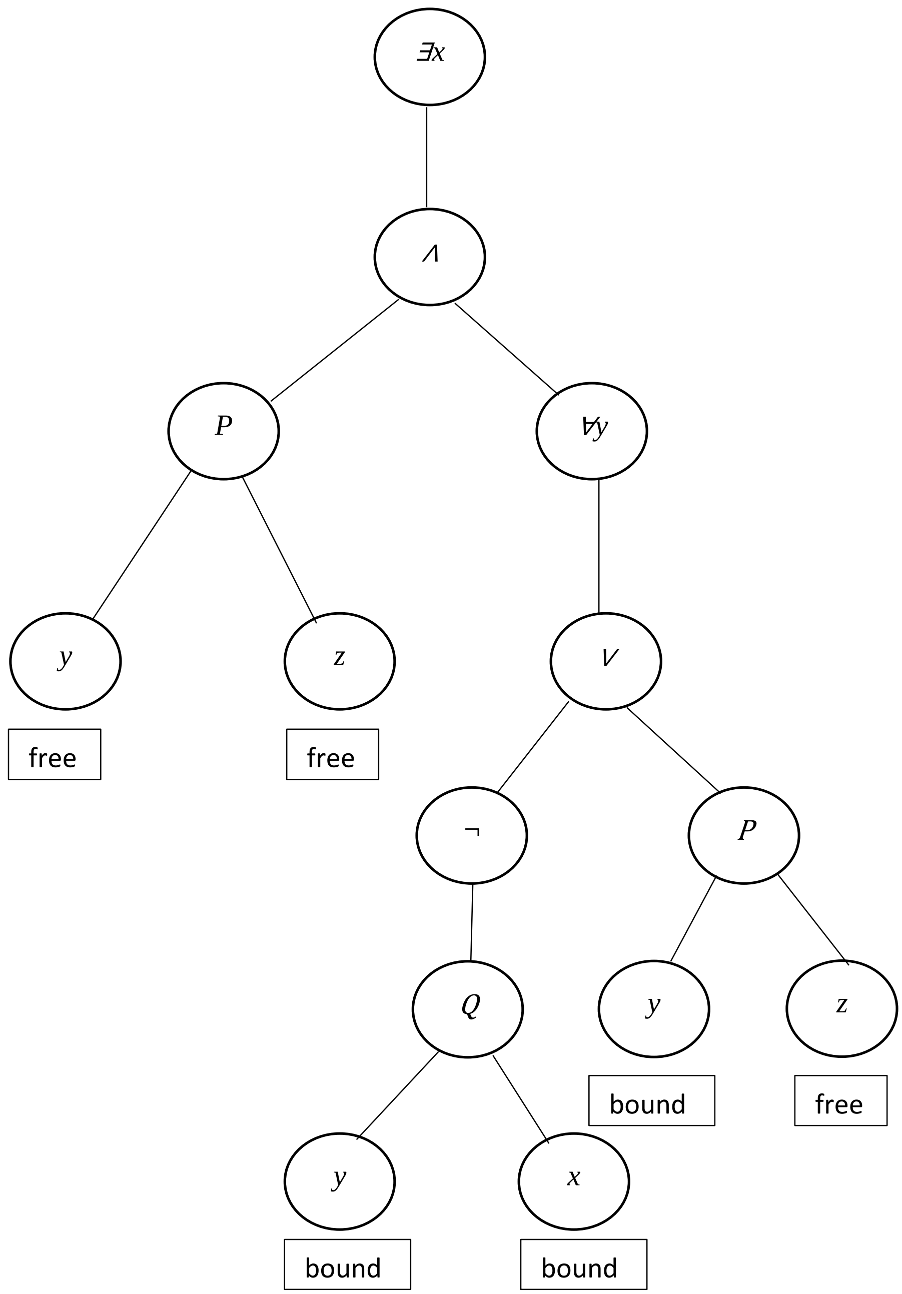
4. (a)



4. (b)

According to the parse tree of φ, all occurrences of z are free, all occurrences of x are bound and left most occurrence of y is free. But other two occurrences of y are bound.

4. (c)

From parse tree diagram, we found *P* variable has bound and free occurrences together.

4. (d)

i.

For $\varphi[w/x]$,
The equation is same a previous one (φ) since there is no free occurrences of x in $\boldsymbol{\varphi}$ that could be replaced by w.

For φ[w/y],
The equation is $\exists x\ (P\ (w, z) \wedge (\forall y\ (\neg Q\ (y, x) \vee P(y, z))))$. Since free occurrence of y is replaced by w.

For $\varphi[f(x)/y]$,
The equation is $\exists x\ (P\ (f(x), z) \wedge (\forall y\ (\neg Q\ (y, x) \vee P(y, z))))$. Since free occurrence of y is replaced by $f(x)$.

For $\varphi[g(y, z)/z]$,
The equation is $\exists x\ (P\ (y, g(y, z)) \wedge (\forall y\ (\neg Q\ (y, x) \vee P(y, g(y, z)))))$. Since all free occurrences of z is replaced by $g(y, z)$.

ii.

There is no free occurrences of x that could be begin the equation.

iii.

The terms w, and $g(y, z)$ are free for y in φ since the free occurrence of y is in $\exists x$ quantifiers. However $f(x)$ is not free for y in φ since x in $f(x)$ is together with the substitution process of $\exists x$.

4. (e)

The scope of $\exists x$ is the formula $P(y, z)$.

4. (f)

The only scope of $\exists x$ is the formula $P(y, z)$. Since the inner formulas are in the quantifier of $\forall x$, we consider that formula as a binds of x.