

# CLASSIFICATION METRICS

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# REGRESSION VS CLASSIFICATION (REVIEW)

- Regression and Classification are both supervised learning methods
- Regression
  - The target or response variable ( $y$ ) is numeric
  - Model predicts the *value* of  $y$
  - Prediction accuracy is measured by how close the predicted value is to the truth, such as

$$(y_i - \hat{y}_i)^2 \text{ or } |y_i - \hat{y}_i|^2$$

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# REGRESSION VS CLASSIFICATION (REVIEW)

- Regression and Classification are both supervised learning methods
- Classification
  - The target or response variable ( $y$ ) is categorical
  - Model predicts category of  $y$  (or in some cases the probability of being in a particular class, given the features)
  - Prediction accuracy can be measured by the error rate,

$$\text{ErrorRate} = \frac{1}{n} \sum_{i=1}^n \mathbf{1}\{y_i \neq \hat{y}_i\}$$

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# ACCURACY

$$\text{accuracy} = \frac{\text{Number of correct decisions made}}{\text{Total number of decisions made}}$$

- Accuracy assumes equal costs for all types of errors
  - When is accuracy not a good fitness metric?
    - When there is class skew (e.g., imbalance)
    - In these cases good accuracy could mean always predicting the majority class!
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# CONFUSION MATRIX

		Predicted Output	
		Positive (PP)	Negative (PN)
True Values	Positive (P)	True Positives TP	False Negatives FN
	Negative (N)	False Positives FP	True Negatives TN

# PRECISION

True  
Values

		Predicted Output	
		Positive (PP)	Negative (PN)
True Values	Positive (P)	True Positives TP	False Negatives FN
	Negative (N)	False Positives FP	True Negatives TN

$$Precision = \frac{TP}{TP + FP}$$

- Of those that are **predicted to be positive**, the percent that are actually positive
- Describes how good the model is at predicting the positive class
- Also call **positive predictive value**

Under what conditions is high **precision** the most appropriate performance objective for a classification model?

# RECALL

True  
Values

		Predicted Output	
		Positive (PP)	Negative (PN)
True Values	Positive (P)	True Positives TP	False Negatives FN
	Negative (N)	False Positives FP	True Negatives TN

$$Recall = \frac{TP}{TP + FN}$$

- Of those that are **actually positive**, the percent that are predicted to be positive
- The proportion of positives that are correctly identified
- Also called **sensitivity** or **true positive rate**

Under what conditions is high **recall** the most appropriate performance objective for a classification model?

# F1 SCORE

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- The precision & recall are often combined into one metric called  $F_1$
- $F_1$  is the **harmonic mean** of the precision and recall

$$F_1 = \frac{2}{\frac{1}{precision} + \frac{1}{recall}} = 2 \left( \frac{precision \cdot recall}{precision + recall} \right)$$

- ▶ The  $F_1$  score favors classifiers with similar precision and recall scores
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# $F_\beta$ SCORE

- The  $F_1$  score can be generalized to give more weight to either the precision or recall

$$F_\beta = (1 + \beta^2) \left( \frac{precision \cdot recall}{(\beta^2 \cdot precision) + recall} \right)$$

$0 < \beta < 1 \Rightarrow$  More weight given to the precision

$\beta > 1 \Rightarrow$  More weight given to the recall

$\beta = 1 \Rightarrow$  Equal weight given to precision and recall ( $F_1$ )

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# OTHER MEASURES

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Recall / Sensitivity /  
True Positive Rate

Predicted Output	
Positive (PP)	Negative (PN)
True Values	Positive (P)
Negative (N)	TP
Positive (P)	FN
Negative (N)	FP
Positive (P)	TN

$$\frac{TP}{TP + FN}$$



Precision /  
Positive Predictive Rate

Predicted Output	
Positive (PP)	Negative (PN)
Negative (N)	Positive (P)
Positive (P)	TP
Negative (N)	FN
Positive (P)	FP
Negative (N)	TN

$$\frac{TP}{TP + FP}$$

Negative Predictive Value

Predicted Output	
Positive (PP)	Negative (PN)
True Values	Positive (P)
Negative (N)	TP
Positive (P)	FN
Negative (N)	FP
Positive (P)	TN

$$\frac{TN}{TN + FN}$$

Specificity /  
True Negative Rate

Predicted Output	
Positive (PP)	Negative (PN)
Negative (N)	Positive (P)
Positive (P)	TP
Negative (N)	FN
Positive (P)	FP
Negative (N)	TN

$$\frac{TN}{TN + FP}$$

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# ROC CURVE AND AUC

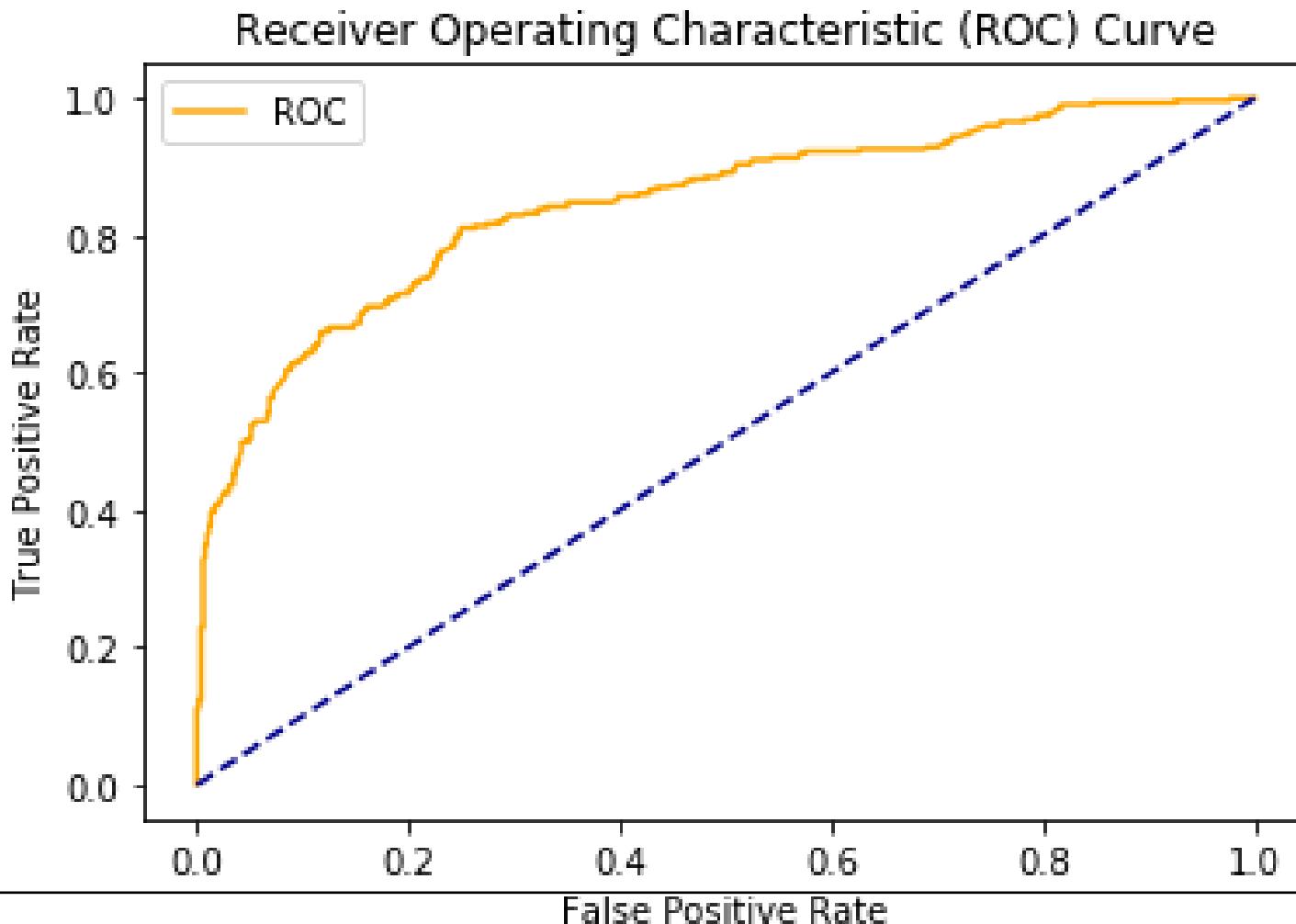
- ROC stands for Receiver Operating Curve
  - Developed in WWII to model false positive and false negative detections of radar operators
  - It plots the True Positive Rate (sensitivity) versus the False Positive Rate (1 - specificity) for different classification thresholds
- AUC stands for Area Under the (ROC) Curve
  - It is a single measure for summarizing the ROC curve
    - $AUC = 1 \Rightarrow$  perfect model
    - $AUC = 0.5 \Rightarrow$  random model

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# BUILDING THE ROC CURVE

- Many classifier actually predict a probability or score rather than a classification
  - Then the classification is made by setting a threshold
    - For example: if  $P(y_i = \text{positive}) > p$  then classify  $y_i$  as positive (for  $0 < p < 1$ )
  - The ROC curve looks at the true positive rate versus the false positive rate for a range of thresholds
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# EXAMPLE ROC CURVE



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# METRICS IN SCIKIT-LEARN

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# FUNCTION NAMES

<https://scikit-learn.org/stable/api/sklearn.metrics.html#>

- `accuracy_score(y_true, y_pred)`  
**(default for many functions)**
- `classification_report(y_true, y_pred)`
- `confusion_matrix(y_true, y_pred)`
- `f1_score(y_true, y_pred)`
- `precision_score(y_true, y_pred)`
- `recall_score(y_true, y_pred)`
- `roc_auc_score(y_true, y_score)` **Note:** (The second argument must be a score (e.g. probability) and not a class prediction)

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# .PREDICT () VS .PREDICT\_ PROBA ()

- For classification estimators, there are two ways to produce predictions
  - `.predict()` will produce an 1D array with n rows with the class prediction
    - Class with the highest probability
  - `.predict_proba()` will produce a 2D array with n rows and n-class columns
    - Each column is the probability of the row observation being in class 0 to n-class
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# CLASSIFICATION REPORT

- This special function will show precision, recall, f1, and accuracy for all class (treat each class as positive class)

print(classification_report(y_test, yhat))				
	precision	recall	f1-score	support
0	0.95	0.99	0.97	980
1	0.91	1.00	0.95	1135
2	0.98	0.91	0.95	1032
3	0.96	0.96	0.96	1010
4	0.97	0.94	0.95	982
5	0.96	0.95	0.96	892
6	0.96	0.98	0.97	958
7	0.95	0.94	0.94	1028
8	0.99	0.91	0.95	974
9	0.93	0.95	0.94	1009
accuracy			0.95	10000
macro avg	0.96	0.95	0.95	10000
weighted avg	0.95	0.95	0.95	10000

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# MULTI-CLASS CLASSIFICATION

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# MULTI-CLASS STRATEGIES

- Some classifiers (e.g., Naive Bayes, KNN) are capable of handling multiple classes directly
  - Other classifiers (e.g., binary logistic regression) are binary classifiers
  - For binary classifiers, there are two strategies for extending to multiple classes:
    - One-versus-all (OvA) (sometimes called One-versus-rest (OvR))
    - One-versus-one (OvO)
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# ONE VERSUS ALL (OVA)

- For each class, train a model that classify that class versus all the rest:
  - Classify **1** versus **not 1**
  - Classify **2** versus **not 2**
  - etc.
- Classify a value into the classifier whose classifier outputs the highest score (e.g., predictive probabilities)

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# ONE VERSUS ONE (OVO)

- Train a binary classifier for every pair of classes
    - 1 vs 2
    - 1 vs 3
    - 2 vs 3
    - 4 vs 5
    - etc
  - Classify value into the class that wins the most duels
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# OVA VERSUS OVO

- Suppose there are  $q$  classes
  - OvO requires training  $q(q-1)/2$  classifiers
    - For the MNIST example this means  $10(10-1)/2 = 45$  classifiers
    - However, each classifier needs only use a small subset of the data
  - In general OvA is the preferred strategy (only trains  $q$  classifiers)
  - But for classifiers that scale poorly with the size of the training set (e.g., SVM) OvO is preferred
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# SOFTMAX FUNCTION

- A mathematical function that converts a vector of real-valued scores into a probability distribution
- Used to convert real-valued outputs to “probabilities”
- Can be used as a way to extend logistic regression
- Often used in multi-class NN

$$p(c_i) = \frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

Probability of class  $c_i$

$Z_i$  is a raw score for the class  
(for example the class logit)

Sum over all classes