

## REVIEW

Absolute Values

$$\text{ex} / | -5 | = 5$$

$$\text{ex} / | 4 | = 4$$

$$\text{ex} / | 0 | = 0$$

Def ^

$$\text{If } x < 0 \text{ then } |x| = -x$$

$$\text{If } x \geq 0 \text{ then } |x| = x$$

$$\text{i.e., } |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Rules

$$\textcircled{1} | -x | = |x| \Rightarrow | -6 | = | 6 |$$

$$\text{ex/ Simplify } \frac{-|x|}{| -x |}$$

$$= \frac{-|x|}{|x|}$$

$$= -1$$

$$\textcircled{2} \sqrt{x^2} = |x|$$

$$\text{ex/ } \sqrt{(-3)^2} = |-3| = 3$$

$$\textcircled{3} |xy| = |x| |y|$$

$$\text{ex/ } |(-3)(-5)| = |-3| |-5| = 3 \cdot 5 = 15$$

$$\textcircled{4} \left| \frac{x}{y} \right| = \frac{|x|}{|y|}$$

Simplify

$$\frac{| -7ab |}{| ab |} = \frac{| -7 | |a| |b|}{| ab |} = 7 |b|$$

$$\text{ex/ Analyse } \frac{x-2}{|x-2|}$$

Case  $\textcircled{1} x \neq 2$

$$\frac{x-2}{|x-2|} = 0 \text{ und}$$

$$\text{Case } \textcircled{2} x < 2 \Rightarrow \frac{x-2}{|x-2|} = \frac{x-2}{-(x-2)} = -1$$

$$|x-2| = \begin{cases} -(x-2) & \text{if } x-2 < 0 \Rightarrow x < 2 \\ x-2 & \text{if } x-2 \geq 0 \Rightarrow x \geq 2 \end{cases}$$

Case  $\textcircled{3}$

$$x > 2$$

$$\frac{x-2}{|x-2|} = \frac{x-2}{x-2} = 1$$

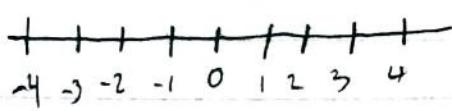
Summary / Conclusion

$$\frac{x-2}{|x-2|} = \begin{cases} -1 & \text{if } x < 2 \\ \text{und} & \text{if } x = 0 \\ 1 & \text{if } x > 2 \end{cases} \quad \begin{cases} -1 & \text{if } x < 0 \\ \text{und} & \text{if } 0 < x < 2 \\ 1 & \text{if } x > 2 \end{cases}$$

## Conventions re' Interval Notation

August 17, 2016

ex/



incl.

$$\{x \mid -1 \leq x \leq 3\} = [-1, 3]$$

ex/



not incl.

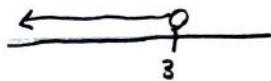
$$\{x \mid -1 < x < 3\} = ]-1, 3[ = (-1, 3)$$

ex/



$$\{x \mid x \geq -5\} = [-5, \infty[ = [-5, \infty)$$

ex



$$\{x \mid x < 3\} = ]-\infty, 3[ = (-\infty, 3)$$

$$\text{Analyse } \frac{x-2}{|x-2|}$$

$$\textcircled{1} |x-2| = -(x-2) \text{ if } x < 2$$

$$\textcircled{2} |x-2| = x-2 \text{ if } x \geq 2$$

$$\textcircled{3} \frac{x-2}{x-2} = 1 \text{ when } x < 2$$

$$\frac{x-2}{|x-2|} = -1 \text{ when } x < 2$$

$$x^2 - 4x - 5 > 0$$

$$x^2 + x - 5x - 5 > 0$$

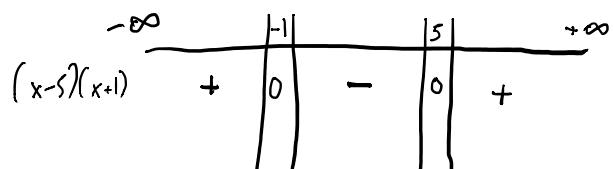
$$x(x+1) - 5(x+1) > 0$$

$$(x-5)(x+1) > 0$$

Find split pts:

$$(x-5)(x+1) = 0$$

$$x = 5, -1$$



$$SS = (-\infty, -1) \cup (5, +\infty)$$

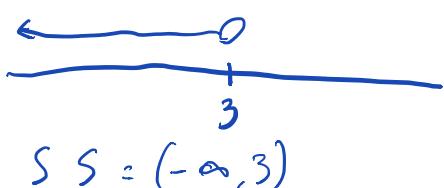
## Linear Inequalities

August 19, 2016

ex/ Solve  $3x - 5 < 4$

$$3x < 9$$

$$x < 3$$



$$SS = (-\infty, 3)$$

ex/  $-3x - 5 < 4$

$$-3x < 9$$

$$x > 3$$

Remember to flip sign

$$SS = (-3, \infty)$$

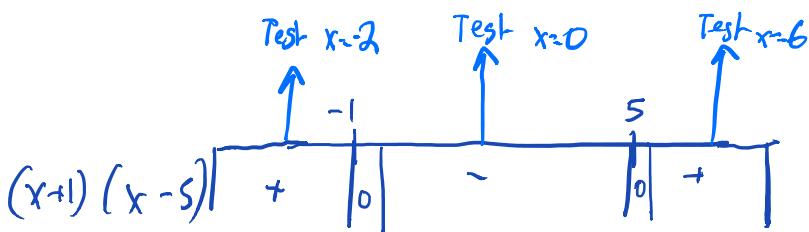
## Non-Linear Inequalities

$x^2 - 4x - 5 > 0$  ↪ only works when its equal to 0

$$(x+1)(x-5) > 0$$

Solve:  $(x+1)(x-5) = 0$

$$x = -1, 5 \in \text{split points}$$



$$SS = (-\infty, -1) \cup (5, \infty)$$

## Summary

- SPIT PT ↪ Table idea only works for expression  $> 0$  or expression  $< 0$

- To find "Spl. Pts", find where:
- or
- expression = 0
  - expression is UND (undefined)

ex/ Solve  $\frac{(x+1)(x-5)}{2x-1} \leq 0$

Spl. Pts:

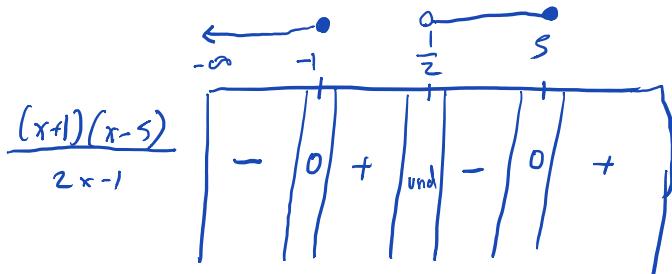
a)  $\frac{(x+1)(x-5)}{2x-1} = 0$

$(x+1)(x-5) = 0$   
 $x = -1, x = 5$

b)  $\frac{(x+1)(x-5)}{2x-1}$  is UND

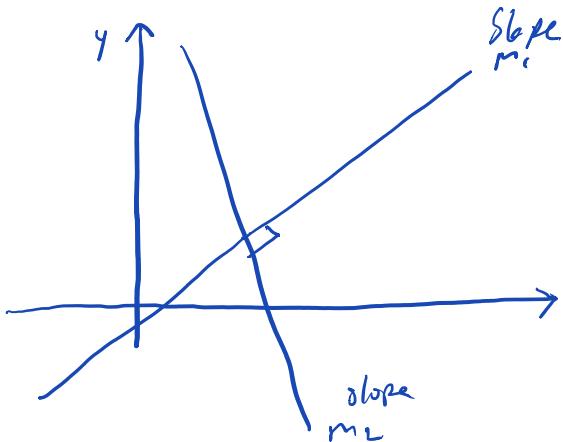
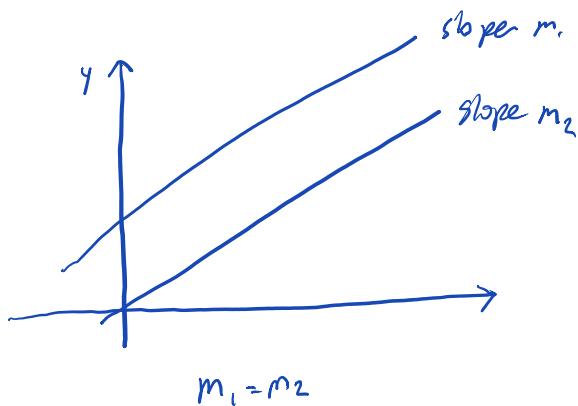
when  $2x-1 = 0$

$x = \frac{1}{2}$



SS =  $(-\infty, -1] \cup [\frac{1}{2}, 5]$

## Straight Lines

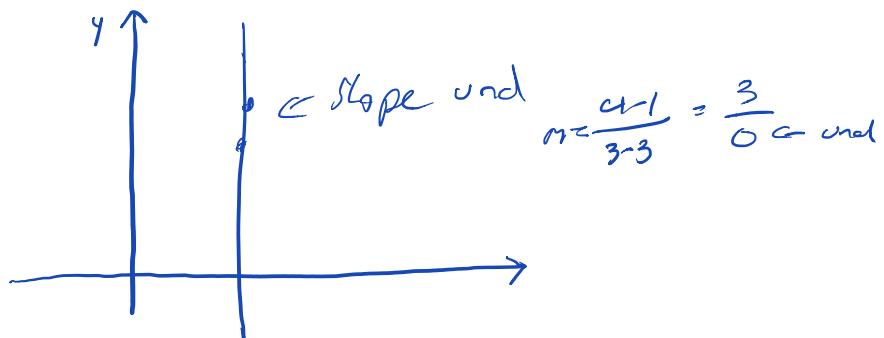


$m_1, m_2 = -1$
$m_1 = -\frac{1}{m_2}$

ex/

$$x=3$$

\* Prove by formula



All lines can be written in the form:

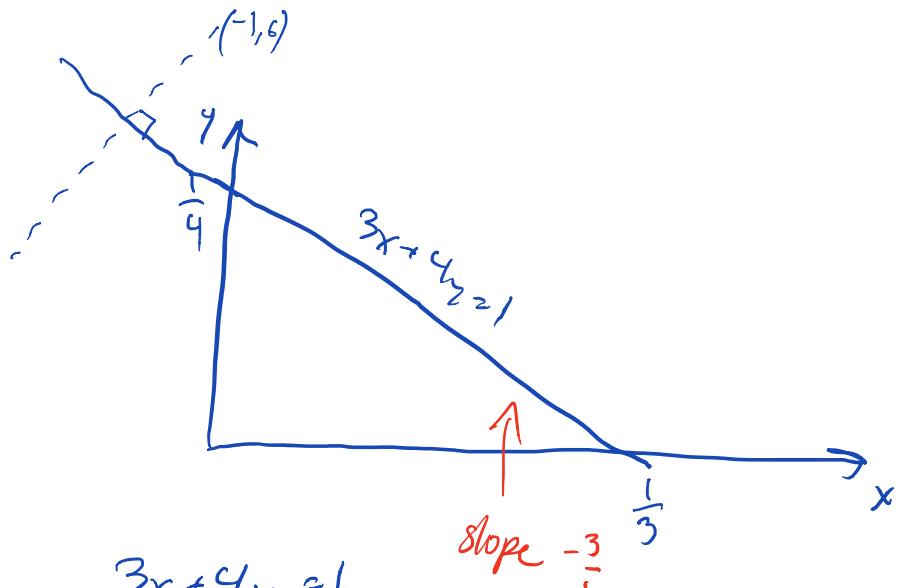
$$ax + by + c = 0$$

Most lines can be written in:

$$y = mx + b \quad \leftarrow \text{slope-intercept form}$$

ex/ Given the line  $3x + 4y = 1$

Find the eqn of the line  $\perp$  to the given line  
and passing through pt P(-1, 6)



$$3x + 4y = 1$$

$$y = \frac{1 - 3x}{4}$$

$$y = -\frac{3}{4}x + \frac{1}{4}$$

Slope  $m$  of required line:

$$m = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

$$y = \frac{4}{3}x + b$$

$$\text{Subst } x = -1, y = 6$$

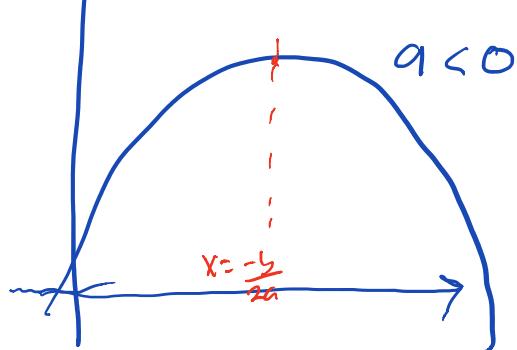
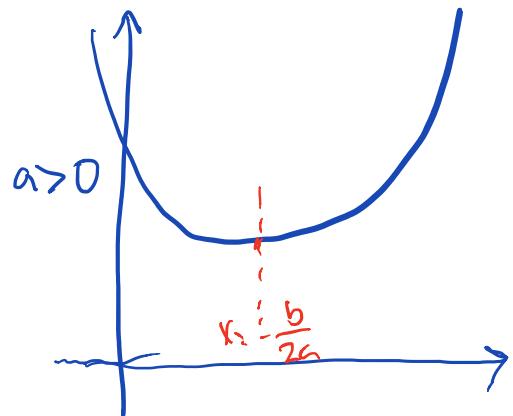
$$6 = \frac{4}{3}(-1) + b$$

$$b = \frac{22}{3}$$

$$\underline{\text{Ans: }} y = \frac{4}{3}x + \frac{22}{3}$$

## Parabolas

$$y = ax^2 + bx + c$$



$$x = -\frac{b}{2a}$$

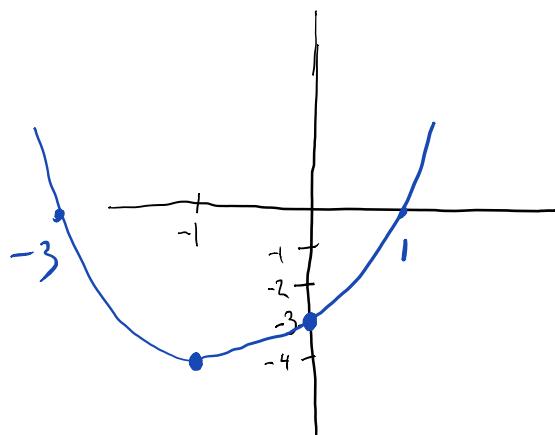
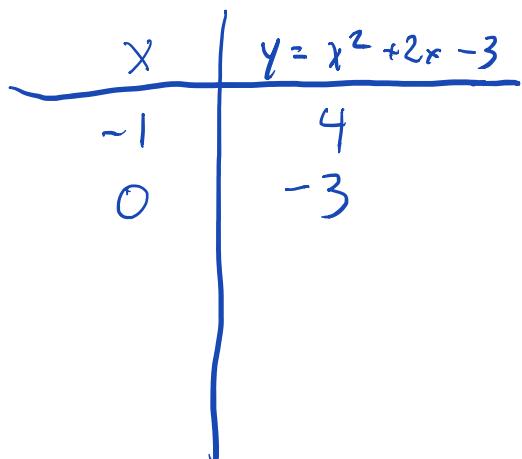
etl Sketch  $y = x^2 + 2x - 3$

$a = 1 \therefore$  parabola up

vertex occurs at  $x = \frac{-b}{2a} = \frac{2}{2(1)} = 1$

when  $x = -1$ ,  $y = (-1)^2 + 2(-1) - 3 = -4$

vertex pt is  $(-1, 4)$



Find x-int

Solve

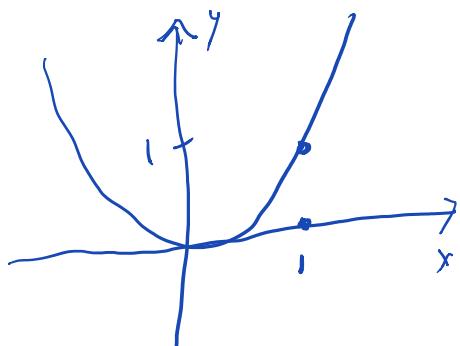
$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

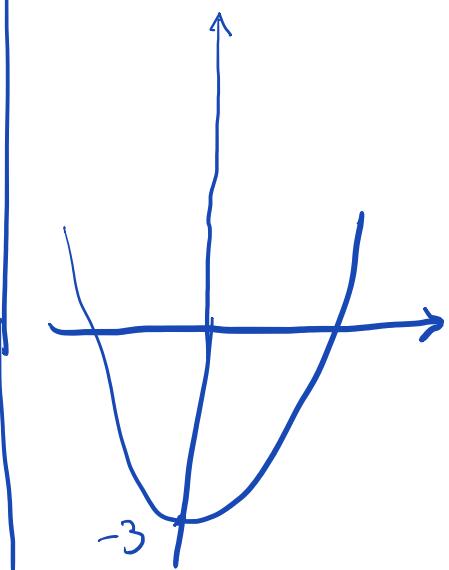
$$x = 1, -3$$

$$\text{Ex/ } y = x^2 - 3$$

Sketch  $y = x^2$

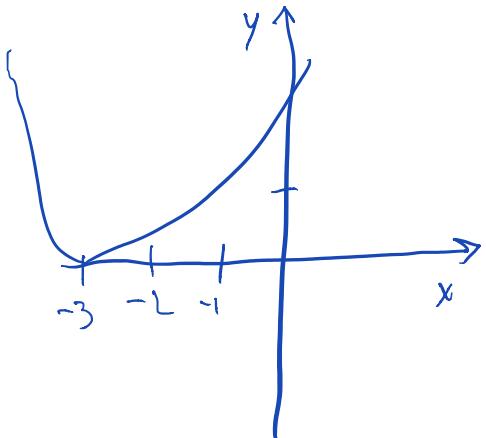


Sketch  $y = x^2 - 3$

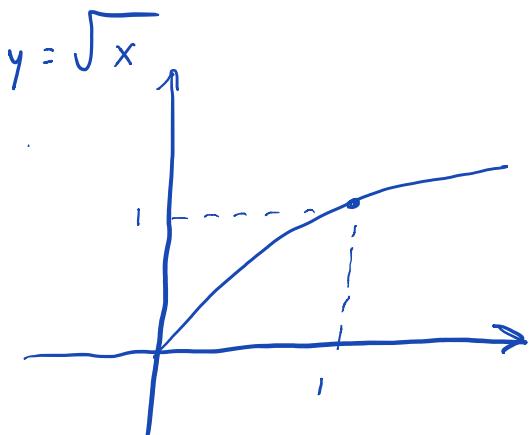
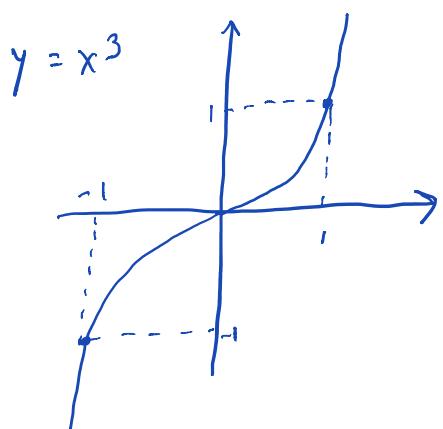
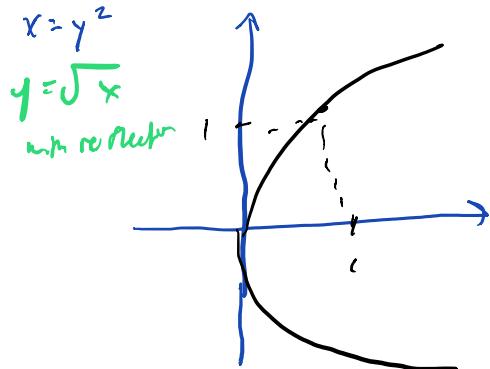
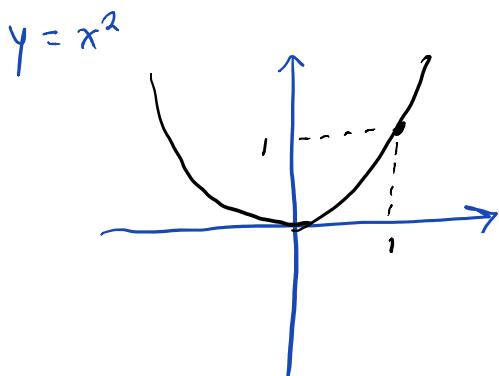


*reflect through  
x-axis*

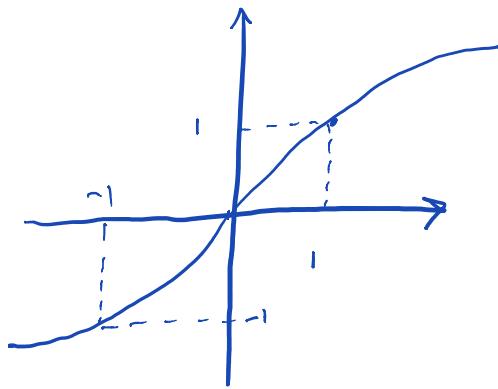
Ex/ Sketch  $y = -(x+3)^2$       Ex/Sketch  $x = y^2$



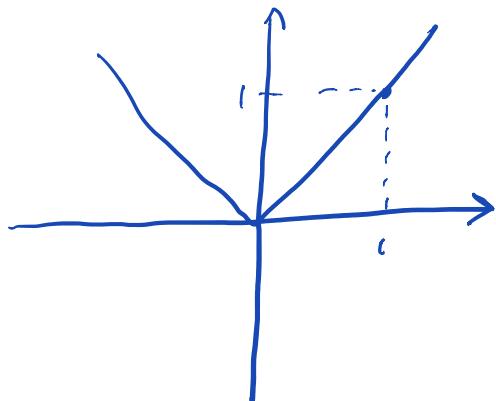
### Other Basic Graphs to know:



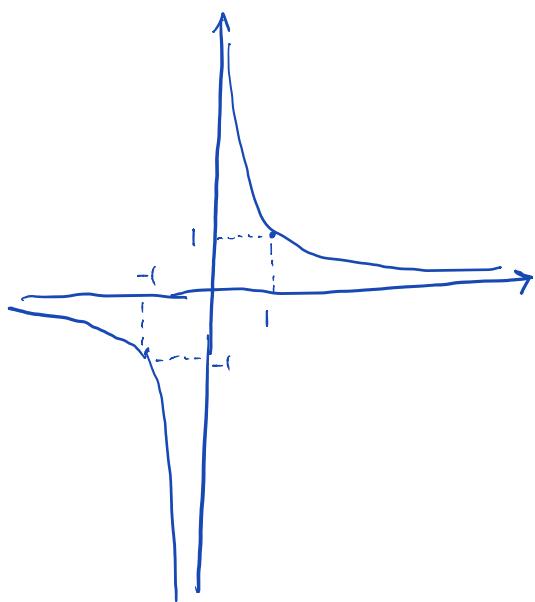
$$y = \sqrt[3]{x} = x^{\frac{1}{3}}$$



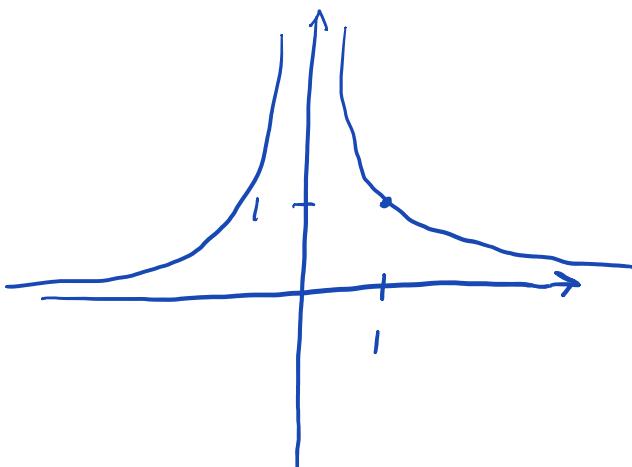
$$y = |x|$$



$$y = \frac{1}{x}$$

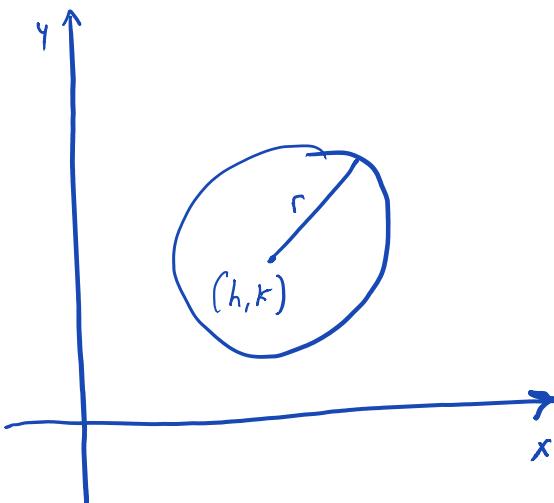


$$y = \frac{1}{x^2}$$



## Circle

$$(x-h)^2 + (y-k)^2 = r^2$$



$$\text{ex/ } x^2 + y^2 = 25$$

\* Sketch graph of  $y = \sqrt{9-x^2}$

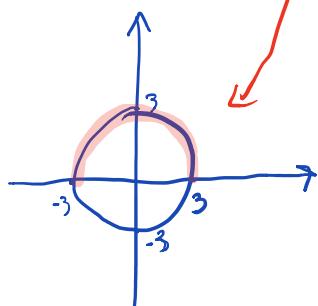
$$\begin{aligned} y &= \sqrt{9-x^2} \\ y &= 3-x \end{aligned}$$

Square both sides

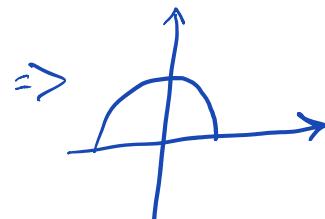
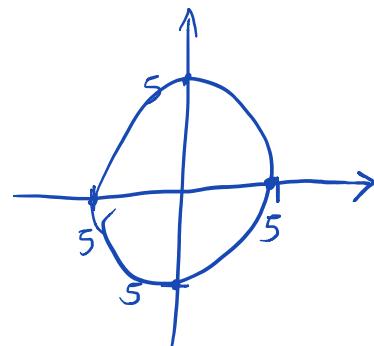
$$y^2 = 9 - x^2$$

$$x^2 + y^2 = 9$$

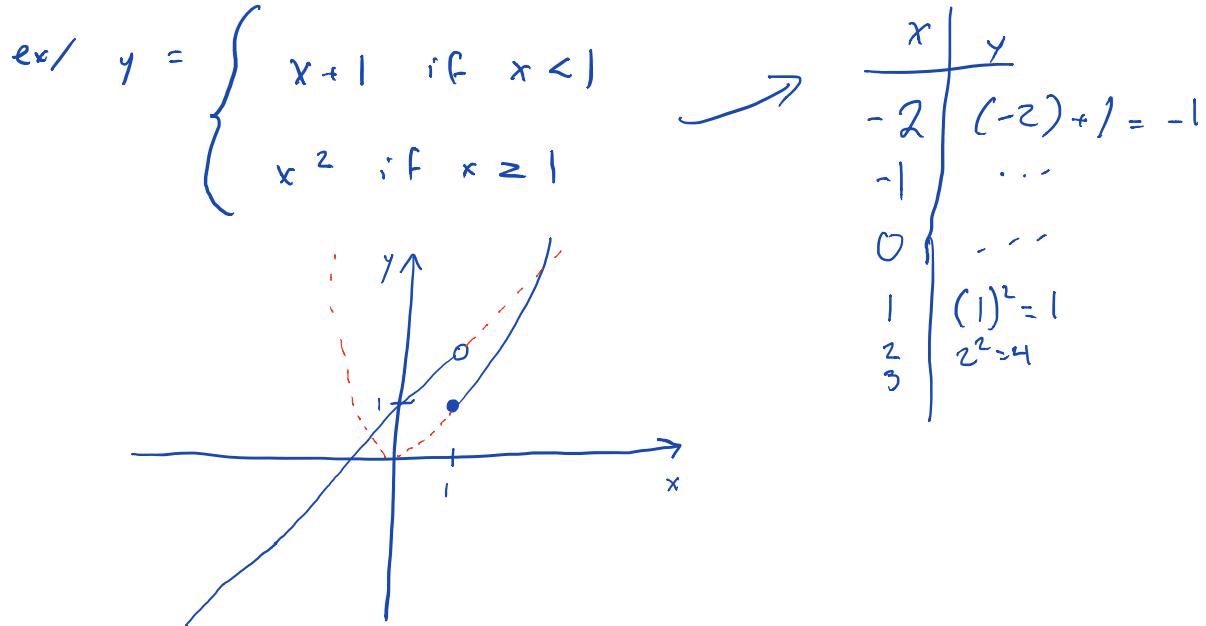
$(y \geq 0)$   $\hookrightarrow$  sketch



Since  $y \in [0, +\infty]$ , we need to just take the top of the graph

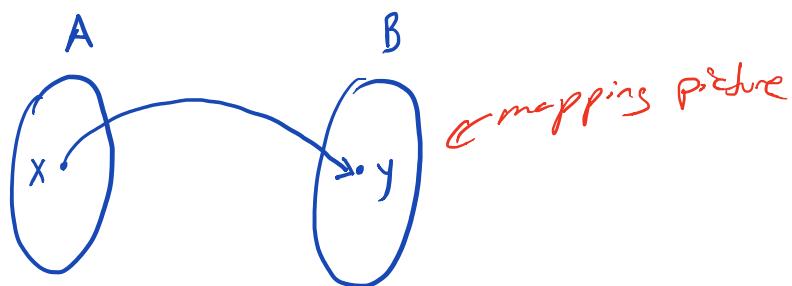


## Piecewise-Defined Functions



## Function Concept

Def: A function is a rule which assigns to each value of  $x$  in a set  $A$  a unique value of  $y$  in some set  $B$



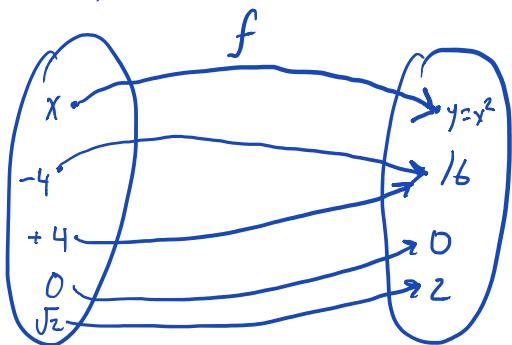
Set  $A$  = Domain

$y$  is the image of  $x$

The set of all images ' $y$ ' is called the Range

ex/ consider the  $f^{\text{cn}}$  defined by the  $y = x^2$  & assume domain is  $\mathbb{R}$

Dom  $\mathbb{R}$  Range =  $[0, \infty)$



$$f = x^2$$

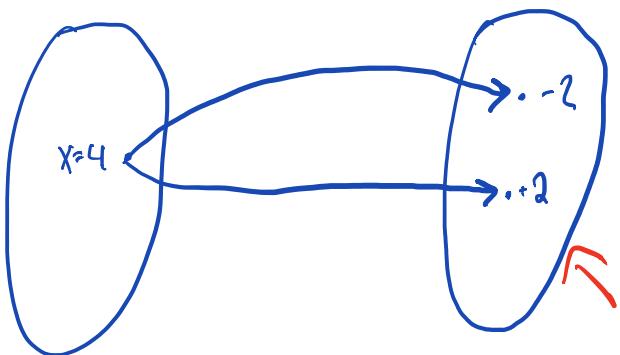
image of  $x$ :  $f(x)$

eg/  $f(2) = 4$   
 $f(x+5) = (x+5)^2$

ex/  $g(x) = \sqrt{\frac{x-1}{x+2}}$   
 what is  $g\left(\frac{\sqrt{x}+1}{5}\right)$ ?

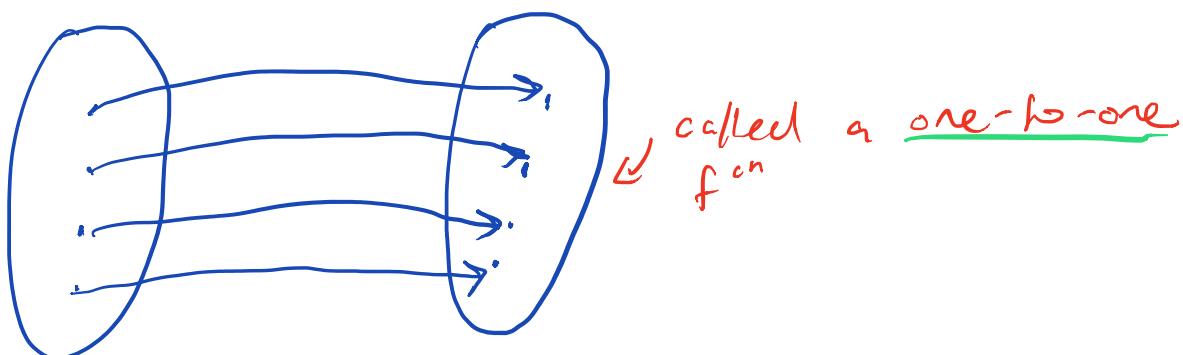
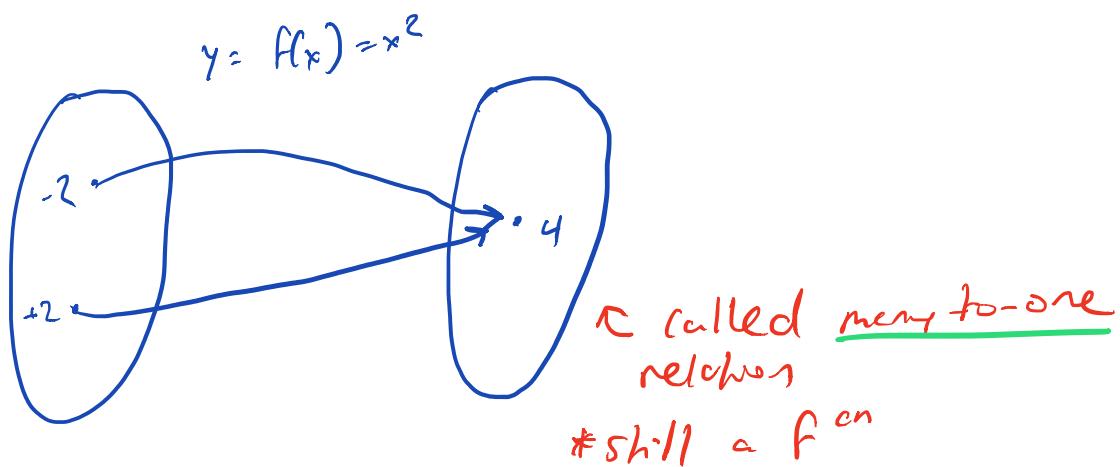
$$g\left(\frac{\sqrt{x}+1}{5}\right) = \sqrt{\frac{\frac{\sqrt{x}+1}{5}-1}{\frac{\sqrt{x}+1}{5}+2}} \quad \text{Simplify}$$

ex/ consider the eq<sup>n</sup>  $y^2 = x$



\* not a unique  $y$ -value  
 :  $y$  is not a  $f^{\text{cn}}$  of  $x$

Called a one-to-many relation



### Domain of $f^{cn}$

$$\begin{array}{l|l} \text{ex/ } f(x) = x^3 + 1 & g(x) = x^3 + 1 \quad (x \geq 5) \\ f(1) = (1)^3 + 1 = 2 & g(1) \text{ UND} \end{array}$$

\* Convention for domains: If the domain of a  $f^{cn}$  is not explicitly given, assume the domain to be all real numbers for which the expression on the RHS (right-hand side) of the defining rule makes sense

$$\text{ex/ } g(x) = \frac{1}{x-2}$$

Assume  $\text{DOM} = \mathbb{R} \setminus \{2\}$  it doesn't care how we communicate

$$\text{ex/ } H(t) = \sqrt{t}$$

Assume  $\text{DOM} = [0, \infty)$

$$\text{ex/ } g(x) = \frac{1}{\sqrt{3-5x}}$$

$$3-5x > 0$$

$$3 > 5x$$

$$\frac{3}{5} > x$$

$$x < \frac{3}{5}$$

$$\text{DOM} = (-\infty, \frac{3}{5})$$

$$\text{ex/ } R(t) = \sqrt{t^2 - 2t - 3}$$

$$t^2 - 2t - 3 \geq 0$$

$$(t+1)(t-3) \geq 0$$

Solve using +/- table

left has harder questions

$$\text{ex/ } |x^4 - 5x^3 - 14x^2| = \begin{cases} -(x^4 - 5x^3 - 14x^2) & \text{if } w < 0 \\ x^4 - 5x^3 - 14x^2 & \text{if } w \geq 0 \end{cases}$$

Find spl. f. pts.

$$x^4 - 5x^3 - 14x^2 = 0$$

$$x^2(x^2 - 5x - 14) = 0$$

$$x^2(x+2)(x-7) = 0$$

Spl. f. pts:  $x = -2, 0, 7$

$$x^2(x+2)(x-7) \begin{array}{c} \text{---} \\ \text{---} \\ + \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ 0 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ 0 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ 7 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ + \end{array}$$

$$|x^4 - 5x^3 - 14x^2| = \begin{cases} x^4 - 5x^3 - 14x^2 & \text{if } x < -2 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = -2 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } -2 < x < 0 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = 0 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } 0 < x < 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x > 7 \end{cases}$$

$$|x^4 - 5x^3 - 14x^2| = \begin{cases} x^4 - 5x^3 - 14x^2 & \text{if } x \leq -2 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } -2 < x < 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x \geq 7 \end{cases}$$

ex/ If  $f(x) = \frac{1}{x}$  evaluate  $\frac{f(x+h) - f(x)}{h}$  & simplify

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

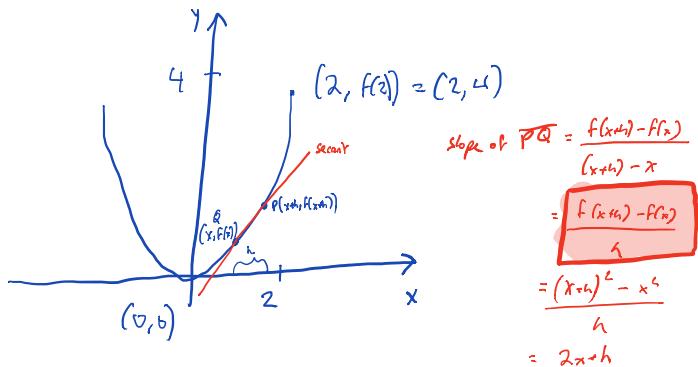
$$= \frac{x - (x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$= \frac{-h}{x(x+h)} \cdot \frac{1}{h}$$

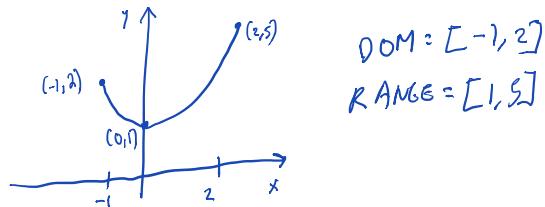
$$= -\frac{1}{x(x+h)}$$

## Graphing Fns

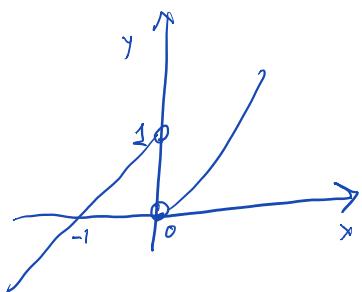
ex/ Sketch  $y = f(x) = x^2$



ex/ Sketch  $g(x) = x^2 + 1 \quad (-1 \leq x \leq 2)$



ex/ Sketch  $y = f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x^2 & \text{if } x > 0 \end{cases}$



$$\text{DOM} = (-\infty, 0) \cup (0, \infty) = \mathbb{R} \setminus \{0\}$$

$$\text{RANGE} = \mathbb{R}$$

ex/ Sketch  $y = h(x) = \frac{|x-1|}{x-1}$

Funs involving abs. values are usually piecewise defined "funs"

$$\text{DOM} = \mathbb{R} \setminus \{1\}$$

$$|x-1| = \begin{cases} -(x-1) & \text{if } x-1 < 0 \Rightarrow x < 1 \\ x-1 & \text{if } x-1 \geq 0 \Rightarrow x \geq 1 \end{cases}$$

Case ① if  $x < 1$

$$h(x) = \frac{-(x-1)}{x-1} = -1$$

Case ② if  $x = 1$

$$h(1) = \frac{|1-1|}{1-1} = \frac{0}{0} \text{ und}$$

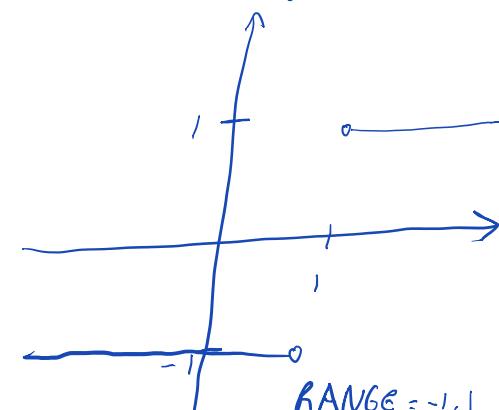
Case ③ if  $x > 1$

$$h(x) = \frac{x-1}{x-1} = 1$$

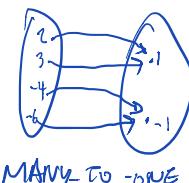
(concl)

$$y = h(x) = \frac{|x-1|}{x-1} = \begin{cases} -1 & \text{if } x < 1 \\ \text{UND if } x=1 \\ +1 & \text{if } x > 1 \end{cases}$$

- Don't need to include this; it's implied

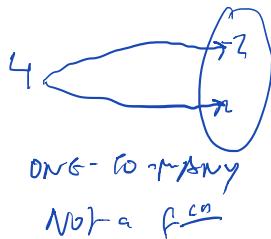
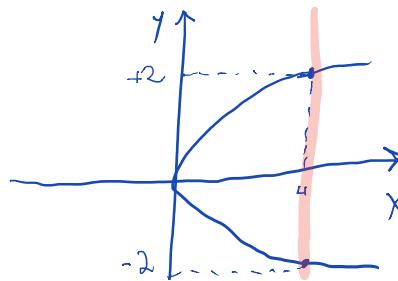


$$\text{RANGE} = -1, 1$$



MANY TO ONE

ex/ consider  $y^2 = x$



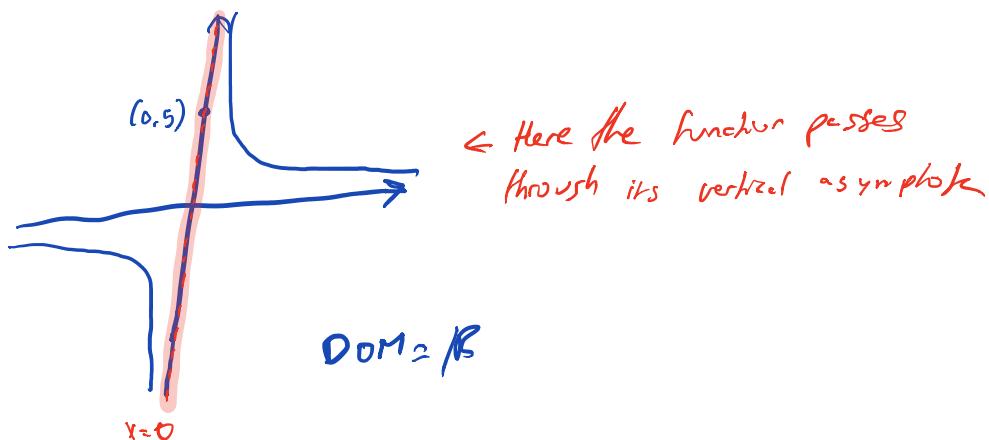
If you can find a vertical line,  $y$  is not a function

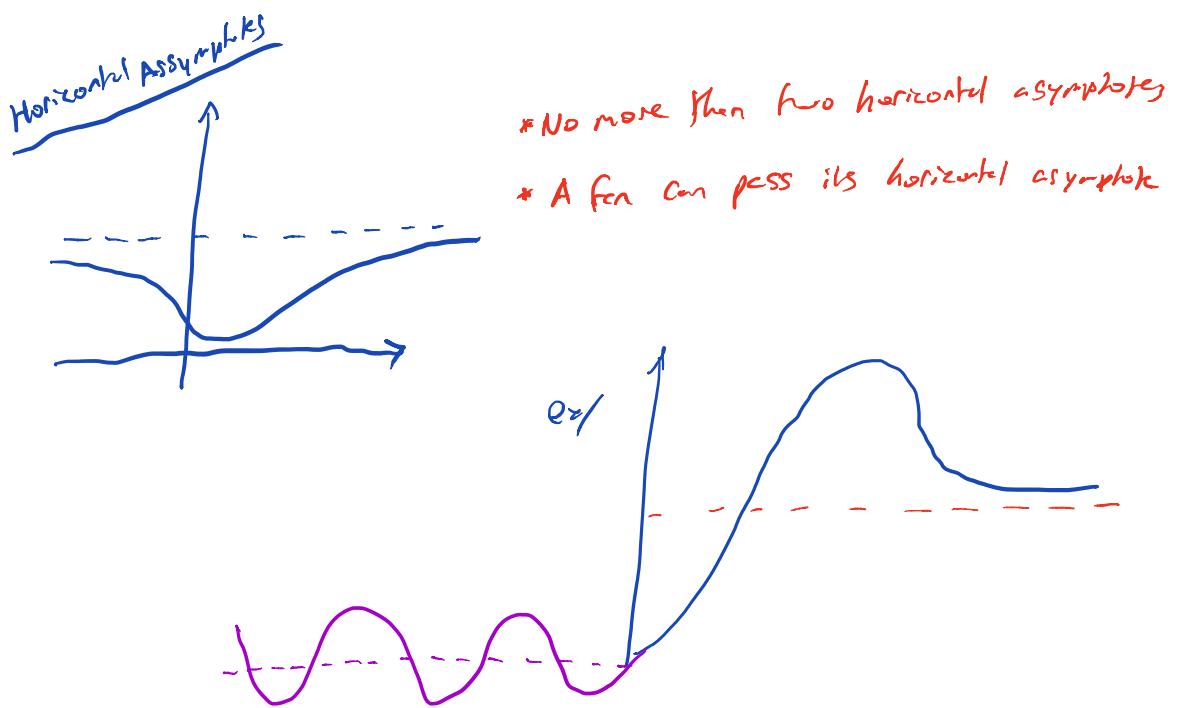
Vertical Asymptote

A function can never pass its vertical asymptote

ex/ consider the function

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$$





### Algebra of $F^{\text{cns}}$

e.g. Suppose  $f(x) = x^2$  &  $g(x) = x^2 - 1$ , define a new  $f^m$  by the rule:

$$y = f(x) + g(x) = x^2 + (x^2 - 1) = 2x^2 - 1$$

Name this  $f^m$  "f+g"  $(f+g)(x) = 2x^2 - 1$

$$(f-g)(x) = f(x) - g(x) = x^2 - (x^2 - 1) = 1$$

$$(f \cdot g)(x) = f(x) \cdot g(x) = x^2 (x^2 - 1) = x^4 - x^2$$

$$(7g)(x) = 7g(x) = 7(x^2 - 1)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{x^2 - 1}$$

### Composition of functions

We can also define a new function by "composing"  $f$  &  $g$  & we name this new function " $f \circ g$ "

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2$$

↑                   ↑  
outside      inside  
function      function

$$(f \circ g)(x) \neq (g \circ f)(x)$$

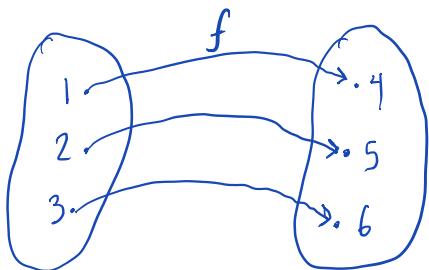
$$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)^2 - 1 = x^4 - 1$$

### INVERSE OF FUNCTIONS

eg/

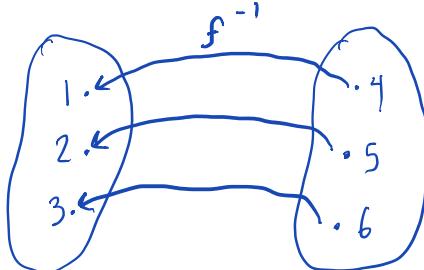
$$DOM = \{1, 2, 3\}$$

$$RANGE = \{4, 5, 6\}$$



$$RANGE = \{1, 2, 3\}$$

$$DOM = \{4, 5, 6\}$$



$$f^{-1}(f(x)) = x$$

for all  $x$  in  $Dom f$

$$f(f^{-1}(x)) = x$$

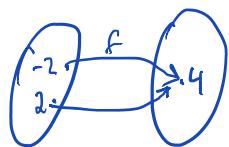
for all  $x$  in  $Dom f^{-1}$

$$\text{ex/ } f^{-1}(f(1)) = f^{-1}(4) = 1$$

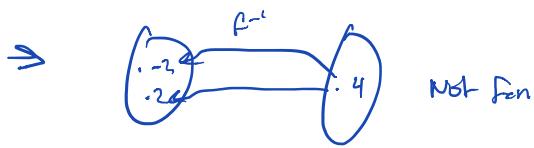
$$\text{ex/ } f(f^{-1}(4)) = f(1) = 4$$

$$\text{ex/ } g(x) = x^2$$

$$\text{Dom} = \mathbb{R} \quad \text{Range} = [0, \infty)$$



If we "reverse arrows" we get



only  $f^{\text{inv}}$  that have a one-to-one can have an inverse fun.

$$\text{ex/ consider } f(x) = \frac{x-2}{x+1} \quad \text{find } f^{-1}$$

$$\text{Let } y = \frac{x-2}{x+1} \quad (x \neq -1)$$

$$x = \frac{y-2}{y+1} \quad (y \neq -1)$$

$$x(y+1) = y-2$$

$$xy + x = y - 2$$

$$xy - y = -x - 2$$

$$y(x-1) = -x - 2$$

$$y = \frac{-x-2}{x-1} \quad (y \neq -1)$$

$$\text{So } f^{-1}(x) = \frac{-x-2}{x-1}$$

Ex/ Consider  $y = \sqrt{2-x}$ , find the inverse  $f^{-1}$

$$y = \sqrt{2-x} \quad \text{DOM: } x \leq 2$$

RANGE:  $y \geq 0$

$$x = \sqrt{2-y} \quad \text{RANGE: } y \leq 2$$

DOM:  $x \geq 0$

$$y = 2-x^2 \quad (x \geq 0) \leftarrow \text{Inverse } f^{-1}$$