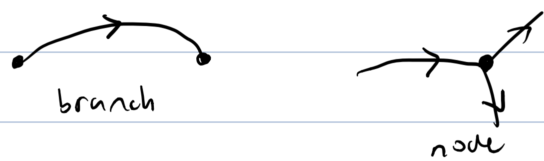
6 - Integrative Activity: Electrical Circuits

October 31, 2017

* Note: not on test 2.

An electrical circuit is composed of <u>branches</u> with a direction for the current and <u>nodes</u> (intersection of at least 3 branches.



There are two types of electrical elements along the circuit:

<u>power source</u> which "force" a current of electrons through the network, and <u>resistors</u>, which require a certain amount of force (or electrical potential) for the current to flow through it.

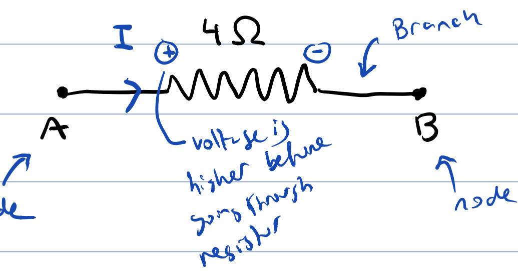
Laws for Electrical Current

The principle governing the "passage" of current through a resistor is <u>Ohm's Law</u>.

The force (electrical potential) required to drive a current I through a resistor with resistance R is proportional to I with the following relationship

$$\underbrace{\Delta V}_{\text{voltage}} = IR$$

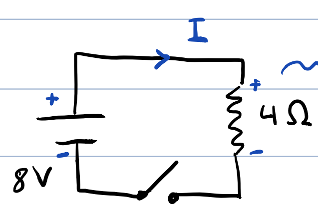
ex,



$$\Delta V = 4I$$

Units:
- ΔV (electrical potential or force or voltage) : volts
- R (resistance) : ohms (Ω)
- I (current) : amps (A)

ex,



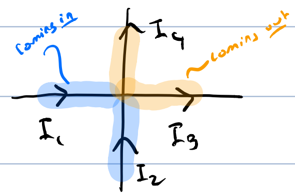
~ note: no nodes, simply a loop

The two principles we will use to assign a current to each branch are called Kirchoff's Laws.

Kirchoff's First Law (Current Law)

At every node, the sum of the currents flowing into the node equals the sum of the currents flowing out.

ex.



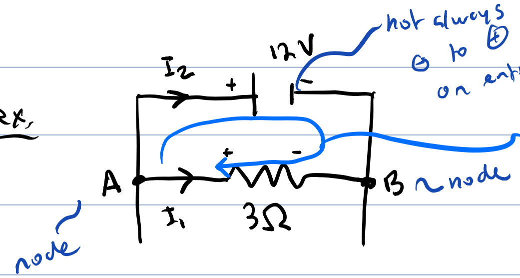
$$I_1 + I_2 = I_3 + I_4$$

Kirchoff's Voltage Law

Over any loop, the total change in voltage (potential) is zero.

Note: A loop is a sequence of branches that begins and ends at the same node.

ex.



not always from ⊖ to ⊕, depends on entire circuit

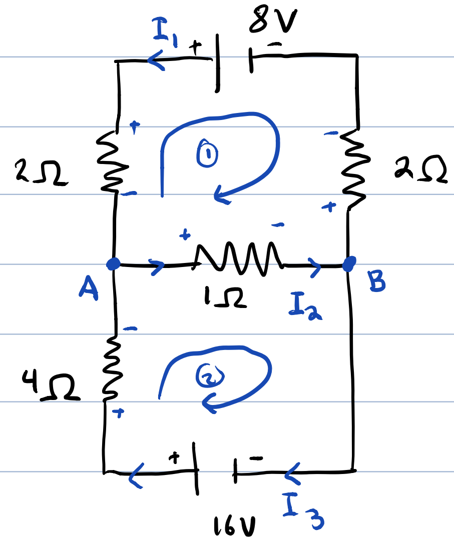
direction that you're analyzing the loop

convention: from ⊖ to ⊕ is ⊕, from ⊕ to ⊖ is ⊖ } drop in potential

$$-12 + 3I_1 = 0$$

$$I_1 = 4\text{ A}$$

ex.



* each branch has a corresponding intensity and current

same node A: $I_3 + I_1 = I_2 \rightarrow ① \; I_1 - I_2 + I_3 = 0$ kirchoff's current law

node B: $I_2 = I_1 + I_3$

loop 1: $2I_1 - 8 + 2I_1 + I_2 = 0$ kirchoff's voltage law

$\rightarrow ② \; 4I_1 + I_2 = 8$

loop 2: $-I_2 + 16 - 4I_3 = 0$

$③ \; I_2 + 4I_3 = 16$

$$\begin{aligned} I_1 - I_2 + I_3 &= 0 \\ 4I_1 + I_2 \quad &= 8 \\ I_2 + 4I_3 &= 16 \end{aligned}$$

Solve using Gauss-Jordan or $\underbrace{X = A^{-1}B \text{ or Cramer's Rule}}_{\text{if the matrix } A \text{ is invertible}}$

ex C (cont'd) November 1

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 1 & 0 & 8 \\ 0 & 1 & 4 & 16 \end{array}\right] \xrightarrow{R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & -4 & 8 \\ 0 & 1 & 4 & 16 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 4 & 16 \\ 0 & 5 & -4 & 8 \end{array}\right] \xrightarrow{R_3 - 5R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 4 & 16 \\ 0 & 0 & -24 & -72 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{24} R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 4 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ could stay at RREF since only 1 sol'n

$I_1 = 4 - 3 = 1$

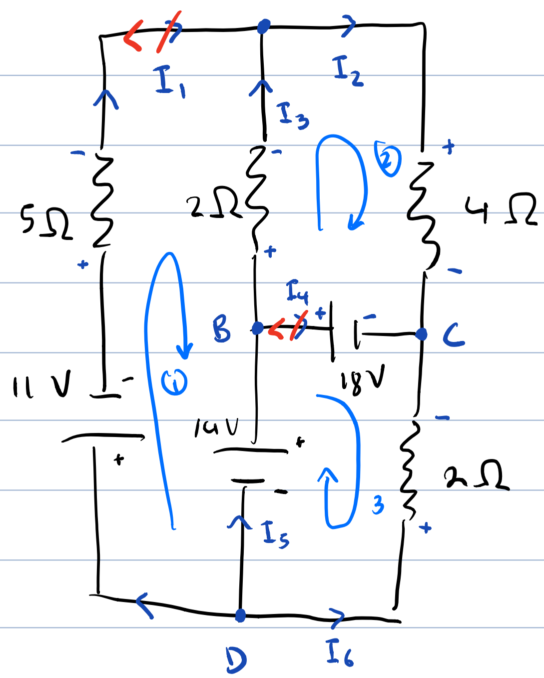
$I_2 \quad 16 - 4 \cdot 3 = 4$

$I_3 = 3$

$\therefore$ Sol $\begin{cases} I_1 = 1 \\ I_2 = 4 \\ I_3 = 3 \end{cases}$

A

ex.



* Bigger loops that combine smaller loops are redundant

node A: $I_1 + I_3 = I_2 \rightarrow I_1 - I_2 + I_3 = 0$

node B: $I_5 = I_3 + I_4 \rightarrow I_3 + I_4 - I_5 = 0$

node C: $I_2 + I_4 + I_6 = 0 \rightarrow I_2 + I_4 + I_6 = 0$

node D: $I_1 + I_5 + I_6 = 0 \rightarrow I_1 + I_5 + I_6 = 0$

loop ①: $-11 - 5I_1 + 2I_3 - 14 = 0 \rightarrow 5I_1 - 2I_3 = -25$

loop ②: $-2I_3 - 4I_2 + 18 = 0 \rightarrow 4I_2 + 2I_3 = 18$

loop ③: $-18 + 2I_6 + 14 = 0 \rightarrow I_6 = 2$

$$
\begin{array}{cccccc}
I_1 & I_2 & I_3 & I_4 & I_5 & I_6 & \\
\end{array}
$$

$$
\left[\begin{array}{cccccc|c}
1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & -1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
5 & 0 & -2 & 0 & 0 & 0 & -25 \\
0 & 4 & 2 & 0 & 0 & 0 & 18 \\
0 & 0 & 0 & 0 & 0 & 1 & 2
\end{array}\right]
$$

$$\vdots$$
$$\downarrow$$

change I_1 direction

$$
\begin{array}{cl}
\left[\begin{array}{ccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 5 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
&
\begin{array}{l}
\\ \\ \\ I_4 \\ \\ \\ \\
\end{array}
\end{array}
$$

ELECTRICAL CIRCUITS

LINEAR ALGEBRA

Abstract. Using a linear system and matrix reduction, we find the current through the different branches of an electric circuit with batteries and resistors.

1. Electrical circuits

An electrical circuit is a system of electrical elements joined together by electric cables. Our circuits will contain batteries which will deliver a constant electrical tension. They will also contain resistors which create resistance to the current.

Definition. 1. A node or junction is a point where at least three pieces of cable meet. A branch of a circuit is a piece of electrical cable that runs between two nodes. A loop in a circuit is a path through branches that starts and ends at the same node.

Since we can wrap around the same region multiple times, each circuit has infinitely many loops. We will consider inner loops. An inner loop is a path going once around a bounded region.

Example 1. Find the nodes, the branches and the bounded regions of the following circuits. Pick one inner loop for each enclosed region.

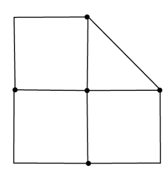
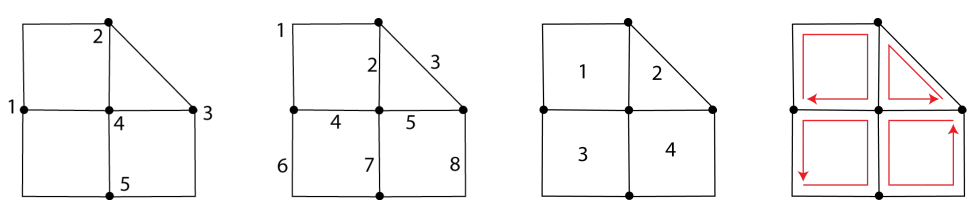
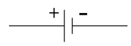
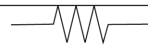
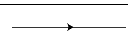
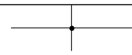


Figure 1. An electrical circuit

Solution. We have five points that meet at least three cables. So we have five nodes. Note that the three "corners" are not nodes as they only have two pieces of cable. We have eight branches and four enclosed regions.



(a) Nodes (b) Branches (c) Regions (d) Inner loops

Battery		Electrical potential $\mathcal{E}$ in volts (V).
Resistor		Resistance R in ohms (Ω).
Current		Current I in amperes (A)
Nodes		An intersection of branches.

1.1. **Water analogy.** To have some intuition about electrical circuits, think of electrical current as water current flowing through pipes. Under this correspondence, a branch becomes a pipe and a battery plays the role of a water pump. Under this comparison, electrical potential relates to gravitational potential. Resistors to constrictions in a pipe.

2. Laws for electrical circuit

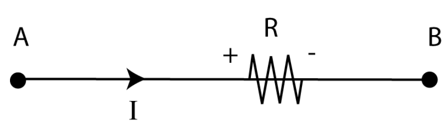
To determine the electrical current flowing through our circuits, we will need the following three laws.

Theorem. 1 (Ohm's Law). *The current in an electrical conductor is directly proportional to the potential applied to it.*

$$\Delta V = IR \textit{ volts.}$$

The constant of proportionality R is called the resistance of this conductor. From Ohm's law, we get that when a current of I amperes passes through a resistor of R ohms, the electrical potential drops by IR.

Example 2. Consider a circuit with the following branch. By Ohm's law the electric po-



tential $V(B)$ at B is lower than the potential $V(A)$ at A. More precisely,

$$V(B) = V(A) - IR$$

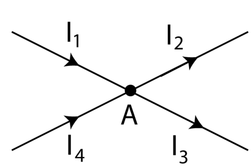
Note that

$$V(B) - V(A) = -IR \qquad V(A) - V(B) = IR$$

And so $\Delta V = \pm IR$. To help keep track of the sign, follow the current and put a $+$ on the first side of the resistor you encounter and a $-$ on the second side. The side with a $+$ has higher potential.

Theorem. 2 (Kirchhoff's Current Law or Kirchhoff's Junction Rule). *At every node, the sum of the currents flowing into a node is equal to the sum of the currents flowing out.*

Example 3. Consider a circuit with the following node A. According to the current law,

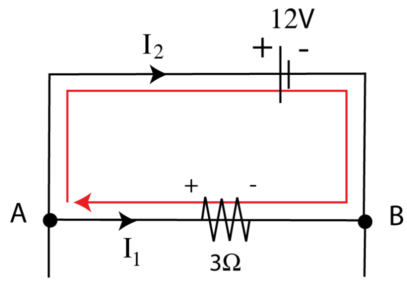


the total current coming into node A should be equal to the total current leaving node A. Hence

$$I_1 + I_4 = I_2 + I_3$$

Theorem. 3 (Kirchhoff's Voltage Law or Kirchhoff's Loop Rule)**.** *Over any loop, the total change in voltage is zero.*

Example 4. Consider the following loop in a circuit. The change in potential over the whole



loop is zero. We look at each conductor separately.

- When we move from A to B on the top branch we pass through a battery. The pole of the battery with the "+" has higher potential. We therefore get a drop in potential of $12V$.
- When we come back to A on the bottom branch we pass by a resistor. According to Ohm's law, the right side of the resistor has lower electrical potential than the left side. Since we are going from right to left, our change in potential is $+3I_1$.

The sum of these changes in potentials should add up to zero and so the voltage rule gives

$$-12V + 3I_1 = 0.$$

2.1. **Water pump analogy and laws of electric circuits.** Ohm's law says that

$$\Delta V = IR$$

where R is called the resistance of the conductor. In a water system, a constriction in a pipe would cause resistance. The tighter the constriction the more resistance it opposes to the flow of water. If we think of potential as corresponding to the gravitational potential energy, Ohm's law says that the potential energy drops after a constriction : passing water through this constriction costs energy. To get the signs right, think of water flowing because of gravity. If you are going along with the current, you are going down and your potential is dropping. If you are going against the current then you are going up and your potential is increasing.

The junction rule says that the the total current coming into a note is equal to the total current leaving a node. This is also true for junction in a water system. The water coming into the junction needs to leave the junction. And so the current coming into the junction is equal to the current leaving it.

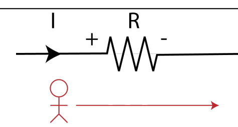
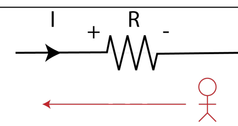
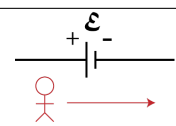
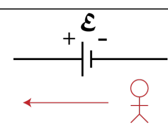
In a water system, the height at the starting point of a loop is equal to the height at the end point. (Remember that a loop starts and ends at the same point.) This means that the potential energy at the end and at the beginning of a loop are equal. Any drop in potential energy along the loop must be compensated by an increase in potential energy somewhere else in the loop.

3. From an electric circuit to a linear system.

Given an electric circuit we can determine the current going through each branch.

3.1. **Strategy.**

(1) For each branch, pick a (random) direction for the current and label it with $I_1, I_2, I_3, \cdots$
(2) For each resistor label with $+$ the first side hit by the current and by $-$ the second one.
(3) For each node, write the equation : incoming current equals outgoing current.
(4) For each enclosed region.
   - Pick a way of travelling through the corresponding inner loop.
   - For each element in the loop, determine the change in potential when moving in the chosen direction. The following table might be helpful.

Element in the loop	Change in potential
	$\Delta V = -IR$
	$\Delta V = IR$
	$\Delta V = -\mathcal{E}$
	$\Delta V = \mathcal{E}$

   - The sum of these changes is zero.

(5) Solve the system.
   - Put the equations for the nodes and for the regions together in a system.
   - Solve the system using augmented matrices.

(6) Assume that the system has a solution $I_1 = a_1, I_2 = a_2, \cdots, I_n = a_n$. If a_k is negative, then a current of $|a_k|$ amperes is going the opposite way than your original arrow.

Remark. 1 (Number of solutions)**.** As long as each loop has some resistance, the system will have a unique solution. In real-life, the cable itself has some resistance and so any real electrical circuit will lead to a system with a unique solution. To see what happens if a loop has no resistance, go to problem 6.

4. Solved examples.

Example 5. Find the electric currents in the circuit of figure 2.

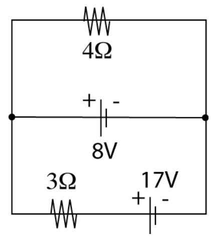
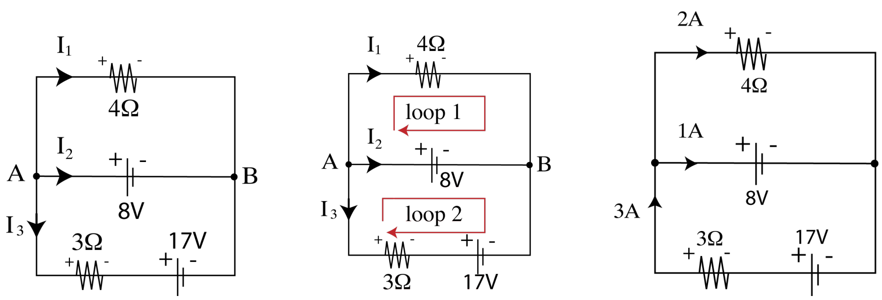


FIGURE 2. Circuit of example 5

We start by picking a direction for the current and labelling all three branches. Lets label the two nodes as well to simplify the notation. For each node we get an equation.



(a) Labeling (b) Picking loops (c) Final answer

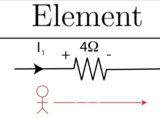
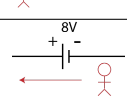
FIGURE 3. For example 5

Node	In	=	Out
A	0	=	$I_1 + I_2 + I_3$
B	$I_1 + I_2 + I_3$	=	0

Note that these two equations are equivalent. Feel free to remove one or keep both in your system.

We have two regions and so we consider two inner loops. We choose to go around each region as in figure 3(b).

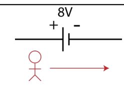
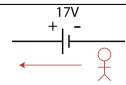
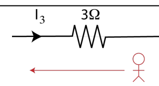
(1) In the first loop, we have two elements : a resistor and a battery.

Element	Change in potential
	$-RI = -4I_1$
	$\mathcal{E} = 8$

While going through the resistor, the loop is going with the current and we get a drop in potential : $\Delta V = -RI = -4I_1$. For the battery, the loop is moving to the side with higher potential, we get an increase in potential of 8 : $\Delta V = 8V$. The sum of these changes must be zero and so we get

$$-4I_1 + 8 = 0 \implies \boxed{-4I_1 = -8}$$

(2) In the second loop, we have three elements.

Element	Change in potential
	$-\mathcal{E} = -8$
	$\mathcal{E} = 17$
	$RI = 3I_3$

The sum of these changes must be zero and so we get

$$-8 + 17 + 3I_3 = 0 \implies \boxed{3I_3 = -9}$$

All together, we have

$$\begin{cases} I_1 + I_2 + I_3 = & 0 \\ -4I_1 = & -8 \\ 3I_3 = & -9 \end{cases}$$

This system is quite simple and you could solve it directly. But let's use matrices anyways.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -4 & 0 & 0 & -8 \\ 0 & 0 & 3 & -9 \end{array}\right] \underset{RREF}{\implies} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

We get $I_1 = 2A$, $I_2 = 1A$ and $I_3 = -3A$. Since I_3 is negative, the current is going the opposite way than your arrow. The final answer is in the picture 3(c).

Example 6. Find the electrical currents in the circuit of figure 4.

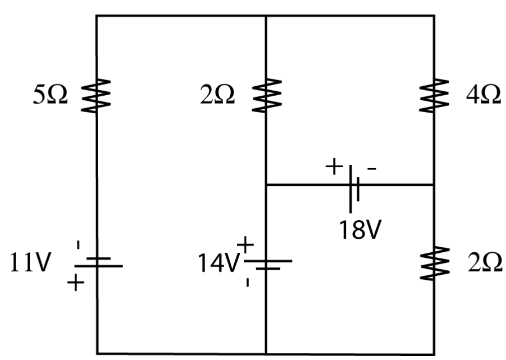
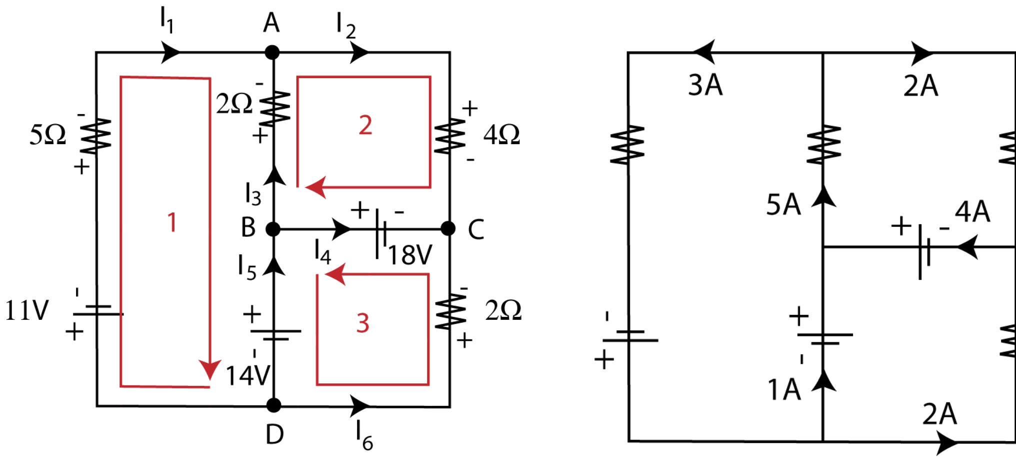


FIGURE 4. The circuit of example 6

We have four nodes, six branches and three inner regions. We label them as in figure 5(a) follows. From our nodes and our loops, we get



(a) The labeling (b) Final answer

FIGURE 5. Figures for example 6.

$$
\begin{array}{lll@{\qquad\qquad}ll}
A & I_1 + I_3 = I_2 & & & \\
B & I_5 = I_3 + I_4 & & 1 & -11 - 5I_1 + 2I_3 - 14 = 0 \\
C & I_2 + I_4 + I_6 = 0 & & 2 & -2I_3 - 4I_2 + 18 = 0 \\
D & 0 = I_1 + I_5 + I_6 & & 3 & -14 - 2I_6 + 18 = 0
\end{array}
$$

Together the nodes and the loops give the linear system

$$
\left\{
\begin{array}{rcrcrcrcrcrcr}
I_1 & - & I_2 & + & I_3 & & & & & & & = & 0 \\
 & & & - & I_3 & - & I_4 & + & I_5 & & & = & 0 \\
 & & I_2 & & & + & I_4 & & & + & I_6 & = & 0 \\
-I_1 & & & & & & & - & I_5 & - & I_6 & = & 0 \\
-5I_1 & & & + & 2I_3 & & & & & & & = & 25 \\
 & - & 4I_2 & - & 2I_3 & & & & & & & = & -18 \\
 & & & & & & & & & - & 2I_6 & = & -4.
\end{array}
\right.
$$

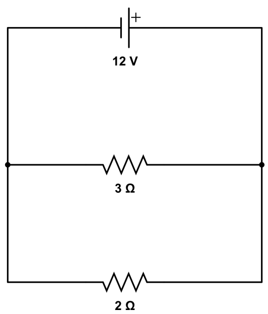
We put it in a matrix and reduce!

$$
\left[\begin{array}{cccccc|c}
1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 & -1 & -1 & 0 \\
-5 & 0 & 2 & 0 & 0 & 0 & 25 \\
0 & -4 & -2 & 0 & 0 & 0 & -18 \\
0 & 0 & 0 & 0 & 0 & -2 & -4
\end{array}\right]
\underset{RREF}{\Longrightarrow}
\left[\begin{array}{cccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 1 & 0 & 0 & 0 & 5 \\
0 & 0 & 0 & 1 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

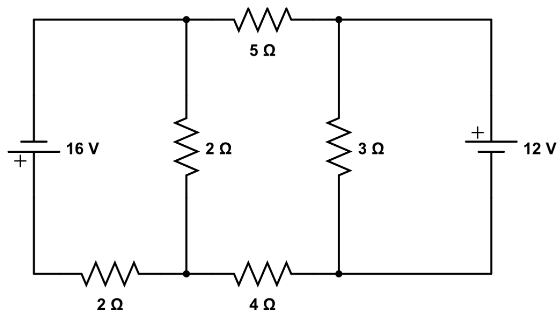
and so $I_1 = -3, I_2 = 2, I_3 = 5, I_4 = -4, I_5 = 1, I_6 = 2$. You need to flip the arrows corresponding to negative signs. The solution is in figure 5(b)

5. Unsolved problems.

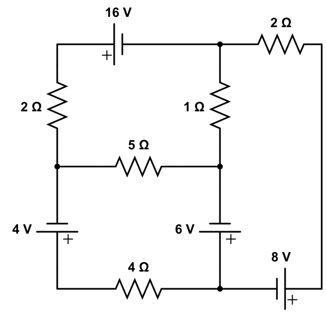
Problem 1. Find the currents in the electrical circuit of figure 6(a).



(a) Circuit for problem 1.



(b) Circuit for problem 2.



(c) Circuit for problem 3.

Figure 6. Circuit for problem 1, 2 and 3.

Problem 2. Find the currents in the electrical circuit of figure 6(b).

Problem 3. Find the currents in the electrical circuit of figure 6(c).

Problem 4. Consider the electrical circuit of figure 7.

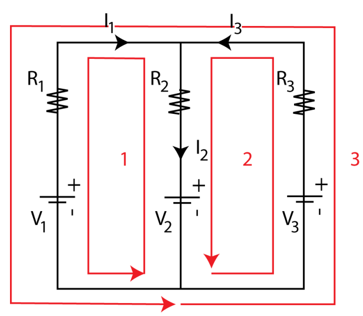


Figure 7. Circuit for problem 4.

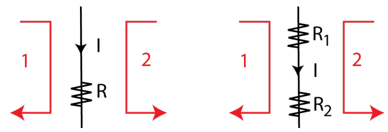
(1) Find the equations associated to the three labeled loops.
(2) Explain why ignoring the third loop when building the linear system will not affect the resulting solutions.

Problem 5. Consider the following two branches of electrical circuits.

(1) For the branch of figure 8(a) find the coefficients of I in the equation of loop 1.
(2) For the branch of figure 8(b) find the coefficients of I in the equation of loop 1.
(3) Assume that in an electrical circuit, we replace the branch of figure 8(a) by the one of figure 8(b) with

$$R = R_1 + R_2.$$

Explain why this change would not affect the currents of the circuit.



(a) One resistor. (b) Two resistors.

FIGURE 8. Circuit for problem 5.

Remark. 2 (Resistors in series)**.** Problem 5 proves that when resistors are placed in series, the total resistance is the sum of the resistance of each resistor.

Problem 6. Consider the electrical circuit of figure 9.

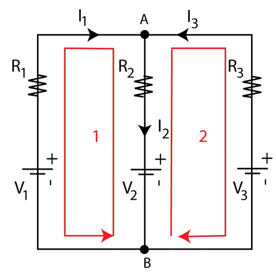


FIGURE 9. Circuit for problem 6.

(1) Set up the system for this circuit.
(2) Explain why we can drop one of the two node equations.
(3) Let A be the coefficient matrix of the smaller system with three equations and three variables.
   (a) Find $\det(A)$.
   (b) Use $\det(A)$ to find conditions on R_1, R_2, R_3 for A to be invertible. How many solutions do we have then?
   (c) Consider the case where $R_1 = R_3 = 0\Omega$, $R_2 = 1\Omega$ and $V_1 = V_3 = 0V$, $V_2 = 1V$. Find all possible currents.
   (d) Consider the case where $R_1 = R_3 = 0\Omega$, $R_2 = 1\Omega$ and $V_1 = V_2 = 0V$, $V_3 = 1V$. Find all possible currents.

Problem 7. Assume that you have a linear system coming from an electric circuit.

(1) Euler's theorem gives that for any connected planar graphs

$$\# \text{ nodes - } \#\text{branches } + \# \text{ inner regions } = 1.$$

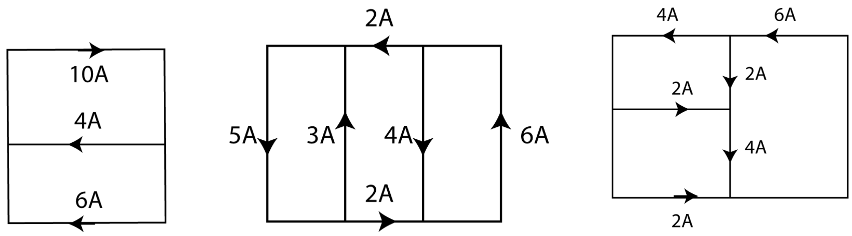
Use this theorem to show that your system has one more equations than variables.

(2) Lets consider the equations coming from the nodes

$$I_{in} - I_{out} = 0$$

   (a) Show that each current I_k appears in exactly two of the node equations.
   (b) Show that the sum of all the node equations gives $0 = 0$.
   (c) Explain why we can always drop any one of the node equations.

6. Answers



(a) Solution for problem 1. (b) Solution for problem 2. (c) Solution for problem 3.

Figure 10. Answers for problem 1, 2 and 3.

Answer to problem 4. (1) We get the following equations

$$\begin{cases} R_1I_1 + R_2I_2 = V_1 - V_2 \\ \quad - R_2I_2 - R_3I_3 = V_2 - V_3 \\ R_1I_1 - R_3I_3 = V_1 - V_3 \end{cases}$$

(2) The third equation is the sum of the first two. It is redundant.

Answer to problem 5. (1) We would get $-RI$

(2) We get $-R_1I - R_2I$.

(3) This change would not affect the equations associated to the nodes. It would only affect the equations for loop 1 and loop 2. But in these two equations, the coefficient of I goes from $-R$ to $-R_1 - R_2 = -R$. And so we get exactly the same system.

Answer to problem 6. (1) We get

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ -I_1 + I_2 - I_3 = 0 \\ R_1I_1 + R_2I_2 = V_1 - V_2 \\ R_2I_2 + R_3I_3 = V_3 - V_2 \end{cases}$$

(2) The second equation is minus the first. They are equivalent.

(3) (a) We have

$$\det A = \begin{vmatrix} 1 & -1 & 1 \\ R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \end{vmatrix} = R_1R_2 + R_1R_3 + R_2R_3$$

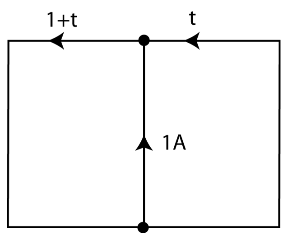
(b) Since all the resistances are positive, we need that two of them be non-zero. We would get a unique solution

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 0 \\ V_1 - V_2 \\ V_3 - V_2 \end{bmatrix}$$

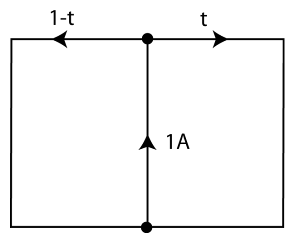
(c) We reduce our system

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

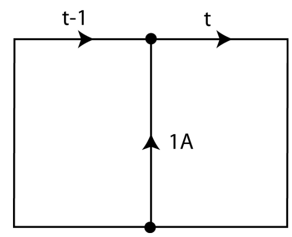
and get infinitely many solutions. $I_2 = -1$ and then $I_1 + I_3 = -1$. We get three cases depending on signs.



(a) When $I_3 \geq 0$.



(b) When $-1 \leq I_3 \leq 0$.



(c) When $I_3 \leq -1$.

FIGURE 11. Possible currents for problem 6.

(4) There are no solutions as we get

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Answer to problem 7. (1) According to the way we build our systems, we get

$$\begin{aligned} \#\text{ equations} &= \#\text{ nodes} + \#\text{ inner regions} \\ \#\text{ variables} &= \#\text{ branches} \end{aligned}$$

and so using Euler's theorem

$$\#\text{equations - }\#\text{variables} = \#\text{ nodes} + \#\text{ inner regions} - \#\text{ branches} = 1$$

We have one more equation than we have variables.

(2) (a) The branch associated to I_k starts at one node, say A, and ends at one node, say B. I_k will only appear in the equations of the nodes A and B.

(b) Since I_k starts at A and ends at B, we will get

$$\begin{array}{ll} A & \cdots - I_k + \cdots = 0 \\ B & \cdots + I_k + \cdots = 0 \end{array}$$

and so the coefficient of I_k in the sum of the equations is 0.

(c) By (b), any node equation will be equal to minus the sum of all the other equations of the nodes.