Gaussian Naive Bayes — *By-Hand* Worked Example

This file walks through a complete **Naive Bayes classification example** where features are assumed to follow **normal (Gaussian) distributions**. All calculations are shown explicitly, with **all conditional means and variances chosen to be different** for clarity.

Problem setup

We consider a **binary classification** problem with classes:

- C_0
- C_1

We observe two continuous features:

$$x = (x_1, x_2)$$

Model assumptions

1. **Class priors**: $P(C_k)$
2. **Conditional independence** (Naive Bayes assumption):

$$p(x_1, x_2 \mid C_k) = p(x_1 \mid C_k)\, p(x_2 \mid C_k)$$

3. **Gaussian likelihoods** for each feature:

$$p(x_j \mid C_k) = \frac{1}{\sqrt{2\pi\sigma_{k,j}^2}} \exp\left(-\frac{(x_j - \mu_{k,j})^2}{2\sigma_{k,j}^2}\right)$$

Given quantities

Class priors

$$P(C_0) = 0.6, \quad P(C_1) = 0.4$$

Feature distributions

Class	Feature	Mean μ	Variance σ^2
C_0	x_1	0.0	1.0
C_0	x_2	0.0	4.0
C_1	x_1	2.0	1.0

C_1	x_2	3.0	1.0

New observation

$$x = (1.0,\ 2.0)$$

Step 1 — Compute likelihoods

Likelihood under C_0

Feature $x_1 = 1.0$

$$p(x_1 \mid C_0) = \frac{1}{\sqrt{2\pi(1)}} \exp\left(-\frac{(2-0)^2}{2}\right) \approx 0.24197$$

Feature $x_2 = 2.0$

$$p(x_2 \mid C_0) = \frac{1}{\sqrt{2\pi(4)}} \exp\left(-\frac{(2-0)^2}{2 \cdot 4}\right) \approx 0.17603$$

Joint likelihood (Naive Bayes)

$$p(x \mid C_0) = 0.24197 \times 0.17603 \approx 0.04259$$

Likelihood under C_1

Feature $x_1 = 1.0$

$$p(x_1 \mid C_1) = \frac{1}{\sqrt{2\pi(1)}} \exp\left(-\frac{(1-2)^2}{2}\right) \approx 0.24197$$

Feature $x_2 = 2.0$

$$p(x_2 \mid C_1) = \frac{1}{\sqrt{2\pi(1)}} \exp\left(-\frac{(2-3)^2}{2}\right) \approx 0.05399$$

Joint likelihood

$$p(x \mid C_1) = 0.24197 \times 0.05399 \approx 0.01306$$

Step 2 — Multiply by priors (unnormalized posteriors)

$$P(C_0 \mid x) \propto 0.04259 \times 0.6 = 0.02556$$

$$P(C_1 \mid x) \propto 0.01306 \times 0.4 = 0.00523$$

Step 3 — Normalize

Sum of unnormalized posteriors:

$$0.02556 + 0.00523 = 0.03079$$

Final posterior probabilities:

$$P(C_0 \mid x) = \frac{0.02556}{0.03079} \approx 0.8302$$

$$P(C_1 \mid x) = \frac{0.00523}{0.03079} \approx 0.1698$$

Final classification

$$\boxed{\text{Predict class } C_0}$$

Notes

- All **means and variances differ** across classes and features to make each Gaussian term visually and numerically distinct.
- Independence is assumed **conditionally on the class**.
- In practice, implementations use **log-probabilities** to avoid numerical underflow.
- This corresponds directly to `sklearn.naive_bayes.GaussianNB`.

End of file.