Vectors

- → magnitude and direction
- → $(\vec{a})$ = vector , $|\vec{a}|$ = magnitude

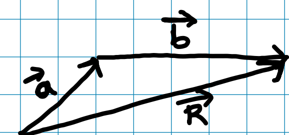
Distance = length of the path taken

Displacement = position of point A relative to point B
(final position) − (initial position)

Displacement ≤ Distance

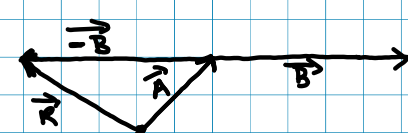
Vector addition:

- Graphically place vectors head to tail



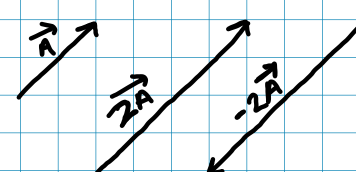
vector substraction:

- $\vec{A} - \vec{B} = \vec{A} + \overrightarrow{-B}$
- A negative vector goes (points) in the opposite direction

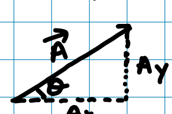


multiplication by scalar

- magnitude is multiplied
- if multiplied by negative, direction is opposite



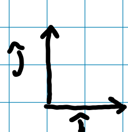
Components



Use Trig : $S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$
Use Pythagorus : $a^2 + b^2 = c^2$

The components are positive or negative based on the direction relative to the referential axis.

Unit vectors



These vectors have a magnitude of 1

- $\hat{i}$ points in x direction
- $\hat{j}$ points in y direction
- $\hat{k}$ points in z direction

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

* Add components of vectors for vector addition

Kinematics

Motion diagram

→ equal intervals of time between each frame

ex: car A vs. car B

□ □ □ □ □ □ Car B is going faste

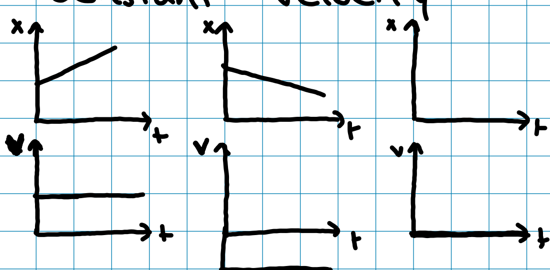
- The bigger the space between each interval, the more distance was travelled for a same time

Acceleration

→ Rate of velocity change

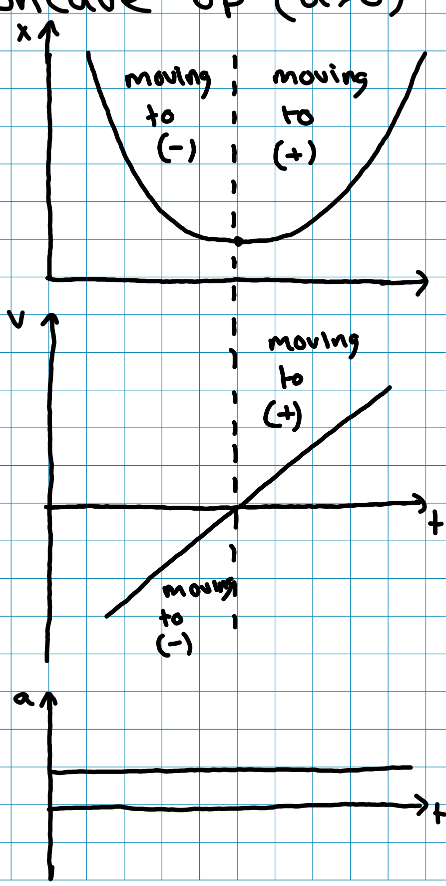
- In 1s, how will my velocity change

Acceleration = 0
Constant velocity

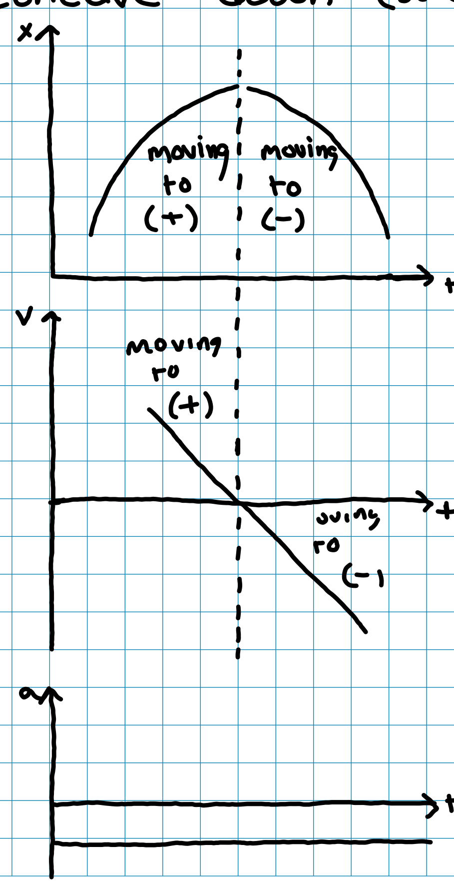


Constant Acceleration

Concave up (a>0)



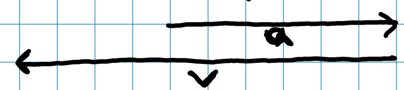
Concave down (a<0)

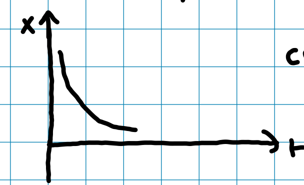


An object moves to the left (-) and is slowing down.

→ positive acceleration

- velocity going to negative direction
- with a positive acceleration, the magnitude of the velocity decreases



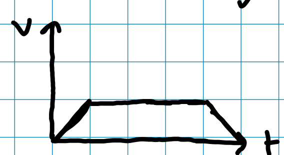


A person rides an elevator to top floor

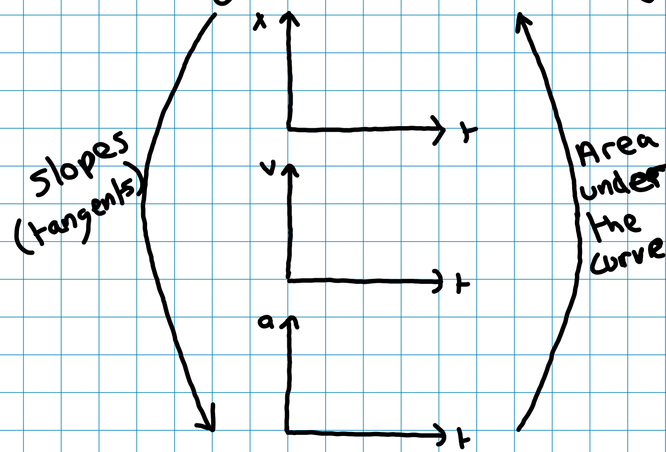
→ positive acceleration

→ constant velocity ($a = 0$)

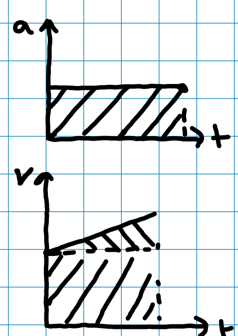
→ negative acceleration (until stop)



Going from one graph to another



Area under the curve



① $a \cdot t = \Delta v$ (negative "a" = (-) area)

Δx = area

Δx = [trapezoid]

② $\Delta x = \frac{(v_i + v_f)\,t}{2}$

Δx = [triangle] + [rectangle]

$\Delta x = \frac{1}{2}(v_f - v_i) \cdot t + v_i \cdot t$

$\Delta x = \frac{1}{2}\,\Delta v \cdot t + v_i\,t$

sub ①

$\Delta x = \frac{1}{2}(a\,t)\,t + v_i \cdot t$

③ $\Delta x = v_i\,t + \frac{1}{2}\,a\,t^2$

Formulas:

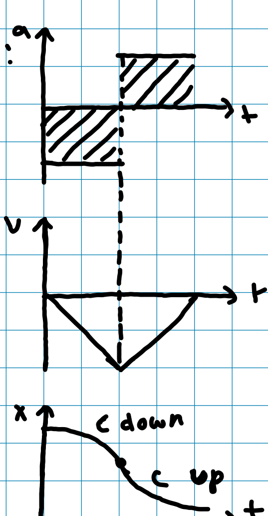
① $\Delta v = a \cdot t$

② $\Delta x = \frac{(v_i + v_f)\,t}{2}$

③ $\Delta x = v_i\,t + \frac{1}{2}\,a\,t^2$

④ $v_f^2 = v_i^2 + 2\,a\,\Delta x$

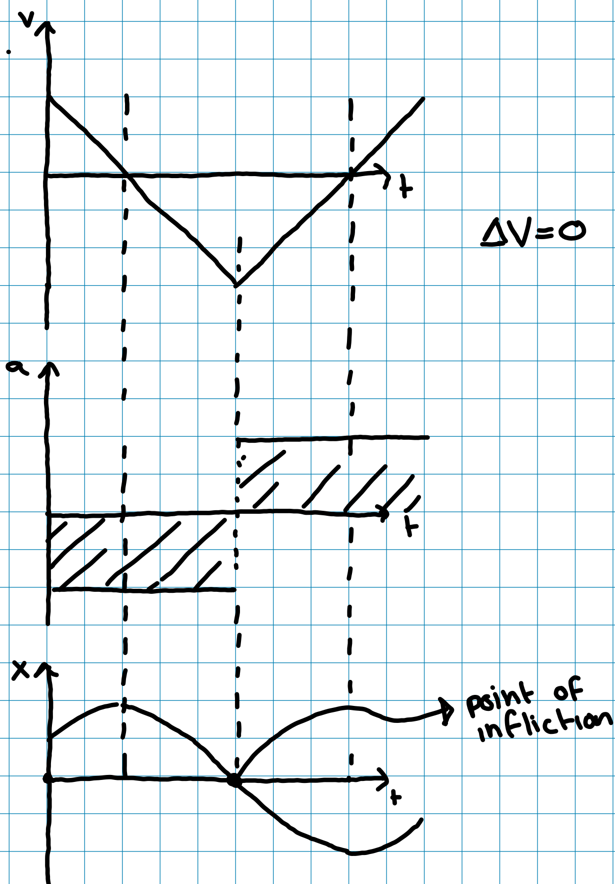
ex:



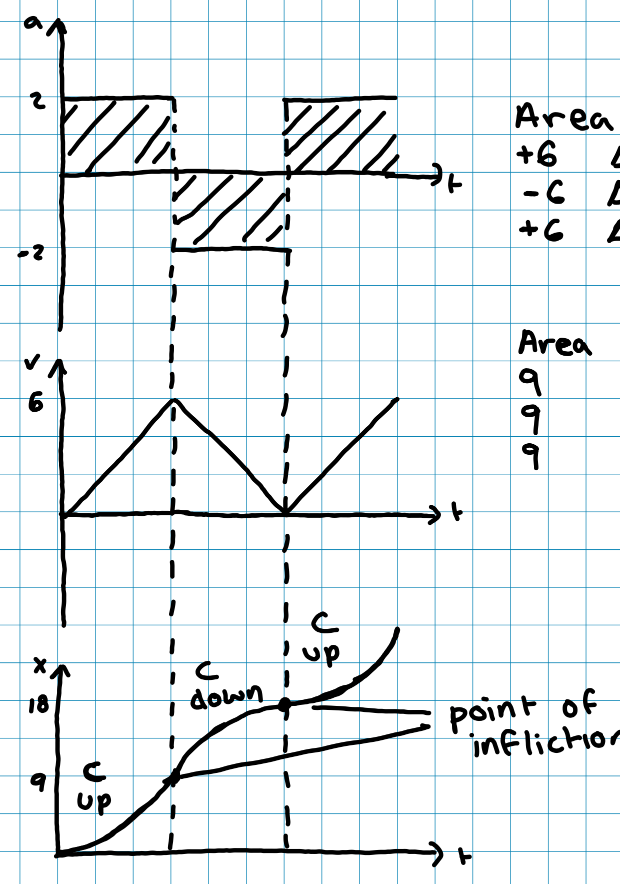
Point of inflection

- → Sign of acceleration changes
- → concave up to concave down

ex.



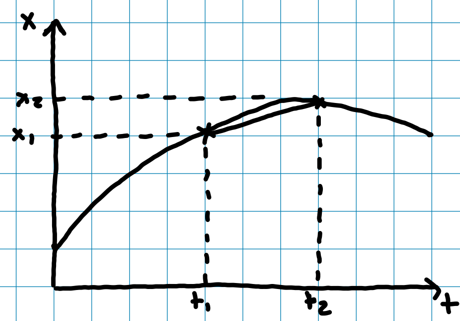
$\Delta V=0$



Area
+6 ΔV
-6 ΔV
+6 ΔV

Area
9
9
9

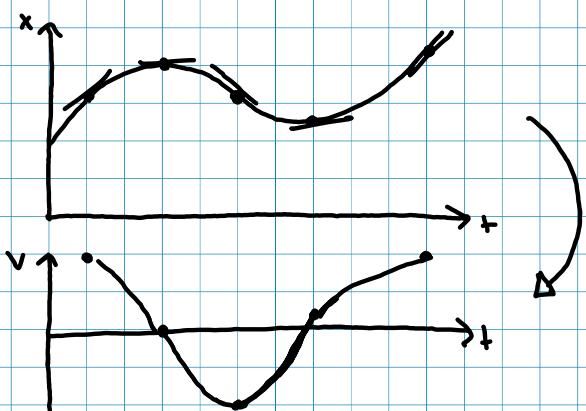
Slope of tangents and secants
(curve is a continuous amount of tangents)



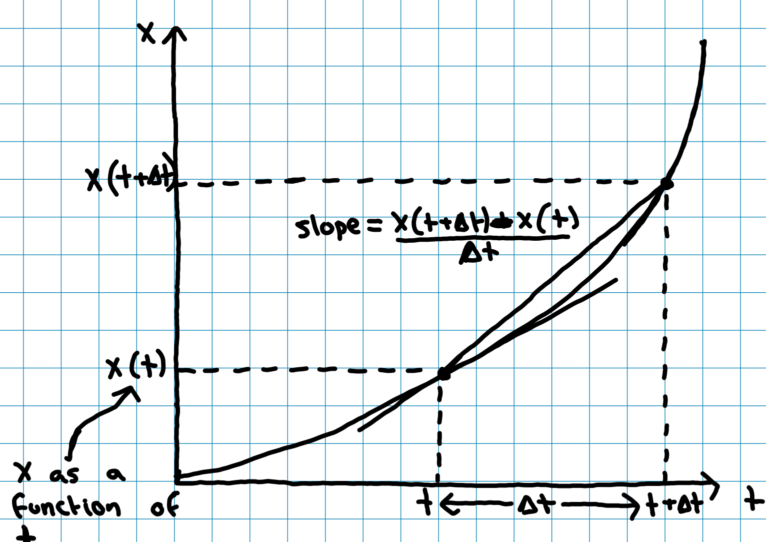
Secant:
- Slope = average velocity

$$V_{avg} = \frac{\Delta x}{\Delta t}$$

- Average velocity = instantaneous velocity at t in between t_1 t_2



Tangent:
- Slope = ~~XXX~~ instantaneous velocity
- V_s : instantaneous velocity at time 1



The more the ~~XXX~~ Δt is small, the more the secant is close to the tangent
- Δt approaches 0, the secant becomes tangent
- The average velocity at one instant is instantaneous

Secant: $V_{avg} = \frac{\Delta x}{\Delta t}$

$$= \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Tangent: $V_s = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$

Free Fall

- → object moving with gravity only
- → no air resistance (vaccume)

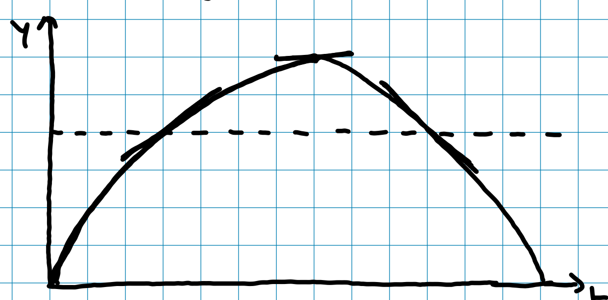
$\vec{a}_g = 9.8\ m/s^2$, vertical downward

Magnitude:

Freefall acceleration (g) → $g = 9.8\ m/s^2$

- → g always pos
- → $a_y = -g$
- → g is not gravity (gravity = force)
  - → g = freefall acceleration
- → g is different on other planets or different areas of a same planet

$\vec{a}_g = -g\hat{j}$



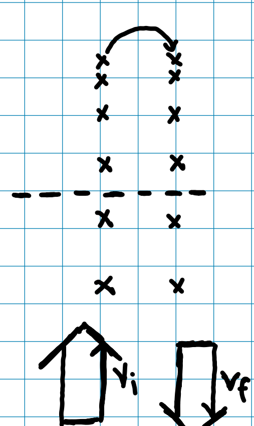
- At max height
  - ↳ $v = 0$
- At same height,
  - ↳ $|V_u| = |V_d|$ → $V_f = -V_i$
- $\Delta t_u = \Delta t_d$

$\vec{a}$ is always pointing down. Throw ball up with v_i

- Initial positive velocity, being slowed down by neg. acceleration
- Turning point : $v = 0$
- Neg acceleration speeds up ball with neg. velocity
- Ball reaches the ground with $|V_f| = |V_i|$ (opposite sign)

2 objects (different masses) dropped from the same height

- Land at the same time with the same velocity



Object goes slower and spends more time at the top half.

- → He has been slowed down on the way up
- → Hasn't been spead up on the way down

At same height:

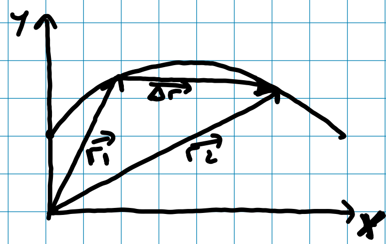
$\Delta y = 0$

$V_f^2 = V_i^2 + 2a\cancel{\Delta y}$

$V_f = -V_i$

Tide (waves):

R2 Motion



$\vec{r_1}, \vec{r_2}$: position vectors

$\vec{\Delta r}$: displacement

$\vec{\Delta r}$ = secant

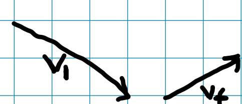
$\vec{V_{avg}} = \frac{\vec{\Delta r}}{t}$ ∴ $\vec{v_{avg}}$ points in the same direction as $\vec{\Delta r}$

Tangent : $\vec{\Delta r}$ at $\lim_{t \to 0}$ = direction motion

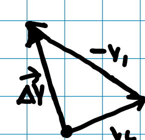
∴ tangent points in the same direction as instantaneous velocity

Direction of acceleration

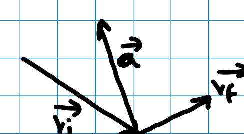
$\vec{a_{avg}} = \frac{\vec{\Delta V}}{\Delta t}$ ∴ $\vec{a}$ points in the same direction as $\vec{\Delta V}$

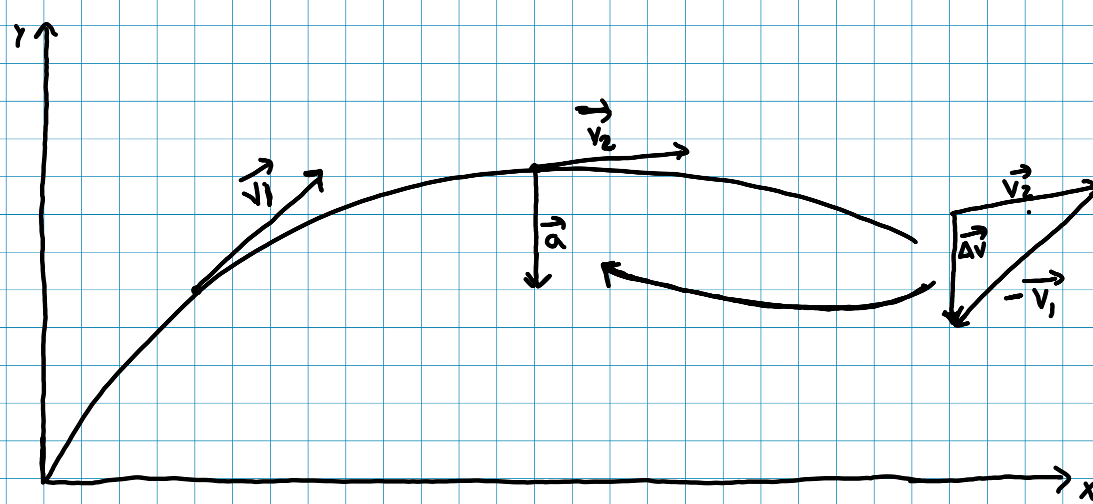


$\Delta V = \vec{v_f} - \vec{v_i}$



$\vec{\Delta V}$ same direction as $\vec{a}$





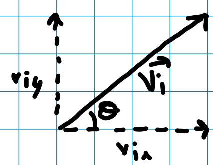
$\vec{a}$ can be decomposed into $a_{//}$ and $a_{\perp}$

- $a_{//}$ (parralel to $\vec{v}$) : changes speed
- $a_{\perp}$ (perpendicular to $\vec{v}$) : changes direction

Projectile motion

- moves in 2D under the force of gravity

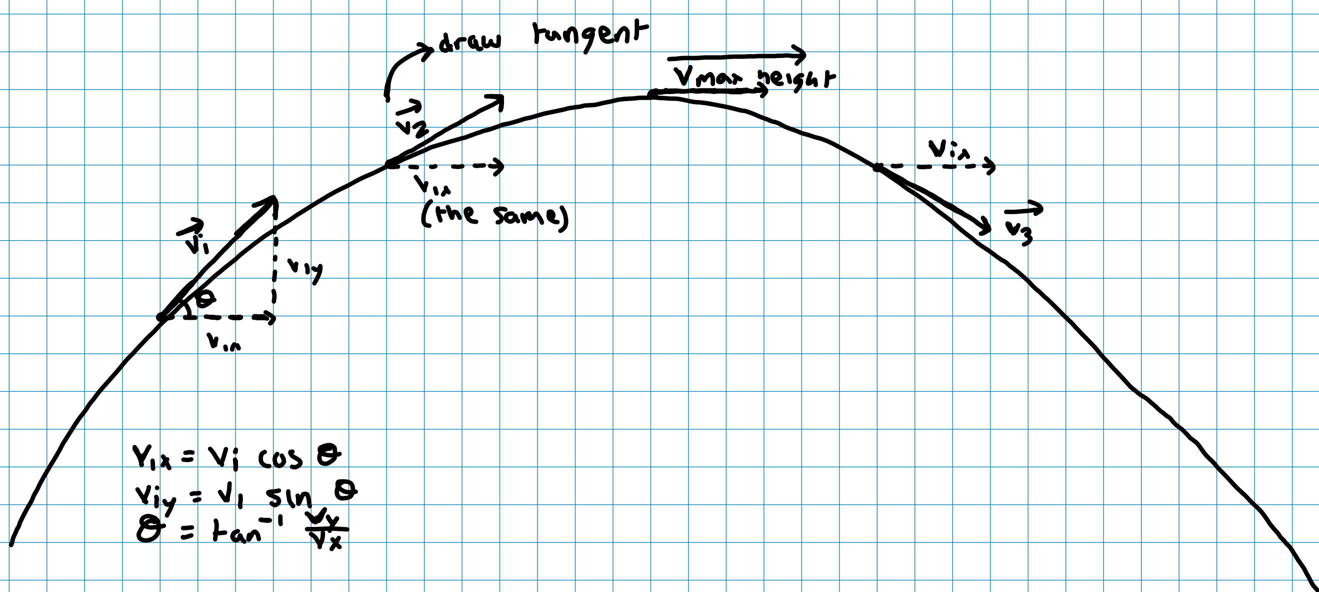
Launch:



$$\vec{V_i} = (V_{ix}\hat{i} + V_{iy}\hat{j})$$

2 motions: → horizontal ($a_x = 0$)
→ vertical ($a_y = -g$)
* t is the same for both motions

* Always make sure signs (+/-) of displacement, velocity and acceleration correspond to referential



$V_{ix} = V_i \cos\theta$
$V_{iy} = V_i \sin\theta$
$\theta = \tan^{-1} \frac{V_y}{V_x}$

To sketch: - V_x is constant
- $\vec{V}$ is tangent

X	Y
$\Delta x = v_{ix}t + \cancel{\frac{1}{2}at^2}$ $a=0$	$\Delta y = v_{iy}t + \frac{1}{2}at^2$

No acceleration, constant v_x

$\Delta x = v_{ix} \cdot t$

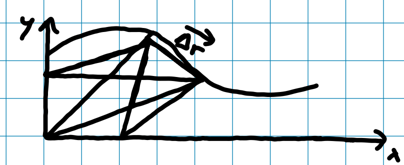
$\Delta y = v_{iy} \cdot t + \frac{1}{2}(-g)t^2$

ex: Shoot 2 objects horizontally from same height
- One at supersonic speed (300 m/s)
- One at 5 m/s

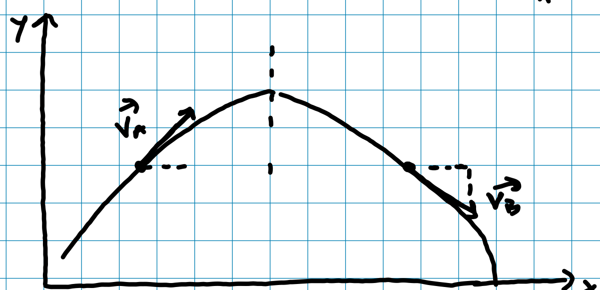
Both reach the ground at the same time
∵ both have $v_{1y} = 0$ and same Δy

ex: Does displacement depend on origin ?

No



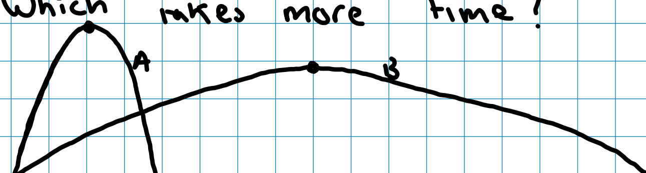
ex



$\vec{v}_A \neq \vec{v}_B$
$v_{Ax} = v_{Bx}$
$v_{Ay} = -v_{By}$

∴ $\vec{v}_A = v_{Ax}\hat{i} + v_{Ay}\hat{j}$
$\vec{v}_B = v_{Ax}\hat{i} - v_{Ay}\hat{j}$

ex. Which takes more time?



$t_{up} = t_{down}$

∵ A is higher ∴ t_A is longer

ex: Boat is moving. From the top of the mast you drop a rock:

The rock lands at the base of the mast

Force

Causes an object to change its motion
Causes an acceleration (constant F $\Rightarrow$ constant a)

Force must have a point of contact (agent)

$F = m \cdot a$
$(N) = (kg)(m/s^2)$ $\quad N = \frac{kg \cdot m}{s^2}$

- F and a $\Rightarrow$ directly proportional
- m and a $\Rightarrow$ indirectly proportional

$\vec{F} = m \cdot \vec{a}$, acceleration points in the same direction as force

An object only responds to force acting on it (by an agent – contanct / long range) at that instant. No memory of force exerted at earlier time.

Inertia:
Tendency to resist change in velocity and keep motion.

- If a force puts an object in motion (gives velocity)
- Force stops
- Object will continue with its motion (no force needed)

Objects don't need force to move, they need force to change their motion.
$\rightarrow$ Force gives an acceleration
$\rightarrow$ Force causes change in velocity
$\rightarrow$ No cause needed for object to move (uniform motion = natural state)

Slowing down an object does require force
Making an object at rest move requires force

- Force has agent
- Point of contact between agent and object (except long range forces : gravity / magnetism)
- Interactions happening now (not at previous instant)

Newton's First Law

→ If Force acting on an object is zero, object at rest stays at rest and an object in motion continues moving in a straight line with constant $\vec{v}$

$$F = m \cdot a$$
$$(0) = m \cdot (0)$$

∴ If there were no air resistance or friction, an object moving, would keep moving in a straight line at constant $\vec{v}$ forever

Newtons Second Law

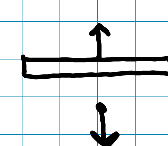
$$\vec{F}_{net} = \Sigma \vec{F} = m \cdot \vec{a}$$

Newtons Third Law

Every action has an equal but opposite reaction
Apply a force on an object, object exerts the same force on the agent.

Forces:

- normal : $\vec{n}$ (surface)



- gravitational : $\vec{F}_g$ (mass attraction)
- Friction : $\vec{F}_s$ (static)
  $\vec{F}_k$ (kinetic)
- tension : $\vec{T}$

Inertial reference frame

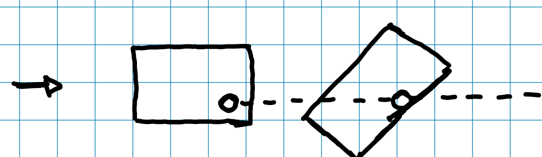
→ reference frame where $a = 0$ (v constant)
→ no acceleration (no brake)

In an inertial reference frame, the body inside keeps the same uniform motion as the reference frame and therefore feels like he's not moving.

If there is an acceleration (change in motion), the body will keep his uniform motion, but the reference frame's will be changing. The body will therefore feel the acceleration (try to resist).

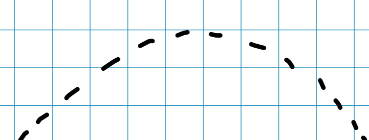
ex: car going fast breaks abruptly
→ car breaks
→ People inside keep going (keep motion)
→ Hit windshield

ex: car makes sharp left turn (not changing speed)
→ Car goes left
→ People keep going straight
→ Hit the right door (leftward force)



Cars have 3 ways to accelerate:
- gas
- break
- turn

ex: What force acts on ball after launch?



- Only the weight of the ball acting vertically down
- No horizontal force ($a = 0$)
- No memory of force of launch

Free body diagram:

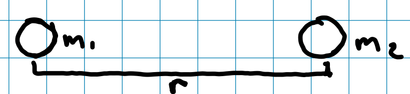
Equilibrium: net force $= 0$
$\hookrightarrow a = 0$

Object either at rest or moving along a straight line

$F_{net\,x} = 0$
$F_{net\,y} = 0$

Gravitational Force (F_g)

- Long range, attractive force, between 2 masses



$$F_g = \frac{G\, m_1 \cdot m_2}{r^2}$$

$G = 6.67 \times 10^{-11}\ N\, m^2/kg^2$ (constant)

$F_g = m \cdot g \quad \Rightarrow \quad g = \frac{G \cdot M}{R^2}$

Normal Force (n)

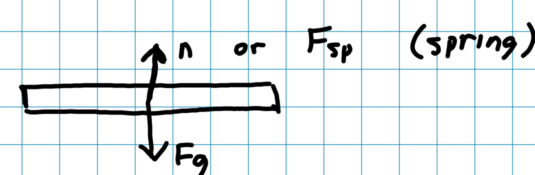
* Must be a surface
- At a molecular level, the springs of the surface when pushed on, push back up
- Force exerted by a surface against object pushing down on the surface

Weight:

- weight is what a scale would measure
- A scale measures the normal force

$\Sigma F_y = m a_y$

$F_n - F_{g(y)} = m a_y$



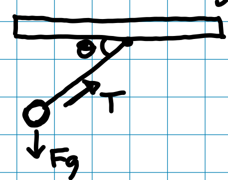
* weight varies with acceleration and g, but mass is always the same

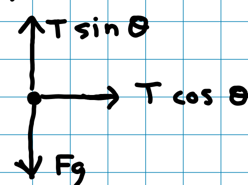
Free body diagram:

- Draw all forces applied on object
- Draw components on x and y axis

$\boxed{\Sigma F_x = m \cdot a_x}$ and $\boxed{\Sigma F_y = m \cdot a_y}$

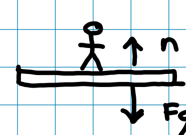
ex: ball hanging from ceilling



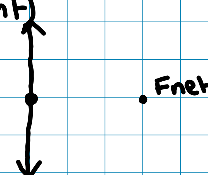


* no $\vec{n}$ ∵ no surface

Scale measure n_{sp}



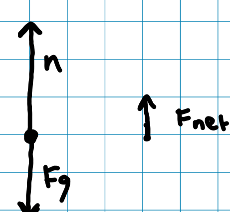
- No acceleration (stationary)
  → $F_{net} = 0$ $(a=0)$ $(\vec{v}$ constant$)$
  $\Sigma F_y = 0$
  $n - F_g = 0$
  $n = F_g$
  $n = m \cdot g$



* $|n| \neq |F_g|$

- Accelerating up
  $\Sigma F_y = n - F_g$
  $n - F_g = m \cdot a_y$
  $n = ma + mg$
  $n = m(a+g)$

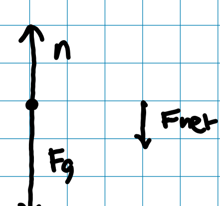
  * $a > 0$ ∴ heavy



Up start / Down stop

- Accelerating down
  $\Sigma F_y = n - F_g$
  $n - F_g = m \cdot a_y$
  $n = ma + mg$
  $n = m(a+g)$

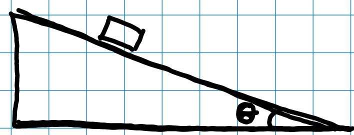
  * $a < 0$ ∴ levity

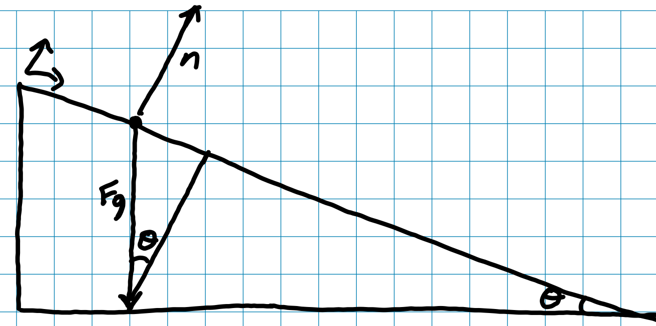


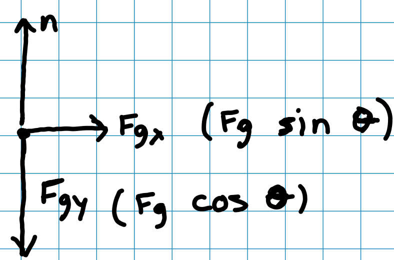
Up stop / Down start

* If $a = -g$ ∴ free fall, $n = 0$ (weightlessness)
  → scale falling under me at same $\vec{a}$, doesn't feel me push
  → Only completely true when no particles in air
  → Also when in orbit

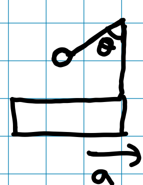
Inclined Plane

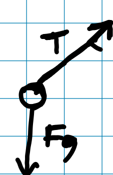


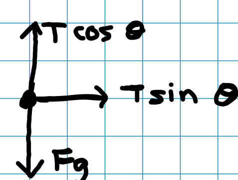




ex: ball suspended from a string is attached to a cart accelerating. What is the acceleration?







$$\underline{\Sigma F_x = m a_x}$$

$$T \sin\theta = m\, a_x$$

$$a_x = \frac{T \sin\theta}{m}$$

$$a_x = \frac{\cancel{T} \sin\theta}{\frac{\cancel{T} \cos\theta}{g}}$$

$$a_x = g \tan\theta$$

$$\underline{\Sigma F_y = m a_y}$$

$$T \cos\theta - F_g = m \underbrace{\cancel{a_y}}_{0}$$

$$T \cos\theta = F_g$$

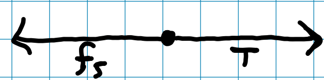
$$T \cos\theta = mg$$

$$\frac{T \cos\theta}{g} = m$$

Friction

Static friction (f_s)

→ Force that keeps an object from slipping



$F_{net} = 0$

Direction: opposite of likely motion (if no f_s)

- $\|f_s\|$ depends on the magnitude of the applied force (push / pull).
- f_s responds to applied force
- Static friction only exists to resist a force so if NO force is applied there is NO friction

$$f_s = T$$

- If an object is being pulled and doesn't move ∴ equilibrium ∴ $\Sigma F = ma$
  $T - f_s = 0$

- The harder you push, more f_s is big (to match push)

* Until you reach f_s max

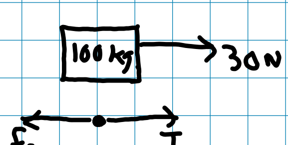
* $f_s\ max = \mu_s \cdot n$ $\Longrightarrow f_s \neq \mu_s \cdot n$ (f_s match push)

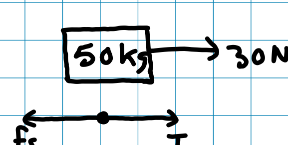
$f_s < f_s\ max$ ∴ rest

$f_s = f_s\ max$ ∴ slip (go to f_k)

$f_s > f_s\ max$ impossible

ex. 2 boxes being pulled with same tension (different masses)
Neither of the boxes move.



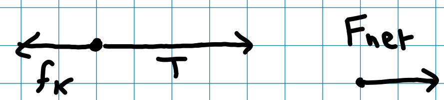


$\underline{\Sigma F = ma}$
$T - f_s = 0$
$T = f_s = 30\ N$

$\underline{\Sigma F = ma}$
$T - f_s = 0$
$T = f_s = 30\ N$ ∴ same f_s

Kinetic friction (f_K)

→ Once object slips, we have kinetic friction

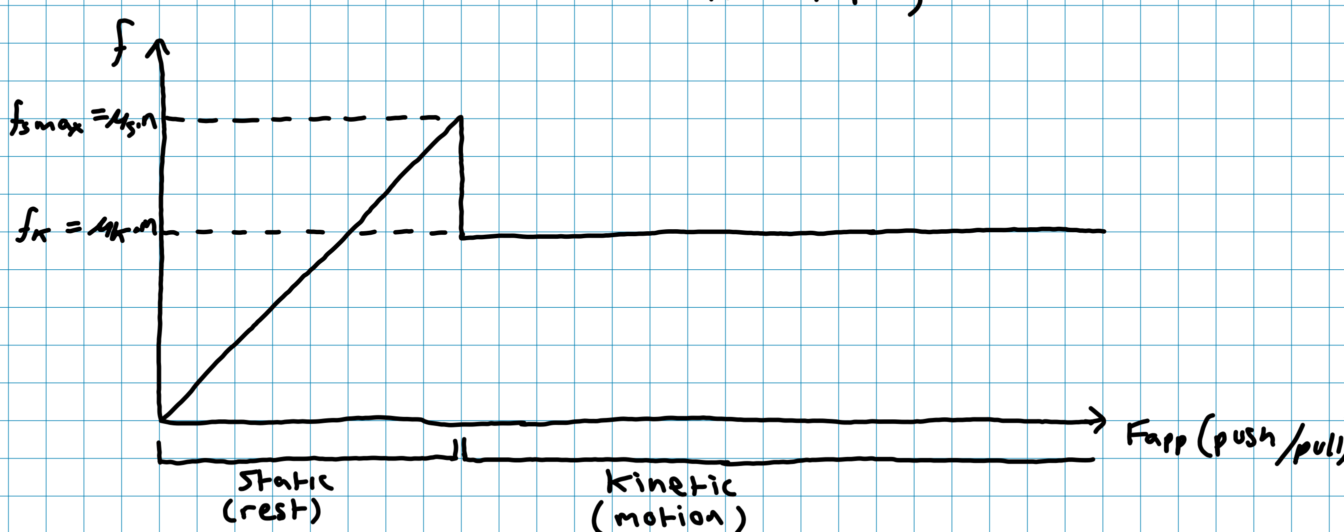


Direction: opposite direction of motion

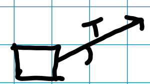
- $\|f_K\|$ constant

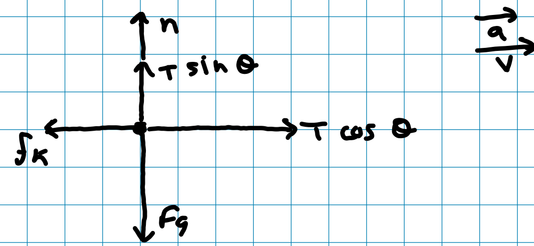
* $f_K = \mu_K \cdot n$

- $f_K < f_s \therefore \mu_K < \mu_s$ (easier to keep object moving than to stop)



ex: a box is being pulled by a ro that angles up. It is accelerating. What is the acceleration?





$\underline{\Sigma F_x = ma_x}$

$T\cos\theta - f_k = ma$

$T\cos\theta - (\mu_k \cdot n) = ma$

$T\cos\theta - \mu_k(-T\sin\theta + mg) = ma$

$T\cos\theta + \mu_k \cdot T\sin\theta - \mu_k \cdot mg = ma$

$\frac{T}{m}(\cos\theta + \mu_k \cdot \sin\theta) - \mu_k \cdot g = a$

$\underline{\Sigma F_y = ma_y}$

$n + T\sin\theta - F_g = 0$

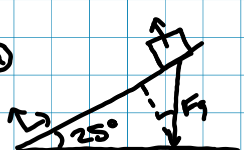
$n = mg - T\sin\theta$

ex: object placed on 2 m long platform. Small angles, object stays put. Tilt exceeds 25°, object slides. At 29°, 2 s to go down.

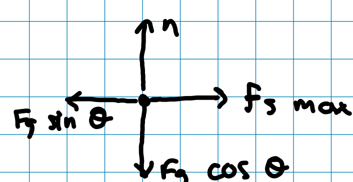
a) coefficient static friction

b) coefficient kinetic friction

a)



f_s max at 25°



$\underline{\Sigma F_x = ma_x}$

$f_{s\,max} - F_g\sin\theta = 0$

$\mu_s \cdot n = F_g \sin\theta$

$\mu_s = \frac{F_g \sin\theta}{F_g \cos\theta}$

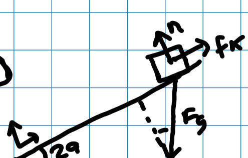
$\boxed{\mu_s = \tan 25}$

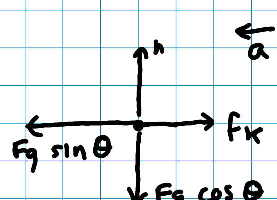
$\underline{\Sigma F_y = ma_y}$

$n - F_g\cos\theta = 0$

$n = F_g\cos\theta$

b)





$\underline{\Sigma F_x = ma_x}$

$f_k - F_g\sin\theta = ma$

$\mu_k \cdot n = ma + F_g\sin\theta$

$\mu_k = \frac{ma + mg\sin\theta}{mg\cos\theta}$

$\boxed{\mu_k = \frac{(-1)}{g\cos 29} + \tan 29}$

$\underline{\Sigma F_y = ma_y}$

$n - F_g\cos\theta = 0$

$n = F_g\cos\theta$

Acceleration:

$\Delta x = \cancel{v_i t} + \frac{1}{2}at^2$

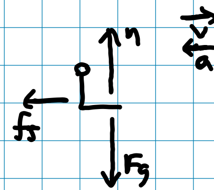
$-2 = 0 + \frac{1}{2}a(2)^2$

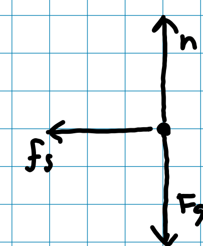
$a = -1 \text{ m/s}^2$

ex. (Determine if object goes to kinetic)
→ Verify if applied force is greater than f_s max

Car traveling 20 m/s stops in 50 m distance (constant $\vec{a}$)
$\mu_k = 0.3$, $\mu_s = 0.5$
Will passenger slide off

acceleration:
$V_f^2 - V_i^2 = 2a\Delta X$
$0 - 20^2 = 2a(50)$
$a = -4\ m/s^2$





$$\frac{\Sigma F_y = ma_y}{n - F_g = 0}$$
$n = mg$

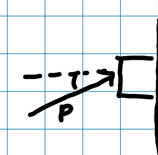
$$\frac{\Sigma F_x = ma_x}{-f_s = m(-4)}$$
$f_s = m(4)$

* If there were no friction, there would be no acceleration. The passenger would continue with constant $\vec{v}$ as car slows down and go through the window

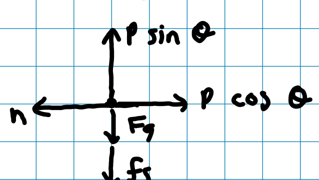
$f_s \neq \mu_s \cdot n$
$f_{s\,max} = \mu_s n$
$f_s \leq \mu_s n$

$m(4) \leq \mu_s (mg)$
$\frac{m\,4}{m\,g} \leq \mu_s \longrightarrow 0.41 \leq \mu_s$ Yes
∴ no slide

ex: Block pushed up against wall force P at θ angle
What interval P, object stays static?



Case 1 (P max)



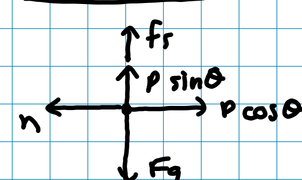
↑ likely motion

$$\frac{\Sigma F_x = ma_x}{P\cos\theta - n = 0}$$
$P\cos\theta = n$

$\boxed{f_s \leq f_{s\,max} \\ f_s \leq \mu_s \cdot n}$

$$\frac{\Sigma F_y = ma_y}{P\sin\theta - F_g - f_s = 0}$$
$P\sin\theta - F_g = f_s$
$P\sin\theta - mg \leq \mu_s \cdot n$
$P\sin\theta - \mu_s P\cos\theta \leq mg$
$P(\sin\theta - \mu_s\cos\theta) \leq mg$
$P \leq \frac{mg}{\sin\theta - \mu_s\cos\theta}$

Case 2 (P_{min})



↓ likely motion

$$\frac{\Sigma F_x = ma_x}{P\cos\theta - n = 0}$$
$P\cos\theta = n$

$\boxed{f_s \leq f_{s\,max} \\ f_s \leq \mu_s \cdot n}$

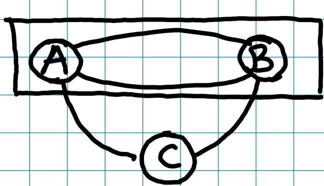
$$\frac{\Sigma F_y = ma_y}{f_s + P\sin\theta - F_g = 0}$$
$f_s = F_g - P\sin\theta$
$mg - P\sin\theta \leq \mu_s \cdot n$
$mg \leq \mu_s P\cos\theta + P\sin\theta$
$mg \leq P(\mu_s\cos\theta + \sin\theta)$
$\frac{mg}{\mu_s\cos\theta + \sin\theta} \leq P$

Newton's 3rd Law

→ interaction between 2 objects

Law: Every force is a member of an action/reaction pair
→ 2 forces act on 2 seperate objects
→ $F_{A \text{ on } B} = -F_{B \text{ on } A}$

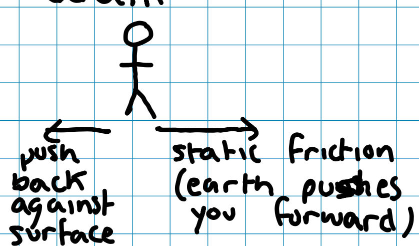
→ For contact and long range forces



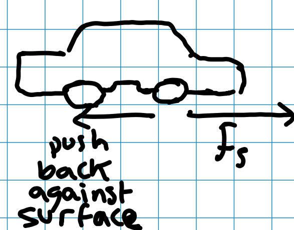
- each object = circle
- identify interactions (surfaces have 2: normal, friction)
- Enclose objects interest
- Draw free-body diagram for each object.

Propulsion (static friction)

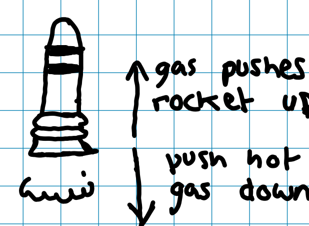
Walk



Drive



Rocket

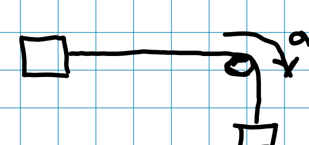


* If there were no friction, we would slide instead of moving (being propulsed) forward.

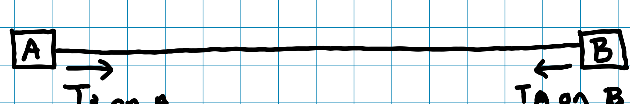
Acceleration constraints

→ 2 objects moving together have the same acceleration

→ Set direction of acceleration as the positive reference frame

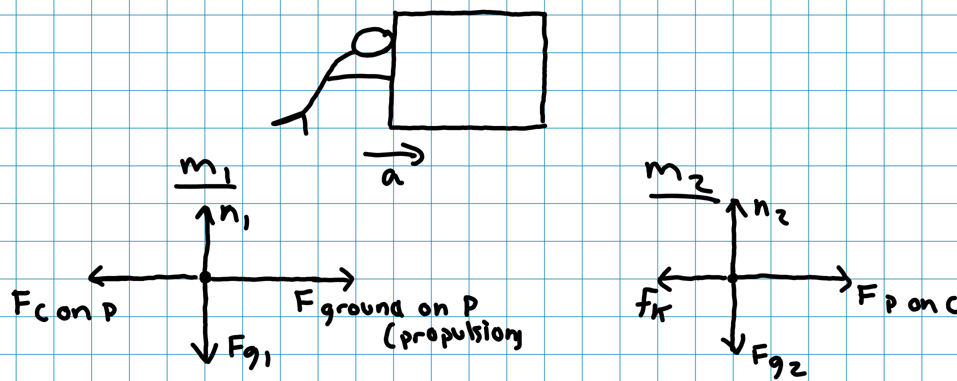


Tension

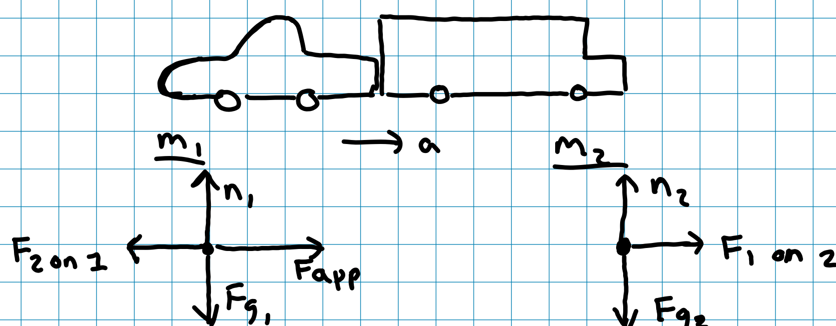


* The tensions are equal to eachother.

ex: Person pushing on a crate



ex. A 1100 kg car pushes a 2100 kg truck. Wheels of the car push against the ground with 4600N.



$$\underline{\Sigma F_x = ma_x}$$

$$F_{app} - F_{2\ on\ 1} = m_1 a$$

$$\underline{\Sigma F_x = ma_x}$$

$$F_{1\ on\ 2} = m_2 a$$

Addition or sub ($\because$ There's a force-pair)

$$F_{app} - (m_2 a) = m_1 a$$

$$F_{app} = (m_1 + m_2)\, a$$

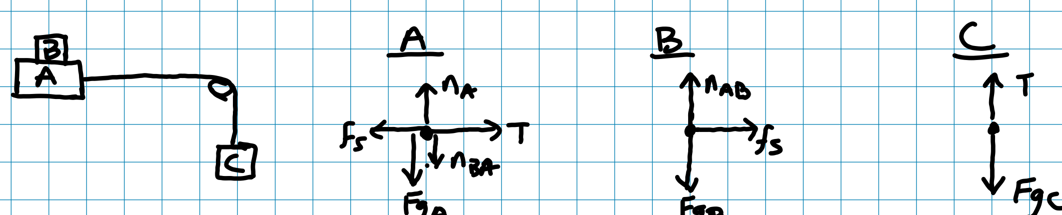
$$a = \frac{4600\ N}{1100\ kg + 2100\ kg} = 1.4375\ m/s^2$$

$$F_{c\ on\ T} = m_2 a$$

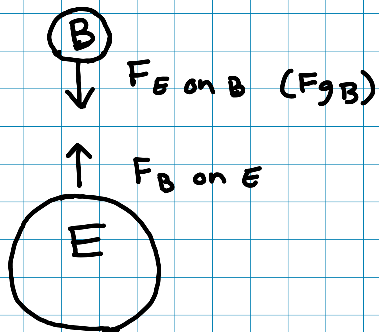
$$= (2100\ kg)(1.4375\ N/kg)$$

$$= 3000\ N$$

ex:



ex: ball and earth gravitational attraction



→ equal forces, so why doesn't the earth fall towards the ball?
→ Different mass ∴ different acceleration

$\underline{\Sigma F_y = m a_y}$

$F_{E \text{ on } B} = m_B \cdot a_B$

$a_B = \dfrac{F_{E \text{ on } B}}{m_B}$

$\underline{\Sigma F_y = m a_y}$

$F_{B \text{ on } E} = m_E \cdot a_E$

$a_E = \dfrac{F_{B \text{ on } E}}{m_E}$

Because $-F_{E \text{ on } B} = F_{B \text{ on } E}$,
∵ m_B is much smaller than m_E
you divide by a much smaller number
so acceleration is much greater

$a_B = -g$

$$a_E = \frac{F_{B \text{ on } E}}{m_E}$$

$$= \frac{-F_{E \text{ on } B}}{m_E}$$

$$= \frac{-(m_B \cdot a_B)}{m_E}$$

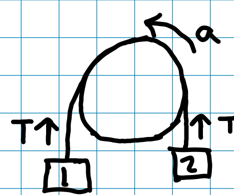
$$a_E = g\left(\frac{m_B}{m_E}\right)$$

* Huge m_E ∴ small a_E

Atwoods Machine

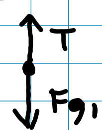
* Use subscripts

ex:



* Set direction of acceleration as positive reference frame

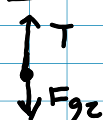
m_1



$\Sigma F = ma$

$F_{g_1} - T = m_1 a$

m_2



$\Sigma F = ma$

$T - F_{g_2} = m_2 a$

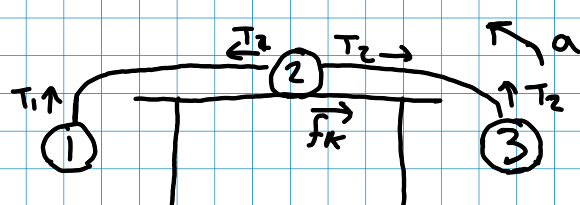
Addition :

$$\begin{aligned} m_1 g - T &= m_1 a \\ T - m_2 g &= m_2 a \\ \hline m_1 g - m_2 g &= (m_1 + m_2) a \end{aligned}$$

$$\boxed{\frac{g(m_1 - m_2)}{(m_1 + m_2)} = a}$$

* Tensions cancel

ex:



m_1

$\Sigma F_y = ma_y$

$F_{g_1} - T_1 = m_1 a$

m_2

$\Sigma F_x = ma_x$

$T_1 - T_2 - f_K = m_2 a$

$\Sigma F_y = ma_y$

$F_{g_2} - n_2 = 0$

m_3

$\Sigma F_y = ma_y$

$T_2 - F_{g_3} = m_3 a$

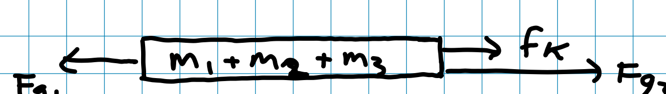
Addition :

$$\begin{aligned} m_1 g - \cancel{T_1} &= m_1 a \\ \cancel{T_1} - \cancel{T_2} - f_K &= m_2 a \\ \cancel{T_2} - m_3 g &= m_3 a \\ \hline \end{aligned}$$

$$m_1 g - m_3 g - (\mu_K m_2 g) = a(m_1 + m_2 + m_3)$$

$$\frac{g(m_1 - m_3 - \mu_K m_2)}{(m_1 + m_2 + m_3)} = a$$

or

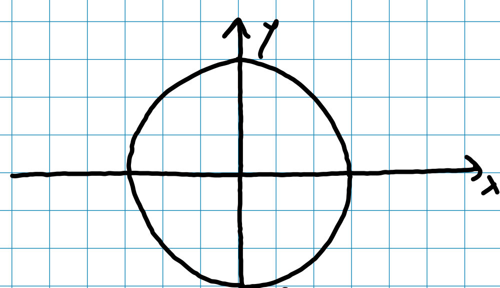


Circular Motion

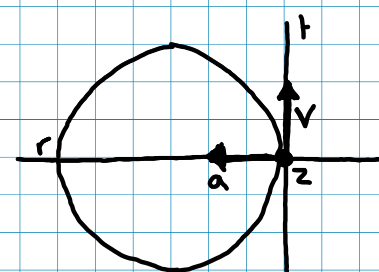
→ Particle moving in a circular path of radius r

→ To be in circular motion there must be a force pointing towards the centre of circle

rtz coordinate system



Not effective

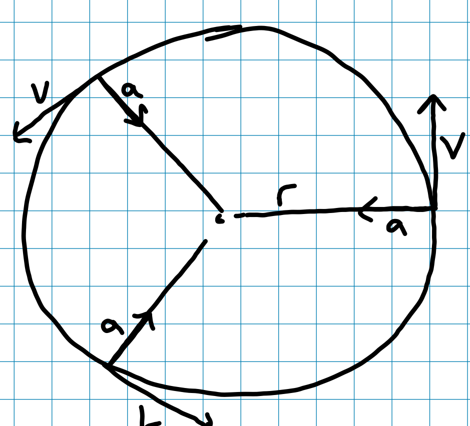


- origin : particle position
- r axis : particle to center
- t axis : tangent to circle (counter-clockwise)
- z axis : ⊥ plane of motion

How acceleration affects velocity:

→ $\vec{a}$ changes $\vec{v}$

- a ⊥ changes direction of $\vec{v}$
- a // changes magnitude of $\vec{v}$ (speed)



There must constantly be an a ⊥ to v pointing towards the centre of the circle

Angular position:

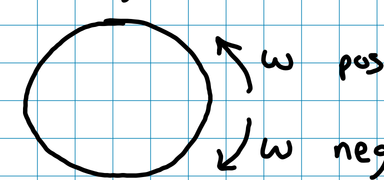
$\boxed{S = r\theta}$

θ = angular position   θ in rad

Angular velocity (ω):

→ Rate at which angular position changes

$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$



$\boxed{\omega = \frac{d\theta}{dt}}$ [rad/s]

* Constant ω ⇒ uniform circular motion (UCM)

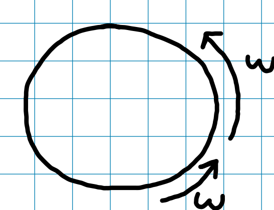
Period (T) = time interval to complete one revolution (2π rad/$360°$)

$\boxed{T = \frac{2\pi}{|\omega|}}$ $\frac{(rad)}{(rad/s)}$ 2π = full revolution

Angular acceleration (α):

→ Rate at which angular velocity changes

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$



$\boxed{\alpha = \frac{d\omega}{dt}}$ [rad/s²]

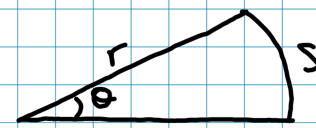
* $\alpha = 0$ ⇒ UCM

Rotational Equations

$\omega_f = \omega_i + \alpha\Delta t$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

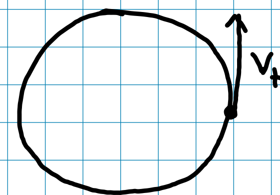
$\Delta x \Rightarrow \Delta\theta$
$v \Rightarrow \omega$
$a \Rightarrow \alpha$

Position :



$\boxed{S = r\theta}$ [m]

Tangential velocity (v_t):



→ Tangent to trajectory

$$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

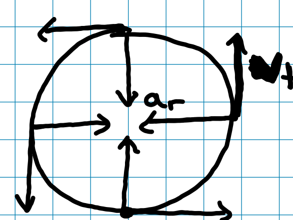
$\boxed{v_t = r \cdot \omega}$ [m/s]

$\boxed{T = \frac{2\pi r}{v}}$ $\frac{\text{circumference}}{\text{velocity}}$

* constant $\|v_t\| \Rightarrow$ UCM

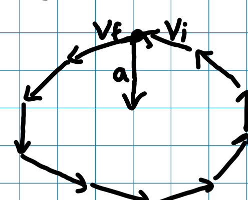
Acceleration :

Centripetal acceleration (a_r):

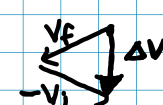


→ Doesn't change speed ($\|v_t\|$)
→ $\vec{a_r} \perp \vec{v_t}$ ∴ changes direction
→ points towards centre (seeks circle)

$$\boxed{a_r = \frac{v^2}{r} = \omega^2 r}$$

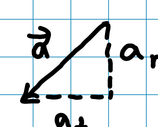


$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_f - v_i}{t}$$

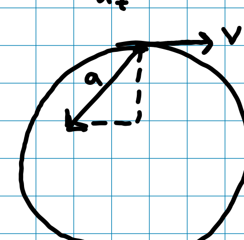


2 accelerations ? ex: Slowing down in circular

- Driving in circle (a_r : $a\perp$ change direction)
- Slow down (a_t : $a //$ change speed)



Tangential acceleration (a_t):



$$a_t = \frac{dv_t}{dt} = \frac{d\omega}{dt} \cdot r = \alpha \cdot r$$

$\boxed{a_t = \alpha \cdot r}$

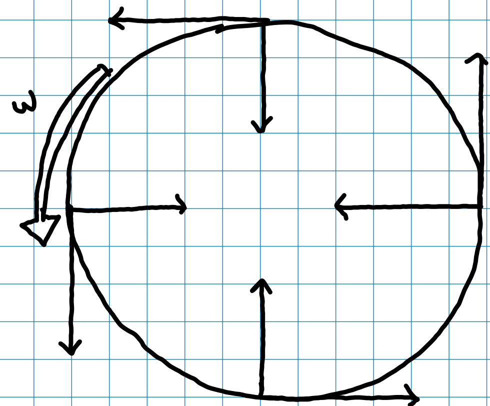
* $a_t = 0 \Rightarrow$ UCM

$\boxed{S = r\ \theta}$ given

$\boxed{v_t = r \cdot \omega}$

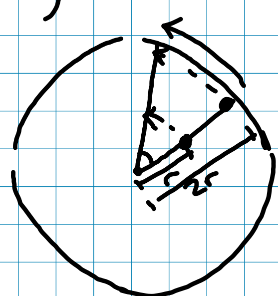
$\boxed{a_t = r \cdot \alpha}$

Uniform Circular Motion



- constant ω
- steady rotation
  - ↳ system fails when rotation isn't steady

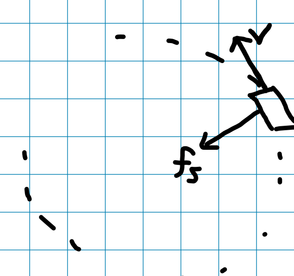
Angular velocity vs Tangential velocity



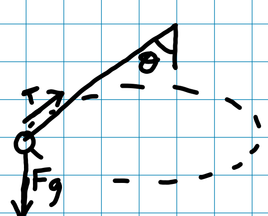
- ω is the same for both particles (same change in angle)
- The higher particle has to travel double the arc-length in the same time ∴ Double ~~xxxxxxx~~ V_t
  $V_t = \omega \cdot r$
- a_r is also doubled : $a_r = \omega^2 r$

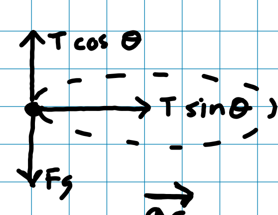
Car in circular motion

- gas/break : a_t
- steering : a_r



Free body diagram

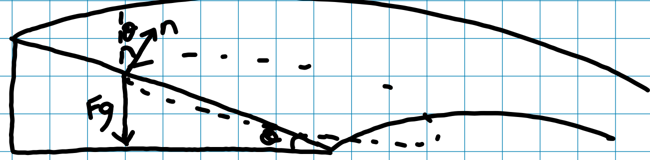


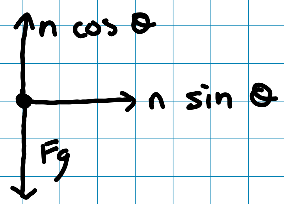


$$\underline{\Sigma F_r = m a_r} \qquad\qquad \underline{\Sigma F_z = m a_z}$$
$$T \sin\theta = m\left(\frac{v^2}{r}\right) \qquad\qquad T\cos\theta - F_g = 0$$
$$T\cos\theta = F_g$$

$$① \div ② \quad \frac{\cancel{T} \sin\theta}{\cancel{T} \cos\theta} = \frac{\cancel{m} \cdot v^2/r}{\cancel{m} \cdot g} \Rightarrow \tan\theta = \frac{v^2}{g \cdot r}$$

Banked rode (NOT INCLINE PLANE)





$$\underline{\Sigma F_r = ma_r}$$
$$n \sin\theta = ma_r$$
$$n \sin\theta = m \cdot v^2/r$$

$$\underline{\Sigma F_z = ma_z}$$
$$n \cos\theta - F_g = 0$$
$$n \cos\theta = F_g$$

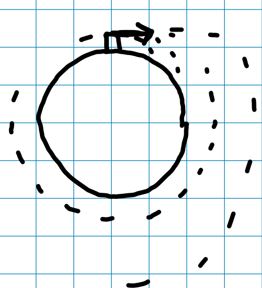
① ÷ ②

$$\frac{n \sin\theta}{n \cos\theta} = \frac{m \; v^2/r}{m \cdot g}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$v = \sqrt{\tan\theta \cdot r \cdot g}$$

Orbit



$$a_r = \frac{v^2}{r}$$
$$V_t = \sqrt{ar}$$

- If V_t is too small, it goes into projectile motion with a flat earth surface
- If V_t is too big, it leaves the orbit (not enough "a" to keep in O)
- Perfect V_t : orbit
  → constantly falling

g depends on r

$$\underline{\Sigma F_r = ma_r}$$
$$F_g = m g_{(r)}$$
$$\frac{G \cdot m \cdot m_E}{r^2} = mg$$
$$\boxed{g = \frac{G \cdot m_E}{r^2}}$$ → acceleration (depends on r)

$$F_g = \frac{G \cdot m \cdot m_E}{r^2}$$

At the surface of the earth (r = radius of earth)
$$g = 9.8 \; m/s^2$$

Critical speed = slowest speed at which particle can stay in circular motion

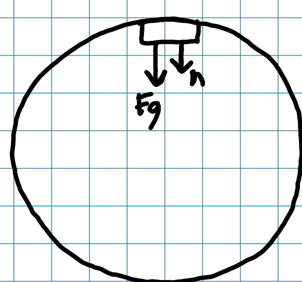
→ orbits

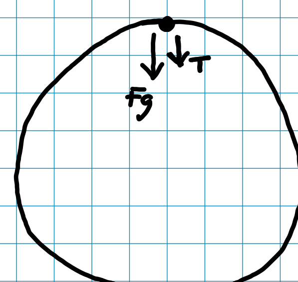
$$\frac{\Sigma F_r = ma_r}{F_g = ma}$$
$$\frac{G\cancel{m}.m_E}{r^2} = \cancel{m} \cdot \frac{v^2}{\cancel{r}}$$
$$v^2 = \frac{G.m_E}{r} \quad \therefore \quad \boxed{V = \sqrt{\frac{G m_E}{r}}}$$

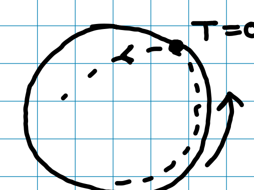
→ Rollercoaster or String

- At max height, if there's the slightest normal force or tension ⇒ can stay in circle
- If particle falls out of circle, it's in QI

↳ Reason: If tension or normal force go to zero before max height, theres no more contact with the edge of the circle and the particle starts to fall





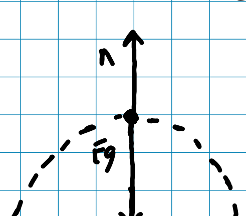


$n \geq 0$
$$\frac{\Sigma F = ma}{F_g + n = ma}$$
$$n = m.\frac{v^2}{r} - mg$$
$$m\left(\frac{v^2}{r} - g\right) \geq 0$$
$$v \geq \sqrt{g.r}$$

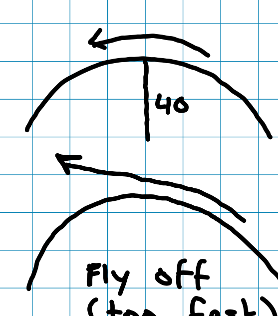
$T \geq 0$
$$\frac{\Sigma F = ma}{F_g + T = ma}$$
$$T = ma - F_g$$
$$ma - F_g \geq 0$$
$$a \geq \frac{mg}{m}$$
$$\frac{v^2}{r} \geq g$$
$$v \geq \sqrt{g.r}$$

ex: car rolling over hill $(r = 40m)$. what is max velocity to not fly of rode ?

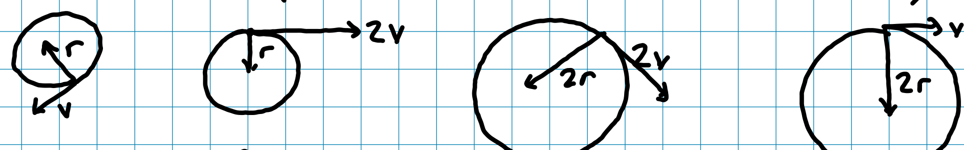
* condition: at max height, must be n
  ↳ $n \geq 0$



$$\frac{\Sigma F_r = ma_r}{F_g - n = ma_r}$$
$$n = mg - ma \geq 0$$
$$\frac{mg}{m} \geq \frac{v^2}{r}$$
$$\sqrt{g.r} \geq v \quad \therefore \quad \boxed{V \leq \sqrt{g.r}}$$

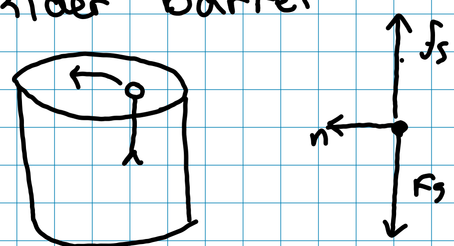


ex: Rank centripetal acceleration from biggest to smallest



$\frac{v^2}{r} = a$ $\quad \frac{(2v)^2}{r} = 4a$ $\quad \frac{(2v)^2}{2r} = 2a$ $\quad \frac{v^2}{2r} = \frac{1}{2}a$

ex: Rider barrel



normal force is centripetal

$\underline{\Sigma F_r = ma_r}$ $\quad \underline{\Sigma F_z = ma_z}$

$n = ma_r$ $\quad f_s = mg$

$f_s \leq f_{s\,max} = \mu_s \cdot n$

$mg \leq \mu_s (m \cdot v^2/r)$

$v \geq \sqrt{gr/\mu_s}$

ex: Where does the net force on M_2 point?

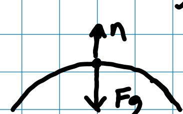
$F_{g2} > F_{g1}$ $\therefore$ point left.

$F_{g1} = \frac{G\, m \cdot 2m}{2^2} = \frac{1}{2} G m^2$

$F_{g2} = \frac{G\, m \cdot 2 \cdot 5m}{4^2} = \frac{5}{8} G m^2$

Weight (n)

→ Car rolling over "hill"

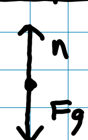


$\downarrow a_r \quad \therefore n < F_g$

$\uparrow a_r \quad \therefore n > F_g$

→ Equator vs North pole

North pole

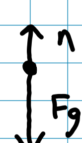


$\underline{\Sigma F_y = ma_y}$

$n - F_g = 0$

$n = mg$

Equator



$\downarrow a_r$ * Set "a" as positive

$\underline{\Sigma F_r = ma_r}$

$F_g - n = ma_r$

$mg - ma = n$

$n = m(g - a)$

ex: 33 r.p.m with $r = 15$ cm. What's angular speed?

$$33 \frac{rev}{min} \cdot \frac{2\pi\ rad}{rev} \cdot \frac{min}{60\ s} = \frac{11}{10}\pi \ rad/s = \omega$$

ex: Record player playing, then turned off. Completes 1200 revs in 40 s.
What is initial angular velocity?
What is angular acceleration?

$$\Delta\theta = 1200\ rev \cdot \frac{2\pi\ rad}{rev} = 2400\pi\ rad$$

$t = 40\,s$

$\omega_f = 0$

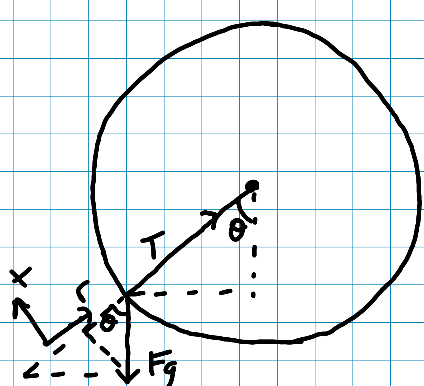
$$\Delta\theta = \left(\frac{\omega_i + \omega_f}{2}\right) \cdot t$$

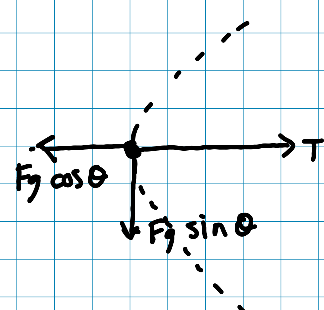
$\omega_i = 120\pi\ rad/s$

$$\Delta\theta = \omega_i \cdot t + \tfrac{1}{2}\alpha t^2$$

$\alpha = -3\pi\ rad/s^2$

ex: Vertical circle
→ Find expression for T



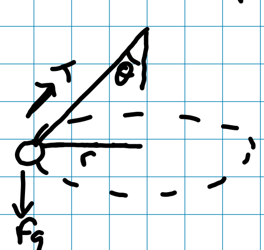


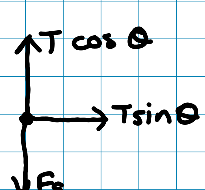
$$\underline{\Sigma F_r = ma_r}$$
$T - F_g \cos\theta = ma_r$

$T = m(g\cos\theta + a_r)$

$T = m\left(g\cos\theta + \frac{v^2}{r}\right)$

ex: conical pendulum





$$\underline{\Sigma F_r = ma_r}$$
$T\sin\theta = ma_r$

$$\underline{\Sigma F_z = Ma_z}$$
$T\cos\theta - F_g = 0$

$T\cos\theta = mg$

① ÷ ② $\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}a_r}{\cancel{m}g}$

$\tan\theta = \frac{v^2}{r \cdot g} \Rightarrow v = \sqrt{r \cdot g \cdot \tan\theta}$

* Same as banked road

Energy and Work

Energy of a CLOSED system.

→ The total energy of a closed system remains unchanged

Transformation of energy:

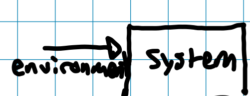
→ Energy can be transformed without changing the total amound
- Can be transformed to kinetic, to potential (gravitational or spring potential), etc.

ex: ball drop
o max potential energy (U_g)
↓
o max kinetic energy (K)

If a system interacts with the outside environment, energy can be transfered

Transfer of energy:

→ Adding or removing energy from the system
→ Transfer of energy can be done 2 ways:
1. Work (mecnanical)
2. Heat (thermal)



ex: push a chair (add energy)
lift a ball (add energy)
slowly put down a ball (remove energy)

Types of energy:

Kinetic : $K = \frac{1}{2} mv^2$

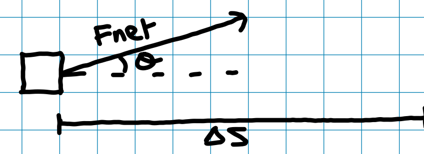
Potential gravitationnal : $U_g = mgy$

Potential spring : U_s

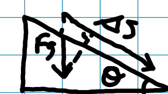
Work (W)

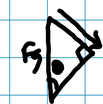
$W = \vec{F} \cdot \vec{\Delta s}$ $\quad [N.m = J]$

$W = F(\cos\theta) \cdot \Delta s$



* If incline plane





$F(\sin\theta) \cdot \Delta s = W$

For there to be work:
→ Net Force $\neq 0$
  (∴ constant v → no work)
→ Displacement
  (∴ hold up a rock for 2 hours → no work - but tiring)
→ Transfer in energy
  (∴ transformation → no work)

Work is the component of the force pointing in the direction of motion multiplied by the displacement

Work is the variation in energy of the system

$\underset{(ext)}{W} = \underset{(system)}{\Delta E}$

$W_{ext} = \Delta K + \Delta U_g + \Delta U_s$
 * sometimes one of them is just 0
  (initial v and final v is same, no spring, no gravity)

$W_{net} = \vec{F}_{net} \cdot \vec{\Delta s}$
$= (F_1 + F_2 + F_3 \ldots) \cdot \vec{\Delta s}$
$= W_1 + W_2 + W_3 \ldots$

$$\left.\begin{aligned} W_{net} &= F \cdot \Delta s \\ &= m \cdot a \cdot \Delta s \\ &= m \frac{(v_f - v_i)}{t} \frac{(v_f + v_i)}{2} t \\ &= \frac{m}{2}(v_f^2 - v_i^2) \\ &= \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 \end{aligned}\right\} W_{net} = K_f - K_i = \Delta K$$

* speed up : pos. work
  slow down : neg. work

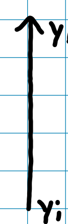
$$\left.\begin{aligned} W_g &= \vec{F_g} \cdot \vec{\Delta s} \\ &= mg \cdot \Delta s \\ &= (-mg)\hat{j} \cdot (y_f - y_i)\hat{j} \\ &= -mgy_f + mgy_i \\ &= -(mgy_f - mgy_i) \end{aligned}\right\} W_{net} = -\Delta U_g$$

Conservation of mechanical energy

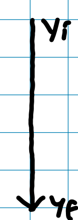
→ When an object is let to fall or slide down:
- → mechanical energy is constant ($\Delta E_{tot} = 0$)
- → $W_{ext} = 0$ (nothing outside of the system adds energy that wasn't previously there)
- → $W = \Delta K + \Delta U = 0$

$\Delta K = -\Delta U$

Free fall:



ΔK: ⊖
ΔU: ⊕



ΔK: ⊕
ΔU: ⊖

Lift up rock at constant speed: In this case, you are adding energy to system so there is work (ext). However the net work, is zero, since the net force is 0 (constant speed)

Sign of work

⊕ → Force in direction of motion (speed up)

ex: Push chair
Lift up cup in air

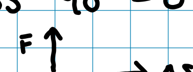
⊖ → Force in opposite direction of motion (slow down)

ex: friction
put cup down (force up)

$\cos 0 = 1$

⊕ work

$\cos 90 = 0$



no work

$\cos 180 = -1$

⊖ work

Using unit vector dot product:

$W = \vec{F} \cdot \vec{\Delta S}$

$F = (3\hat{i} - 5\hat{j})\ N$
$\Delta S = (-6\hat{i} + 4\hat{j})\ m$

$W = (3\hat{i} - 5\hat{j})N \cdot (-6\hat{i} + 4\hat{j})m$
$= (-18 - 20)\ Nm$
$= -38\ J$

$\hat{i} \cdot \hat{i} = |\hat{i}| \cdot |\hat{i}| \cdot \underbrace{\cancel{\cos 0}}_{1}$

$\hat{i} \cdot \hat{j} = |\hat{i}| \cdot |\hat{j}| \cdot \underbrace{\cancel{\cos 90}}_{0}$

multiply only $\hat{i}$ with $\hat{i}$ and $\hat{j}$ with $\hat{j}$, others cancel out ($\hat{i}$ with $\hat{j}$)

W_{Force} vs. W_{net} vs W_{ext}

W_{Force} = the work done by one force
$W = \vec{F} \cdot \vec{\Delta s}$

ex: the work done by gravity when it pulls an object in free fall

~~ex~~

W_{net} = the work done by all forces combined acting on object
because we use F_{net} (ΣF) we can say it is = ma
From this we can derive that $W_{net} = \Delta K$
and if speed is constant throughout, theres no acceleration $\therefore \Sigma F = 0 \therefore W = 0$

$$W_{net} = \Sigma\vec{F} \cdot \vec{\Delta s}$$
$$= W_1 + W_2 + W_3$$
$$= \Delta K$$

W_{ext} = the work is the energy added to the system from an outside force

$$W_{ext} = \Delta E_{system}$$
$$= \Delta K + \Delta U_g + \Delta U_s$$

examples:	W_{Force}	W_{net}	W_{ext}
1. Gravity on block F_g ↓ □ y_i □ y_f	$W = \vec{F_g} \cdot \vec{\Delta s}$	$W_{net} = \Delta K$ $= -\Delta U_g$ = to W_{Force} (only 1 force)	$W_{ext} = 0$ total energy constant $-\Delta U_g = \Delta K$ all U_g converted ΔK
2. 2 people push fridge F_1 → F_2 →	work of only one of the forces	work of both forces combined $W_{net} = F_{net} \cdot \Delta S$	Both forces are adding to sys from outside
3. F_a → □ ← F_f doesn't move	no work ($\Delta S = 0$) $W = F \cdot \Delta S$	$\Sigma F = 0$ $W_{net} = F_{net} \cdot \Delta S = 0$	zero
4. Lift rock constant v ↑F ↑Δr ↓F_g $\Sigma F = ma$ $F - F_g = 0$ $F = F_g$	positive work only of hand Force direction displacement	$\Sigma F = F - F_g = 0$ $W_{net} = 0$ constant v $\therefore \Delta K = 0$ $W_{net} = \Delta K = 0$	external force hand $W_{ext} = \Delta K + \Delta U$ $W_{ext} = \Delta U$ adding potential ⊕ energy to system
5. constantly let rock come down ↑F ↓Δr ↓F_g	negative work push up (resist gravity) displacement down	$\Sigma F = 0$ $\Delta K = 0$ $W_{net} = 0$	$W_{ext} = \Delta K + \Delta U$ $= \Delta U$ removing potential ⊖ energy from sys
6. Friction f_k ← → ΔS	only friction (neg)	can be pos or neg ΣF vs displacement $W = \Delta K$ (slow down)/speed up)	

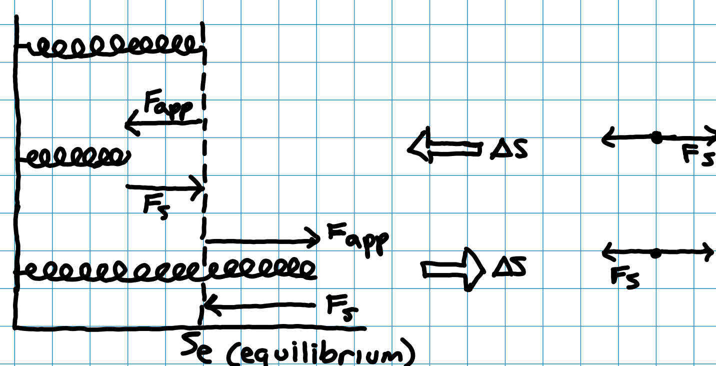
Springs

$\vec{F}_s = -K\vec{\Delta S}$ [K is spring constant]

- the negative sign tells us that $\vec{F}_s$ always seeks to return to the equilibrium position (restoring force) and so it points in the opposite direction as $\vec{\Delta S}$
- because a spring, when stretched or compressed, always seeks to return to its equilibrium position, there is energy stored in the coils of the spring.

$U_s = \frac{1}{2} K \Delta S^2$

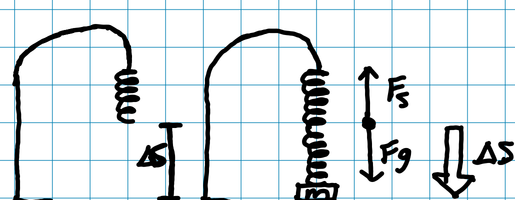
$W_s = -\Delta U$



$F_s = 0$

$\Sigma F_x = \cancel{ma}$
$F_{app} - F_s = 0$ ∵ spring is not moving
$F_{app} = F_s$

$\Delta S = (S_f - S_e)$

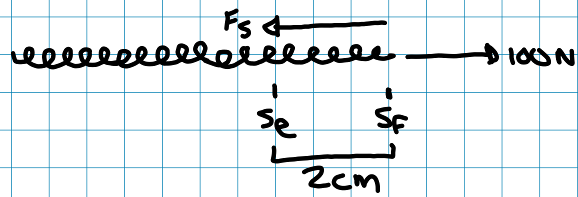


$\Sigma F_y = ma_y$
$F_g - F_s = 0$
$F_g = K\Delta S = F_s$
$K = \frac{mg}{\Delta S}$

Sign of work for spring:
- Man compresses spring
  - $\vec{F}$ and $\vec{\Delta S}$ have same direction ∴ ⊕
  - Add energy (ΔE +)

⊛ Start every work/energy problem with $W_{ext} = \Delta E = \Delta K + \Delta U_g + \Delta U_s$

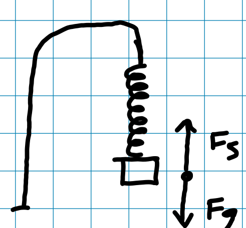
ex: 20 cm spring attached to wall. Stretched to 22 cm with force 100N. Same spring suspended with 5kg mass attached. How long is stretched spring?



$F_s = K \Delta S$

$100 = k(0.02)$

$k = 5000$ N/m



$\underline{\Sigma F_y = ma_y}$

$F_s - F_g = 0$

$F_s = mg$

$F_s = K \Delta S$

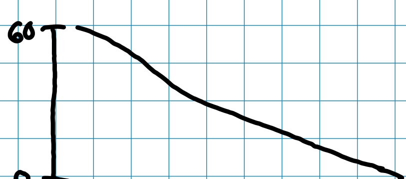
$mg = K(S_f - S_e)$

$S_f = \frac{mg}{K} + S_e$

$S_f = 0.2098$ m

$= 21$ cm

ex: Skier at top of 60 m high slope



a) No resistive force, what is her speed

$W_{ext} = 0$

$\Delta K = -\Delta U$

$K_f - \cancel{K_i} = -(\cancel{U_f} - U_i)$

$\frac{1}{2}mv^2 = mgy$

$v = \sqrt{2gy} = 34.3$ m/s

b) Resistive force, what energy lost if $m = 60$ kg and $V_f = 25$ m/s

$W_{ext} = \Delta K + \Delta U$

$= (K_f - \cancel{K_i}) + (\cancel{U_f} - U_i)$

$= \frac{1}{2}mv_f^2 - mgy_i$

$= -16530$ J

Note: However $W_{net} \geq 0$

$\therefore \Delta K \geq 0$

c) No resistive force, at end of hill, run into spring ($K = 2000$ N/m). Find compression of spring

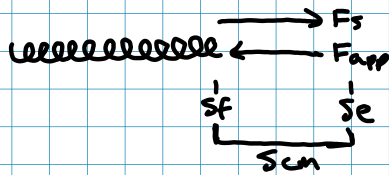
$W_{ext} = 0$

$0 = \cancel{\Delta K} + \Delta U_g + \Delta U_s$

$= (\cancel{U_{gf}} - U_{gi}) + (U_{sf} - \cancel{U_{si}})$

$= \frac{1}{2}K\Delta S^2 - mgy$

$\Delta S = \sqrt{\frac{2mgy}{K}} = 5.94$ m ex: 2 kg block attached to spring ($k = 600$ N/m) and compressed 5 cm. Block released from rest. What is blocks speed at equilibrium position.



$W_{ext} = 0$

$0 = \Delta K + \Delta U_s$

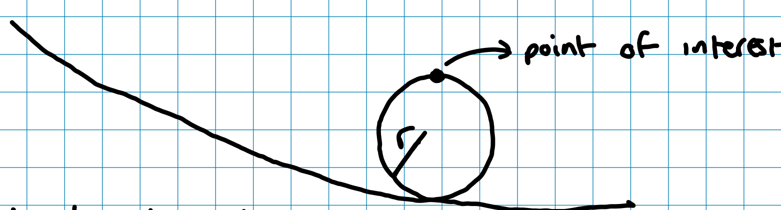
$0 = K_f - \cancel{K_i} + \cancel{U_{sf}} - U_{si}$

$\frac{1}{2} m v^2 = \frac{1}{2} k \Delta s^2$

$v = \sqrt{\frac{k \Delta s^2}{m}}$

$v = 0.87$ m/s

ex: What is min height we can release the ball for it to complet the loop



circular dynamics

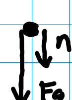
$\Sigma F_r = m a_r$

$F_g + n = m a_r$

$n = m(a_r - g)$

$m\left(\frac{v^2}{r} - g\right) \geq 0$

$v \geq \sqrt{g \cdot r}$



$n \geq 0$

conservation energy

Note: U_f is not 0

At top of loop $y_f = 2r$

$W_{ext} = 0$

$\Delta K = -\Delta U$

$K_f - \cancel{K_i} = -(U_f - U_i)$

$\frac{1}{2} m v^2 = -m g y_f + m g y_i$

$\frac{1}{2} m v^2 = -m g (2r - y_i)$

$v^2 = -2g(2r - y_i)$

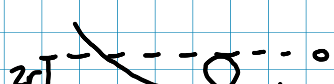
$-2\cancel{g}(2r - y_i) \geq \cancel{g} r$

$2r - y_i \leq \frac{r}{-2}$

$2r + \frac{r}{2} \leq y_i$

$\boxed{y_i \geq 2.5 r}$

OR set top of loop as zero



$\Delta K = -\Delta U$

$v^2 = 2 g y_i$

$2 g y_i \geq g r$

$y_i \geq \frac{r}{2}$

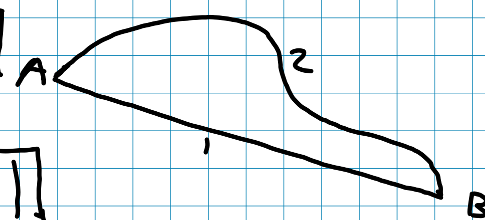
height $\geq \frac{r}{2} + 2r$

Conservative vs Nonconservative force

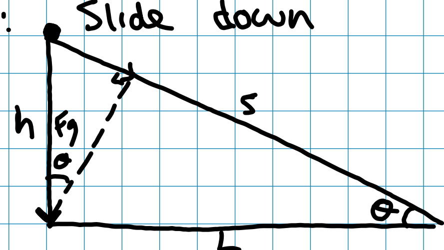
→ conservative forces have the ability to store work as potential energy
$W_c = -\Delta U$

- conservative force : $\Delta E = 0$
- non-conservative : $W = \Delta E$ ex : friction

- conservative force is path independant $\boxed{W_1 = W_2}$
- non-conservative force is path dependant
  → longer the path, the more friction $\boxed{|W_1| < |W_2|}$



Proof: Slide down



$$W_g = \vec{F_g} \cdot \vec{\Delta s}$$
$$= F_g \cdot \Delta s \cdot \sin\theta$$
$$= F_g \, \Delta s \cdot \left(\frac{h}{\Delta s}\right)$$
$$= mgh$$
∴ path independant

$$W_{fk} = \vec{f_k} \cdot \vec{\Delta s}$$
$$= f_k \cos 180 \cdot \Delta s$$
$$= -\mu_k \cdot n \cdot \Delta s$$
$$= -\mu_k (mg \cos\theta) \, \Delta s$$
$$= -\mu_k \cdot mg \left(\frac{L}{\Delta s}\right) \Delta s$$
$$= -\mu_k \cdot mg \cdot L$$
∴ path dependant

Power Output

$P = \frac{\Delta E}{\Delta t} = \frac{dE}{dt}$ [J/s = W]

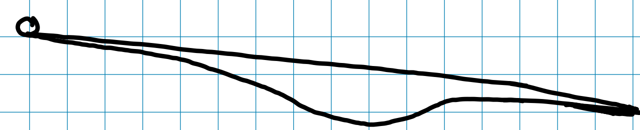
$P = \vec{F} \cdot \vec{v}$ [N.m/s = J/s]

Note: $kWh = 10^3 \frac{J}{s} \cdot 3600\,s$
⇒ Joules (energy)

ex: block shot up 40° slope with speed v ⇒ reaches height h
if shot up 20° slope with speed v, what height will it reach? (no friction)

h ∵ path independant

ex:

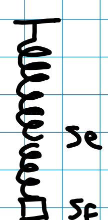


a) Which ball reaches end of track first?
→ on curved track

b) which ball has greater speed at the end?
→ both have same

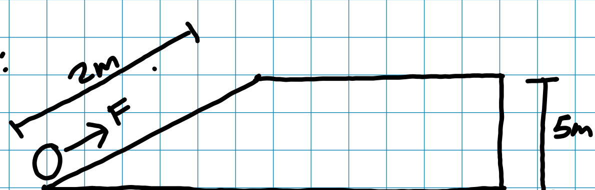
* Ball on curved track had greater speed accross the whole trajectory

ex



Stretched and released.
When it comes back down, it will come to the same height
∵ conservation of energy

ex:



end velocity is zero

$$\vec{W}_{ext} = \cancel{\Delta K} + \Delta U$$
$$\vec{F} \cdot \vec{\Delta S} = 0 + (mgy_f - \cancel{mgy_i})$$
$$F = \frac{mgy_f}{\Delta S}$$
$$F = 12.5 \text{ N}$$

ex. block slides down ramp from height h reaches speed v. What height must it be released to reach 2v?

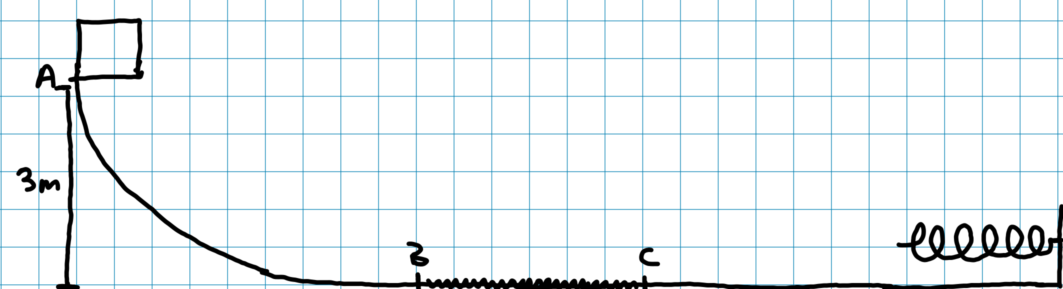
$$K_f - K_i = -U_f + U_i$$
$$\tfrac{1}{2}mv^2 = mgh$$
$$v = \sqrt{2gh}$$
$$2v = \sqrt{2g(4h)} \quad \therefore 4h$$

ex. hockey puck slides with $v = 4\,m/s$, comes to 1m high hill. Will it make it to the top?

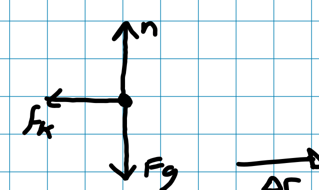
$$W_{ext} = 0$$
$$K_f - K_i = -mgy_f + \cancel{mgy_i}$$
When will $K_f = 0$
$$-\tfrac{1}{2}mv_i^2 = -mgy_f$$
$$y_f = \frac{v_i^2}{2g} = \frac{16}{2g}$$

$y_f < 1$ ∴ hockey puck will not make it

ex: 10 kg block released from point A. Frictionless track between B and C (6 m length). Block hits spring ($K = 2250\,N/m$) and compresses spring 0.3 m. What is μ_k between B and C?



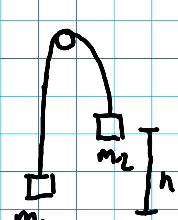
$$W_{ext} = W_{f_k}$$
$$\cancel{\Delta K} + \Delta U_g + \Delta U_s = \vec{f_k} \cdot \vec{\Delta r}$$
0 rest to rest
$$\cancel{U_{gf}} - U_{g_i} + U_{sf} - \cancel{U_{si}} = f_k \cdot \cos 180 \cdot \Delta r$$
$$\tfrac{1}{2}K\Delta s^2 - mgy = -\mu_k \cdot mg \cdot \Delta r$$
$$\frac{-\tfrac{1}{2}K\Delta s^2 + mgy}{mg\,\Delta r} = \mu_k$$
$$\mu_k = 0.328$$



$$\Sigma F_x = ma_x$$
$$f_k = ma_x$$

$$\Sigma F_y = ma_y$$
$$n - F_g = 0$$
$$n = mg$$

ex:



In a problem like this one the total energy of system considers both masses

$$W_{ext} = \Delta E$$
$$= E_f - E_i$$
$$= (U_{f1} + U_{f2} + K_{f1} + K_{f2}) - (U_{i1} + U_{i2} + K_{i1} + K_{i2})$$

Momentum (ρ)

$$\boxed{\vec{p} = m\vec{v}}$$

$F_{net} = \frac{dp}{dt}$ ∴ if $F_{ext} = 0$
$p_f = p_i$ (momentum is conserved)

Newton's 2nd :
$F = ma$
$F = m\left(\frac{\Delta v}{\Delta t}\right) = \frac{m \Delta v}{\Delta t} \Rightarrow \frac{p}{t}$ ∴ $F = \frac{\Delta p}{\Delta t}$

* Always treat momentum as a vector

Isolated system $(F_{net} = 0)$

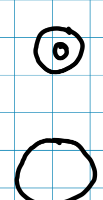
→ Momentum is conserved $\vec{p_f} = \vec{p_i}$
* It is the total momentum of the system that is conserved, not momentum of a single object
$\vec{p_f} = \vec{p_i}$ treat as vector (x, y)

∴ $p_{1ix} + p_{2ix} + p_{3ix} \ldots = p_{1fx} + p_{2fx} + p_{3fx} \ldots$
∴ $p_{1iy} + p_{2iy} + p_{3iy} \ldots = p_{1fy} + p_{2fy} + p_{3fy} \ldots$

Chose system where momentum is conserved

ex: Drop a ball, bounce back

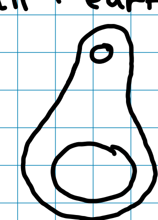
Set ball as system:



p not conserved
b/c $F_{floor\ on\ ball}$ changes direction of velocity
$\vec{p} = m\vec{v}$

∴ momentum of ball is not conserved
momentum of system is

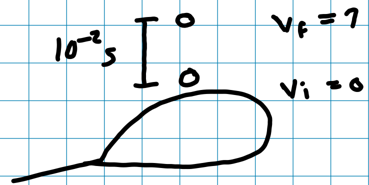
Set ball + earth as system:



p conserved
Initially at rest
∴ $\vec{p} = p_{ball} + p_{earth} = 0$
↓ $F_{E\ on\ B}$
↑ $F_{B\ on\ E}$
At contact
↑ $F_{floor\ on\ B}$
↓ $F_{B\ on\ floor}$
(pushing earth away)

ex: Tennis ball initially at rest is struck and moves straight upward to a height of 5 m. Find average force during the collision. Mass of tennis ball is 10 g and the duration of collision is 10^{-2} s.

Part 1 (impact)

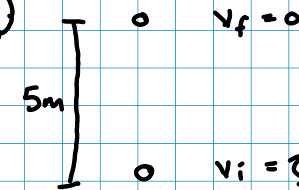


$$F_{net} = \frac{\Delta p}{t}$$

* momentum not conserved
→ external force
→ initial velocity = 0 ∴ momentum = 0
Then it moves

$V_{fI} = V_{iII}$

Part 2



Part 2 :

$$\Delta E = 0$$
$$K_i = U_f$$
$$\tfrac{1}{2} m v_i^2 = m g y_f$$
$$v_i = \sqrt{2gh}$$

Part 1 : $v_f = \sqrt{2gh}$

$$F_{net} = \frac{\Delta p}{t} = \frac{p_f - \cancel{p_i}^{\,0}}{t} = \frac{m\sqrt{2gh}}{t} = 9.9 \text{ N}$$

ex: Perfectly Inelastic Collision

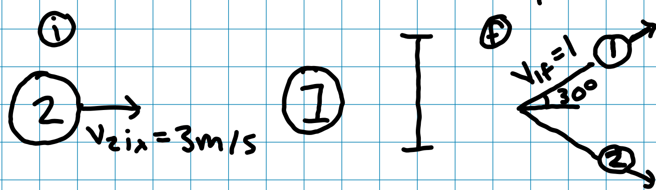
$\boxed{m_1} \rightarrow v_{1i}$ $\boxed{m_2} \rightarrow v_{2i}$ ⓘ

$\boxed{m_1 + m_2} \rightarrow v_f$ ⓕ

$$F_{ext} = 0$$
$$\vec{p}_f = \vec{p}_i$$
$$p_{fx} = p_{1ix} + p_{2ix}$$
$$(m_1 + m_2) v_f = m_1 v_{1ix} + m_2 v_{2ix}$$

$$\boxed{v_f = \frac{m_1 v_{1ix} + m_2 v_{2ix}}{m_1 + m_2}}$$

ex: 2-D collision
Ball 1 (rest) is struck by an identical ball 2 (3 m/s). After collision, ball 1 moves 1 m/s at angle 30°.
What is final velocity of ball 2?



$$\underline{\vec{P}_{TOTf} = \vec{P}_{TOTi}}$$

X

$P_{fx} = P_{ix}$

$P_{1fx} + P_{2fx} = \cancel{P_{1ix}}^{0} + P_{2ix}$

$\cancel{m}(v_{1f} + v_{2f}) = \cancel{m} v_{2i}$

$v_{2f} = 3 - 1\cos 30$

$$\boxed{v_{2f} = 2.1 \ m/s}$$

Y

$P_{fy} = \cancel{P_{iy}}^{0}$

$P_{1f} + P_{2f} = 0$

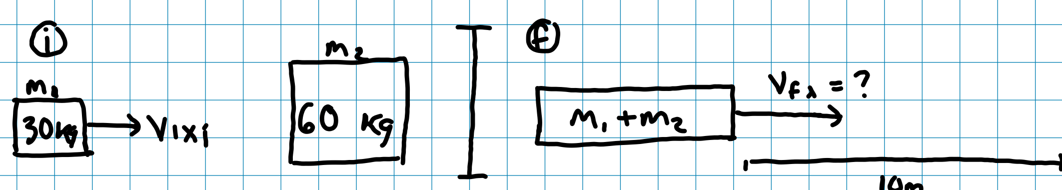
$m v_{1f} = -m v_{2f}$

$1 \sin 30 = -v_{2f}$

$$\boxed{v_{2f} = -1/2 \ m/s}$$

$\therefore \vec{v}_{2f} = (2.1\hat{i} - 0.5\hat{j})\ m/s$

ex: 30 kg sack strikes 60 kg ice skater (rest). After impact, they travel 10 m together before coming to rest. $\mu_k = 0.5$. What is v_i of sack?



Part 2 (work)

$W_{net} = \Delta K$

$\vec{f}_k \cdot \vec{\Delta r} = \cancel{K_f}^{0} - K_i$

$-\mu_k \cancel{(m_1+m_2)} g \Delta r = -1/2 \cancel{(m_1+m_2)} v_i^2$

$$\boxed{v_i = \sqrt{2 \Delta r \mu_k g}}$$

$v_{iII} = v_{fI} = 10 m/s$

Part 1 (conserv. momentum)

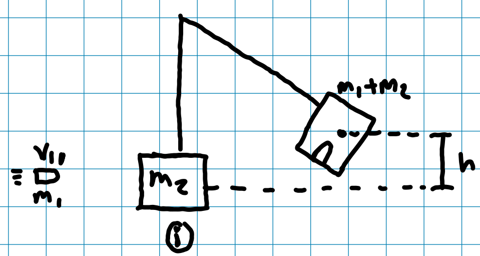
$\vec{P}_{TOTf} = \vec{P}_{TOTi}$

$(m_1 + m_2) v_{fx} = m_1 v_{1ix} + \cancel{m_2 v_{2ix}}^{0}$

$$\boxed{v_{ix1} = \frac{(m_1 + m_2) v_{fx}}{m_1}}$$

$= 30 m/s$

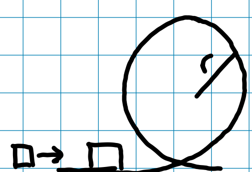
Ballistic Pendulum

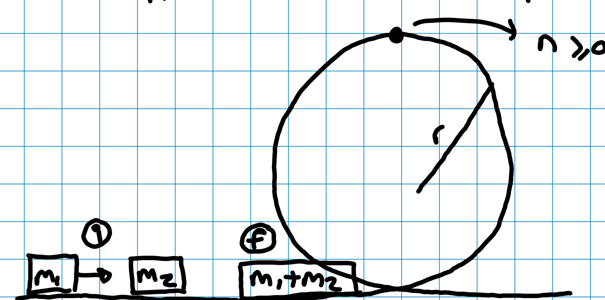


Part 1 : perfectly inelastic collision (conservation of momentum)

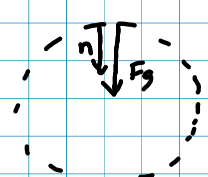
Part 2 : conservation of energy

ex: Block m_1 slides with $v = V_{1i}$
Strikes m_2 (perfect inelastic) who is at rest
What value of value of V_{1i} minimum to loop-the-loop?





Part 3 (circular dynamics)



$$\Sigma F_r = ma_r$$
$$n + mg = ma_r$$
$$n = m(a_r - g) \geq 0$$
$$\frac{v^2}{r} \geq g$$
$$\boxed{v_i \geq \sqrt{rg}}$$

$V_{i\,III} = V_{f\,II}$

Part 2 (work)

$$(K+U)_f = (K+\cancel{U})_i^{\,0}$$
$$\tfrac{1}{2}(\cancel{m_1+m_2})(rg) + (\cancel{m_1+m_2})g(2r) = \tfrac{1}{2}(\cancel{m_1+m_2})V_i^2$$
$$V_i^2 = rg + 4rg$$
$$\boxed{V_i = \sqrt{5gr}}$$

$V_{i\,II} = V_{f\,I}$

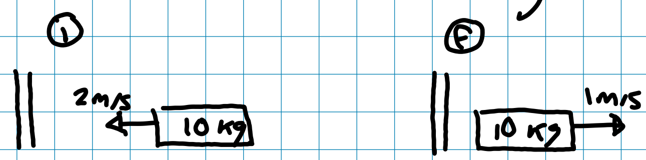
Part 1 (collision)

$$\vec{P}_f = \vec{P}_{TOT\,i}$$
$$(m_1+m_2)V_f = m_1 V_{1i} + \cancel{m_2 V_{2i}}^{\,0}$$
$$\frac{(m_1+m_2)V_f}{m_1} = V_i$$

sub $V_f = \sqrt{5gr}$

$$\boxed{V_i = \frac{(m_1+m_2)}{m_1}\sqrt{5rg}}$$

ex: what is the carts change in momentum?



$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$
$= mV_{fx} - mV_{ix}$
$= m(1\hat{\imath} - -2\hat{\imath})\ m/s$
$= 30\ kg \cdot m/s$

ex: 2 balls of same mass (1 rubber, 1 clay) are thrown at a wall. The rubber ball bounces, the clay ball sticks. Which ball has a larger change in momentum?

clay: $\xrightarrow[mv]{p_i}$ $\quad p_f$ 0

rubber: $\xrightarrow[mv]{p_i}$ $\quad \xleftarrow[m(-v)]{p_f}$

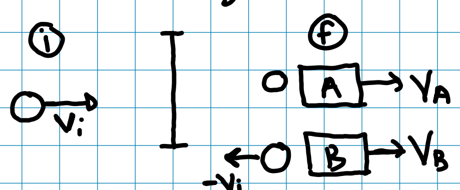
$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$\Delta\vec{p}_c = 0 - (mv_i)\hat{\imath} = -mv_i\,\hat{\imath}$

$\Delta\vec{p}_r = (-mv_i)\hat{\imath} - (mv_i)\hat{\imath} = -2mv_i\,\hat{\imath}$

∴ The rubber ball has a greater $\|\Delta\vec{p}\|$

ex: Objects A and B have same mass. When ball collides with A, the ball ends up at rest. When ball collides with B, the ball rebounds to the left. Who has greater velocity after the collision? A or B?



$\vec{p}_f = \vec{p}_i$ (of system)

A: $(\cancel{\vec{p}_{ball}} + \vec{p}_A)_f = m_{ball} \cdot V_i$
$m_A \cdot V_A = m_{ball} \cdot V_i$
$\boxed{V_A = \dfrac{m_{ball} \cdot V_i}{m_A}}$

$\Big\}\ \vec{p}_A = \vec{p}_{ball\ i}$

B: $(\vec{p}_{ball} + \vec{p}_B)_f = m_{ball} \cdot V_i$
$m_{ball}(-V_i) + m_B \cdot V_B = m_{ball} \cdot V_i$
$m_B \cdot V_B = m_{ball} \cdot V_i - -m_{ball} \cdot V_i$
$\boxed{V_B = \dfrac{2\,(m_{ball} \cdot V_i)}{m_B}}$

* if the ball bounces backwards, object B must move faster to compensate and conserve momentum.

$m_B = m_A \quad \therefore \quad V_B > V_A$

ex: 2 particles move to the right. Particle 1 catches up to particle 2 and collides with it (and sticks to it). They continue with v_f. What is v_f compared to v_1 and v_2?

$v_1 > v_2$ (b/c 1 caught up to 2)

$p_f = p_i$

$(m_1+m_2)v_f = m_1v_1 + m_2v_2$

$\underbrace{m_1 v_f + m_2 v_f} = \underbrace{m_1 v_1 + m_2 v_2}$ and $v_1 > v_2$

$\downarrow$

$m_1v_2 + m_2v_2 < m_1v_1 + m_2v_2 < m_1v_1 + m_2v_1$

$\therefore$

$m_1v_2 + m_2v_2 < m_1v_f + m_2v_f < m_1v_1 + m_2v_1$

$v_2\cancel{(m_1+m_2)} < v_f\cancel{(m_1+m_2)} < v_1\cancel{(m_1+m_2)}$

$\boxed{v_2 < v_f < v_1}$

ex: An explosion in a pipe shoots out 3 pieces. A 6 g piece comes out the right end with v_1. A 4 g piece comes out the left end with $v_2 = 2v_1$. From which end did the third piece come out from?

Pieces start at rest $\therefore \vec{p_i} = 0$

$\vec{p_f} = \vec{p_i}$

$m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} = 0$

$6v + 4(-2v) + m_3\vec{v_3} = 0$

$-2v + m_3\vec{v_3} = 0$

$\therefore \boxed{m_3v_3 = +2v = \vec{p_3}}$

$\therefore$ to the right.

ex: 2 carts at rest on a track (mass m and 2m)
If you push ~~[illegible]~~ one cart with F and during time t, and then the other one with the same force and time, which will have greater momentum?

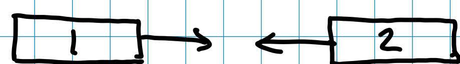
$$F = ma$$
$$= m\frac{dv}{dt}$$

$$F = \frac{dp}{dt} \qquad \therefore \; dp = F.dt$$

same F and dt $\therefore$ same dp

both start at rest $\therefore$ p is the same

ex: A compact car and truck collide and stick together. Which has larger change in momentum?

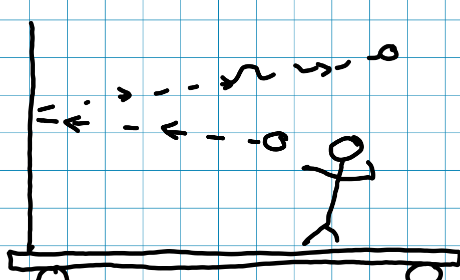


NIII

$$dp = F.dt$$

same F and dt so, once again change in momentum is the same.

ex: On a cart initially at rest. Throw a ball at the wall that bounces back. Where does the cart move?



(i) $p = 0$

(f) $p_{ball} = MV_f$

$p_{cart} = -(p_{ball})$ to conserve momentum

$\therefore$ cart moves left.

Torque

τ (torque)
I (moment of inertia)
α (angular acceleration)

Comparing Linear and Angular

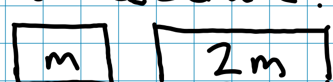
	Linear	Angular
• position	x [m]	θ [rad]
• velocity	v [m/s]	ω [rad/s]
• acceleration	a [m/s²]	α [rad/s²]
• resistance to acceleration (intrisic)	m [kg]	I [kg m²] $= mr^2$
• Ability to impart acceleration	$F = ma$	$\tau = I\alpha$ [Nm]

Resistance to acceleration

Linear

Force F produces acceleration a on m.

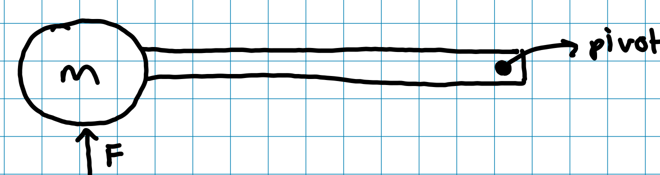
Which is easier to accelerate?



The one with the lower mass

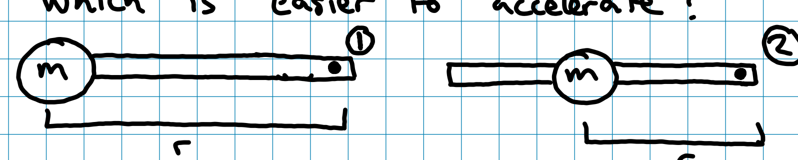
Rotational

Force F produces Torque τ, which induces an acceleration α



I depends on average, how is mass distributed

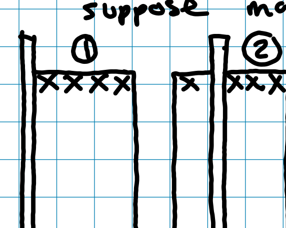
Which is easier to accelerate?



$$I_1 > I_2$$

b/c mass distributed over longer r
∴ One with smaller I_2 is easier

Which door is easier to open?
suppose mass of door = 4 molecules



$I_1 > I_2$

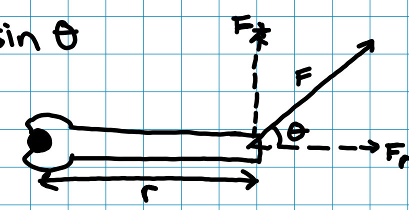
on average, masses on door 2 are closer to pivot ∴ smaller I
∴ easier to accelerate.

Torque (to twist)

- consider r, t, z
- Torque is an ability to impart a rotational acceleration

τ depends on F_t (tangential) $= F \sin\theta$

τ depends on r



$$\boxed{\tau = r \cdot F \sin\theta}$$

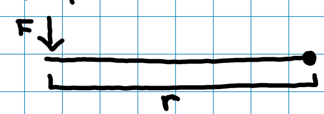
Note: Different "r"

$I = m\underline{r}^2 \Rightarrow r =$ distribution of mass relative to pivot (doesn't matter where force is applied)

$\tau = \vec{r} \cdot \vec{F} \Rightarrow r =$ place of force relative to pivot

ex: What is easiest way to open a door?

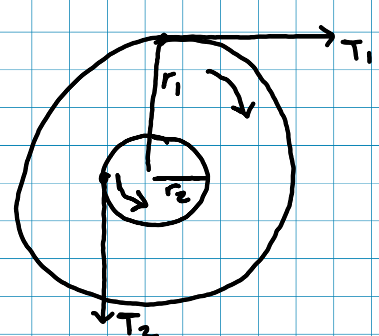
Apply force as far away from pivot (greatest r)



$\tau = r\, F \sin\theta$ $\quad \uparrow r$

$\therefore \downarrow F$ demanded

ex:



$T_1 = 5$ N, $r_1 = 2$ m

$T_2 = 8$ N, $r_2 = 1.2$ m

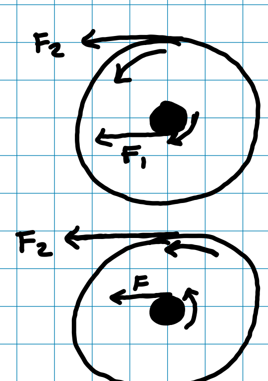
Find net Torque

$$\begin{aligned}\tau_{net} &= \tau_1 + \tau_2 \\ &= -r_1 T_1 \sin(90) + r_2 T_2 \sin(90) \\ &= -10 \text{ Nm} + 9.6 \text{ Nm} \\ &= -0.4 \text{ Nm}\end{aligned}$$

* Tension is <u>ALWAYS</u> tangential ($\sin 90 = 1$)

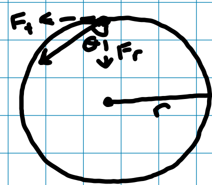
* counter-clockw $= \oplus$, clock w $= \ominus$

ex:



F_1 and F_2 point in same direction (translational) but induce different results (rotation)

Now, rotation is the same counter-clock w direction.



$$\tau = r\,F_t$$
$$= r\,(m a_t)$$
$$= r\,(m r \alpha)$$
$$= m r^2 . \alpha$$
$$\boxed{\tau = I\ \alpha}$$
$$F = m\ a$$

$a_t = r\alpha$

$I = mr^2$

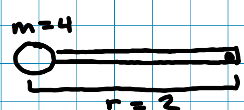
I (moment of inertia)

$I = mr^2$ for a single particle located distance r

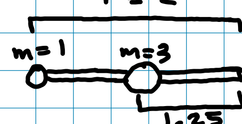
→ Higher r ⇒ higher I

For a collection of particles (N) : $I = \sum_{i=1}^{n} I_i$

ex.



$I = mr^2 = 16$ kg m^2

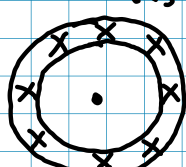


$I = m_1 r_1^2 + m_2 r_2^2 = 8.69$ kg m^2

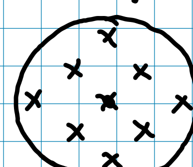
* on average, closer particles to pivot so smaller I

Which has smaller I? Ring or Disc? (suppose mass = 8 molecules)

ring



disc

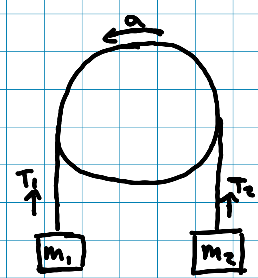


I disc $<$ I ring

b/c on average, molecules are closer to pivot.

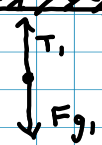
Atwoods Machine

→ Pulley has mass and I



m_1

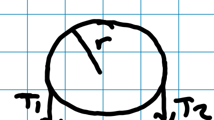
$\cancel{\Sigma F_x = m a_x}$



$\Sigma F_y = m a_y$

$m_1 g - T_1 = m_1 a$

pulley



$\tau = I\alpha$

$T_1 r(\sin 90) - T_2 r(\sin 90) = I\alpha$

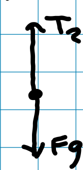
$r\,(T_1 - T_2) = I\alpha$

$T_1 - T_2 = \frac{I\alpha}{r}$

$\{a_t = r\alpha\}$

$T_1 - T_2 = I\frac{a}{r^2}$

m_2



$\Sigma F_y = m a_y$

$T_2 - m_2 g = m_2 a$

① + ② + ③

$m_1 g - \cancel{T_1} = m_1 a$

$\cancel{T_1} - \cancel{T_2} = I\frac{a}{r^2}$

$\cancel{T_2} - m_2 g = m_2 a$

$g(m_1 - m_2) = a\left(m_1 + m_2 + \frac{I}{r^2}\right)$

$$\boxed{a = \frac{g(m_1 - m_2)}{m_1 + m_2 + I/r^2}}$$

Note: $I_{pulley} = \frac{1}{2} M_p r^2$

$\frac{I}{r^2} = \frac{M_p}{2}$

$\therefore\; a = \frac{g(m_1 - m_2)}{m_1 + m_2 + M_p/2}$

Note: $T_1 - T_2 = I\frac{a}{r^2}$

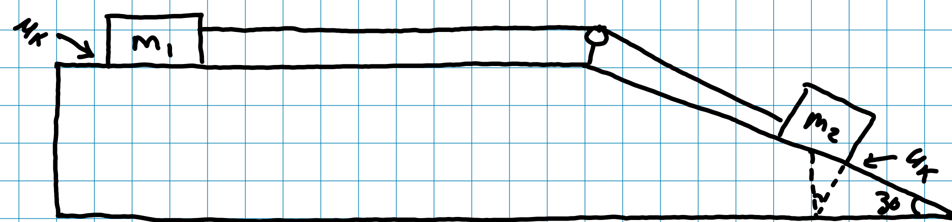
$T_1 - T_2 = \frac{M_p}{2} \cdot a$

for M_p extremely small

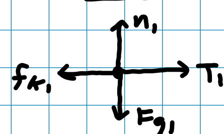
$\therefore\; T_1 - T_2 \approx 0$

$T_1 \approx T_2$

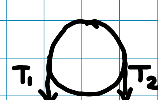
ex:



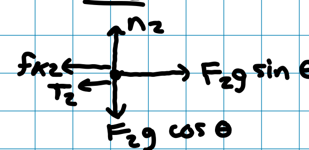
$\underline{m_1}$



$\underline{\text{pulley}}$



$\underline{m_2}$



$\underline{\Sigma F_y = ma_y}$

$n_1 = m_1 g$

$\underline{\Sigma F_x = ma_x}$

$T_1 - \mu_k n_1 = m_1 a$

$T_1 - \mu_k m_1 g = m_1 a$

$\tau = I\alpha$

$\tau_2 - \tau_1 = I\alpha$

$T_2 r(\sin 90) - T_1 r(\sin 90) = I\frac{a}{r}$

$(T_2 - T_1) r = I\frac{a}{r}$

$T_2 - T_1 = I\frac{a}{r^2}$

$\underline{\Sigma F_x = ma_x}$

$m_2 g \sin\theta - \mu_k n_2 - T_2 = m_2 a$

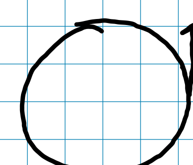
$m_2 g \sin\theta - \mu_k m_2 g \cos\theta - T_2 = m_2 a$

$m_2 g(\sin\theta - \mu_k \cos\theta) - T_2 = m_2 a$

① + ② + ③

$$\cancel{T_1} - \mu_k m_1 g = m_1 a$$
$$\cancel{T_2} - \cancel{T_1} = I\frac{a}{r^2}$$
$$\underline{m_2 g(\sin\theta - \mu_k \cos\theta) - \cancel{T_2} = m_2 a}$$
$$g\left(m_2(\sin\theta - \mu_k \cos\theta) - m_1 \mu_k\right) = a\left(m_1 + m_2 + I/r^2\right)$$

ex: Upside down bicycle. Spin wheel (r = 0.381). Drop water fly off to h = 54 cm. On next turn, drop goes up h = 51 cm. Determine magnitude of angular acceleration.



$\underline{\text{conservation energy}}$

$E_f = E_i$

$\cancel{(K+U)_f} = (k + \cancel{U})_i$

$mgy_f = \frac{1}{2} m v_1^2$

$v_1 = \sqrt{2gy_f}$

$\underline{\text{circular dynamics}}$

$\omega = \frac{v_f}{r} = \frac{\sqrt{2gy_f}}{r}$

$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

$\frac{2gh_f}{r^2} - \frac{2gh_i}{r^2} = 2\alpha(2\pi)$

$\frac{2g}{r^2}(h_f - h_i) \cdot \frac{1}{4\pi} = \alpha$

$\alpha = \frac{g(h_f - h_i)}{2\pi r^2}$

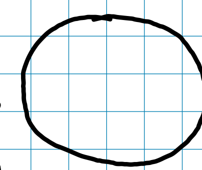
Let

$h_i = 0.54$ m

$h_f = 0.51$ m

$r = 0.38$ m

ex: The combination of an applied force and a force of friction produces a torque of 36 Nm. The applied force is applied for 6s. The angular speed goes from 0 to 10 rad/s. The applied force is removed. It takes 60s for the wheel to come to rest.



a) Find the moment of inertia of the wheel
b) The magnitude of torque due to friction
c) How many revolutions of the wheel during total 66s

a) $\left.\begin{array}{l} \alpha = \frac{\Delta\omega}{t} = \frac{10-0}{6} \\ \tau = 36 \end{array}\right\} \tau = I\alpha \Rightarrow I = \frac{\tau}{\alpha} = \boxed{21.6 \ kgm^2}$

b) Friction:

$\left.\alpha = \frac{\Delta\omega}{t} = \frac{-10}{60} = -1/6 \right\} \tau = I\alpha = 21.6 \cdot \frac{-1}{6} = -3.6$

$\therefore \boxed{3.6 \ Nm}$

c) $\Delta\theta = \frac{(\omega_1+\omega_2)t}{2}$

$\Delta\theta \left\{\begin{array}{l} \text{Part 1}: \frac{(10)\cdot(6)}{2} = 30 \text{ rad} \\ \text{Part 2}: \frac{(10)\cdot 60}{2} = 300 \text{ rad} \end{array}\right\} 330 \text{ rad} \cdot \frac{\text{rev}}{2\pi \text{ rad}} = \boxed{\frac{165}{\pi} \text{ revs}}$

Rotational Kinetic Energy

→ Not all particles on rotating body have the same speed ($v_t = r\omega$ ∴ depends on r)

For 1 particle : $K = \frac{1}{2} m v^2$

The entire body $K = \sum_{i=1}^{n} K_i$

$$= \sum \tfrac{1}{2} m_i v_i^2$$
$$= \sum \tfrac{1}{2} m_i (r_i \omega_i)^2$$
$$= \sum \tfrac{1}{2} (m_i r^2) \omega_i^2$$
$$= \tfrac{1}{2} (\sum m_i r_i^2) \omega_i^2$$
$$= \tfrac{1}{2} I \omega^2$$

$I \cdot \sum m r^2$

$$\boxed{K = \tfrac{1}{2} I \omega^2}$$

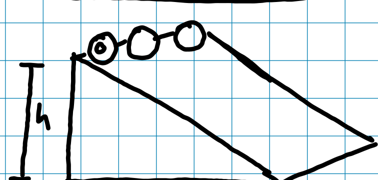
$K = \frac{1}{2} m v^2$

* Note : If there is a translational and a rotational movement, add $K_{trans} + K_{rot}$

ex: 3 objects are placed on an incline. They are released from rest. Which object reaches the ground first?

$$\boxed{\begin{array}{l} I_{spore} = \frac{2}{5} M r^2 \\ I_{disc} = \frac{1}{2} M r^2 \\ I_{cylindre} = m r^2 \end{array}}$$

Note: $I = \underset{\text{constant}}{c} M r^2$



$$E_f = E_i$$
$$(K + \cancel{U})_f = (\cancel{K} + U)_i$$
$$K_{trans} + K_{rot} = m g y_i$$
$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega^2 = m g y_i$$
$$\tfrac{1}{2} m v^2 + \tfrac{1}{2} (c m r^2) \left(\tfrac{v}{r}\right)^2 = m g y_i$$
$$\tfrac{1}{2} m v^2 + \tfrac{1}{2} c m \cancel{\tfrac{r^2}{r^2}} \cdot v^2 = m g y_i$$
$$\tfrac{1}{2} \cancel{m} v^2 (1 + c) = \cancel{m} g y_i$$
$$\boxed{v = \sqrt{\frac{2 g y_i}{1 + c}}}$$

$I = c m r^2$
$v = r\omega$
$\omega = \frac{v}{r}$

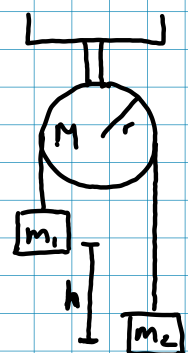
∴ bigger the "c"
∴ bigger the divider
∴ smaller the speed

* Problems involving velocity, acceleration ⇒ use Dynamics
* Problems involving speed ⇒ use Energy

ex: A 15 kg object and a 10 kg object are joined by a cord that passes over a pulley of mass 3 kg and radius 10 cm. The objects start from rest 3 m apart.

$I_{pulley} = \frac{1}{2} M r^2$

Determine speed of 2 objects as m_1 hits the ground



$$E_f = E_i$$

$$(K+U)_f = (\cancel{K}^{0} + U)_i$$

$$K_{1f} + K_{2f} + K_{pulley_f} + \cancel{U_{1f}}^{0} + U_{2f} + \underbrace{\cancel{U_{pull\,f}} = U_{1i} + \cancel{U_{2i}}^{0} + \cancel{U_{pull\,i}}}_{\text{same}}$$

$$\frac{1}{2}(m_1+m_2)V_f^2 + \frac{1}{2} I \omega_f^2 + m_2 g h = m_1 g h$$

$$\frac{1}{2}(m_1+m_2)V_f^2 + \frac{1}{2}\left(\frac{1}{2} M r^2\right)\left(\frac{V}{r}\right)^2 = (m_1 - m_2) g h$$

$$\frac{1}{2}\left(m_1 + m_2 + M/2\right) V_f^2 = (m_1 - m_2) g h$$

$$V_f = \sqrt{\frac{2(m_1 - m_2) g h}{(m_1 + m_2 + M/2)}}$$

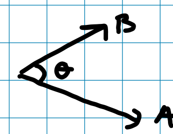
Cross Product

→ Dot product of 2 vectors gives : scalar
→ Cross product of 2 vectors gives : vector

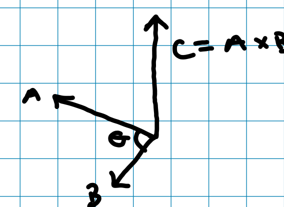
$\vec{A} \times \vec{B} = \vec{C}$ (importance to what comes first A or B)

- Magnitude of C

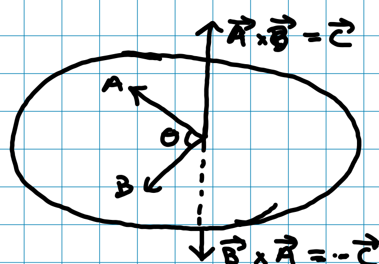
$\|\vec{C}\| = AB \sin\theta$

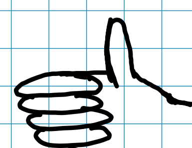


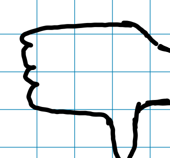
- Direction of C



$\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$ according to right hand rule :
→ point fingers along $\vec{A}$
→ curl fingers towards $\vec{B}$
* Rotate wrist to make this possible
→ Thumb points along $\vec{C} = \vec{A} \times \vec{B}$



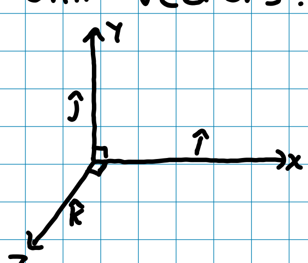


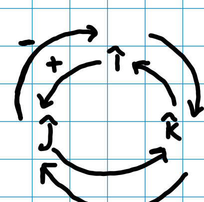


$\therefore\ \vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$

Property : If $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = AB \cancel{\sin(0)}_{0} = 0$

Unit Vectors :





$\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{i} = -\hat{k}$

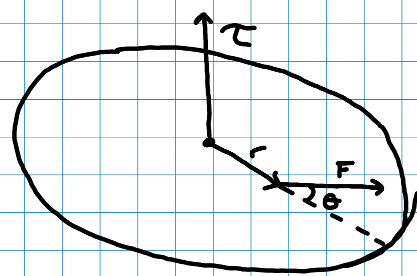
here : $\hat{i} \times \hat{i} = 0$
b/c angle is 0 $\therefore \sin(0) = 0$

ex: if $A = 2\hat{i} - 4\hat{j}$, $B = 3\hat{i} + \hat{j}$, Find $\vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = (2\hat{i} - 4\hat{j}) \times (3\hat{i} + \hat{j})$$
$$= \cancel{6\hat{i}\hat{i}} + 2\hat{i}\hat{j} - 12\hat{j}\hat{i} - \cancel{4\hat{j}\hat{j}}$$
$$= 2\hat{k} - 12(-\hat{k})$$
$$= 14\hat{k}$$

Torque as a vector product

$\tau = \vec{r} \times \vec{F}$



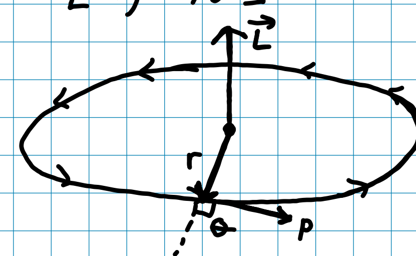
τ is in the z direction (r+z)

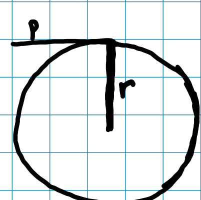
Angular Momentum

→ linear momentum : $\vec{p} = m\vec{v}$

→ Angular momentum : $\vec{L} = \vec{r} \times \vec{p}$ [kg m²/s]

$\vec{L}$ points in z direction





$$\vec{L} = \vec{r} \times \vec{p}$$
$$L = rp \cdot \sin(90)$$
$$= r(mv)$$
$$= r\, m(r\omega)$$
$$= mr^2 \cdot \omega$$
$$L = I \cdot \omega$$

$v = r\omega$

$\therefore$ $\boxed{\vec{L} = I\vec{\omega}}$

$p = mv$

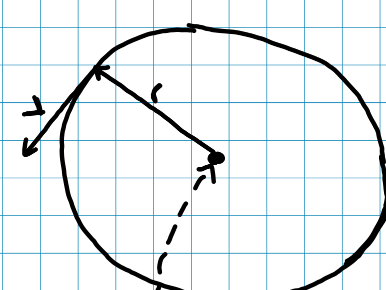
Analagous equations:

$\boxed{\vec{\tau} = I\vec{\alpha}}$ $\qquad$ $\boxed{\vec{\tau} = \frac{dL}{dt}}$ $\qquad$ $\boxed{\vec{L} = I\vec{\omega}}$

$F = ma$ $\qquad$ $F = \frac{dp}{dt}$ $\qquad$ $p = mv$

$$L = I\omega$$
$$\frac{dL}{dt} = \frac{d}{dt}(I\omega)$$
$$\frac{dL}{dt} = I \cdot \left(\frac{d\omega}{dt}\right)$$
$$\frac{dL}{dt} = I\alpha$$
$$\boxed{\tau = \frac{dL}{dt}}$$

$\tau = I\alpha$



L points out of the page

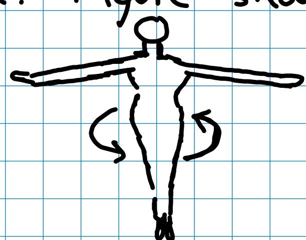
Conservation of Angular Momentum

→ If $\tau_{net} = 0$, angular momentum is conserved (of system)

$$\tau_{net} = \frac{dL}{dt} = 0 \quad \therefore \vec{L} = \text{constant}$$

$$F_{net} = \frac{dp}{dt} = 0 \quad \therefore \vec{p} = \text{constant}$$

ex: Figure skater / carousel



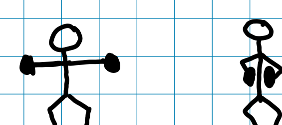
Decrease radius (distribution) $\therefore$ decrease I

L constant

$L = I\omega$ (I ↓, ω ↑)

ω increases to conserve angular momentum.

ex. Student sits on rotating stool with 2 weights (3.00 kg each). His arms are extended and weights are 1m from axis. Rotates at $\omega = 0.75$ rad/s. The moment of inertia of the student + stool is 3 kg m². The student pulls weights inward at 0.3 m from axis. Find the new angular speed.



$\tau_{net} = 0 \quad \therefore L_f = L_i$

$L = I\omega$

→ $I = I_{person} + I_{weight\,1} + I_{weight\,2}$

$I = 3 + m_1 r^2 + m_2 r^2$

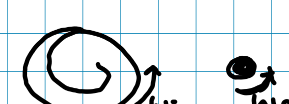
$I = 3 + 2mr^2$ ← same weight / distance

$L_f = L_i$

$I_f \omega_f = I_i \omega_i$

$$\omega_f = \frac{I_i}{I_f} \cdot \omega_i = \frac{(3 + 2m r_i^2)}{(3 + 2m r_f^2)} \cdot \omega_i = 1.91 \text{ rad/s}$$

ex: Neutron Star ($r_i = 10^4$ km, $T_i = 30$ days, $r_f = 6$ km)



Find T (period) given that I proportional to r^2

$\boxed{\omega = \frac{2\pi}{T}}$

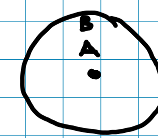
$\tau_{ext} = 0 \quad \therefore L_f = L_i$

$I_f \omega_f = I_i \omega_i$

$$r_f^2 \cdot \frac{2\pi}{T_f} = r_i^2 \cdot \frac{2\pi}{T_i} \quad \therefore T_i = \frac{r_i^2 \cdot 2\pi \cdot T_f}{r_f^2 \cdot 2\pi} = \frac{r_i^2}{r_f^2} \cdot T_f$$

Conceptual

ex: 2 coins rotate on turntable. Coin B is twice as far as A (from the axis).



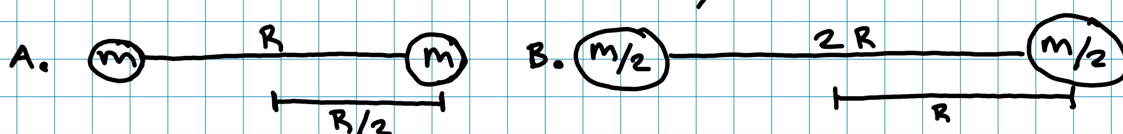
- Which has greater angular velocity?
  → Same ω
- Which has greater tangential velocity?
  → B $\because$ $v = r\omega$

ex: A rotating object has some rotational kinetic energy. If the angular speed is doubled, what happens to K?

$$K = \tfrac{1}{2} I \omega^2$$

$$\tfrac{1}{2} I (2\omega)^2 = 4\left(\tfrac{1}{2} I \omega^2\right) = 4 \times K$$

ex: Which dumbbell has a larger I

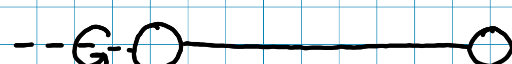


$$I = m\left(\frac{R}{2}\right)^2 = \frac{mR^2}{4}$$

$$I = (m/2)R^2 = \frac{mR^2}{2}$$

$$\therefore I_B > I_A$$

Which has a greater I, if it is rotating on itself?

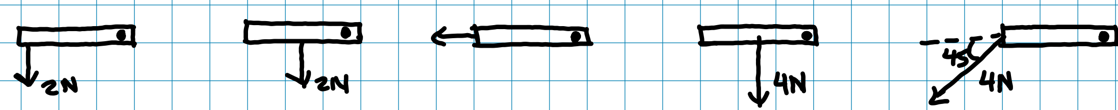


$R = 0$

$$\therefore I_A = I_B = 0$$

ex: Rank torques :

a) b) c) d) e)

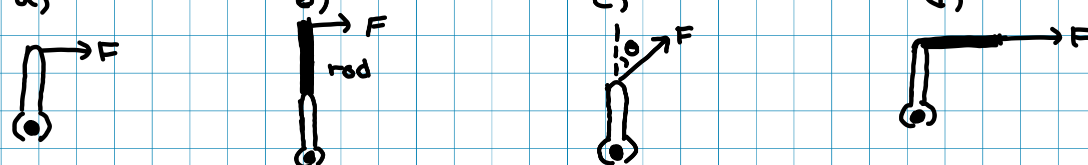


$\tau = rF\sin\theta =$ $2r$ $\quad 2\cdot\frac{r}{2} = r$ $\quad 2r\sin(0) = 0$ $\quad 4\frac{r}{2} = 2r$ $\quad 4r\sin(45) = \frac{4r}{\sqrt{2}}$

$\therefore\ \tau_e > \tau_a = \tau_d > \tau_b > \tau_c$

ex: Which arrangement is most effective to losen a nut?

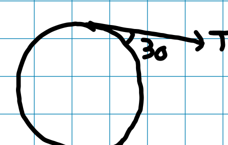
a) b) c) d)



$\tau = rF\sin\theta =$ $rF\sin(90) = rF$ $\quad \uparrow rF\sin(90) = \uparrow rF$ $\quad rF\sin(45) = \frac{rF}{\sqrt{2}}$ $\quad rF\sin(90) = rF$

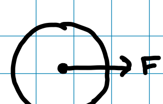
$\therefore\ \tau_b > \tau_a = \tau_d > \tau_c$

ex: pulley or wheel :



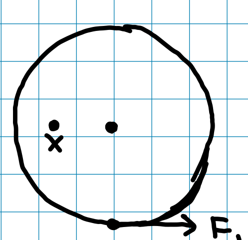
$\Rightarrow$ Angle is ALWAYS going to readjust to $90°$

ex:

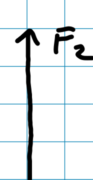


$\tau = 0$ b/c $F \parallel r$

ex:



What Force at X will make net torque zero?



* Note: longer than F_1 b/c r_1 is longer than r_2