Project 2

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Part 1

Answer to the problem goes here.

1. Exercise 2.1.1 answer here.

   (a). $\forall x\,(P(x) \to A(m,x))$
   (b).$\exists x\,(P(x) \wedge A(x,m))$
   (c).$A(m,m)$
   (d).$\neg\exists x\,(S(x) \wedge \forall y\,(L(y) \to B(x,y)))$
   (e).$\neg\exists y\,(L(y) \wedge \forall x\,(S(x) \to B(x,y)))$
   (f).$\forall x\,\forall y\,(L(x) \wedge S(y) \to \neg B(y,x)))$

2. Exercise 2.1.3 answer here.

   InBox(x) = x is in the box
   Animal(x) = x is an animal
   Cat(x) = x is a cat
   Dog(x) = x is a dog
   Red(x) = x is red
   Boy(x) = x is a boy
   Prize(x) = x is a prize
   Won(y, x) = x won y

   (a). $\forall x\,(Red(x) \to InBox(x))$
   (b).$\forall x\,(InBox(x) \to Red(x))$
   (c).$\neg\exists x\,(Animal(x) \wedge Cat(x) \wedge Dog(x))$
   (d).$\forall x\,(Prize(x) \to \exists y\,(Boy(y) \wedge Won(y,x)))$
   (e).$\exists y\,(Boy(y) \wedge \forall x\,(Prize(x) \to Won(y,x)))$

3. Exercise 2.3.9 (k) answer here.

1.		$\forall x\,(P(x) \wedge Q(x))$	premise
2.	x_0	$P(x_0) \wedge Q(x_0)$	$\forall x$ e 1
3.		$P(x_0)$	$\wedge$ e$_1$ 2
4.		$\forall x\,P(x)$	$\forall x$ i 2–3
5.	x_0	$P(x_0) \wedge Q(x_0)$	$\forall x$ e 1
6.		$Q(x_0)$	$\wedge$ e$_2$ 5
7.		$\forall x\,Q(x)$	$\forall x$ i 5–6
8.		$\forall x\,P(x) \wedge \forall x\,Q(x)$	$\wedge$ i 4, 7

Exercise 2.3.9 (l) answer here.

1.		$\forall x\, P(x) \lor \forall x\, Q(x)$	premise
2.	x_0		
3.		$\forall x\, P(x)$	assumption
4.		$P(x_0)$	$\exists x$ e 3
5.		$P(x_0) \lor Q(x_0)$	$\lor\,\mathrm{i}_1$ 4
6.		$\forall x\, Q(x)$	assumption
7.		$Q(x_0)$	$\exists x$ e 6
8.		$P(x_0) \lor Q(x_0)$	$\lor\,\mathrm{i}_2$ 7
9.		$P(x_0) \lor Q(x_0)$	$\lor$ e 1, 3–5, 6–8
10.		$\forall x\, (P(x) \lor Q(x))$	$\forall x$ 2–9

Exercise 2.3.9 (m) answer here.

1.		$\exists x\, (P(x) \land Q(x))$	premise
2.	x_0	$P(x_0) \land Q(x_0)$	assumption
3.		$P(x_0)$	$\land\,\mathrm{e}_1$ 2
4.		$\exists x\, P(x)$	$\exists x$ i 3
5.		$Q(x_0)$	$\land\,\mathrm{e}_2$ 2
6.		$\exists x\, Q(x)$	$\exists x$ i 5
7.		$\exists x\, P(x) \land \exists x\, Q(x))$	$\land$ i 4, 6
8.		$\exists x\, P(x) \land \exists x\, Q(x))$	$\exists x$ e 1, 2–7

4. Exercise 2.4.1

   According to the page 133 to 135, we found truth value of $\forall x\, \forall y\, Q(g(x, y), g(y, y), z)$ depends on the valuation of its free variables.

   Let, we choose A to be the set of integers, $g^M(a, a)$ is the result of a from a and third integers (a, b, c) is in Q^M if and only if, c equals the product of a and b. That way $g(y, y)$ is interpreted as 0. So the valuation of 0 equals to z. Thus, $l(z) = 0$ the formula holds in our model whereas for l $(z) = 1$ is false.

5. Exercise 2.4.6

   According to the page 139 to 140,

   Let choose a model A with integer set. We define (n, m) $x \in P^M$ if and only if n is $n \leq m$. Evidently the interpretation of P is reflexive and transitive: $n \leq m$, $m \leq k$, and $n \leq k$. However $2 \leq 3$ and $3 \neq\leq 2$ shows that the interpretation is not symmetric.

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We interpret $P(x, y)$ as x is a brother of y. This relationship is transitive and symmetric but not reflexive.

We define $A = a, b, c$ and $P^M = (a, a), (b, b), (c, c), (a, c), (a, b), (b, a), (c, a)$. While the interpretation is reflexive and symmetric, we have (b, a) and (c, a) are in P^M. Thus we need $(b, c) \in P^M$ to secure transitivity of P^M. Since this is not the case, we infer that the interpretation of P^M is not transitive.