]\!]$

$[\![\text{Ted likes the shawarma}]\!]$ is true iff
$\langle s, t\rangle = \langle [\![\text{the shawarma}]\!], [\![\text{Ted}]\!]\rangle \in [\![\text{like}]\!] = \{\langle x, y\rangle\colon y \text{ likes } x\}$

(iii) definite descriptors — DP $\rightarrow$ D NP

$[\![\text{D NP}]\!]$ = contextually salient individual d s.t. $d \in [\![\text{NP}]\!]$

$[\![\text{the shawarma}]\!]$ = contextually salient shawarma s in set $[\![\text{shawarma}]\!]$

Model-theoretic semantics exercises

1. State the type of model object each word or phrase is.

   (a) the desk — ELEMENT/INDIVIDUAL
   (b) Prince — ELEMENT/INDIVIDUAL
   (c) chair — SET OF INDIVIDUALS
   (d) sleep — SET OF INDIVIDUALS
   (e) hit — SET OF ORDERED PAIRS
   (f) write — SET OF INDIVIDUALS or SET OF ORDERED PAIRS
   (g) colorful — SET OF INDIVIDUALS
   (h) the person wearing the colorful shirt — ELEMENT/INDIVIDUAL

2. To create a toy model, all we need to do is define a universe of objects and then specify relations between them — i.e. we first create a complete set of individuals and a complete set of ordered pairs; then, we create subsets of these objects, giving those subsets names. For example, say we have a model universe which only contains the elements a, b, c and we have one thing that happens in this world which is burger flipping, and suppose a and c happen to be the ones who do that; then, the complete set of individuals is $\{a, b, c\}$ and the burger-flippers are $\{a, c\}$. A true utterance with respect to this model would be 'the individual a burger-flips'.

   (a) Extend the world in the example above to make b do something with its model life; make b a burger-eater.

      . create the set $[\![\text{burger-eater}]\!] = \{b\}$

   (b) Create a world of three individuals where there are suspicions between them. Make people of varying degrees of suspicion, where two people are very suspicious (i.e. they suspect something about the two other people) and the other one is slightly suspicious (suspicious of only one person). Then, define a set called *paranoid* containing the very suspicious people.

      . make universe / set of individuals $= \{x, y, z\}$
      . make $[\![\text{suspect}]\!] = \{\langle y, x\rangle, \langle z, x\rangle, \langle x, y\rangle, \langle z, y\rangle, \langle y, z\rangle\}$
      . make $[\![\text{paranoid}]\!] = \{x, y\}$

   (c) Create a world where there are four runners and one walker, and each person runs with at least one other person, but no person runs with every other person. (You can assume *run with* denotes a set of ordered pairs.)

      . make universe / set of individuals $= \{w, x, y, z\}$
      . make $[\![\text{walk}]\!] = \{w\}$
      . make $[\![\text{run}]\!] = \{w, x, y, z\}$
      . make $[\![\text{run with}]\!] = \{\langle y, x\rangle, \langle x, y\rangle, \langle z, w\rangle, \langle w, z\rangle\}$

   (d) Redefine the world directly above to where there is one person everybody runs with.

      . make $[\![\text{run with}]\!] = \{\langle y, x\rangle, \langle x, y\rangle, \langle y, z\rangle, \langle z, y\rangle, \langle w, y\rangle, \langle y, w\rangle\}$
      . everyone runs with y

   (e) For the two previous questions, is there a sentence which could be true in both worlds?

      Yes. The sentence is *Everyone runs with someone.*