Neutral Objects: Don't attract or repel

Charging: Charge objects by contact (rubbing)
↳ More rubbing → more charge
↳ Charged objects attract neutral ones

- Force exerted by charged objects is inversely proportional to distance

- Only two kinds of charges (+ve & -ve), like charges repel, opposites attract
↳ Charge is transferred by contact.

- Insulators - Do not conduct charge (charge doesn't move)

- Conductors - Charge easily flows through.

- Everything is made up of: Protons +e
Neutrons 0
Electrons -e

- Fundamental unit of charge: $e = 1.6 \times 10^{-19}$ C

- Charge is quantized: $Q = N_p e - N_e e = (N_p - N_e) e$
(is always an integer multiple of e) AND IS ALWAYS CONSERVED

- When two neutral objects are rubbed together, electrons are transfered from lower to higher electron affinity (how tightly a nucleus holds onto electrons)
↳ Friction breaks molecular bonds

Insulators	vs	Conductors
- High electron affinity		- Low electron affinity
- Electrons are localised		- Electrons redistribute throughout material "Sea of electrons"

- Charged object - Neutral object attraction:
When a charged object is brought close to a neutral one, the electrons in the neutral object are pushed away, creating dipoles

Polarisation: The seperation of +ve and -ve charges in a neutral object

In Insulators: The atoms themselves become polarised forming electric dipoles

Charging by Induction: Placing a charged object near a neutral conductor

Grounding: Connect a conductor to a larger conductor ($\overline{\underline{\underline{\under

Scalar Fields: Math functions that assign values to every point in space

→ Gravitational Field: Alteration space due to mass
→ Electric Field: Alteration of space due to charge

Vector Field: Math function that assigns a vector to every point in space

SF: $T(x,y,z) = 24^{\circ}C + \frac{0.1^{\circ}C}{3m} y$

VF: $\vec{w}(x,y,z) = (1m/s + 0.05m/s\ z)\hat{k}$

<u>Mathematical description of Electric Fields</u>

1) Some charge creates alterations in space

2) If a test charge q' is placed in this field then it experiences a force exerted by the field

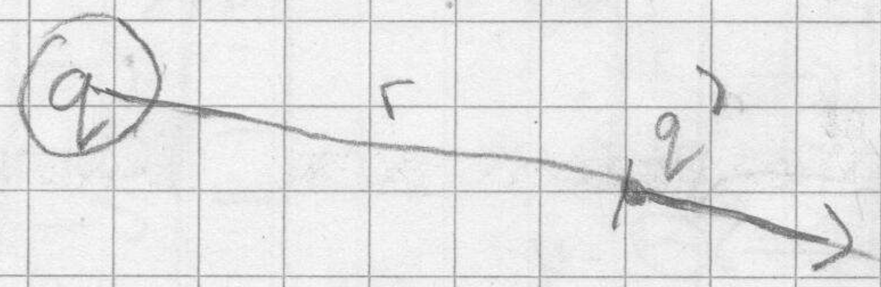
3) <u>The electric field created by the source charges is defined as</u>

$$\boxed{\vec{E}(x,y,z) = \frac{\vec{F}_{on\,q'}(x,y,z)}{q'} = \frac{F_{on\,q'}(x,y,z)}{q'}\hat{r}}$$

If a real charge Q is placed inside an electric field then the force exerted on the charge by the field is

$$\boxed{\vec{F}_{on\,Q} = Q\vec{E}}$$

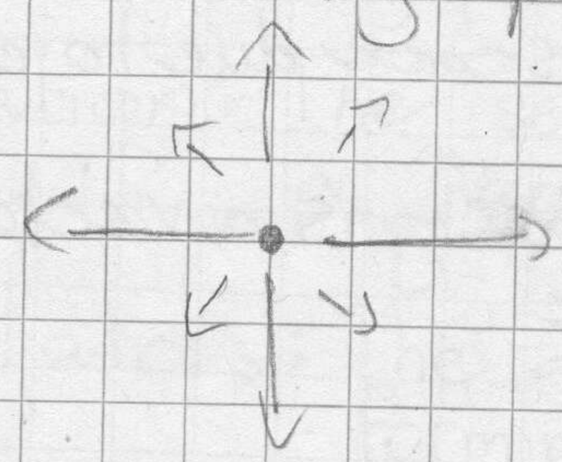
source



$$\vec{F}_{q\,on\,q'} = \frac{kqq'}{r^2}\hat{r}$$

Electric field Vector Diagrams

- Each point in space has an electric field vector associated with it
- If we take enough points, we can draw a vector field diagram



- The length of the vector represents the magnitude of the electric field at that point

Electric field line diagrams

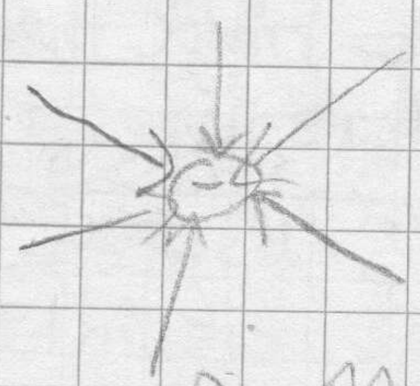
- Field line diagrams are lines that follow the pattern of field vector diagrams

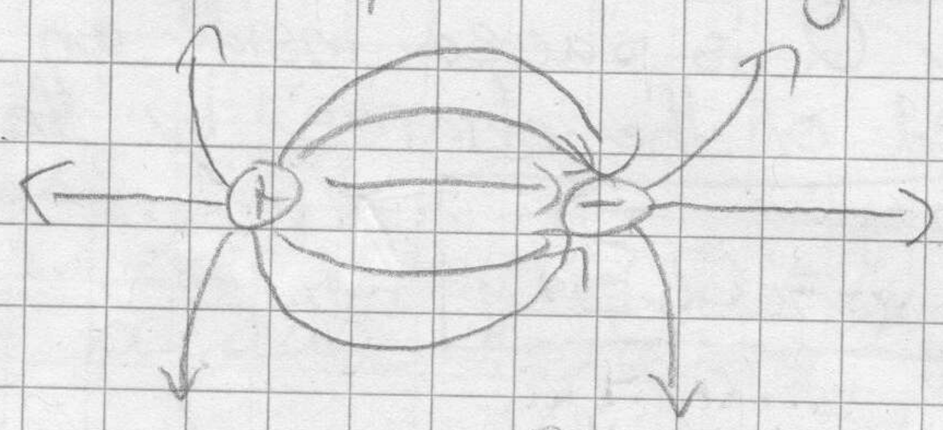
① Field lines are continuous curves drawn tangent to the field vectors and conversely the field vectors are tangent to the field lines.

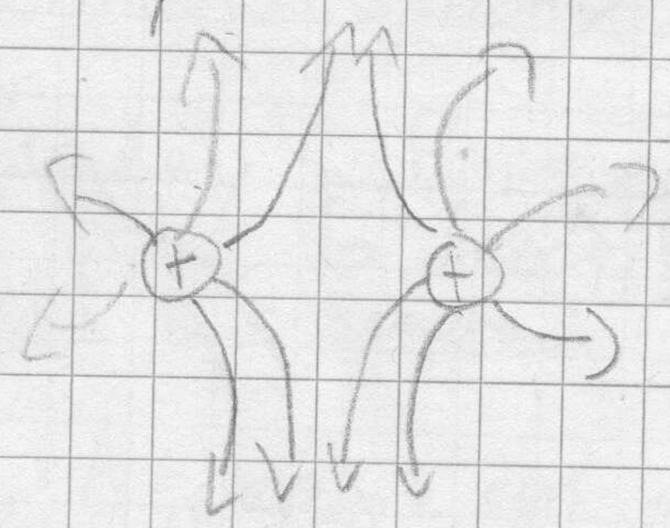
② Density of the lines corresponds to field strength

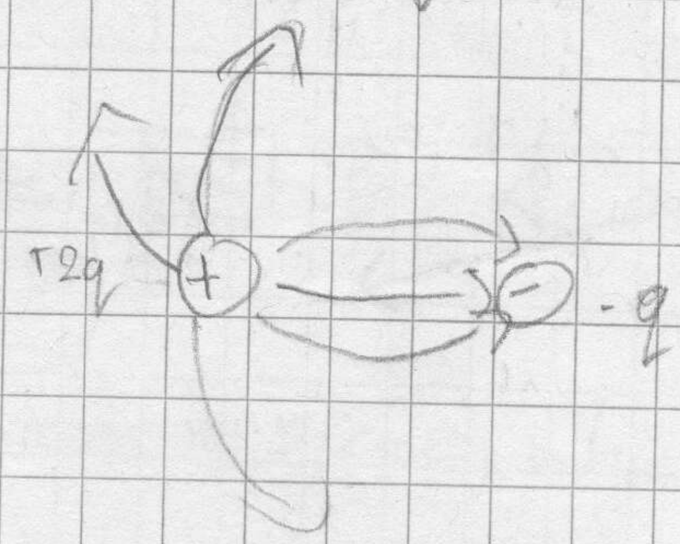
③ Lines never cross

④ Electric field lines start at positive charges and end at negative charges

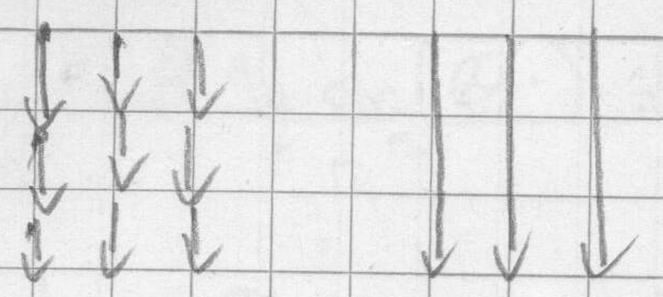




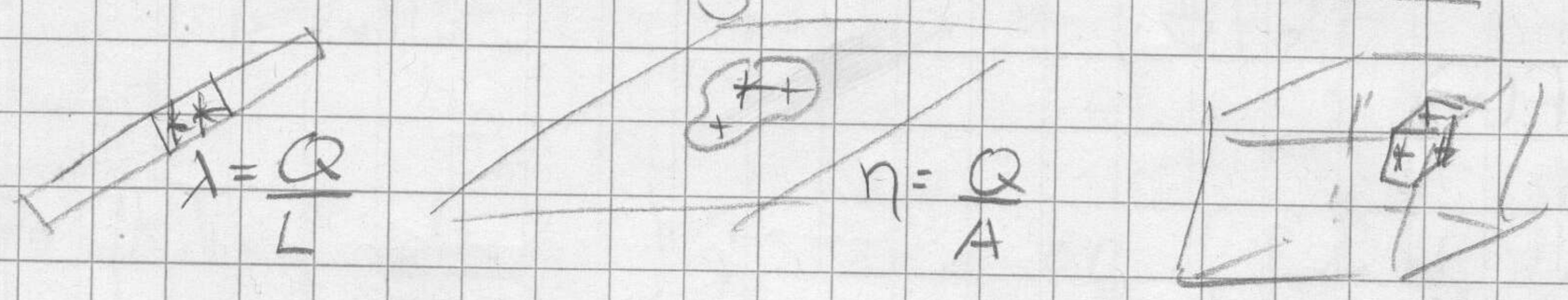




Uniform Electric Field: Field which has the same length and direction at every point in space

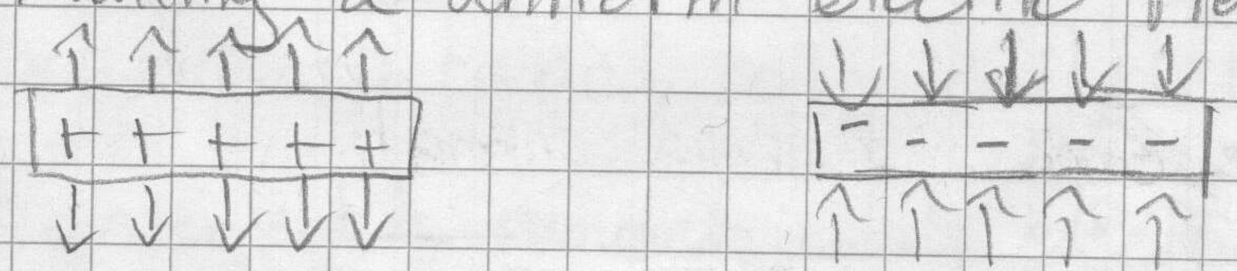


Continuous Charge Distributions



Def'n	Symbol	Units
Linear charge Density	λ	C/m
Surface charge Density	σ or η	C/m²
Volume charge Density	ρ	C/m³

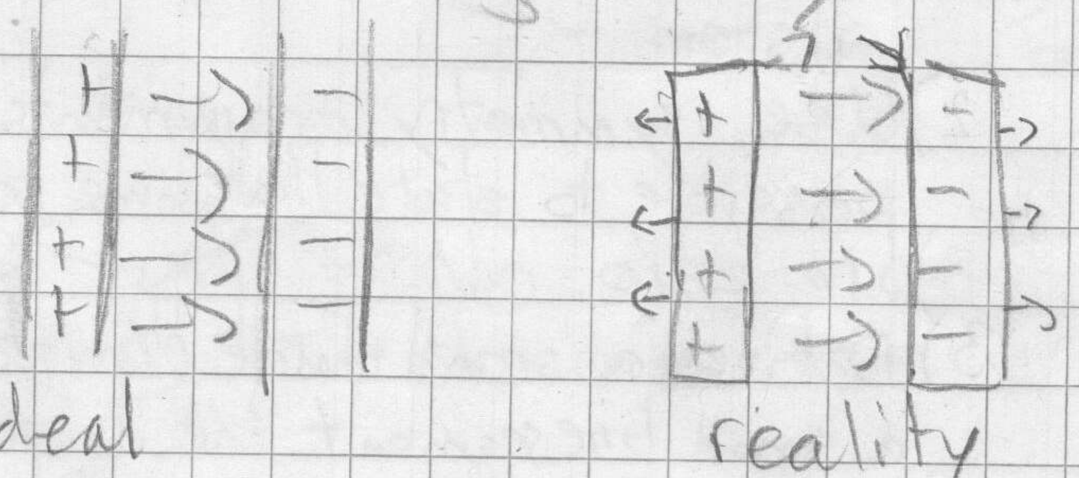
Making a uniform electric field: Infinite plane of charge



$$|\vec{E}| = \frac{\eta}{2\varepsilon_0}$$

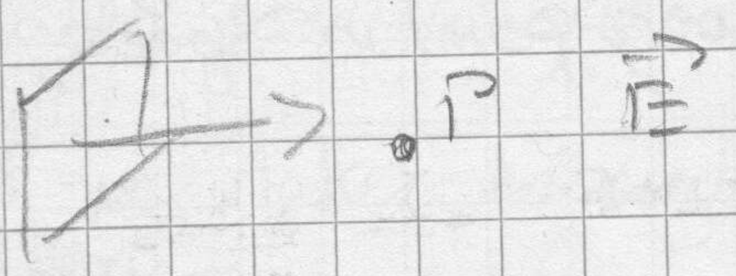
Ex: Capacitors

Parallel-Plate Capacitor: Two electrodes of equal magnitude of surface charge density but opposite signs



$$\vec{E}_{inside} = \left(\frac{\eta}{\varepsilon_0}\right)$$

Electric Field of a Continuous Field

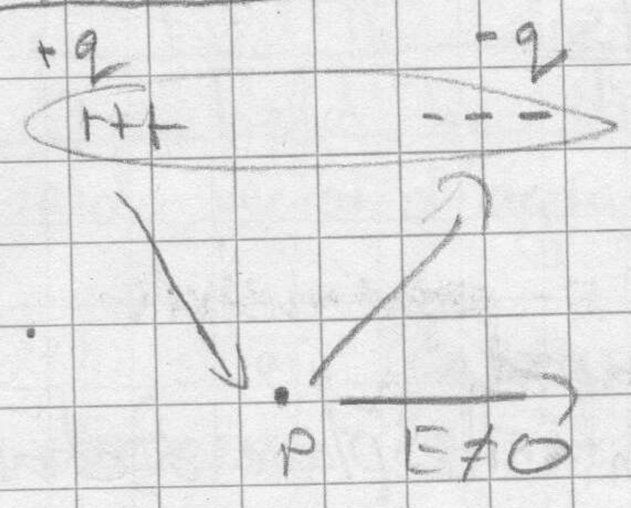


$$\vec{E} = k_e \left(\frac{q}{r^2}\right)\hat{r}$$

$$\vec{E} \approx \sum \left[k_e \left(\frac{\Delta q}{r^2}\right)\hat{r}\right]$$

$$\vec{E} = \lim_{n\to\infty} \sum_{i=0}^{n} F(x)$$

Considerations:



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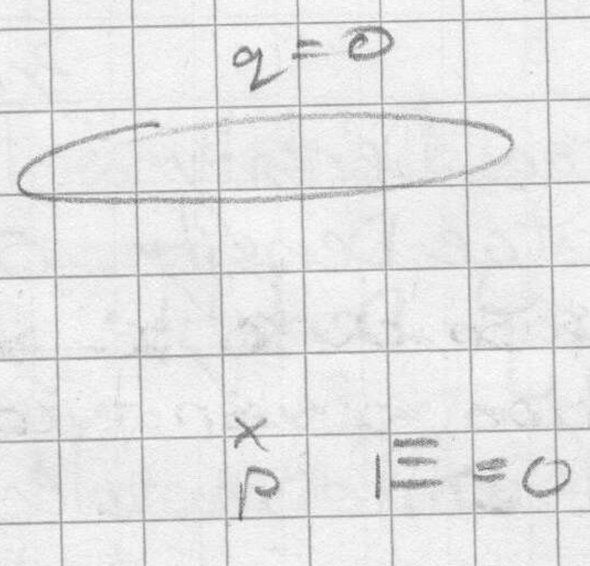
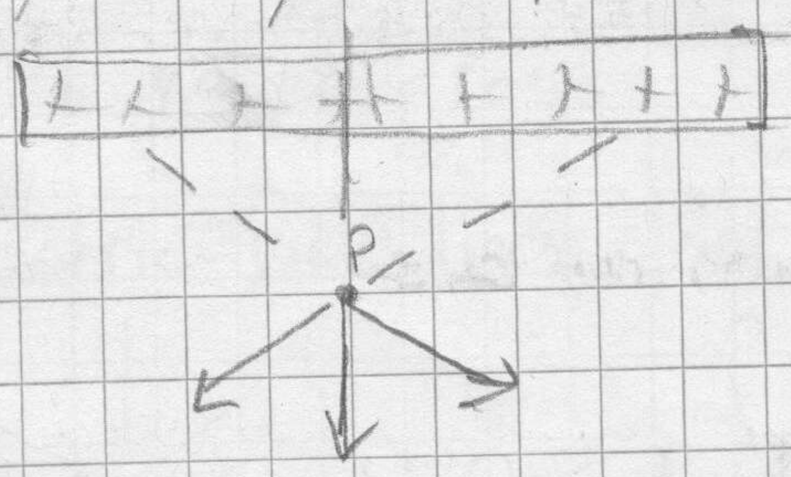
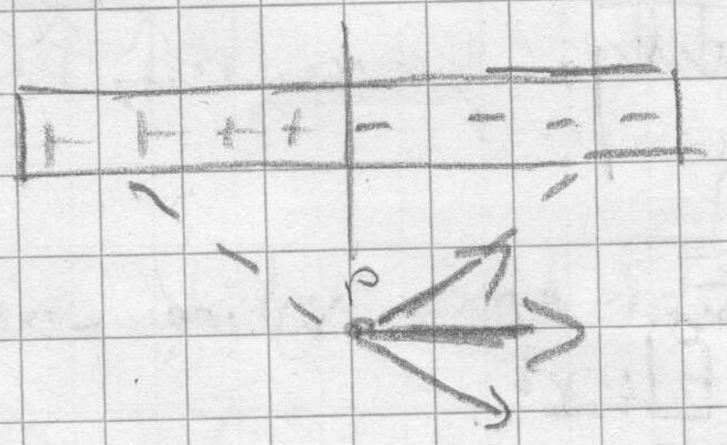
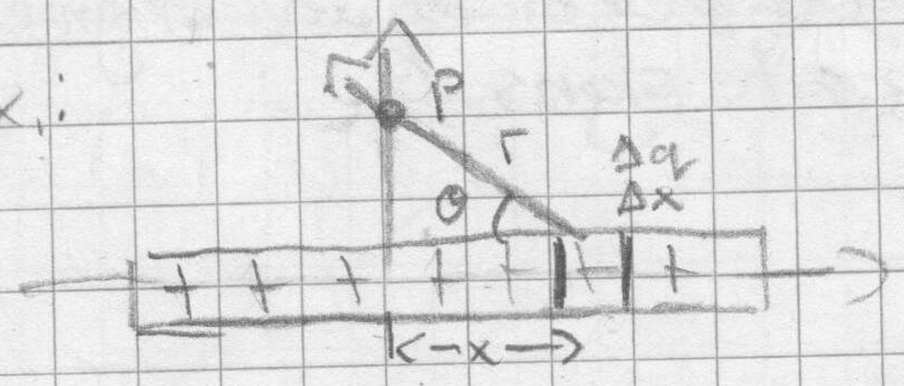


Symmetry is exploitable





Ex.:



$$\Delta \vec{E}_i = \frac{k\Delta q}{r_i^2}\hat{r}_i$$

$$\Delta E_{xi} = -\frac{k\Delta q}{r_i^2}\sin(\theta_i)\hat{i}$$

$$\Delta E_{yi} = \frac{k\Delta q \sin(\theta_i)}{r_i^2}\hat{j}$$

1) Define a coordinate system
2) Use symmetry arguments, if possible, to state that some components are zero
3) Define a small charge element Δq, a small linesegment and its position
4) Write down an expression for all relevant terms
5) Identify all variables that would change if you take a different segment (here $\theta, r, \hat{r}, x$). Put subscript i

$$\Delta E_{x_i} = -\frac{k\Delta q}{r_i^2}\cos(\theta_i) = \frac{-k\lambda x\Delta x}{(x^2+y^2)^{3/2}} \qquad \Delta q = \lambda \Delta x$$

$$\Delta E_{y_i} = \frac{k\Delta q \sin(\theta_i)}{r_i^2} = \frac{k\lambda y \Delta x}{(x^2+y^2)^{3/2}} \qquad \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\text{or } \sin\theta = \frac{x}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

let $\Delta x, \Delta y \to 0$

$$E_y = \int_{-L}^{L} \frac{k\lambda y\, dx}{(x^2+y^2)^{3/2}} \qquad \int \vec{\nabla} f\, dt$$

$$\vec{E} = \left(k\lambda y \int_{-L}^{L} \frac{dx}{(x^2+y^2)^{3/2}}\right)\hat{j} \qquad \text{or } \int F_x\, dx + F_y\, dy$$

<u>Symmetry</u>

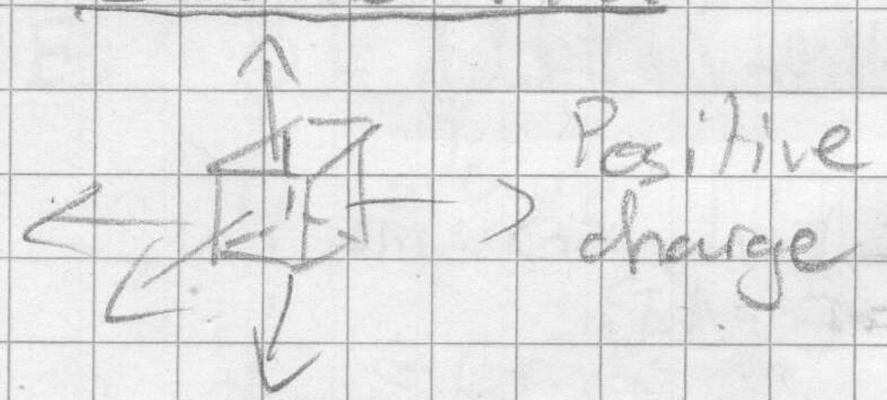
An object is said to be symmetric if a geometric transformation doesn't cause physical dmg.

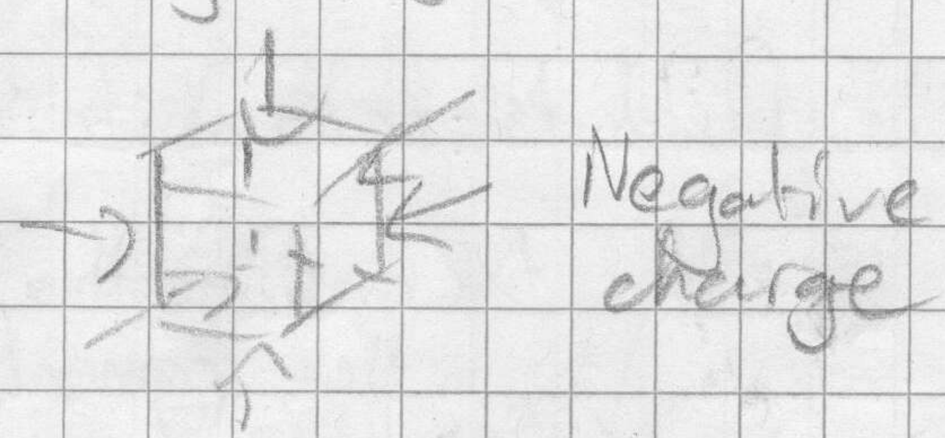
Ex.: Translational, rotational, reflectional symmetry

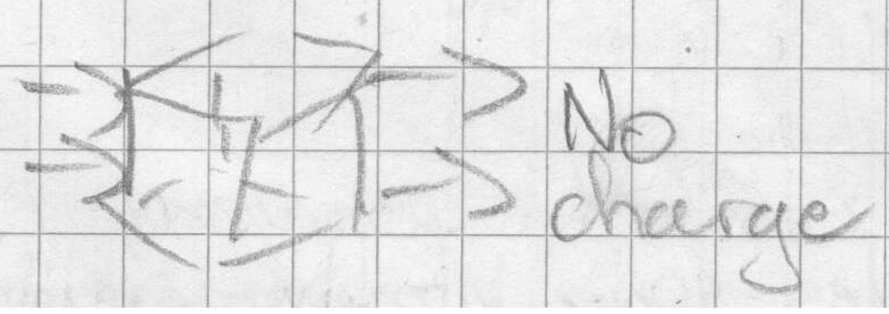
- If an object is symmetric, its electric field must have the same symmetry

<u>Flux</u>: Amount of "something" passing through an area

<u>Electric flux</u>:







Flux in General

What effects the amount of Flux

1) Area of net
2) Density of lines
3) Angle of net

Electric Flux: Depends on the field strength on the surface.

$$\Phi_E \propto E$$
$$\propto A$$
related θ

To capture the dependencies for electric flux, we define an area vector where: magnitude -> Area
Direction -> Normal perpendicular to surface

Electric Flux of a constant electric field and planar surface

$$\Phi = \vec{E} \cdot \vec{A}$$
$$= EA\cos(\theta)$$
$$= E_x A_x + E_y A_y + E_z A_z$$

What if the field is non-uniform electric field?

Consider this segment flat, so the flux in the segment is $\Delta\Phi = \vec{E}_i \cdot \Delta\vec{A}_i$

So $\Phi = \sum_i \vec{E}_i \cdot \Delta A$

Closed surface: 2D surface that divides space into two distinct
↳ Area vector points out

Integral over closed surface

$$\Phi = \oint_{surface} \vec{E} \cdot d\vec{A}$$

Gauss' Law: $\Phi = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

What is it good for?

- Links source charge and electric field
- For continuous charge distributions, we can find electric fields using the law and symmetries

B

When can it be applied?

-Peter- Requires high degree of symmetry

↳ Although the law is always true, the surfaces need to satisfy:

1) The magnitude of the electric field along the surface is constant (or zero)

2) $\vec{E} \cdot d\vec{A}$ simplifies due to the angles between the vectors being constant.

Ex: Spherical, cylindrical or planar symmetries

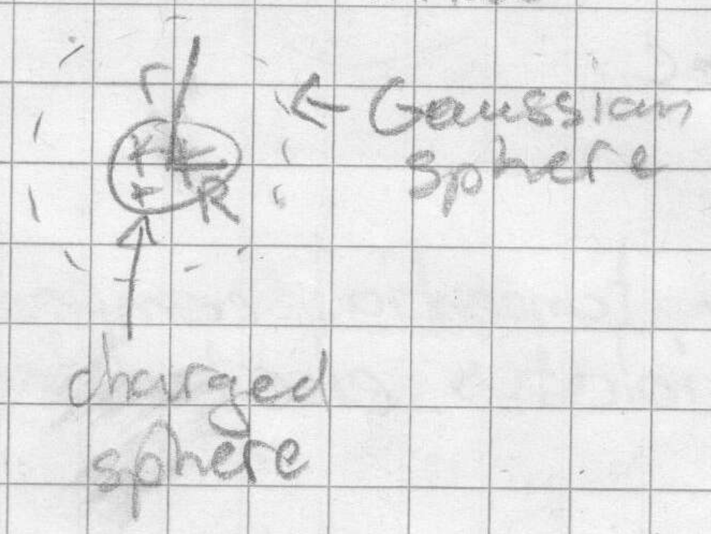
Method: Use symmetry arguments to determine the direction of the field

- Choose Gaussian surface that exploits symmetry and simplifies dot product
- Apply Gauss's surface

Ex: $\Phi = \oint_{\text{Gauss surface}} \vec{E}(r) \cdot d\vec{A} = \frac{Q}{\epsilon_0} = \oint E(r)\, dA$

$= E(r) \oint dA$

$= E(r)\, 4\pi r^2$



$E(r) = \frac{Q}{4\pi r^2 \epsilon_0}$

$= \frac{kQ}{r^2}$

$$\Phi = \oint \vec{E}(r) \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$E(r)\, 4\pi r^2 = \frac{Q \frac{r^3}{R^3}}{\epsilon_0}$$

$$E(r) = \frac{Qr}{4\pi R^3 \epsilon_0}$$

$$= \frac{Q \cancel{4\pi} r k}{\cancel{4\pi} R^3}$$

$$= \frac{Qrk}{R^3}$$

<u>Electric field Inside A conductor</u>

- The charges inside feel a force $\vec{F} = q\vec{E}$
- So the conductor becomes polarized, and an electric field is created inside the conductor in the opposite direction to the exterior electric field
- The electric fields inside and outside the conductor reach <u>Electrostatic Equilibrium</u>, so $\vec{E}_{in} = 0$

<u>Conductors in Electrostatic Equilibrium</u>

- Since $\vec{E} = 0$, then $\Phi = 0$
- $\Phi = \frac{q_{in}}{\epsilon_0}$ means no charge inside conductor
- Therefore all charge is on the surface.
- So what about $\vec{E}$ on the surface?
  - ↳ $\vec{E} \perp$ to the surface since any tangential component would create a current, which violates electrostatic equilibrium assumption

- But what if there's a hole?

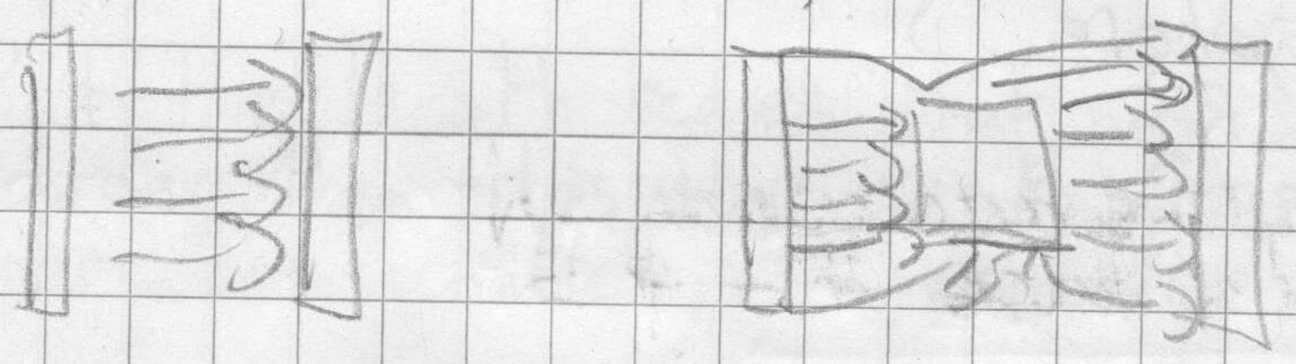
$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = 0$$

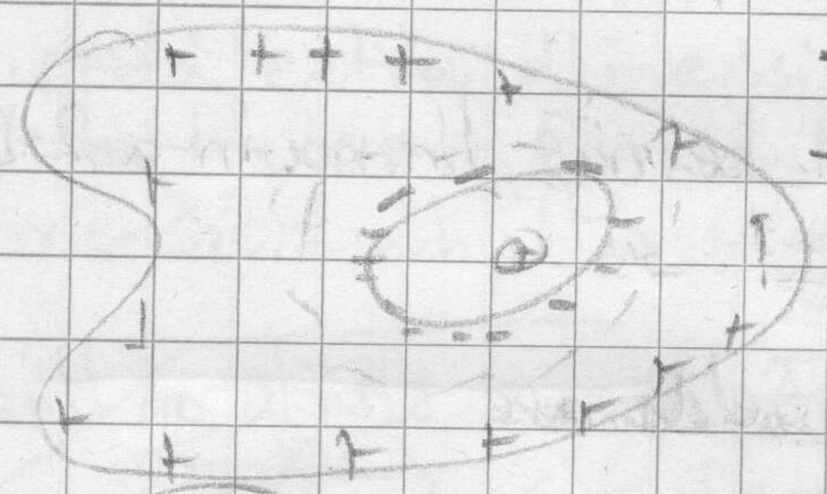
No charge on the inside

"Faraday Cage"

- When a conducting box with a hole in it is placed in an electric field, it blocks out the field.

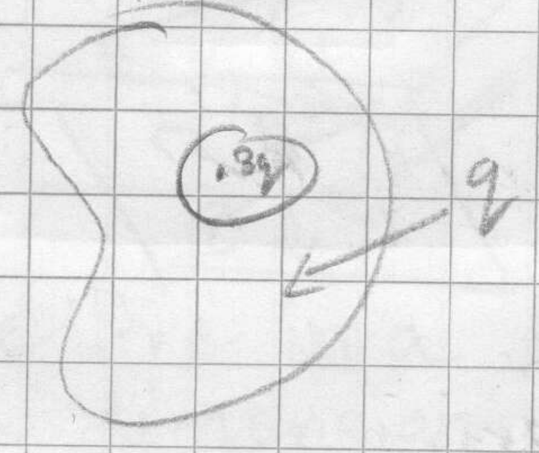


- But what if there's a point charge inside?



- The inside stays neutral
- The inner hole gets a charge opposite and equal to the point charge

Ex.:



$$Q_{inside} = -3q$$

$$Q_{tot} = q = Q_{in} + Q_{out}$$

$$Q_{out} = q - Q_{in} = q + 3q = 4q$$

Electric Potential Energy

In an isolated system $\Delta E_{mech} = \Delta K + \Delta U = 0$

K: Kinetic Energy
U: Potential Energy

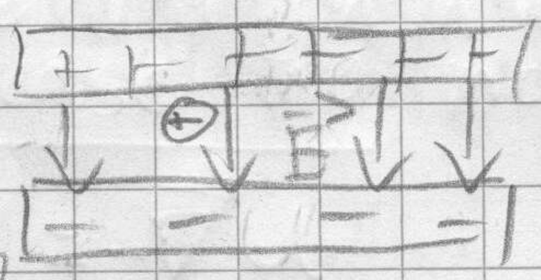
Work: Transfer of energy - $\boxed{W = \vec{F} \cdot \Delta \vec{r}}$

Uniform Forces and Uniform Fields

Gravitational force vs electric force

- Near surface of Earth $\vec{F}_g = m\vec{g}$

$\vec{g}$ is the gravitational field



In a parallel plate capacitor, a particle experiences force towards -ve charge

$\vec{F}_e = q\vec{E}$

$\vec{E}$ is electric field

Work done by gravitational (electric) fields

$$W_{grav} = \vec{F}_g \cdot \Delta \vec{y}$$
$$= F_g \Delta y \cos(0°)$$
$$= mg|y_f - y_i|$$
$$= mgy_i - mgy_f$$

$$W_{elec} = \vec{F}_e \cdot \Delta \vec{r}$$
$$= F_e \Delta r \cos(0°)$$
$$= qE|s_f - s_i|$$
$$= qEs_i - qEs_f$$

So $\boxed{U_{el} = qEs}$

↳ Electric potential energy of a point charge in an uniform electric field

For positive charges

higher U_{elec}
+ve charge
Lower U_{elec}

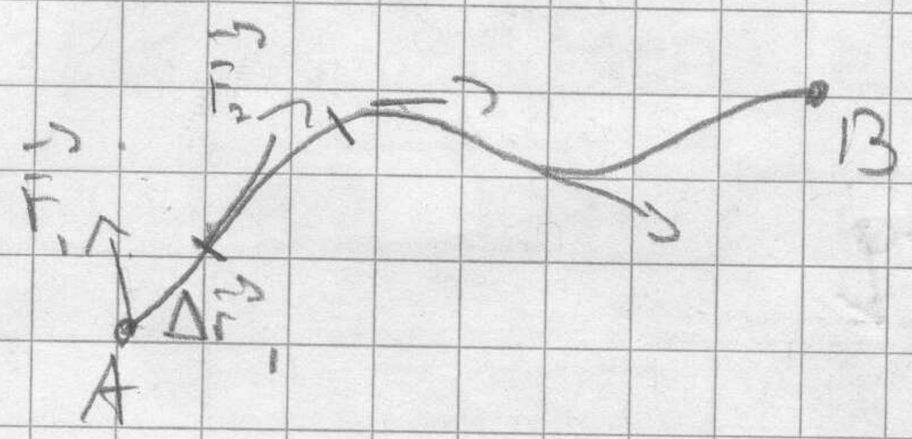
For negative charges

lower U_{elec}
-ve charge
Higher U_{elec}

- Electric force is conservative (path independent)

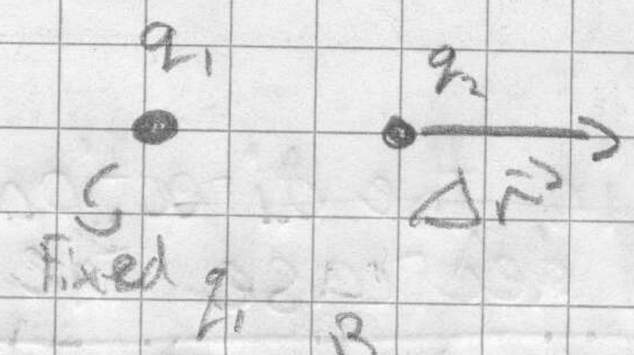
What is the electric potential energy stored between two point charges?

$\cancel{W_{el} = \vec{F}_{el} \cdot \Delta\vec{r}}$ → not a uniform field



$$W = \sum_i \Delta W_i$$

$$W = \int_{A_r}^{B} \vec{F} \cdot d\vec{s}$$



$$W = \int_A^B F\,dr$$

$$= \int_A^B \frac{kq_1q_2}{r^2}\,dr$$

$$W = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

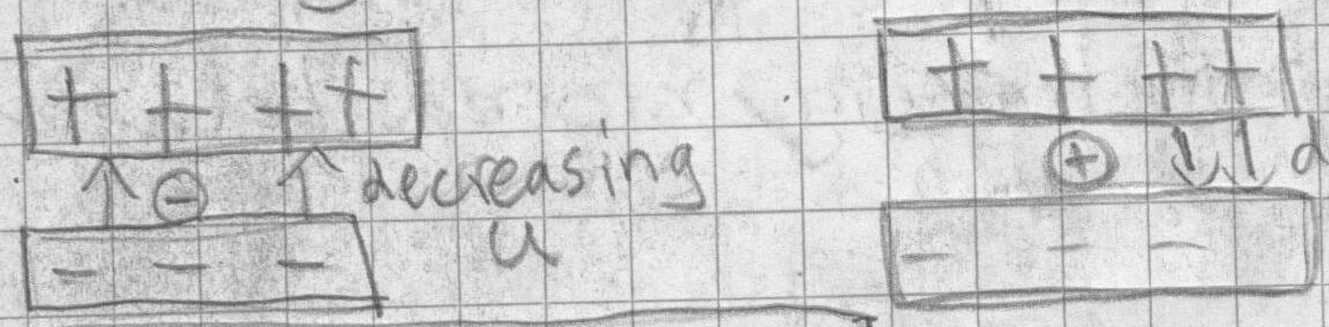
$$\boxed{U_{el} = \frac{kq_1q_2}{r}}$$

1) ⊕ ⊕ → decreasing U_{el}
2) ⊕ decreasing U_{el} ⊕
3) ⊖ ⊖ → decreasing U_{el}

When multiple point charges are present, the potential energy is

$$\boxed{U_{el} = \sum_{i<j} \frac{kq_iq_j}{r_{ij}}}$$

Finding forces from Potential Energies



$$\boxed{\vec{F}_{el} = -\frac{dU_{el}}{ds}\hat{s}}$$

So if we have a pt charge (+q)

$$\boxed{\vec{F}_{el} = -\frac{dU_{el}}{ds}\hat{r}}$$

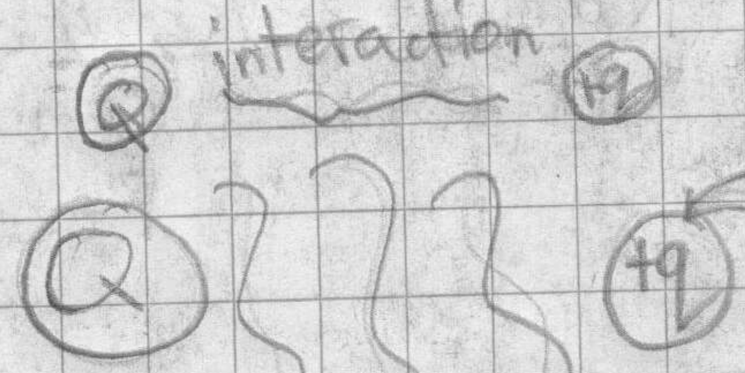
The force points in the direction in which the electric potential energy will decrease the most

$$\boxed{\vec{F}_{el} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = -\vec{\nabla}U}$$

Electric Potential, V

We know $\vec{E}(x,y,z) = \frac{\vec{F}_{ang}}{q}$

Similarly,



$$\boxed{V = \frac{U_{el\,Q+q}}{q}}$$

Energy interaction

Alters space to allow interaction

$$\boxed{V(x,y,z) = \frac{U_{q+source}(x,y,z)}{q}}$$

Scalar field

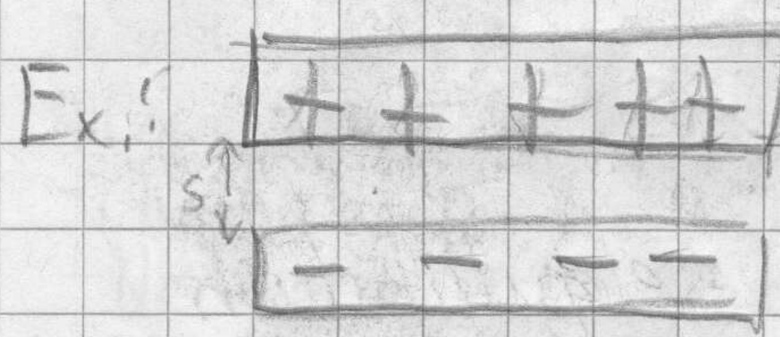
So $\boxed{U = qV}$

- Electric potential is sometimes just called the <u>potential</u>

- Potential is usually talked about as potential difference $\Delta V \rightarrow J/C$

- Potential difference is usually called <u>voltage</u>

Ex:



$U_{el} = qEs$

$V = \frac{U_{el}}{q} = Es$

$\Delta V = Es$

$\vec{E} = -E\hat{s}$

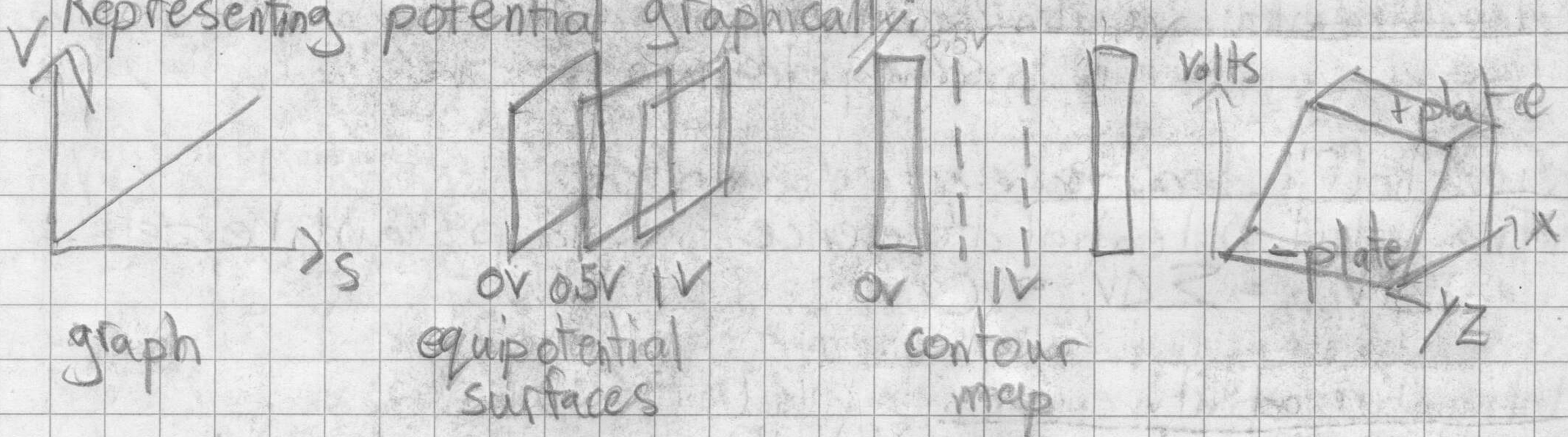
so $\vec{E} = -\frac{dV}{ds}\hat{s}$

<u>Connections between electric fields and electric potential</u>

We have $\vec{F}_{el} = -\vec{\nabla} U$, $\vec{F}_{el} = q\vec{E}$ and $U_{el} = qV$

so $\vec{E} = -\vec{\nabla} V$

Representing potential graphically:



<u>Electric Potential of a point charge</u>

Suppose there's some point charge q. Then its electric potential is

$$V = \frac{U_{q\,on\,q'}}{q'} = \frac{\frac{kqq'}{r}}{q'} = \boxed{\frac{kq}{r}}$$

Connecting electric potential and electric Fields

From work, we know $\Delta U_{el} = -\int_i^f \vec{F} \cdot d\vec{s}$

We also know that: $\vec{F}_{el} = q\vec{E}$ and

$U_{el} = qV$

So we get $\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

Potential in a conductor

- E-field inside a conductor in electrostatic equilibrium

$\vec{E} = -\vec{\nabla} V = 0 \quad \Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

$\therefore$ V is constant

$\therefore$ The potential is the same at all points on/in the conductor in electrostatic equilibrium

Conductors in contact

- Putting conductors in contact will put them at the same potential

physical grounding wire

Relevance to circuits: Any two points connected by a wire are at the same potential

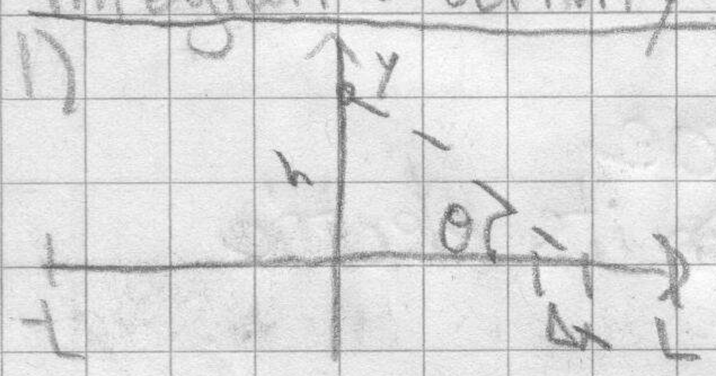
Kirchhoff's loop rule

- The total potential difference in a loop should be zero

i.e. $\Delta V_{loop} = \sum \Delta V_i = 0$

Integrative activity examples

1)



$\lambda(x) = 10x$

$\Delta V = \frac{k}{r}\Delta q = \frac{k(10x)\Delta x}{\sqrt{x^2+h^2}}$

$V = \int_{-L}^{L} \frac{10kx\,dx}{\sqrt{x^2+h^2}} = 0$

Capacitance

- Recall conductors in electric fields:
  - ↳ Charges in the conductor move → Create an $\vec{E}$ that cancels the outside
  - ↳ Conductor reaches electrostatic equilibrium ($\vec{E} = 0\hat{r}$)

Battery: device that prevents conductor from reaching equilibrium by making charges flow through it

↓

- Source of constant potential difference
- Operates on chemical reaction
- Chemical energy → Electric potential energy
- Not a source charge
- Maintains constant ΔV between terminals
- Gives U_{el} to e^- already in the wire

Electric Current

- Connecting a conducting wire across battery terminals results in a ΔV between the 2 ends of the wire
  - ↳ Therefore an electric field is created in the wire
- Electrons move from low to high potential, battery raises them to higher potential
  - ↳ Flow is called current

Parallel Plate Capacitors

- Between two parallel plates of equal and opposite charge,

$$E = \frac{\eta}{\epsilon_0}$$

- We also know $\eta \equiv \frac{Q}{A}$ and $\Delta V = Ed$, so we get:

$$\Delta V = \frac{Q}{A\epsilon_0} d$$

Charging a capacitor

- Consider connecting two uncharged plates to a battery:

  - At first, the potential across the wire isn't the same
  - There's an electric field inside the wire due to the battery
  - Charges are accelerated until the field from their displacement cancels out the initial field
  - In the end, the field inside the conductor is zero, so ΔV is constant

Other perspectives

Energy

- Decrease in chemical energy → Increase in U_{el} stored between plates

Loop rule

- Potential increase when going across battery → has to be an equivalent potential drop across the capacitor

Capacitance

Recall: $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ and $E = \frac{\Delta V}{d}$

$\therefore Q = \frac{A\epsilon_0}{d} \Delta V$

where the capacitance $C \equiv \frac{Q}{\Delta V} = \frac{A\epsilon_0}{d}$ for a parallel plate capacitor

Therefore, the amount of charge that accumulates on the plates is proportional to ΔV

↳ The constant for this is called capacitance

↳ capacitance is a physical property of the capacitor

- Units for capacitance for Farads (F) where $1F = 1 \frac{C}{V}$

Capacitors, what are they good for?

- Once a capacitor is charged, we can remove the battery and the charge remains
- If connected, it can now create a current

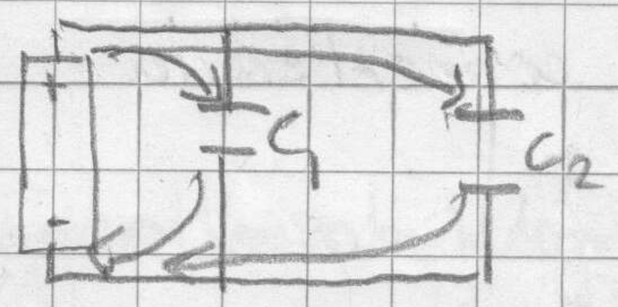
Applications:

- Short bursts of high current (camera flash, defibrillators)
- Current at regularly timed intervals (wipers)
- Current for a fixed amount of time (hush button on smoke detectors)

Capacitors in circuits

- In circuits, capacitors are drawn as: −|⊢
- Can be joined in parallel or series

In parallel

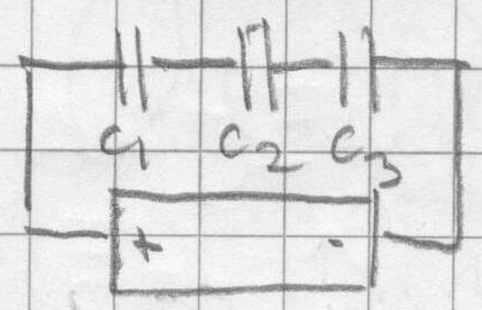


Capacitors in parallel have the same ΔV across them

$$C_{eq} = \frac{Q}{\Delta V_c} = \frac{Q_1 + Q_2}{\Delta V_c} = \frac{Q_1}{\Delta V_c} + \frac{Q_2}{\Delta V_c} = C_1 + C_2$$

$\therefore$ In parallel circuits, $C_{eq} = C_1 + C_2 + \ldots$

In Series



Capacitors in series have the same charge

$$\frac{1}{C_{eq}} = \frac{\Delta V_c}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$\therefore$ For circuits in series, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

Electrostatic Breakdown

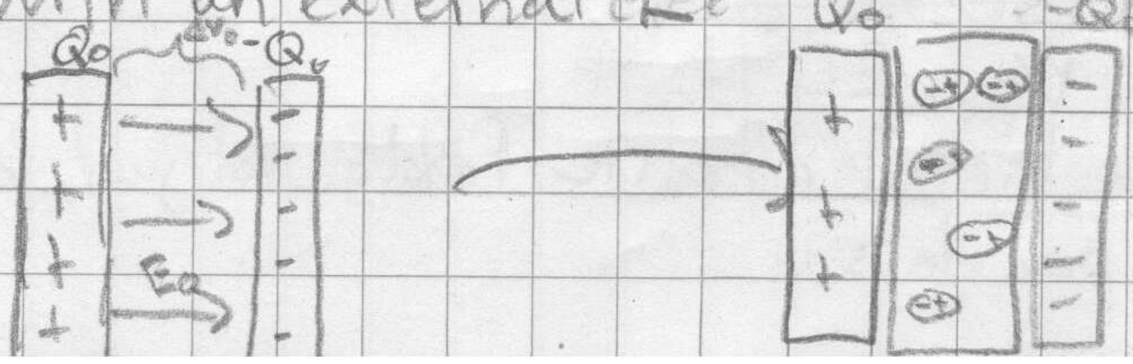
- Each material can sustain a maximum $\vec{E}$ before breakdown (sparking)
- This maximum $\vec{E}$ is called the dielectric strength (3×10^6 V/m in air)

Building Better Capacitors

- How can capacitance be increased:
  1) Decrease seperation
     Problem: Electrostatic breakdown
  2) Increase plate area:
     Problem: Big electronics aren't popular

Dielectrics

- Insulating material who's polar molecules can align themselves with an external $\vec{E}$

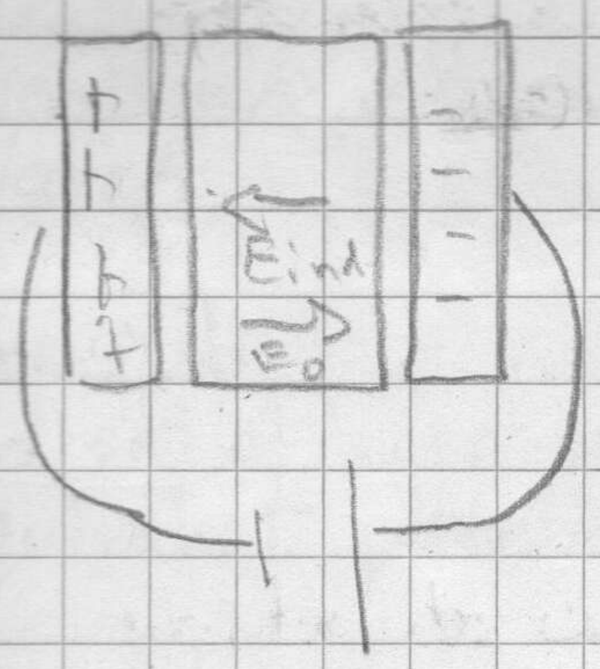


$E = E_0 - E_{in}$ $\Rightarrow E$ decreases

$C = \frac{Q}{\Delta V}$ $\Rightarrow \Delta V$ decreases

$\Rightarrow C$ increases

- Now when the capacitor is connected,



- ΔV doesn't change since its connected to battery
- So E doesn't change, and r doesn't change so Q has to increase
$\Rightarrow C$ increases

<u>Dielectric constant</u>: $K = \frac{E_0}{E}$

$\Delta V_0 = E_0 d$

$\Delta V = Ed = \frac{E_0 d}{K} = \frac{\Delta V_0}{K}$

so $K = \frac{\Delta V_0}{\Delta V}$

$C = \frac{Q}{\Delta V_c} = \frac{Q_0}{(\Delta V_0/K)} = K\frac{Q_0}{(\Delta V_c)_0} \Rightarrow C = KC_0$

<u>Energy stored in a Capacitor</u>

- Imagine charging an uncharged capacitor to a total charge Q
- By charging them, the plates gain energy, where

$dU = \Delta V\, dq$ for infinitesmal charge change

$dU = \frac{q}{C} dq$

$\int_{U(Q=0)}^{U(Q)} dU = \frac{1}{C}\int_0^Q q\, dq$

$U = \frac{1}{2C} Q^2 = \frac{1}{2} C(\Delta V)^2$

<u>Energy stored in an electric field</u>

$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left[\frac{A\epsilon_0}{d}\right][Ed]^2 = \frac{1}{2}[Ad]E^2\epsilon_0 = \frac{1}{2}V\epsilon_0 E^2$

we define energy definition $u_E = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

↳ True for all electric fields. Not just inside capacitors

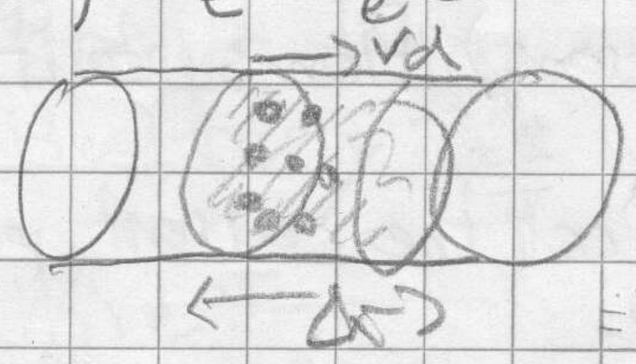
Current and Resistance

- Charge carriers: Charges that move in a conductor
  ↳ These are electrons in metals

- Electrons undergo random thermal motions
- When an electric field is applied, the electrons move in one direction at the drift speed, $v_d = 10^{-4}$ m/s

- Electron current, i_e: Number of electrons per second that pass through a cross-section of wire

or, $N_e = i_e \Delta t$ $n_e \equiv$ electrons/m^3



$\therefore N_e = n_e V = n_e A \Delta x = n_e A v_d \Delta t$

$$\Rightarrow \boxed{i_e = n_e A v_d}$$

Current and Current Density

- Conventional current direction: +ve to -ve
- What actually happens: electrons move from -ve to +ve

Current I is the total amount of charge Q that passes through a wire, thus:

$$\boxed{I \equiv \frac{dQ}{dt}}$$ Where I is in Amperes (A)
$1A \equiv 1 C/s$

- When I is constant: $\boxed{Q = I \Delta t}$

- Current is related to electron current:

rate of charges ← $\boxed{I = \frac{Q}{\Delta t} = \frac{eN_e}{\Delta t} = e i_e}$ → rate of electrons

Given this relationship, $I = e i_e = e n_e A v_d$

So we define current density J as

$$\boxed{J \equiv \frac{I}{A} = n_e e v_d}$$ where J is in A/m^2

Charge Conservation and Current

- Current is the amount of charge moving through a wire per second

Both these lightbulbs are equally bright

Why?

- To change I, n_e or v_d has to change
- The lightbulbs can't change n_e w/out violating conservation of mass and conservation of charge.
- For v_d, consider a water wheel analogy - the water wheel (lightbulb) doesn't decrease the rate at which water (electrons) flows into it

- But lightbulbs use energy, so they don't affect the current passing through them, but the current in the whole wire

Conductivity and Resistivity

- The properties of a material affect v_d
- Therefore the choice of wire material affects current density
- Simply,

$$\boxed{J = \sigma E}$$

where σ is the conductivity of the wire

From this:

1) Current is caused by an electric field
2) Current is linearly dependent on E
3) The choice of wire is also important

- Resistivity is thus: $\boxed{\rho = \frac{1}{\sigma}}$

Resistance and Ohm's Law

- Consider a wire of certain length L. Since E is constant in a constant diameter conductor,

$$E = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L}$$

From before, we know $I = JA = A\sigma E = \frac{A}{\rho} E = \frac{A}{\rho L} \Delta V$

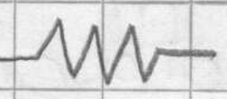
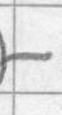
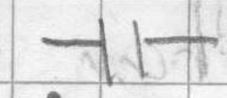
- We define resistance of a constant diameter wire:

$$R = \frac{\rho L}{A} \text{ where } R \text{ is in ohms } (\Omega) = V/A$$

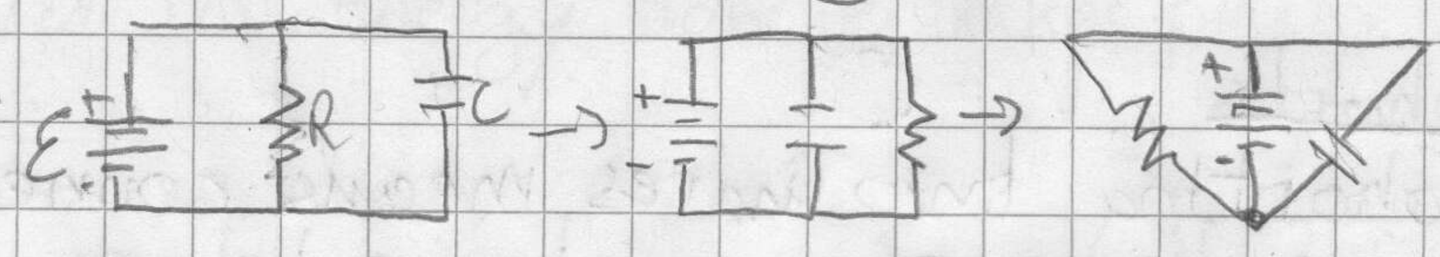
This gives us ohm's law: $I = \frac{\Delta V}{R}$

<u>Note</u>: Resistivity and Resistance are different. Resisitivity describes only the <u>material</u>. Resistance describes a specific <u>piece</u> of the material with a specific <u>geometry</u>.

Circuit Drawing

- Battery:
- Wire:
- Resistor: 
- Bulb: 
- Junction:
- Capacitor: 
- Switch:

*We can sketch and bend the wires however we like without changing the circuit.

Ex: 

Potential Across Circuit

- Each element in a circuit has a potential difference between its two sides
- Power supplies (or batteries) have a clearly labeled positive and negative (ie. Higher potential vs. Lower potential. This doesn't change.
- For resistors, the positive and negative side depends on the direction of current flow.
- Current flows from +ve to -ve
- We say there is a "potential drop" across the resistor, in the direction of current flow.

Resistors in series

- Two resistors that have no junctions between are said to be in <u>series</u>. The current through each resistor is the same.

$$\Delta V = IR_{eq} = I(R_1 + R_2 + \ldots + R_n) = IR_1 + IR_2 + \ldots + IR_n = \Delta V_1 + \Delta V_2 + \ldots \Delta V_n$$

- The current through resistors in series are equal, while the total ΔV is the sum of the ΔVs across each resistor

Resistors in parallel

- Two resistors that are connected to each other via only wires at both ends are said to be in <u>parallel</u>. The ΔV across each resistor is the same.

$$\Delta V = I R_{eq}$$

$$I = I_1 + I_2 + \dots + I_n = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \dots + \frac{\Delta V}{R_n}$$

$$I = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right)$$

$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

Switches

Switches either complete or break a part of a circuit

Shorts

- Shorting two wires means connecting them together with a wire. Hence zero resistance between those points. If the current has an option to go through zero resistance paths it will

Electromotive Force (EMF) ← Not actually a force

- EMF: Maximum potential difference that can be obtained by a device that does work on charge carriers (like battery/power supplies)

↳ $\Delta V = \frac{W_{chem}}{q} \equiv \mathcal{E}$ ← The rating on a battery

- For an ideal battery, $\mathcal{E} = \Delta V$ at the terminals
- In reality, a battery (or power supply) is made of materials that create internal resistance. → $\therefore \mathcal{E} \neq \Delta V$ at the terminals

$$\therefore \Delta V = \mathcal{E} - I r \quad \leftarrow \text{internal resistance}$$

$$\text{where } I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R + r}$$

Power Output of a battery

- Power is the energy output per unit time:

$$P = \frac{dE}{dt}$$

For batteries: $P = \frac{dU_{el}}{dt}$

$$P = \frac{dq}{dt}\mathcal{E}$$

$$P = I\mathcal{E}$$

Power dissipated in a resistor

- When charges move in a material, they collide with particles in the material
- These collisions transfer energy to the particles, and thus to the material
- This means that energy is being taken away from the current
- Power is the rate at which this energy transfer takes place

The change in energy of some charge dq is:

$dE = dq\,\Delta V$

$dE = I\,dt\,\Delta V$

$\frac{dE}{dt} = I\Delta V$

$P =$

$$\boxed{P = I\Delta V = I^2R = \frac{\Delta V^2}{R}}$$

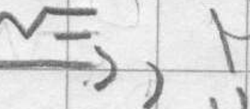
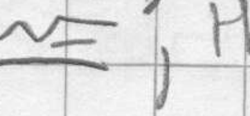
Kirchhoff's Rules

- Sometimes a circuit can't be represented by a single resistor Req. Kirchhoff's rules allow us to analyze the circuit

Kirchhoff's junction rule: $\sum I_{in} = \sum I_{out}$

" Loop rule: $\Delta V_{loop} = \sum \Delta V_i = 0$

Potential gain/loss across a resistor

- If  , then $-\Delta V$
- If  , then ΔV

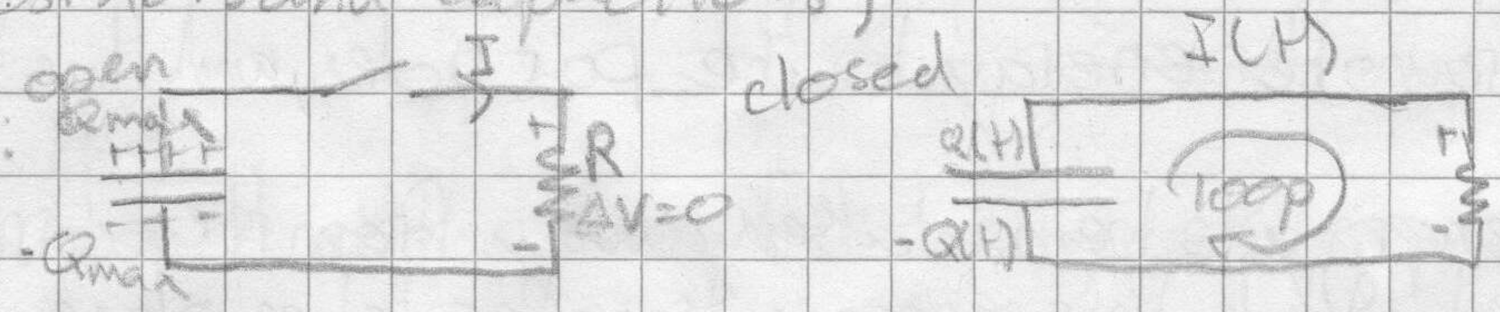
Using Kirchhoff's rules

- Label every junction w/ a letter
- Choose directions for your currents arbitrarily (if you're wrong you'll get a -ve current)
- Apply junction rule at every junction
- Apply loop rule to as many loops as necessary
- Be sure to label clearly
- Get solving.

RC Circuits

- It can be useful to control the rate at which a capacitor is charged or discharged. Simplest way is to use RC circuits (resistors and capacitors)

Eg:



$$\Delta V_C - \Delta V_R = 0$$

$$\frac{q(t)}{C} - I_{(t)} R = 0$$

$$\frac{q_c(t)}{C} - R\frac{dq_c}{dt} = 0$$

$$\int_{q(t')=q(t)}^{q} \frac{dq}{q} = \int_{t'=0}^{t} -\frac{1}{RC}\,dt$$

$$q(t) = q(0) = Q_{max}$$

$$\ln q_t - \ln Q_{max} = \frac{-t}{RC}$$

$$q(t) = Q_0 e^{-t/RC}$$

let $\tau = RC$

$$q(t) = Q_0 e^{-t/\tau}$$

$$I(t) = -\frac{q(t)}{dt} = \frac{Q_0}{RC} e^{-t/RC} = \frac{\Delta V}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

Magnetism

Magnetic Poles

- The only time we have seen forces on magnets so far is when there are other magnets present
- Earth is like a giant magnet: Geo North -> magnetic south
  Geo South -> magnetic north.

Monopoles and dipoles

- Cutting magnets in half results in two weaker magnetic dipoles
- A north pole w/out a south pole (or vice versa) is called a magnetic monopole
  ↳ haven't been observed

- Magnetism and currents are linked -> "right hand rule" for direction

Magnetic field

- Magnetic field is denoted with $\vec{B}$
  ↳ Created at all points in space surrounding current carrying wire
  ↳ Is a vector at each point
  ↳ Exerts a force on the poles. At the north pole, in direction of $\vec{B}$
  At south is opposite.

Representing Magnetic fields

- Magnetic field lines outside a magnet go from the N pole to the S pole
- Magnetic field lines have no beginning or end! They are a closed loop

Are there 2 kinds of magnetic fields?

- Magnetic fields due to: permanent magnets
  - current carrying wires
- Turns out they're both magnetism

Source of Magnetic Fields: A moving point charge

- A current causes magnetic fields
- Biot-Savart Law:

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \hat{r})}{r^2}$$

Sources of magnetic fields: A current carrying wire

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$$

$$q\vec{v} \rightarrow \Delta Q\vec{v} = \Delta Q\frac{\Delta\vec{s}}{\Delta t}$$

$$= \frac{\Delta Q}{\Delta t}\vec{s}$$

$$= I\Delta\vec{s}$$

If we consider a current segment small enough to be treated like a moving charge point

$$\vec{B} = \frac{\mu_0}{4\pi}I\left(\frac{\Delta\vec{s}\times\hat{r}}{r^2}\right)$$

$$d\vec{B} = \frac{\mu_0}{4\pi}I\frac{d\vec{s}\times\hat{r}}{r^2} \rightarrow \boxed{\vec{B}_{wire} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}}$$

← Extended segment of wire

Magnetic field due to a long current carrying wire:

$$\vec{B}_{wire} = \frac{\mu_0 I}{2\pi d}\{\text{direction using RHR}\}$$

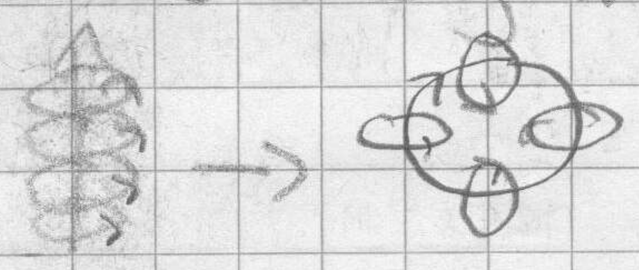
d: distance from wire

For a segment of a complete current loop: $\vec{B} = \frac{\mu_0 I}{4\pi R}\theta$

For a complete loop: $\vec{B} = \frac{\mu_0 I}{2R}$

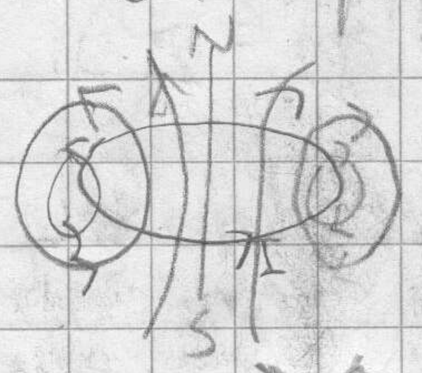
Current loops as magnetic dipoles

- If you take a long wire and bend it into a loop, you get

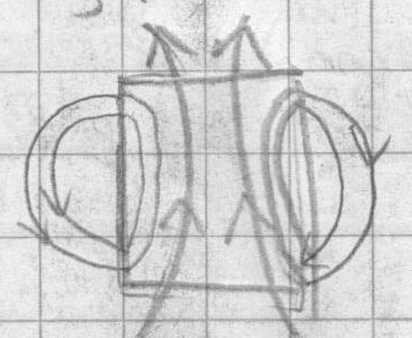


The magnetic field along the axis is perpendicular to the plane of the loop

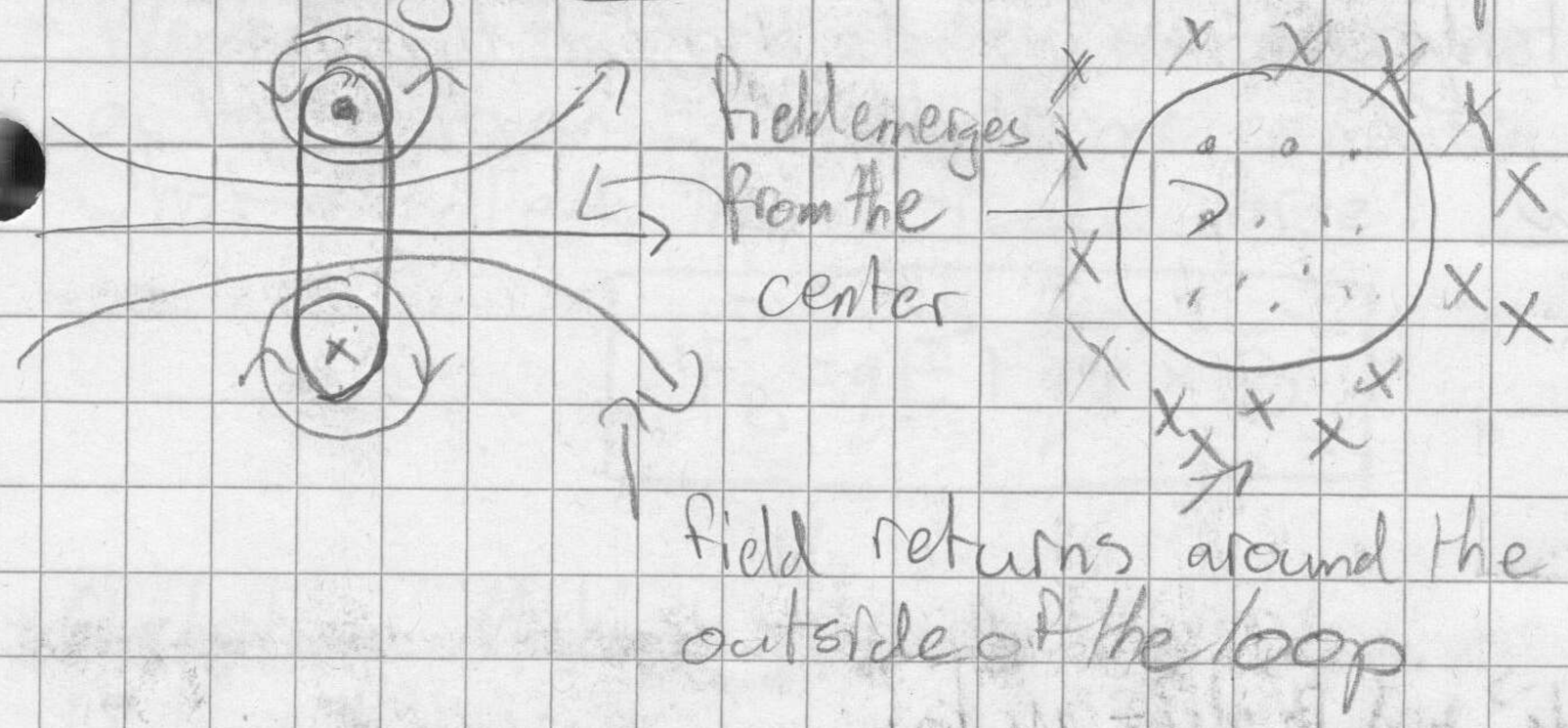
So for a loop we get



which resembles the magnetic field of a magnet:



The Magnetic field of a Current loop



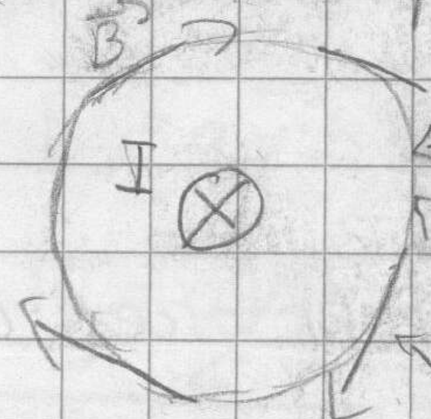
Ampere's law

- Another way to relate the magnetic field to the current is Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{inc}$$

- We call the path an "Amperian Loop"
- I_{inc} is the current inside the loop

Current loops



Circular Amperian Loop a distance d from the wire

On the loop $\|B\|$ is constant

So: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{inc}$

$B \oint ds = \mu_0 I_{inc}$ $(\vec{B} \parallel d\vec{s})$

$B(2\pi d) = \mu_0 I_{inc}$

$$\boxed{B = \frac{\mu_0 I_{inc}}{2\pi d}}$$

Solenoids

- A solenoid is a helical coil of wire with a current passing through
- We assume that our solenoids are tightly wrapped enough that the coils are perpendicular to the axis of the solenoid

- As we add more loops to the solenoid, the field inside becomes more uniform and parallel to the solenoid's axis, and the field outside tends to zero

Magnetic Force on a Moving Charge

- Ampere noticed that two current carrying wires exert forces on each other
- If we consider that one wire produces a magnetic field and the other is a moving charge, we infer a moving charge experiences a forces in a magnetic field

- $|F_B| \propto q, v$
- $|F_B| \propto |\vec{B}|$

- When they're parallel, $|F_B| = 0$
- The direction of $\vec{F_b}$ is always $\perp$ to $\vec{B}$ and $\vec{v}$
- The force on a +ve charge is opposite to that of a -ve charge

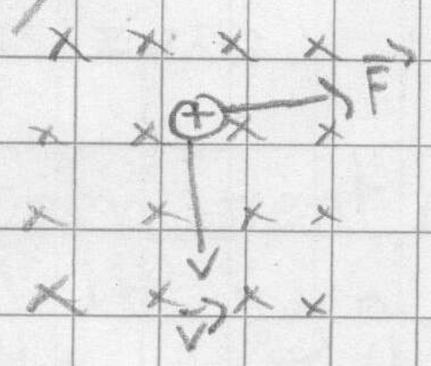
Together, we get $\boxed{\vec{F_B} = q\vec{v} \times \vec{B}}$

Lorentz Force

- We're still talking about charges, so it will also feel a force in an $\vec{E}$-field. If we combine those two forces we get what is called the Lorentz Force:

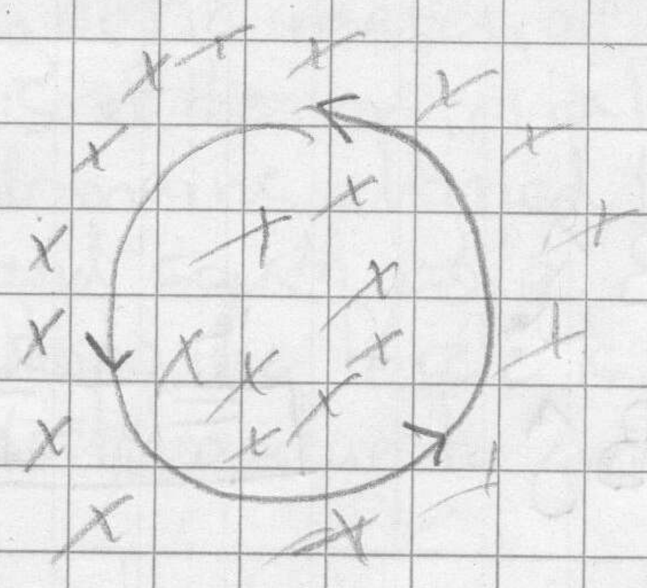
$$\boxed{\vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}}$$

Cyclotron Motion



Regardless of the direction of the motion, $\vec{F} \perp \vec{v}$

So it becomes

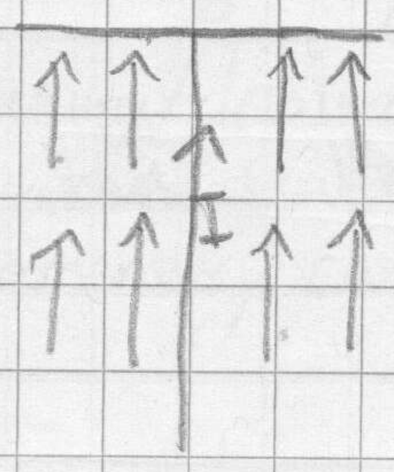


$\vec{F} = q\vec{v} \times \vec{B}$ → A constant force perpendicular to the motion results in circular motion

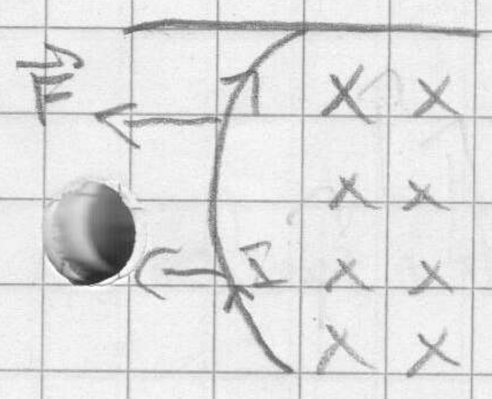
$F = qvB = \frac{mv^2}{R}$

$$\boxed{R = \frac{mv}{qB}}$$

Magnetic Force on a Current Carrying Wire



There's no force on a current-carrying wire parallel to a magnetic field.

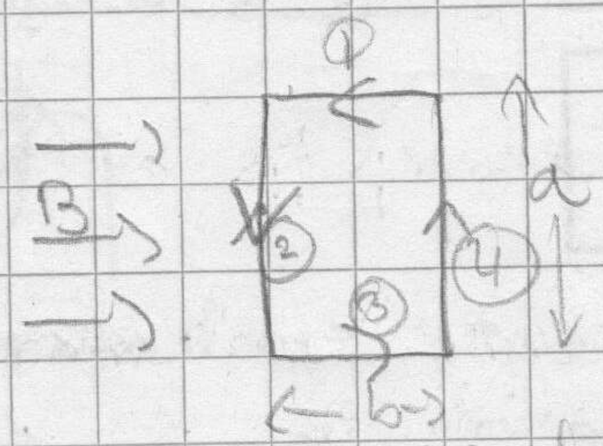


- A current $\perp$ to the field experiences a force in the direction of the RHR
- If a wire of length l contains a current $I = q/\Delta t$, it means a charge q must move along its length in a time $\Delta t = l/v \Rightarrow Il = qv$
- Since $\vec{F} = q\vec{v} \times \vec{B}$, $\boxed{\vec{F}_{wire} = I\vec{l} \times \vec{B}}$

Torque on a rectangular current carrying loop

What is the net-force on this current carrying loop in a uniform B-field?

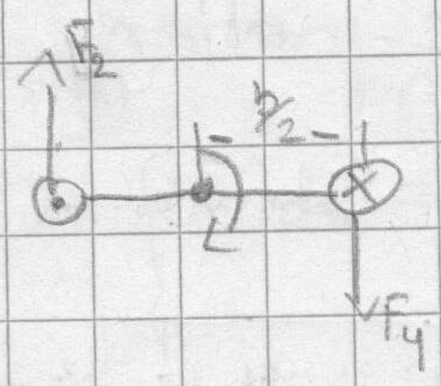
No net force



Does this mean there's no effect on the loop?

$F = IaB$

NO



$\vec{\tau}_2 = [|r| F \sin\alpha \, \hat{j}] = \frac{b}{2} IaB \hat{j}$

$\vec{\tau}_4 = [|r| F \sin\alpha \, \hat{j}] = \frac{b}{2} IaB \hat{j}$

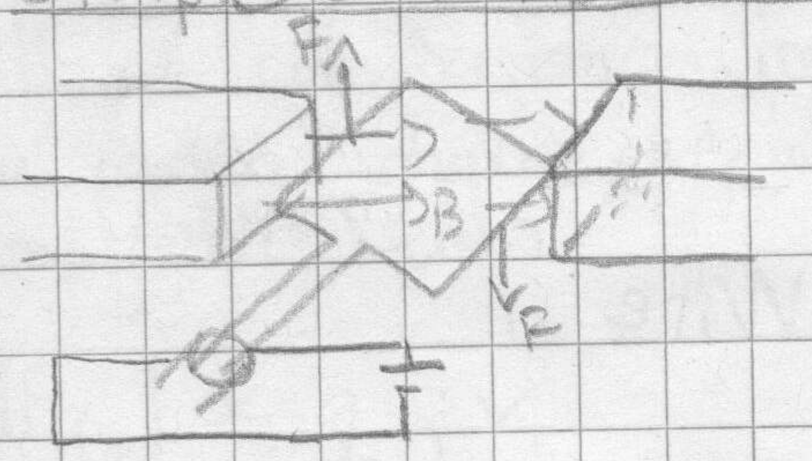
$\vec{\tau}_{net} = I ab B \hat{j}$

$= I \underbrace{A}_{\text{Area}} B \hat{j}$

If circuit is tilted:

$\boxed{\vec{\tau}_{net} = I \vec{A} \times \vec{B}}$

Simple electric Motors



Magnetism - Atomic Level

- There is a current in any atom
- Electrons around the nucleus
- Spin of particles
- Every atom is like a small magnet

Ferromagnets

- In most materials, the atomic magnetic dipoles point in random directions

- This results in a net magnetic field of zero (a piece of material that is not magnetic)
- In iron (and some other materials) the atoms interact in such a way that the dipoles tend to point in the same direction (A Ferromagnet)
- The B-fields of these dipoles add up to make a strong magnet
- However, a chunk of iron is made up of small magnetic domains
- The domains point in random directions, meaning the net B-field is zero

Permanent Magnetics

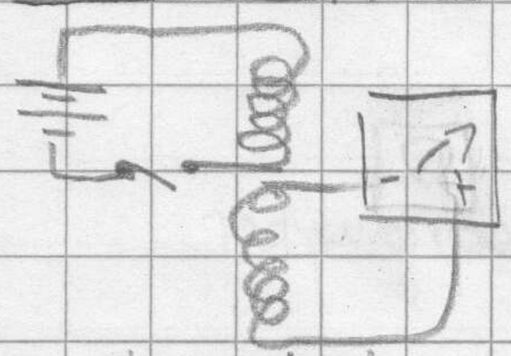
- Putting a ferromagnetic material into a magnetic aligns the magnetic domains of the materials
- With the magnetic domains aligned, the material is now a magnet with a north and south pole.
- The polarity is such that the force is attractive
- Analogous to polarization due to an electric field.

Whether or not a material remains in a state w/ its magnetic domains aligned depends on its crystalline structure.
If they do stay aligned then it is a Permanent Magnet

Faraday's Law - Electromagnetic Induction

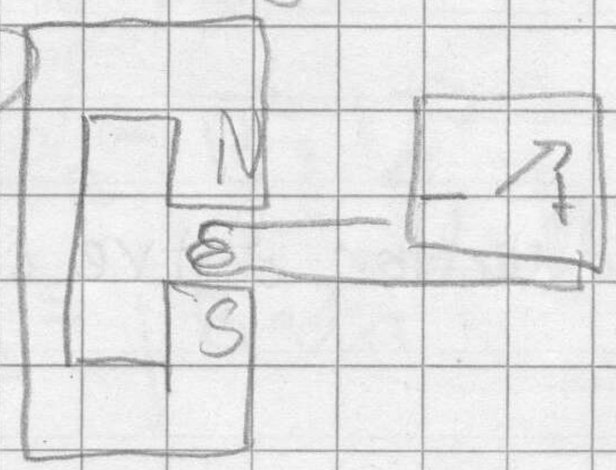
Observations

(1)



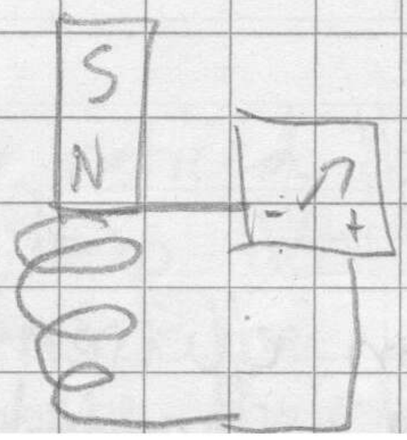
Opening/closing switches creates momentary current

(2)



Pushing/pulling the coil in a $\vec{B}$ creates current

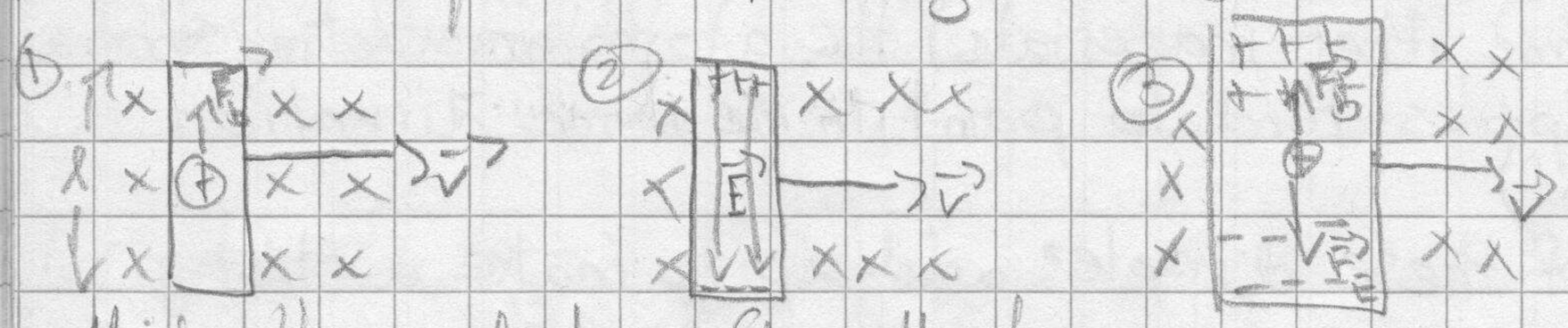
(3)



Moving a bar magnetic through coil creates current

Motional EMF

- Consider a positive charge in a conductor in a B-field



Moving the conductor exerts a force on ⊕

Since the charge is moving $\vec{E}$-field is created

Eventually $F_b = F_e$, charges stop moving and current ceases

Charge seperation by moving a conductor in a B-field:

$\boxed{\text{motional EMF}}$

versus a <u>chemical</u> EMF in a battery

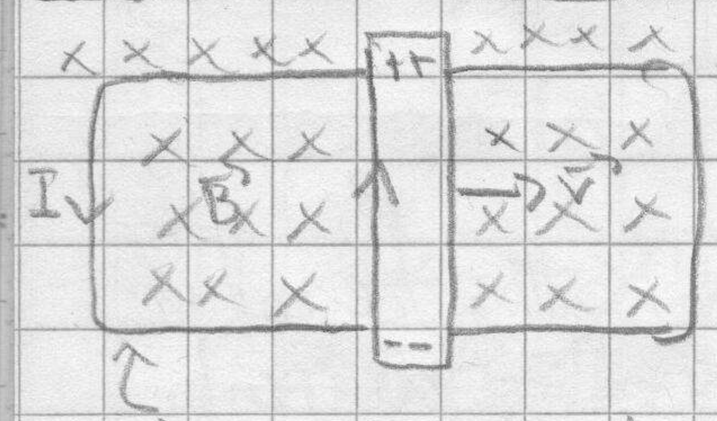
$F_b = F_e$

$qvB = qE$

$vB = \frac{\Delta V}{l}$ ← Can use $\Delta V = Es$ for capacitors because there's no E-field outside conductor

$\Delta V = vlB$

Induced Current in a Circuit



← Circuit with motional EMF as battery

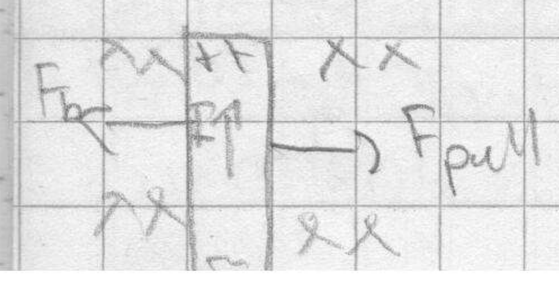
Induced current moves in direction of +ve charges in conductor

$\Delta V = vlB$

$IR = vlB$

$I = \frac{vlB}{R}$.

- Current in B-field ⇒ Force on current carrying wire
- $\vec{F}_b$ on motional EMF → Force is needed to keep it moving at constant v.



$$F_{pull} = F_b = I l B$$
$$= \frac{v l B}{R} l B$$

$$F_{pull} = \frac{v l^2 B^2}{R}$$ ← In the direction of motion of the bar

Eddy Currents

- Consider pulling a sheet of metal through a B-field
- Two "whirlpools" of current begin to circulate in the solid metal called eddy currents

The magnetic force on the eddy currents is a retarding force
↳ form of magnetic braking

Magnetic Flux

Similarly to Electric Flux,

$$\Phi_B = \vec{A} \cdot \vec{B}$$ Units: Webster (Wb) = 1 Tm²

Area vector Magnetic field

Non-uniform Field

Need to add up flux at parts:

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Lenz's Law

- In 1834, Heinrich Lenz noticed that there's an induced current in a closed, conducting loop iff the B-flux through the loop is changing. The direction of the induced current is such that the induced current opposes the change in flux

Using Lenz's Law

Tactics:

1. Find direction of applied B-field
2. Find how flux changes
3. Find the direction of an induced current that opposes the change in flux:
   - Increasing flux: points opposite to B-field
   - Decreasing flux: points in same direction as B-field
   - Steady flux: No induced B-field

Ways to change magnetic flux:

1) The B-field through the loop changes
2) The loop changes in area or orientation relative to the field
3) The loop moves into or out of the magnetic field

Faraday's Law

- For a current to be produced by Lenz's Law, there needs to be an EMF:

$$I_{induced} = \frac{\mathcal{E}}{R}$$

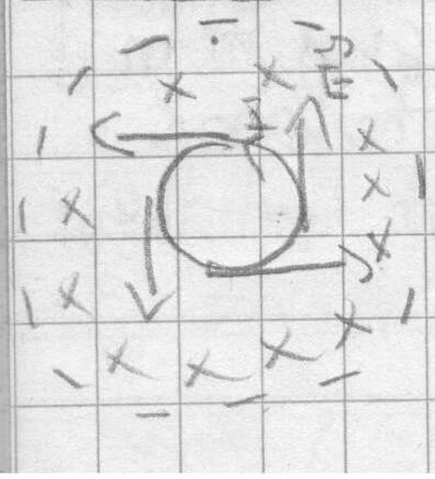
$$\boxed{\text{Faraday's Law: } \mathcal{E} = \left|\frac{d\Phi_B}{dt}\right|, \text{ direction from Lenz's Law}}$$

Multiple Loops

- If there's more than one loop (i.e. a solenoid), the induced EMF is multiplied by the number of turns.

$$\boxed{\mathcal{E} = N\left|\frac{d\Phi_B}{dt}\right|}$$

Induced Fields



- B-field is increasing in figure
- This creates an induced current counterclockwise by Lenz's Law
- Something has to act on the charge carriers to make them move so we infer that there must be an <u>induced electric field</u> tangent to the loop at all points

- When the B-field is increasing in a region of space, we can define a closed loop which is $\perp$ to the B-field
- Faraday law specifies:

$$\boxed{\oint \vec{E} \cdot d\vec{s} = A \left| \frac{dB}{dt} \right|}$$

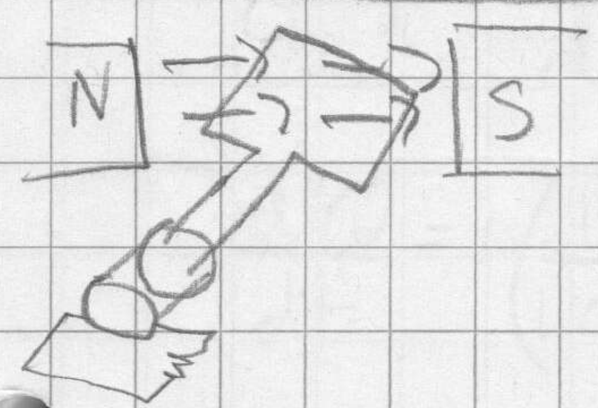
Faraday's Law Rewritten

- Combine Faraday and Lenz's Laws:

$$\boxed{\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \Phi_m}$$

↳ $\mathcal{E}$ opposes change in flux

AC Generators

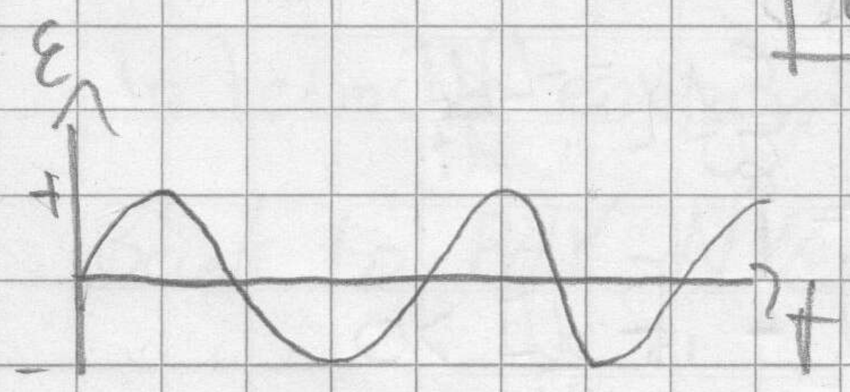


$$\Phi_B = \vec{A} \cdot \vec{B}$$
$$= AB\cos\theta$$

$$\Phi_B = AB\cos(\omega t)$$

$$\mathcal{E} = N \left| \frac{d}{dt} \Phi_B \right| = N \left| \frac{d}{dt} AB\cos\omega t \right| = \omega ABN \sin\omega t$$

$$\boxed{\mathcal{E} = \omega ABN \sin\omega t}$$



← Alternating current

Self-Inductance

- When a switch is closed there's a change in flux → induced current opposite to the original current

- There isn't actually 2 currents: More like opposition to change in current
  ↳ Back EMF
- This phenomenon is called self-inductance

Inductance

- The induced EMF is proportional to change in Φ_B:

$$\mathcal{E} \propto \frac{d\Phi_B}{dt}$$

and $\Phi_B \propto I$

The induced EMF is proportional to the rate of change of current:

$$\boxed{\mathcal{E} = L \left| \frac{dI}{dt} \right|}$$

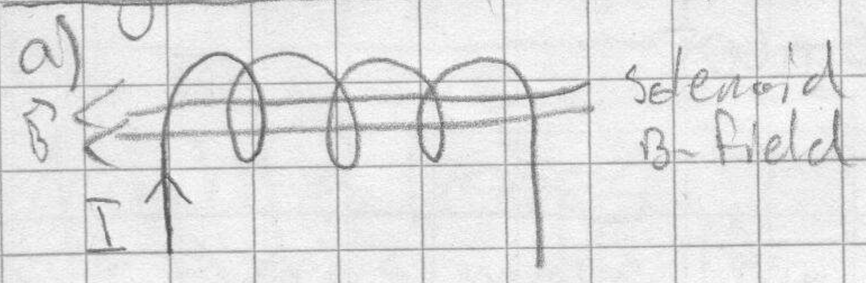
↳ Proportionality constant called <u>inductance</u>

According to Lenz's Law, induced EMF opposes the change in flux

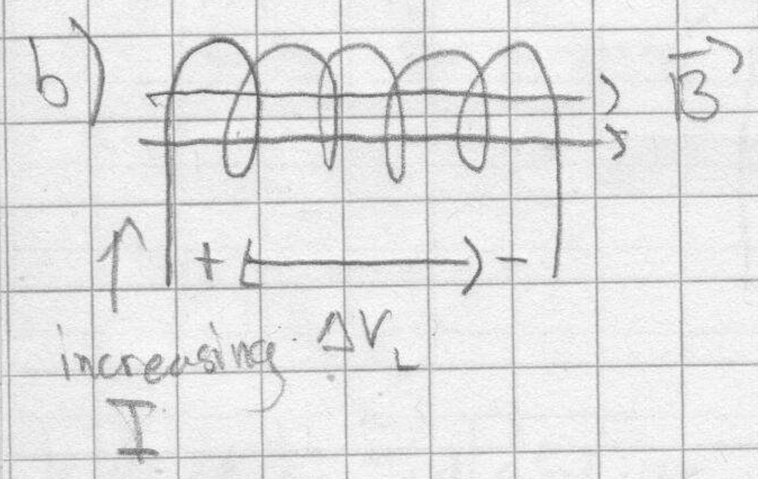
$$L = \frac{\mathcal{E}}{\left| \frac{dI}{dt} \right|}$$

Sign of Potential Difference Across an Inductor

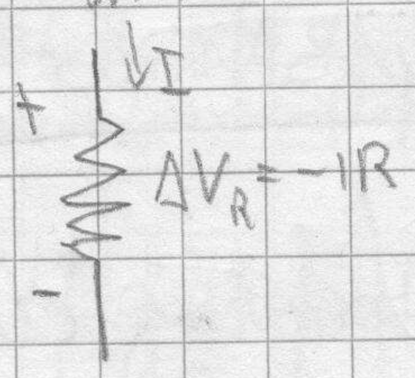
a)



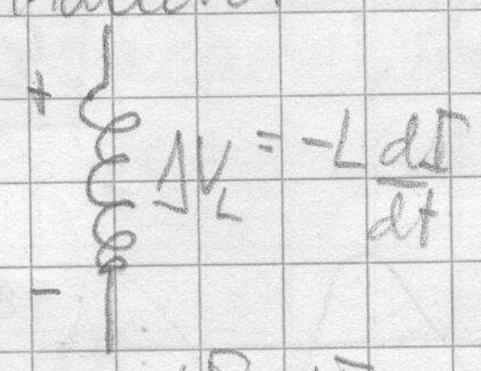
b)



Resistor



Inductor



ΔV_L − if $\frac{dI}{dt} > 0$

ΔV_L + if $\frac{dI}{dt} < 0$

Inductance of a Solenoid

From Faraday's law, $\mathcal{E} = -N\frac{d\Phi_B}{dt}$

For inductors, $|\mathcal{E}| = \left| L\frac{dI}{dt} \right|$

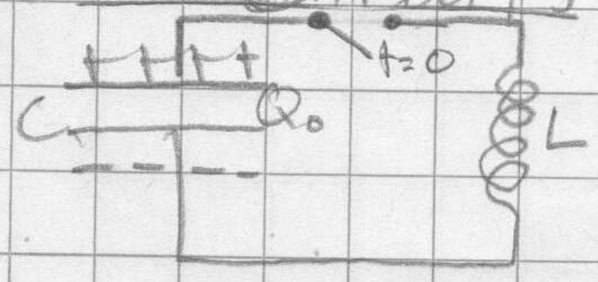
In a solenoid, $\vec{B} \perp \vec{A}$ so $\Phi_B = BA$

$$B = \mu_0 \frac{N}{l} I$$

Together, $\left|N\frac{d\Phi_B}{dt}\right| = \left|\mu_0 \frac{N^2 A}{l}\frac{dI}{dt}\right|$

so $$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

LC Circuits



KLR: $\Delta V_C + \Delta V_L = 0$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \qquad I = -\frac{dQ}{dt} \Rightarrow \frac{dI}{dt} = -\frac{d^2Q}{dt}$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt} = 0$$

$$\frac{d^2Q}{dt} = -\left(\frac{1}{LC}\right)Q$$ ← Simple Harmonic Motion!

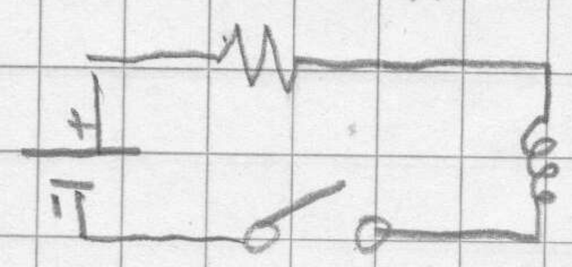
$$\omega = \sqrt{\frac{1}{LC}} \rightarrow Q(t) = Q_0 \cos\omega t$$

$$I = -I_{max}\sin(\omega t) \qquad I_{max} = \omega Q_0$$

In terms of energy: $U_{el} \rightarrow U_B \rightarrow U_{el}$

Back to PHY-NYC: Just like waves, if a current is applied at the right frequency, can achieve resonance

RL Circuits



KLR: $\mathcal{E} - RI - L\frac{dI}{dt} = 0$

$$\frac{dI}{RI-\mathcal{E}} = \frac{dt}{L}$$

$$\frac{dI}{RI-\mathcal{E}} = -\frac{dt}{L}$$

$$\ln\left(\frac{1-\frac{\mathcal{E}}{R}}{-\frac{\mathcal{E}}{R}}\right) = -\frac{Rt}{L}$$

$$\boxed{I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)} \quad \text{where } \tau = \frac{L}{R}$$

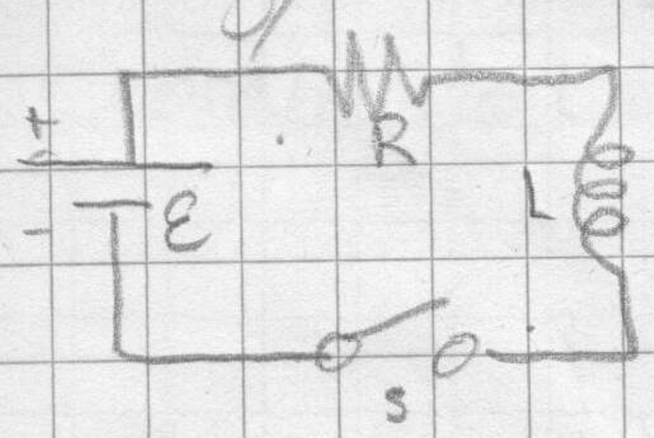
If we suddenly take away the source of EMF and leave just the RL circuit.

$$\text{KLR: } -\mathcal{E}_R + \mathcal{E}_L = 0$$

$$-RI - L\frac{dI}{dt} = 0$$

$$\boxed{I = I_0 e^{-t/\tau}}$$

Energy Stored In an Inductor



$$\text{KLR: } \mathcal{E} = IR + L\frac{dI}{dt}$$

$$\underbrace{\mathcal{E}I}_{P} = \underbrace{I^2R}_{P_R} + \underbrace{LI\frac{dI}{dt}}_{P_L}$$

$$P_L = LI\frac{dI}{dt}$$

$$U_L = \int P_L\,dt = \int LI\,dI \Rightarrow \boxed{U_L = \frac{1}{2}LI^2}$$

Energy Density of a B-Field

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l} I^2 = \frac{1}{2\mu_0} Al\left(\frac{\mu_0 N I}{l}\right)^2$$

$$U_L = \frac{1}{2\mu_0} Al B^2$$

Energy Density $\rightarrow$ $\boxed{u_B = \frac{1}{2\mu_0} B^2}$.