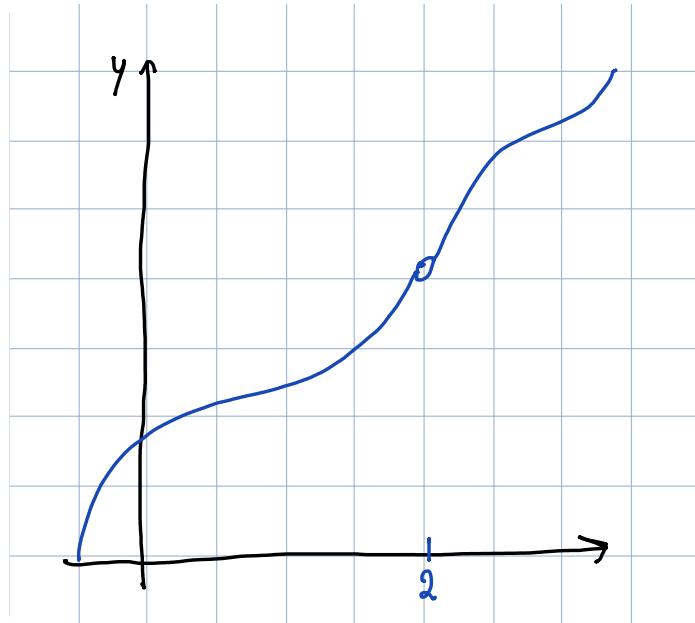
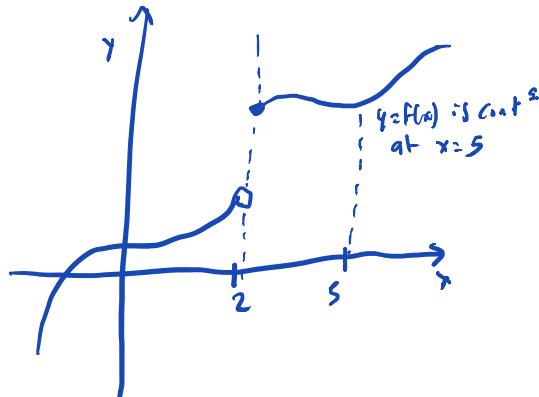


Continuity

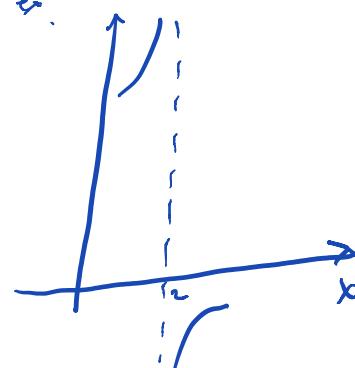


$y = f(x)$ is not continuous at $x=2$

ex.

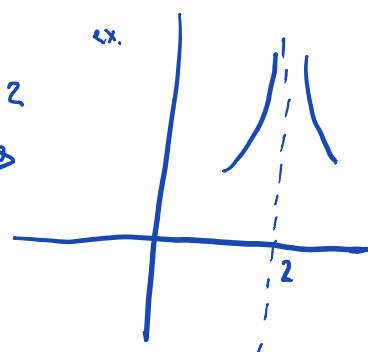


ex.

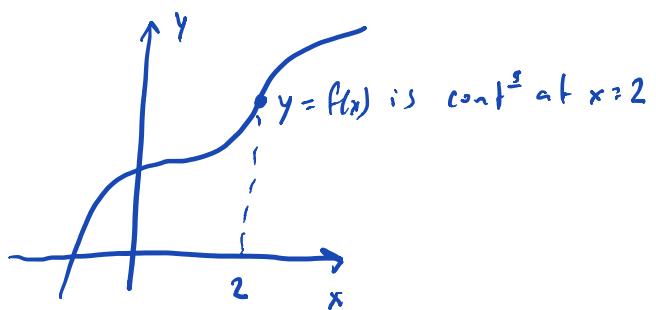


\Rightarrow
 $y = f(x)$ is not continuous at $x=2$

ex.

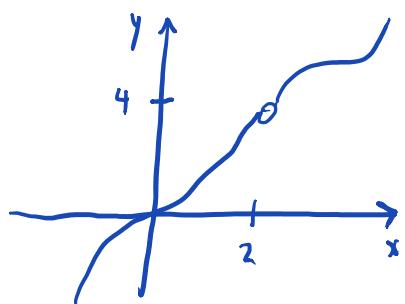


Q4.



Mathematical Defⁿ

①

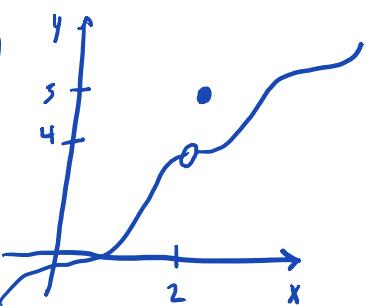


Observations:

① $f(2)$ UND

② $\lim_{x \rightarrow 2}$ exists ($= 4$)

②

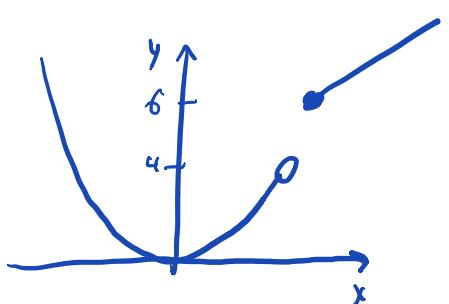


Observations:

① $f(2)$ defn ($= 5$)

② $\lim_{x \rightarrow 2} f(x)$ exists ($= 4$)

③

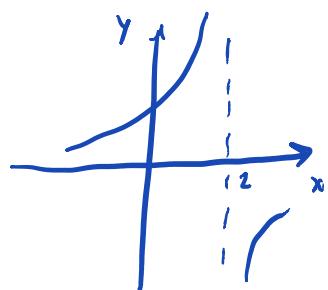


Observations:

① $f(2)$ defined ($= 6$)

② $\lim_{x \rightarrow 2} f(x)$ DNE

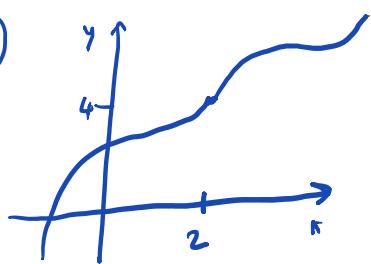
④



observations:

- ① $f(2)$ und
- ② $\lim_{x \rightarrow 2} f(x)$ ohne

⑤



observations:

- ① $f(2)$ defined ($= 4$) ✓
- ② $\lim_{x \rightarrow 2} f(x)$ exists ($= 4$) ✓
- ③ $\lim_{x \rightarrow 2} f(x) = f(2)$ ✓

Defⁿ

We say that a $f \in y = f(x)$ is continuous at $x=a$ if:

① $f(a)$ is defined

② $\lim_{x \rightarrow a} f(x)$ exist

③ $\lim_{x \rightarrow a} f(x) = f(a)$

ex. $g(t) = \frac{1}{1-t^2}$ is g const^s at $t=-1$?

use (3pt) defⁿ

① $g(-1) = \frac{1}{1-(-1)^2} = \frac{1}{0}$ UND

so g is not const^s at $t=-1$

Ex. $g(t) = \frac{t^4 - 1}{1 - t^2}$ is g const^s at $t = -1$?

use (3 pt) defⁿ

$$\textcircled{1} \quad g(-1) = \frac{-1 - 1}{1 - (-1)^2} = \frac{0}{0} \text{ UND}$$

so g is not cont^s at $t = -1$

Ex. Use defⁿ of cont^s to determine if $y = f(x)$ is cont^s at $x = 1$ where

$$f(x) = \begin{cases} 3x - 1 & \text{if } 0 < x \leq 1 \\ x + 2 & \text{if } x > 1 \end{cases}$$

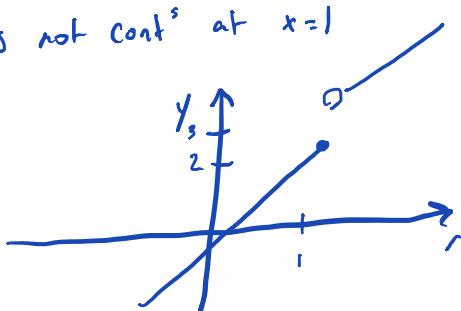
$$\textcircled{1} \quad f(1) = 3(1) - 1 = 2 \quad (\text{defined}) \quad \checkmark$$

\textcircled{2} $\lim_{x \rightarrow 1} f(x)$ exist?

$$\lim_{x \rightarrow 1^-} f(x) = 3x - 1 \stackrel{\text{sh}}{=} 3(1) - 1 = 2 \quad \left. \begin{array}{l} \lim_{x \rightarrow 1} f(x) \text{ DNE} \end{array} \right\}$$

$$\lim_{x \rightarrow 1^+} f(x) = x + 2 \stackrel{\text{sh}}{=} 1 + 2 = 3$$

Concl f is not cont^s at $x = 1$



Called a jump discontinuity at $x = 1$ because $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

$$\text{ex. } f(x) = \begin{cases} 1-x^2 & \text{if } x < -3 \\ 4 & \text{if } x = -3 \\ x-5 & \text{if } x > -3 \end{cases}$$

September 12, 2016

(1) $f(-3) = 4$ (defined) ✓

(2) $\lim_{x \rightarrow -3} f(x)$ exist?

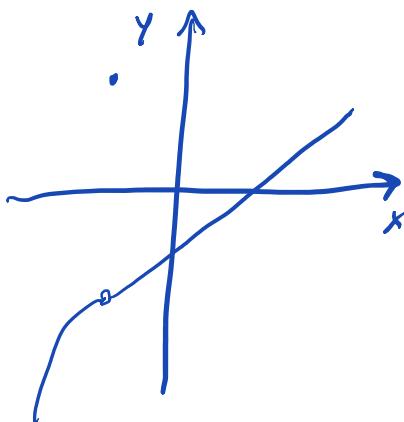
$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} (1-x^2) \stackrel{\text{oh}}{=} -8$$

$$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3^+} (x-5) = -8$$

$$\text{so } \lim_{x \rightarrow -3} f(x) = -8 \text{ (ex. 3.15)} \checkmark$$

(3) $\lim_{x \rightarrow -3} f(x) \neq f(-3) \Rightarrow$ so f is not cont'd at $x = -3$

Called removable discontinuity at $x = -3$ (since $\lim_{x \rightarrow -3} f(x)$ exists)



Summary

① A f^{cn} $y = f(x)$ has a jump discontinuity at $x = a$

$$\text{if } \lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$$

② A f^{cn} $y = f(x)$ has a removable discontinuity at $x = a$

if $y = f(x)$ is not cont^s at $x = a$ but $\lim_{x \rightarrow a} f(x)$ exists.

Defⁿ

- A polynomial expression is an algebraic expression that can be written in the form:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0 \quad (n = 0, 1, 2, \dots \text{ and } a_n \neq 0)$$

- A polynomial f^{cn} is a f^{cn} defined by a single polynomial expression. (i.e. not piecewise defined, involves poly expressions) and with DOMAIN = \mathbb{R} (i.e. no restrictions in the domain)

ex. $f(x) = -3x^4 + 5x + 1$ is a poly f^{cn}

ex. $h(x) = \begin{cases} x^2 + 2x + 1 & \text{if } x < 2 \\ x^3 - 4x + 3 & \text{if } x \geq 2 \end{cases}$ is not considered to be a poly f^{cn}

ex. $g(x) = -3x^5 + 7x + 2 \quad (0 \leq x \leq 3)$ is not a poly f^{cn}

ex. Is the poly $f(x) = -3x^4 + 5x + 1$ cont's at $x = 1$?

① $f(1) = -3(1)^4 + 5(1) + 1 = 3$ (defined)

② $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (-3x^4 + 5x + 1) \stackrel{\text{ch}}{=} -3(1)^4 + 5(1) + 1 = 3$ (exists)

③ $\lim_{x \rightarrow 1} f(x) = f(1)$

so f is cont's at $x = 1$

In fact, the poly $f(x) = -3x^4 + 5x + 1$ cont's at $x = a'$
for any choice of a'

If $y = P(x)$ is a poly fcn & 'a' is any fixed no.
then $y = P(x)$ is conts at $x = a'$

So a poly fcn has no pts of discontinuity or breaks in its graph

Ex. Consider

$$g(x) = \begin{cases} x^3 + 4x + 1 & \text{if } x < 3 \\ x^2 + 2x - 8 & \text{if } x \geq 3 \end{cases}$$

- This fcn is cont's at every $x = a'$ for any $a' \neq 3$
since each "piece" is built from a poly expression
- The only place where g may not be cont's is at $x = 3$. Here we apply 3 pt defn?

① $g(3) = 3^2 + 2(3) - 8 = 23$

② $\lim_{x \rightarrow 3^-} g(x) = \lim_{x \rightarrow 3^-} (x^3 + 4x + 1) \stackrel{\text{ch}}{=} 3^3 + 4(3) + 1 = 40$

$$\lim_{x \rightarrow 3^+} g(x) = \lim_{x \rightarrow 3^+} (x^2 + 2x - 8) \stackrel{\text{ch}}{=} (3)^2 + 2(3) - 8 = 23$$

so g is not cont's at $x = 3$

Ex/ Consider $f(x) = x^2$ ($0 \leq x \leq 2$)

Is $f(x)$ cont's at $x=2$?

① $f(2) = 2^2 = 4$

② $\lim_{x \rightarrow 2} f(x)$ exist?

$$\lim_{t \rightarrow 2^-}$$

$$\lim_{x \rightarrow 2^+} f(x) = \text{DNE}$$

Technically $f(x) = x^2$ ($0 \leq x \leq 2$) is not cont's at $x=2$

However, note:

① ✓

② $\lim_{t \rightarrow 2^-} f(t) = \lim_{x \rightarrow 2^-} x^2 = 2^2 = 4$ (exists)

③ $\lim_{x \rightarrow 2^-} f(x) = f(2)$

$y = f(x)$ is left cont's at $x=2$

so $y = f(x) = x^2$ ($0 \leq x \leq 2$) is not cont's at $x=2$, but it is LEFT-CONT'S at $x=2$

In gen'l, any $f^{(n)}$ is said to be LEFT-CONT'S at $x=a'$

if: ① $f(a)$ is defined

② $\lim_{x \rightarrow a^-} f(x)$ exists

③ $\lim_{x \rightarrow a^-} f(x) = f(a)$

Same for RIGHT-CONT'S

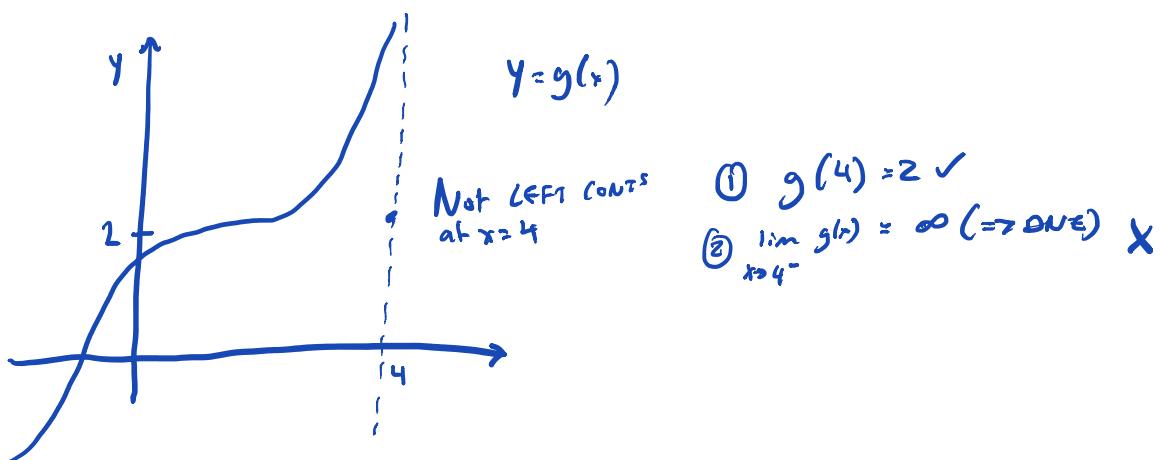
$y = f(x) = x^2$ ($0 \leq x \leq 2$) is RIGHT-CONTINUOUS at $x=0$

since: ① $f(0) = 0$ defined

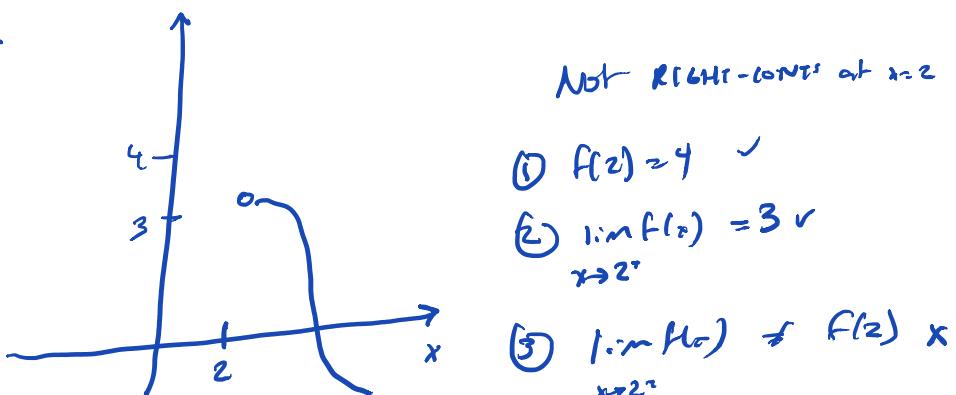
$$\textcircled{2} \quad \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0$$

$$\textcircled{3} \quad \lim_{\substack{x \rightarrow 0^+}} f(x) = f(0)$$

ex.



ex.



RULES OF CONT'S AT PT $x = 'a'$

Assume $y = f(x)$ & $y = g(x)$ are both cont's at $x = 'a'$

- ① $y = f(x) + g(x)$ is also cont's at $x = 'a'$
- ② $y = f(x) - g(x)$ "
- ③ $y = f(x) \cdot g(x)$ "
- ④ $y = Kf(x)$ " (K is a constant)
- ⑤ $y = \frac{f(x)}{g(x)}$ " (provided $g(a) \neq 0$)

ex. let $f(x) = \sqrt{x}$ & $g(x) = x-4$

Both f & g are cont's at $x=9$

so

- ① $y = f(x) + g(x) = \sqrt{x} + (x-4)$ is also cont's at $x=9$
- ② $y = f(x) - g(x) = \sqrt{x} - (x-4)$ is "
- ③ $y = f(x) \cdot g(x) = (\sqrt{x})(x-4)$ is "
- ④ $y = \sqrt[3]{x} f(x) = -\sqrt[3]{x} \sqrt{x}$ is "
- ⑤ $y = \frac{f(x)}{g(x)}$ is "

ex. $y = \frac{x^2+4x+1}{x^3+8}$ where is this func cont's?

$$y = f(x) = x^2 + 4x + 1 \text{ is a poly func & so cont's anywhere}$$

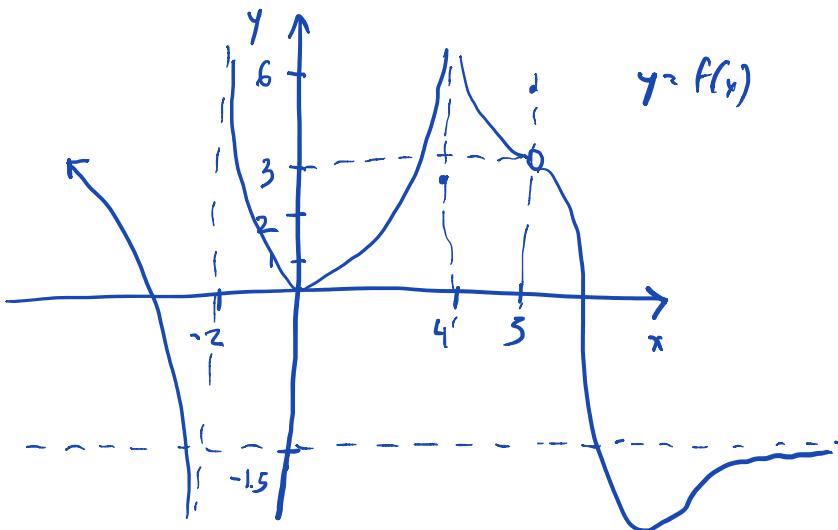
$$y = g(x) = x^3 + 8 \text{ is } "$$

$y = \frac{x^2+4x+1}{x^3+8}$ is not defined at $x=-2$

$y = \frac{f(x)}{g(x)} = \frac{x^2+4x+1}{x^3+8}$ is cont's everywhere except at $x=-2$

Review ex.

September 15, 2016



$$\textcircled{1} \lim_{x \rightarrow -\infty} f(x) = +\infty \quad \text{Not DNE}$$

$$\textcircled{2} \lim_{x \rightarrow +\infty} f(x) = -1.5$$

$$\textcircled{3} \lim_{x \rightarrow -2^-} f(x) = -\infty$$

$$\textcircled{4} \lim_{x \rightarrow -2^+} f(x) =$$

$$\textcircled{5} \lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\textcircled{6} \lim_{x \rightarrow 4^-} f(x) = +\infty$$

$$\textcircled{7} \lim_{x \rightarrow 4^+} f(x) = +\infty$$

$$\textcircled{8} \lim_{x \rightarrow 4} f(x) = +\infty \leftarrow \text{not a finite no.} \quad \text{DNE}$$

$$\textcircled{9} \lim_{x \rightarrow 5} f(x) = 3$$

$$\textcircled{10} f(5) = 6$$

$$\textcircled{11} f(4) = 3$$

\textcircled{12} Is f cont^s at x = 4?

If not, what fails in def?

" , is the discontinuity removable

Not cont^s.

$\lim_{x \rightarrow 4}$ = DNE

Not removable, not jump discontinuity

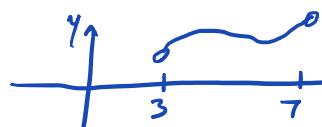
\textcircled{13} Does f have any removable discontinuity?

yes at x = 2

Terminology

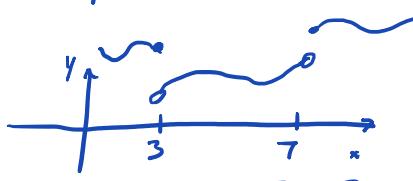
- (1) $y = f(x)$ is cont^s everywhere on the interval $(-\infty, \infty)$
 \therefore cont^s according to 3-pt defn at any ω chosen in $(-\infty, \infty)$
- (2) $y = f(x)$ is cont^s everywhere on the interval $(3, 7)$
 \therefore cont^s according to 3-pt defn at any ω chosen in $(3, 7)$
- (3) $y = f(x)$ is cont^s everywhere on the closed interval $[3, 7]$
 \therefore cont^s according to 3-pt defn at any ω chosen in $(3, 7)$
and must be right-cont^s at $x=3$ and left-cont^s at $x=7$

ex.



$y = f(x)$ is cont^s on $(3, 7)$

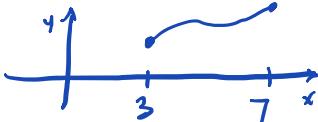
ex.



Is $y = f(x)$ is cont^s on closed $(3, 7)$?

yes

ex.



$y = f(x)$ is cont^s on $[3, 7]$

INTERMEDIATE VALUE THEOREM (IVT)

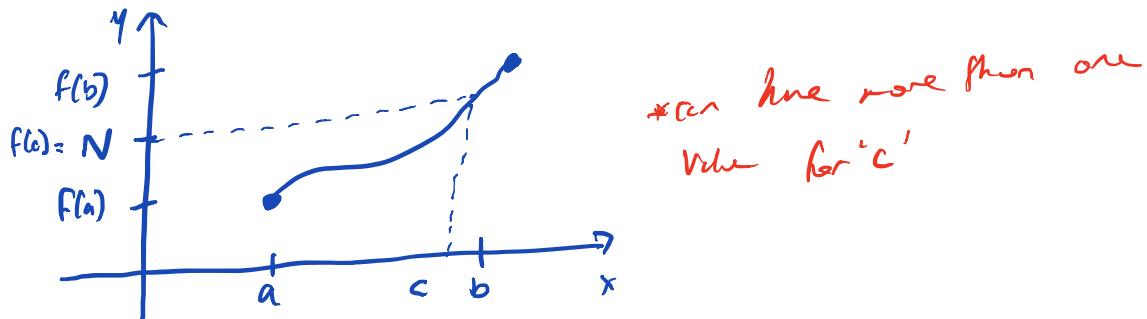
If ① $y = f(x)$ is cont's on $[a, b]$

② $f(a) \neq f(b)$

③ N is any number between $f(a)$ and $f(b)$

Then

there is a no. 'c' in (a, b) so that $f(c) = N$



Ex. Show that the eqn $4x^3 - 6x^2 + 3x - 2 = 0$ has a root between 1 & 2.
(i.e. a zero) between 1 & 2.

$$\text{Let } f(x) = 4x^3 - 6x^2 + 3x - 2$$

① $f(x)$ is cont's on $[1, 2]$ (Since $f(x)$ is a poly. $\text{f}(\text{a})$)

$$② f(1) = 4(1)^3 - 6(1)^2 + 3(1) - 2 = -1$$

$$f(2) = 4(2)^3 - \dots = 12$$

$$f(1) \neq f(2)$$

③ we can chose $N=0$ (falls in between $f(1) = -1$ and $f(2) = 12$)

by concl. of IVT, there is a value c in $(1, 2)$

$$\begin{aligned} f(c) &= 0 (= N) \\ \text{i.e. } 4c^3 - 6c^2 + 3c - 2 &= 0 \end{aligned}$$