Unit Vector

$\hat{r} = \frac{a\hat{i} + b\hat{j}}{\sqrt{a^2+b^2}}$ or $\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

Forces, fields and Kinematics

$\vec{F} = m\vec{a} = q\vec{E}$ $\quad \vec{a} = \frac{q\vec{E}}{m}$ $\quad v_f^2 = 2a\Delta x = 2a\left(\frac{q\vec{E}}{m}\right)$

Potential and Potential Energy

$\Delta U = q\Delta V$ $\quad \Delta V = -\vec{E}\cdot d\vec{s} = -E\Delta s = Ed$ $\quad \vec{E} = \frac{-dV}{ds}$ or $\frac{-\Delta V}{\Delta s}$

$\Delta V = keq\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

Continuous charge distribution

- set up the integral
- $dq = \lambda dx$
- $r = \sqrt{x^2+h^2}$

use symmetry

- always use the $\hat{r}$

for an arc: $\hat{r} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

$ds = Rd\theta$

$\lambda = B\sin\theta$

$q = \int \lambda dx$ $\quad \vec{E} = \lambda\int d\vec{E} = \frac{ke}{r^2}\int dq = \frac{ke\lambda}{r^2}\int dx\,\hat{r}$

Electric flux

$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = \frac{q_{in}}{\varepsilon_0}$

Uniform electric field on planes

$E = \left|\frac{\eta}{2\varepsilon_0}\right|$

Field on cylinders:

$\eta = \frac{Q}{2\pi a L}$ $\quad \lambda = \frac{Q}{\ell}$

Capacitance + circuits

$\Delta U = q\Delta V \Rightarrow$ $\quad \mathcal{E} = RI$ $\quad E = \frac{\eta}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$ $\quad \Delta V = Ed = \left(\frac{Q}{\varepsilon_0 A}\right)d$

$Q = \Delta V\left(\frac{\varepsilon_0 A}{d}\right)$ $\quad C = \frac{Q}{\left(\frac{Qd}{\varepsilon_0 A}\right)} = \left(\frac{\varepsilon_0 A}{d}\right)$ as $C = \frac{Q}{\Delta V}$

In series $\quad C_{eq} = \left(\sum_i \frac{1}{C_i}\right)^{-1}$

In parallel $\quad C_{eq} = \sum C_i$

$1\text{ kWh} = 1000\text{W} \times 3600\text{s}$

$= 3.6\times10^6\text{ J} = 3.6\text{ MJ}$

inner resistance: $I = \mathcal{E}/R_{eq}$

$\Delta V = RI$, where $R \notin R_{inner}$

Dielectrics : non-conductive (insulator/ semi-conductor)

$\vec{E} = \frac{\vec{E_0}}{K}$ $\quad C = kC_0$ $\quad \Delta V = \frac{V_0}{k}$ $\quad$ if capacitor is not connected Q is constant

if capacitor connected $\quad \Delta V$ is constant : Q and C change

$C = kC_0$

$Q = kQ_0$ $\quad$ capacitance always increase when inserting a dielectric

Current and resistors

$I = \frac{dQ}{dt}$ $\quad I_{avg} = nqAv_d$

Model of conduction

$a = \frac{dv}{dt}$ $\quad v_d = a\tau$ $\quad \tau$ = mean time betw. collisions

σ : conductivity

$J = \frac{I}{A} = nqv_d$ $\quad \vec{J} = \sigma\vec{E}$ $\quad R = \frac{\rho l}{A}$ $\quad \vec{E} = \rho\vec{J}$

$P = I\mathcal{E}$ produced by battery

$P = I\Delta V$ spent by circuit $\to$ $P = I^2R$ or $\frac{(\Delta V)^2}{R}$

In series : $R_{eq} = \sum_i R_i$ $\quad$ In // : $R_{eq} = \left(\sum_i \frac{1}{R_i}\right)^{-1}$

Kirchhoff

Loop rule : $\sum_{\text{closed loop}} \Delta V = 0$ $\quad$ conservation of energy The sum of potential differences across all elements around any closed loop is zero

Junction rule :

$\sum_{\text{junction}} I = 0$ $\quad$ Conservation of charge

At a junction, the sum of the currents is zero

LC circuit

$q(t) = \frac{CV\sin(\omega_0 t)}{1 - LC\omega_0^2}$

$I(t) = \frac{\omega_0 CV\cos(\omega_0 t)}{1 - LC\omega_0^2}$

$\omega = \frac{1}{\sqrt{LC}}$

$q(t) = Q\cos(\omega t)$

Magnetism

$T = \frac{2\pi}{\omega}$

- Force on a curved wire is the same as the force on a straight wire
- Force on a closed loop is zero

$\vec{\tau} = I\vec{A}\times\vec{B}$ $\quad C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Magnetic flux for a closed surface $= 0$ $\quad \Phi_B = \vec{B}\cdot d\vec{A} = 0$

Electromagnetic force : $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ $\quad qE = qvB$ $\quad E = vB$

$\Delta V = El = vBl$ $\quad \theta = \omega t$

Induced $\mathcal{E}$ / I $\quad \mathcal{E} = -N\frac{d\Phi_B}{dt}$ $\quad \Phi_B = AB\cos\theta = AB\cos(\omega t)$

$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI^2 = \frac{1}{2}mv^2$ $\quad -\frac{d}{dt}\Phi_B = \omega AB\sin(\omega t) = \mathcal{E}$