frobenius_class_example

February 24, 2023

0.1 Frobenius example - August exam 2022

Consider the 2nd order linear ordinary differential equation with non-constant coefficients

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} + 16x^7y = 0.$$

Equations of this form can be solved using Frobenius method.

1. Assume solutions take the form

$$y = x^m\sum_{n=0}^{\infty} a_n x^n.$$

Subsititue this expansion into the above equation and show that this yields

$$\sum_{n=0}^{\infty}(m+n-4)(m+n)a_n x^{m+n-1} + 16\sum_{n=0}^{\infty} a_n x^{m+n+7} = 0.$$

(7 marks)

2. By examining the $n = 0$ term of the second expression given in part (a) show that m can take the values 0 or 4. (3 marks)

3. Using the second expression given in part (a) (or otherwise) show that the coefficients a_n must satisfy the recurrence relation

$$a_n = -\frac{16a_{n-8}}{(m+n-4)(m+n)}$$

(7 marks)

4. Finally, show that the general solution takes the form

$$y = A\cos(x^4) + B\sin(x^4).$$

(8 marks)

0.2 Note - we'll go through the solution to this problem on the board in the Monday lecture.