PART → 1

1.1.1:

(a). $p \rightarrow q$

where,
p = The sun shines today
q = It won't shine tomorrow

(d). $p \rightarrow (q \wedge \neg r) \vee (\neg q \wedge r)$

where,
p = Request occurs
q = It will eventually be Ae.
r = The requesting process won't be

(h). $(p \wedge \neg q) \vee (\neg p \wedge q)$

where,
p = It will rain.
q = It will shine

(l). $p \rightarrow q \vee r$

where,
p = Dick met Jan yesterday
q = They had a cup of coffee tog.
r = They took a walk in the park

1.2.1:

(c).

1.	$(p \land q) \land r$	premises
2.	$p \land q$	$\land e_1$ 1
3.	r	$\land e_2$ 1
4.	p	$\land e_1$ 2
5.	q	$\land e_2$ 2
6.	$q \land r$	$\land i$ 5,3
7.	$p \land (q \land r)$	$\land i$ 4,6

(f).

1.	$p \land q$	Assum.
2.	p	$\land e_1$ 1
3.	$(p \land q) \to q$	$\to i$ 1,2

(h).

1.	p	prem.
2.	$p \to q$	Assum.
3.	q	$\to e$ 1,2
4.	$(p \to q) \to q$	$\to i$ 2-3

(j).

1.	$q \to r$	premises
2.	$p \to q$	Assum.
3.	p	Ass.
4.	q	$\to e$ 2,3
5.	r	$\to e$ 1,4
6.	$p \to r$	$\to i$ 3-5
7.	$(p \to q) \to (p \to r)$	$\to i$ 2-6

(1).

1.	$p \to q$	premises
2.	$r \to s$	prem.
3.	$p \vee r$	Assum.
4.	p	Assum.
5.	q	$\to e$ 1,4
6.	$q \vee s$	$\vee i_1$ 5
7.	r	Assum.
8.	s	$\to e$ 2,7
9.	$q \vee s$	$\vee i_2$ 8
10.	$q \vee s$	$\vee e$ 3,4-6,7-9
11.	$p \vee r \to q \vee s$	$\to i$ 3-10

(p).

1.	$p \to q \wedge r$	premises
2.	p	Assum.
3.	$q \wedge r$	$\to e$ 1,2
4.	q	$\wedge e_1$ 3
5.	$p \to q$	$\to i$ 2-4
6.	p	Assum
7.	$q \wedge r$	$\to e_1$
8.	r	$\wedge e_2$ 3
9.	$p \to r$	$\to i$ 6-8
10.	$(p \to q) \wedge (p \to r)$	$\wedge i$ 5,9

(x).

1.	$p \to (q \vee r)$	premises
2.	$q \to s$	prem.
3.	$r \to s$	prem.
4.	p	Assum.
5.	$q \vee r$	$\to e$ 1,4
6.	q	Assum
7.	s	$\to e$ 2,6
8.	r	Assum.
9.	s	$\to e$ 3,8
10.	s	$\vee e$ 5, 6-7, 8-9
11.	$p \to s$	$\to i$ 4-10

1.2.2

(b). 1. $\neg p \vee \neg q$ premise

2. $\neg p$ Assum.

3. $p \wedge q$ Assum.

4. p $\wedge e_1$ 3

5. $\bot$ $\neg e$ 2,4

6. $\neg(p \wedge q)$ PBC

7. $\neg q$ Assum.

8. $p \wedge q$ Assum.

9. q $\wedge e_2$ 8

10. $\bot$ $\neg e$ 7,9

11. $\neg(p \wedge q)$ PBC

12. $\neg(p \wedge q)$ $\vee e$ 1,2-6,7-11

(e). 1. $p \to q \vee r$ premise

2. $\neg q$ premise

3. $\neg r$ premise

4. p Assum.

5. $q \vee r$ $\to e$ 1,4

6. q Assum.

7. $\bot$ $\neg e$ 2,6

~~8. $\neg p$ $\bot$~~

8. r Assum

9. $\bot$ $\neg e$ 3,8

10. $\neg p$ $\bot e$ ~~q~~

11. $\neg p$ $\neg i$ 4-10

(d). 1. $\neg p$ premise

2. $p \vee q$ premise

3. p Assum

4. $\bot$ $\neg e$ 1,3

5. q $\bot e$ 4

6. q Assum

7. q copy 5

8. q $\vee e$ 2,3-5,

(g). 1. $p \wedge \neg p$ premise

2. p $\wedge e_1$ 1

3. $\neg p$ $\wedge e_2$ 1

4. $\bot$ $\neg e$ 2,3

5. $\neg(p \to q) \wedge (p \to q)$ $\bot e$ 4

1.2.3

(n). 1. $p \wedge q$ premise

2. $\neg p \vee \neg q$ Assum.

3. $\neg p$ Assum

4. p $\wedge e_1$ 1

5. $\bot$ $\neg e$ 3,4

6. $\neg q$ Assum

7. q $\wedge e_2$ 1

8. $\bot$ $\neg e$ 6,7

~~9. $\neg(\neg p \vee \neg q)$~~

~~$\neg e$~~

9. $\bot$ $\vee e$ 2, 3-5, 6-8

10. $\neg(\neg p \vee \neg q)$ $\neg i$ 2-9

4

1. $\neg q \vee q$ LEM
2. $\neg q$ Assum.
3. q Assum
4. $\bot$ $\neg e$ 2,3
5. r $\bot e$ 4
6. $q \to r$ $\to i$ 3-5
7. $(p \to q) \vee (q \to r)$ $\vee i_2$ 6
8. q Assum.
9. p Assum.
10. q copy 8
11. $p \to q$ $\to i$ 9-10
12. $(p \to q) \vee (q \to r)$ $\vee i_1$ 11
13. $(p \to q) \vee (q \to r)$ $\vee e$ 1, 2-7, 8-12

1.4.27

(a). According to Soundness theorem, If $\phi_1, \phi_2 \ldots, \phi_n \vdash \psi$ valid, then $\phi_1, \phi_2, \ldots, \phi_n \vDash$ holds. we do prove 1.2.3.9 (see please), where we found the exerceise is valid.

Since $\vdash (p \to q) \lor (q \to p)$ is proved and valid, then $\vDash (p \to q) \lor (q \to p)$ holds.

(b). $\vDash ((q \to (p \lor (q \to p))) \lor \neg(p \to q)) \to p$

we know that,

$\exists (p_1, p_2, p_3 \ldots, p_n)$ where $\phi(p_1, p_2, p_3 \ldots, p_n) = T$ then we called the formula is satisfiable.

And,
$\forall\, p_1, p_2, p_3 \ldots, p_n$ with $\phi(p_1, p_2, p_3 \ldots, p_n) = T$ then we say the formulee is valid

we say the formula is satifiable and valid

So, If the formula is satifiable and valid then ϕ holds.

since, the formula contains three T (true) value and one $\perp$ (false) value. This is is $\boxed{\text{Satifiable}}$ $\boxed{\text{Analysis trouth table}}$

And, as all p and q values aree not troue (T)

This is not valid. The formula doesn't $\boxed{\neg \text{holds}}$.

1.5.15

(g)

Step 1: $(\top \to q') \wedge (\top \to s') \wedge (w \to \bot) \wedge (p \wedge q \wedge s \to u) \wedge$
$(u \to s) \wedge (\top \to r') \wedge (r \to p)$

Step 2: $(\top \to q') \wedge (\top \to s') \wedge (w \to \bot) \wedge (p \wedge q'' \wedge s'' \to u)$
$\wedge (u \to s) \wedge (\top \to r') \wedge (r \to p)$

Step 3: $(\top \to q') \wedge (\top \to s') \wedge (w \to \bot) \wedge (p''' \wedge q'' \wedge s'' \to u''')$
$\wedge (u''' \to s''') \wedge (\top \to s') \wedge (r''' \to p''')$

Since $\bot$ is not marked, the formula is $\boxed{\text{Satisfiable}}$

(a).

Step 1: $(p \wedge q \wedge w \to \bot) \wedge (t \to \bot) \wedge (r \to p) \wedge (\top \to r')$
$\wedge (\top \to q') \wedge (u \to s) \wedge (\top \to u')$

Step 2: $(p \wedge q'' \wedge w \to \bot) \wedge (t \to \bot) \wedge (r \to p) \wedge (\top \to r')$ $\wedge (\top \to u')$
$\wedge (\top \to q') \wedge (u \to s)$

Step 3: $(p''' \wedge q'' \wedge w \to \bot) \wedge (t \to \bot) \wedge (r''' \to p''') \wedge (\top \to r')$ $\wedge (\top \to u')$
$\wedge (\top \to q') \wedge (u''' \to s''')$

Thus, $\bot$ is not marked, the formula is $\boxed{\text{Satisfiable}}$

According to page 66, 67 (Horn formula and Algorithm.

1.5.17

According to page 65,

Let,

HORN: $(A \wedge B \wedge C \rightarrow D) \wedge (E \rightarrow F)$

$= \{\neg(A \wedge B \wedge C) \vee D\} \wedge (\neg E \vee F)$

CNF $= (\neg A \wedge \neg B \wedge \neg C \vee D) \wedge (\neg E \vee F)$

$= \{\neg(A \wedge B \wedge C) \vee D\} \wedge (\neg E \vee F)$

HORN $= (A \wedge B \wedge C \rightarrow D) \wedge (E \rightarrow F)$

So, HORN $\equiv$ CNF (proved)