Raffi Holter CALCULUS I August 18, 2016

REVIEW

Absolute Values

ex/ $|-5| = 5$

ex/ $|4| = 4$

ex/ $|0| = 0$

Def^n

If $x < 0$ then $|x| = -x$

If $x \geq 0$ then $|x| = x$

i.e. $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

Rules

① $|-x| = |x|$ ex/ $|-6| = |6|$

ex/ Simplify $\dfrac{-|x|}{|-x|}$

$= \dfrac{-\cancel{|x|}}{\cancel{|x|}}$

$= -1$

② $\sqrt{x^2} = |x|$

ex/ $\sqrt{(-3)^2} = |-3| = 3$

③ $|xy| = |x|\,|y|$

ex/ $|(-3)(-5)| = |-3|\,|-5| = 3 \cdot 5 = 15$

④ $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$

ex/ Simplify

$\dfrac{|-7ab|}{|a|} = \dfrac{|-7|\cancel{|a|}|b|}{\cancel{|a|}} = 7|b|$

ex/ Analyze $\dfrac{x-2}{|x-2|}$:

Case ① $x = 2$

$\dfrac{x-2}{|x-2|} = 0$ und

Case ② $x < 2 \rightarrow \dfrac{x-2}{|x-2|} = \dfrac{x-2}{-(x-2)} = -1$

$|x-2| = \begin{cases} -(x-2) & \text{if } x-2 < 0 \Rightarrow \text{if } x < 2 \\ x-2 & \text{if } x-2 \geq 0 \Rightarrow \text{if } x \geq 2 \end{cases}$

Case ③

$x > 2$

$\dfrac{x-2}{x-2} = \dfrac{x-2}{x-2} = 1$

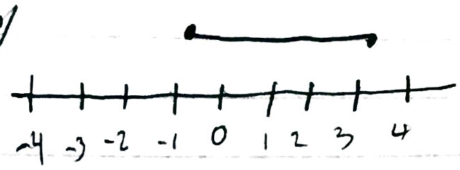
Summary / Conclusion

$\dfrac{x-2}{|x-2|} = \begin{cases} -1 & \text{if } x < 2 \\ \text{Und} & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$ or $\begin{cases} -1 & \text{if } x < 2 \\ 1 & \text{if } x > 2 \end{cases}$

Convention re: Interval Notation

August 17, 2016

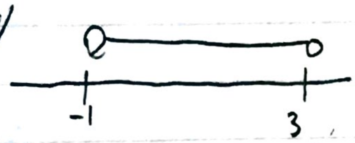
ex/



incl.

$$\{x \mid -1 \leq x \leq 3\} = [-1, 3]$$

ex/



not incl.

$$\{x \mid -1 < x < 3\} =]-1, 3[= (-1, 3)$$

ex/

$$\{x \mid x \geq -5\} = [-5, \infty[= [-5, \infty)$$

ex

$$\{x \mid x < 3\} =]-\infty, 3[= (-\infty, 3)$$

Analyse $\frac{x-2}{|x-2|}$

① $|x-2| = -(x-2)$ if $x-2 < 0$
$x < 2$

② $|x-2| = x-2$ if $x-2 \geq 0$
$x \geq 2$

③ $x-2=0$
$x=2$

$\frac{x-2}{-(x-2)} = -1$ when $x < 2$

$x^2-4x-5>0$

$x^2+x-5x-5>0$

$x(x+1)-5(x+1)>0$

$(x-5)(x+1)>0$

find split pts:

$(x-5)(x+1)=0$

$x=5, -1$

	$-\infty$	-1		5	$+\infty$
$(x-5)(x+1)$	$+$	0	$-$	0	$+$

$SS = (-\infty, -1) \cup (5, +\infty)$

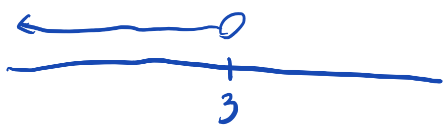
Linear Inequalities

August 19, 2016

ex/ Solve $3x - 5 < 4$

$3x < 9$

$x < 3$



$SS = (-\infty, 3)$

ex/ $-3x - 5 < 4$

$-3x < 9$

$x > 3$ Remember to flip sign

$SS = (-3, \infty)$

Non-Linear Inequalities

$x^2 - 4x - 5 > 0$ ← only works when its equal to 0

$(x+1)(x-5) > 0$

Solve: $(x+1)(x-5) = 0$

$x = -1, 5$ ← split points

Test $x = -2$ Test $x = 0$ Test $x = 6$

	-1		5	
$(x+1)(x-5)$	+	0	−	0	+

$SS = (-\infty, -1) \cup (5, \infty)$

Summary

- Split pt +/- Table idea only works for expression > 0 or expression < 0

- To find "split pts", find where:

or
a) expression = 0
b) expression is UND (undefined)

ex/ Solve $\frac{(x+1)(x-5)}{2x-1} \le 0$

Split pts:

a) $\frac{(x+1)(x-5)}{2x-1} = 0$

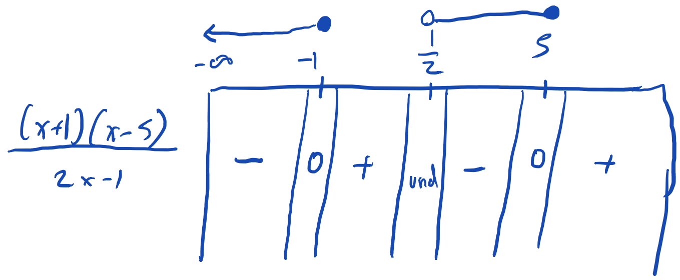
$(x+1)(x-5) = 0$

$x = -1$, $x = 5$

b) $\frac{(x+1)(x-5)}{2x-1}$ is UND

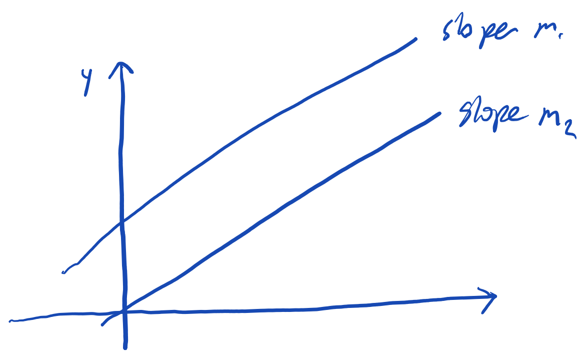
when $2x-1 = 0$

$x = \frac{1}{2}$

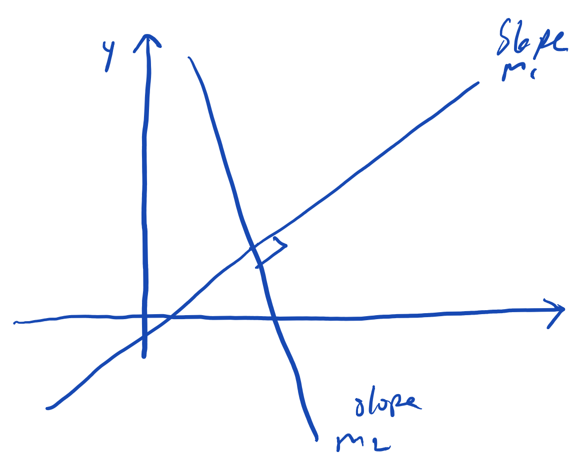


$SS = (-\infty, -1] \cup (\frac{1}{2}, 5]$

Straight Lines



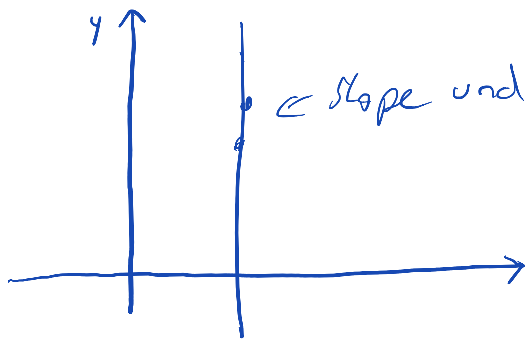
$m_1 = m_2$



$m_1 m_2 = -1$

$m_1 = -\frac{1}{m_2}$

ex/ $x=3$



* Prove by formula

$m = \frac{4-1}{3-3} = \frac{3}{0}$ ← und

All lines can be written in the form:

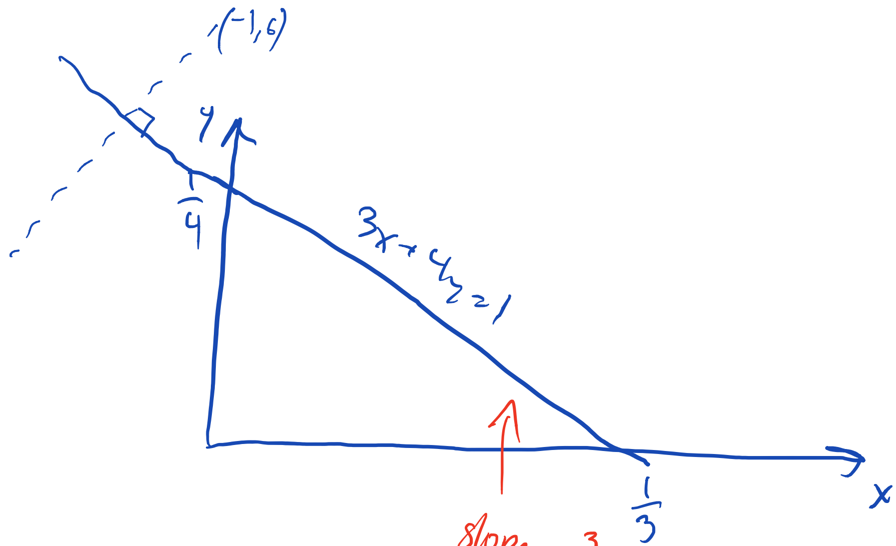
$$ax + by + c = 0$$

Most lines can be written in:

$$y = mx + b$$ ← slope - intercept form

ex/ Given the line $3x + 4y = 1$

Find the eqn of the line $\perp$ to the given line and passing through pt $P(-1, 6)$



$$3x+4y=1$$
$$y=\frac{1-3x}{4}$$
$$y=-\frac{3}{4}x+\frac{1}{4}$$

Slope m of required line:

$$m=-\frac{1}{-\frac{3}{4}}=\frac{4}{3}$$

$$y=\frac{4}{3}x+b$$

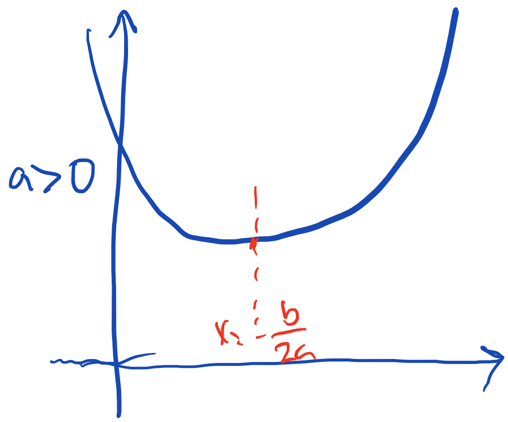
Subst $x=-1$, $y=6$

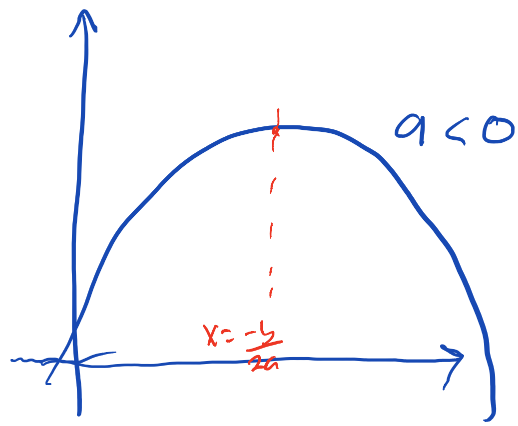
$$6=\frac{4}{3}(-1)+b$$
$$b=\frac{22}{3}$$

Ans: $y=\frac{4}{3}x+\frac{22}{3}$

Parabolas

$y = ax^2 + bx + c$





$$\boxed{x = \frac{-b}{2a}}$$

eg) Sketch $y = x^2 + 2x - 3$

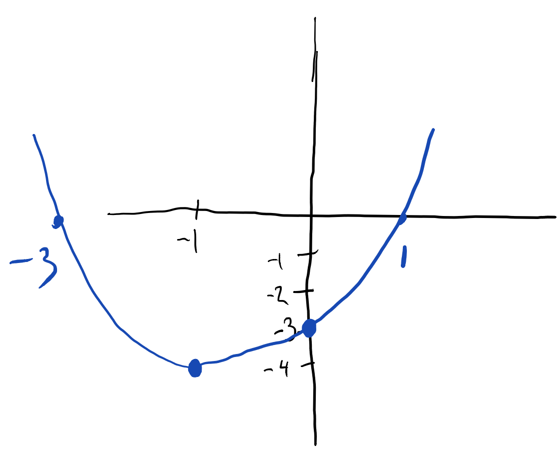
$a = 1 \therefore$ parabola up

vertex occurs at $x = \frac{-b}{2a} = \frac{2}{2(1)} = 1$

when $x = -1$, $y = (-1)^2 + 2(-1) - 3 = -4$

vertex pt is $(-1, 4)$

x	$y = x^2 + 2x - 3$
-1	4
0	-3



Find x-int

Solve

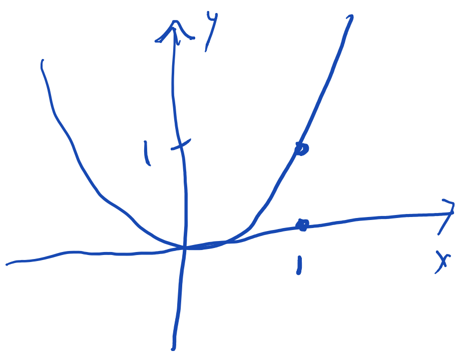
$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

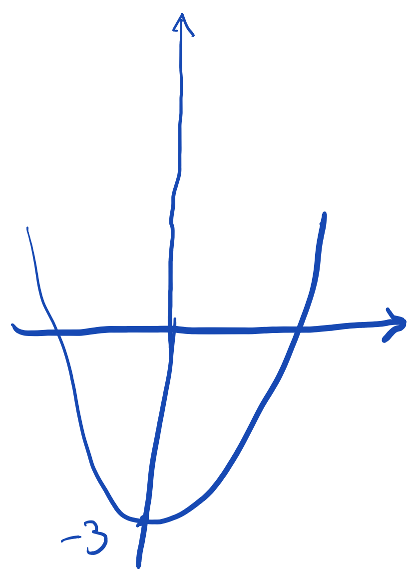
$x = 1, -3$

ex/ $y = x^2 - 3$

Sketch $y = x^2$

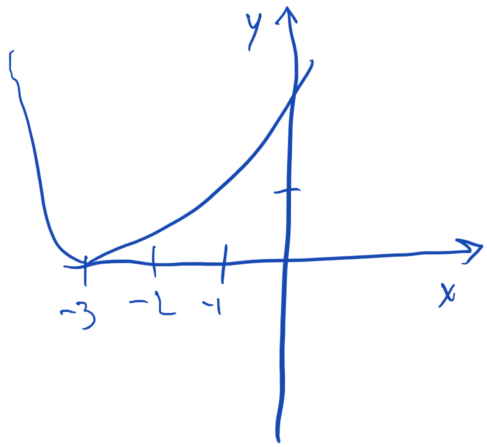


Sketch $y = x^2 - 3$



ex/ Sketch $y = -(x+3)^2$

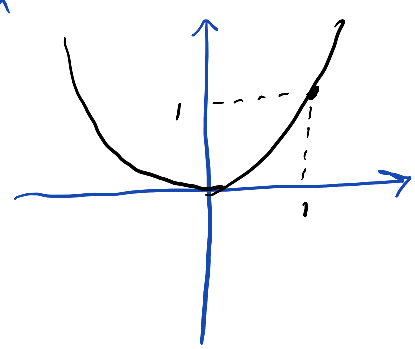
reflect through x-axis



Ex/ Sketch $x = y^2$

Other Basic Graphs to know:

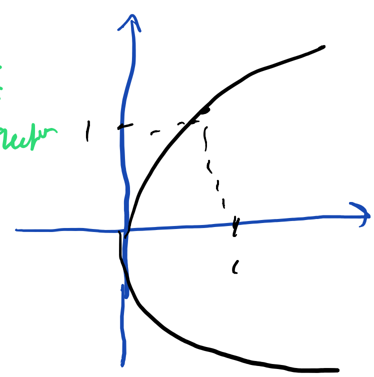
$y = x^2$



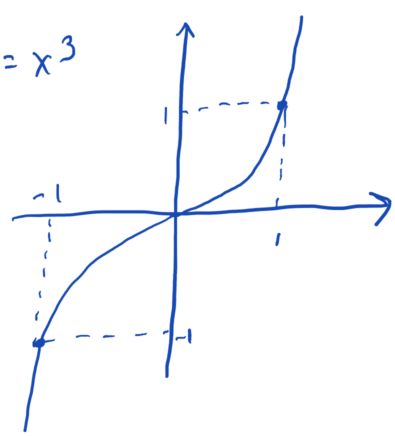
$x = y^2$

$y = \sqrt{x}$

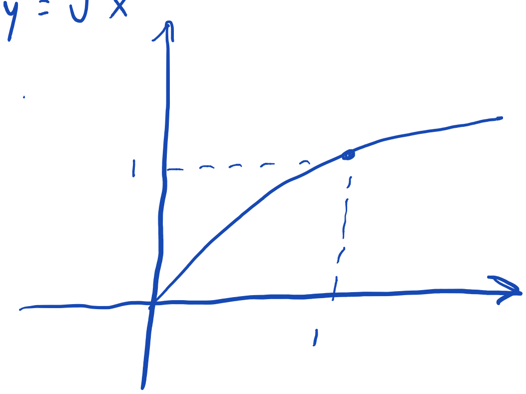
with reflection



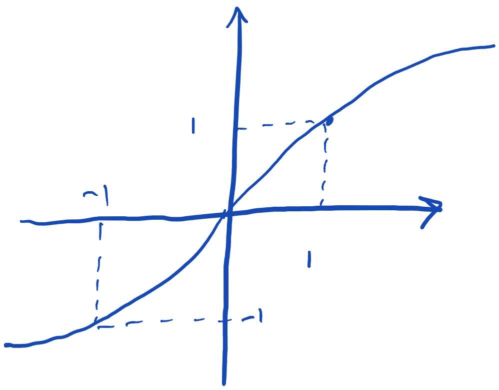
$y = x^3$



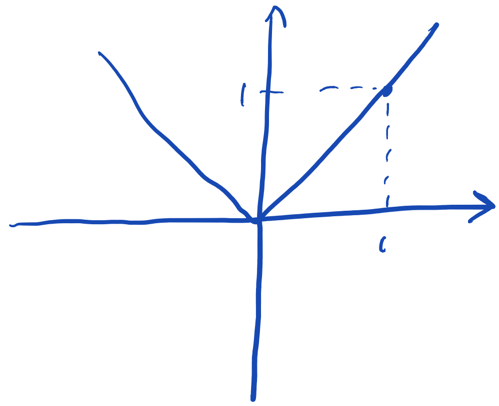
$y = \sqrt{x}$



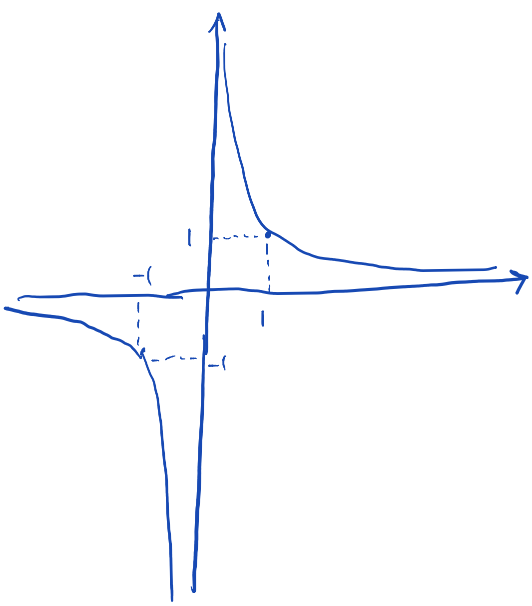
$y = \sqrt[3]{x} = x^{\frac{1}{3}}$



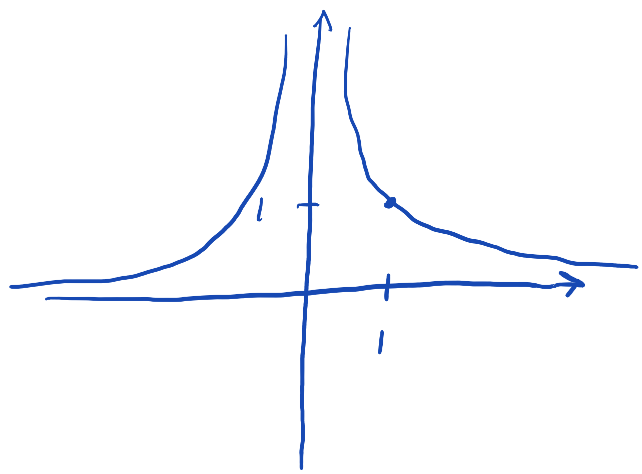
$y = |x|$



$y = \frac{1}{x}$

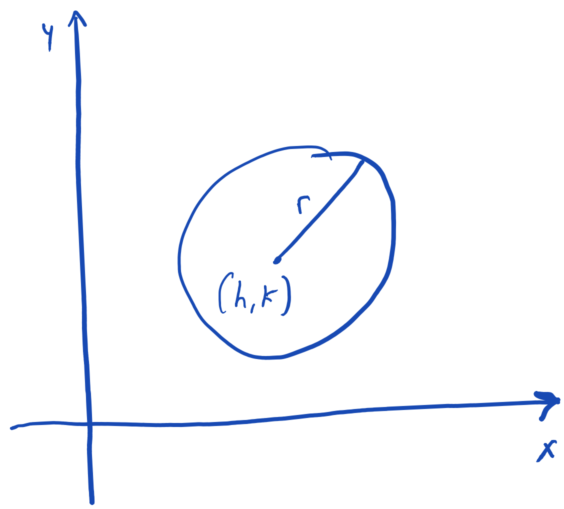


$y = \frac{1}{x^2}$

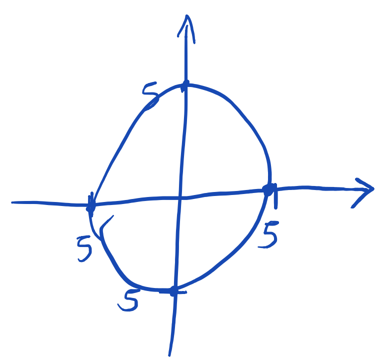


Circle

$$(x-h)^2+(y-k)^2=r^2$$



ex/ $x^2+y^2=25$



ex/ Sketch graph of $y=\sqrt{9-x^2}$

$\cancel{y=\sqrt{9}-\sqrt{x^2}}$

$\cancel{y=3-x}$

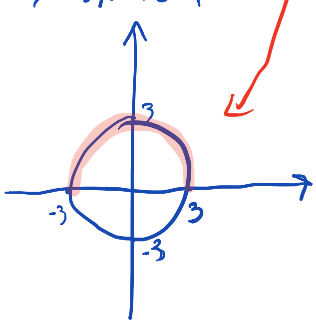
Square both sides $y=+$

$y^2=9-x^2$

$x^2+y^2=9$

$(y\ge 0)$ ↳ Sketch

Since $y\in[0,+\infty[$, we need to just take the top of the graph



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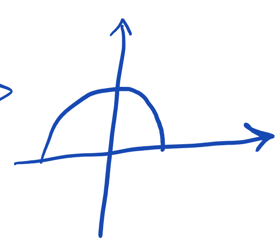
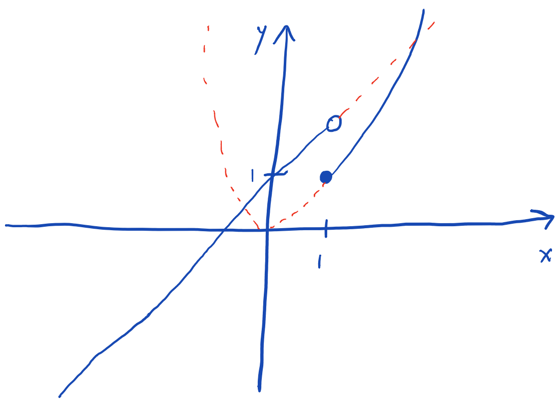


Piecewise-Defined Function

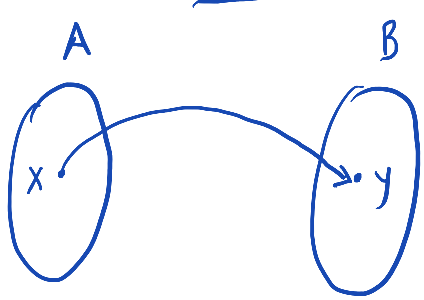
ex/ $y = \begin{cases} x+1 & \text{if } x<1 \\ x^2 & \text{if } x \geq 1 \end{cases}$ $\rightarrow$

x	y
-2	$(-2)+1=-1$
-1	...
0	...
1	$(1)^2=1$
2	$2^2=4$
3	



Function Concept

Def^n: A function is a rule which assigns to each value of x in a set A a unique value of y in some set B



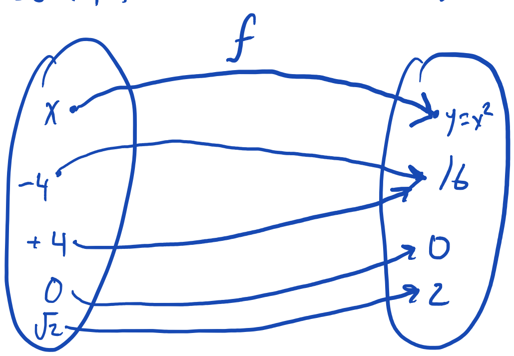
mapping picture

Set A = **Domain**

y is the **image** of x

The set of all images 'y' is called the **Range** ex/ Consider the f^{cn} defined by the $y = x^2$ & assume domain is $\mathbb{R}$

Dom $\mathbb{R}$ Range = $[0, \infty)$



$f = x^2$

image of 'x' : $f(x)$

ex/ $f(2) = 4$

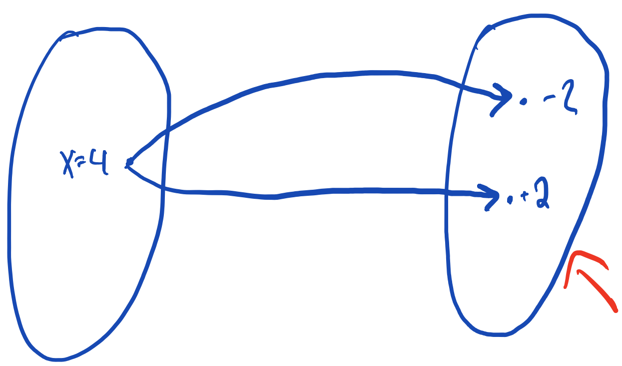
$f(x+5) = (x+5)^2$

ex/ $g(x) = \sqrt{\frac{x-1}{x+2}}$

What is $g\left(\frac{\sqrt{x}+1}{5}\right)$?

$$g\left(\frac{\sqrt{x}+1}{5}\right) = \sqrt{\frac{\frac{\sqrt{x}+1}{5} - 1}{\frac{\sqrt{x}+1}{5} + 2}}$$ Simplify

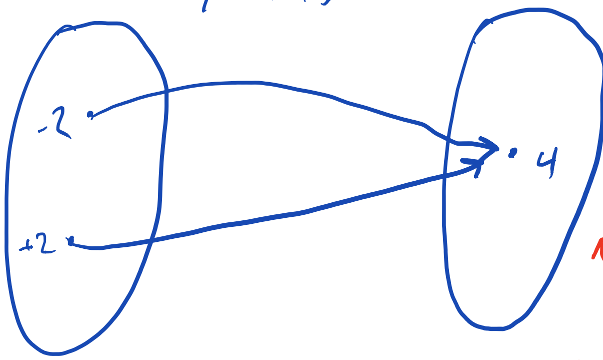
ex/ Consider the eq^n $y^2 = x$



* not a unique y-value $\therefore$ y is not a f^{cn} of x

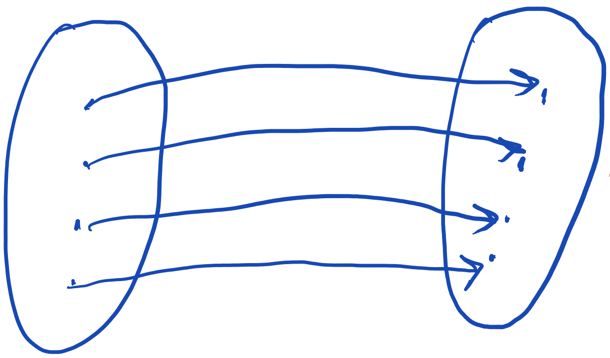
called a one-to-many relation

$y = f(x) = x^2$



← called many-to-one relation

* still a f^{cn}



called a one-to-one f^{cn}

Domain of f^{cn}

ex/ $f(x) = x^3 + 1$	$g(x) = x^3 + 1 \quad (x \geq 5$
$f(1) = (1)^3 + 1 = 2$	$g(1)$ UND

* Convention for domains: If the domain of a f^{cn} is not explicitly given, assume the domain to be all real numbers for which the expression on the RHS (right hand side) of the defining rule makes sense

ex/ $g(x) = \frac{1}{x-2}$

Assume DOM $= \mathbb{R} \setminus \{2\}$ *doesn't care how we communicate it

ex/ $H(t) = \sqrt{t}$

Assume DOM $= [0, \infty)$

ex/ $g(x) = \frac{1}{\sqrt{3-5x}}$

$3 - 5x > 0$

$3 > 5x$

$\frac{3}{5} > x$

$x < \frac{3}{5}$

DOM $= (-\infty, \frac{3}{5})$

Test has harder questions

ex/ $R(t) = \sqrt{t^2 - 2t - 3}$

$t^2 - 2t - 3 \geq 0$

$(t+1)(t-3) \geq 0$

Solve using +/- table

ex/ $$|x^4 - 5x^3 - 14x^2| = \begin{cases} -(x^4 - 5x^3 - 14x^2) & \text{if } w < 0 \\ x^4 - 5x^3 - 14x^2 & \text{if } w \geq 0 \end{cases}$$

<u>Find split pts</u>

$$x^4 - 5x^3 - 14x^2 = 0$$

$$x^2(x^2 - 5x - 14) = 0$$

$$x^2(x+2)(x-7) = 0$$

Split pts: $x = -2, 0, 7$

	$-\infty$	-2		0		7	$+\infty$
$x^2(x+2)(x-7)$	$+$	0	$-$	0	$-$	0	$+$

$$|x^4 - 5x^3 - 14x^2| = \begin{cases} x^4 - 5x^3 - 14x^2 & \text{if } x < -2 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = -2 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } -2 < x < 0 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = 0 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } 0 < x < 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x = 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x > 7 \end{cases}$$

$$|x^4 - 5x^3 - 14x^2| = \begin{cases} x^4 - 5x^3 - 14x^2 & \text{if } x \leq -2 \\ -(x^4 - 5x^3 - 14x^2) & \text{if } -2 < x < 7 \\ x^4 - 5x^3 - 14x^2 & \text{if } x \geq 7 \end{cases}$$

ex/ If $f(x) = \dfrac{1}{x}$ evaluate $\dfrac{f(x+h) - f(x)}{h}$ & simplify

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

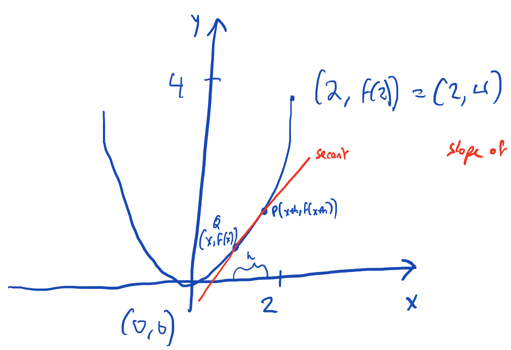
$$= \frac{x-(x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$= \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-1}{x(x+h)}$$

Graphing Fcns

ex/ Sketch $y = f(x) = x^2$



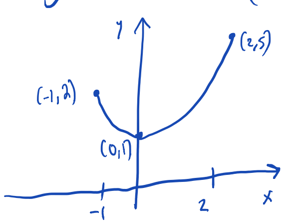
slope of $\overline{PQ} = \dfrac{f(x+h) - f(x)}{(x+h) - x}$

$$= \boxed{\frac{f(x+h) - f(x)}{h}}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= 2x + h$$

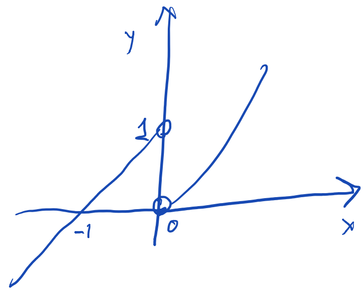
ex/ Sketch $g(x) = x^2 + 1 \quad (-1 \leq x \leq 2)$



DOM: $[-1, 2]$

RANGE: $[1, 5]$

ex/ Sketch $y = f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x^2 & \text{if } x > 0 \end{cases}$



DOM = $(-\infty, 0) \cup (0, \infty) = \mathbb{R} \setminus \{0\}$

RANGE = $\mathbb{R}$

ex/ Sketch $y = h(x) = \dfrac{|x-1|}{x-1}$

Fcns involving abs. values are usually piecewise defined" fcns

Dom = $\mathbb{R} \setminus \{1\}$

$|x-1| = \begin{cases} -(x-1) \text{ if } x-1 < 0 \Rightarrow \text{if } x < 1 \\ x-1 \quad \text{if } x-1 \geq 0 \Rightarrow \text{if } x \geq 1 \end{cases}$

Case ① if $x < 1$

$h(x) = \dfrac{-\cancel{(x-1)}}{\cancel{x-1}} = -1$

Case ② if $x = 1$

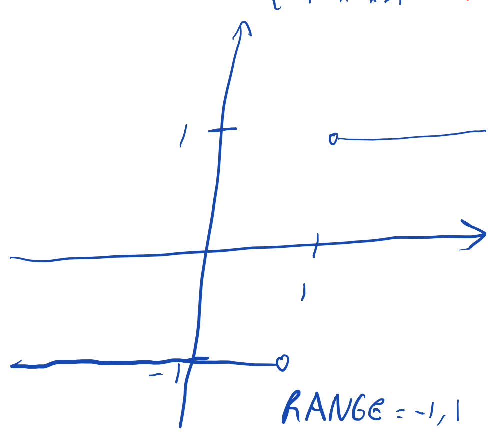
$h(1) = \dfrac{|1-1|}{1-1} = \dfrac{0}{0}$ UND

Case ③ if $x > 1$

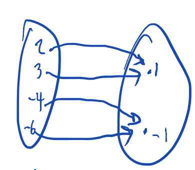
$h(x) = \dfrac{x-1}{x-1} = 1$

Concl

$y = h(x) = \dfrac{|x-1|}{x-1} = \begin{cases} -1 & \text{if } x < 1 \\ \text{UND} & \text{if } x = 1 \\ +1 & \text{if } x > 1 \end{cases}$ – Don't need to include this, it's implied

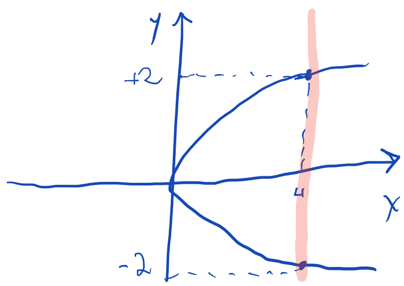


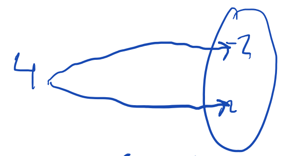
RANGE = $-1, 1$



MANY-TO-ONE

ex/ Consider $y^2 = x$





ONE-TO-MANY
Not a f^{cn}

If you can find a vertical line, y is not a f^{cn} of x

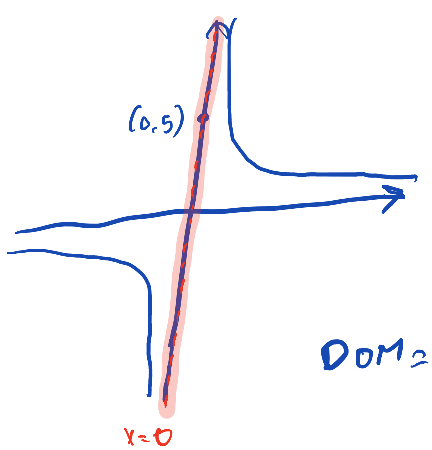
A WORD REGARDING ASYMPTOTES

Vertical Assymptotes

A function can usually never pass its vertical assymptote

ex/ Consider the fcn

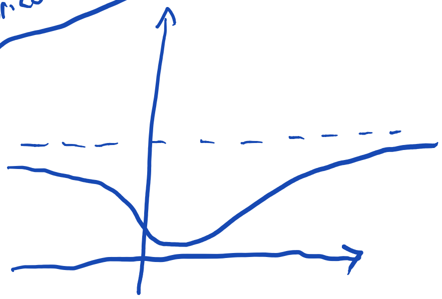
$$f(x)\begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$$



← Here the function passes through its vertical asymptote

$DOM = \mathbb{R}$

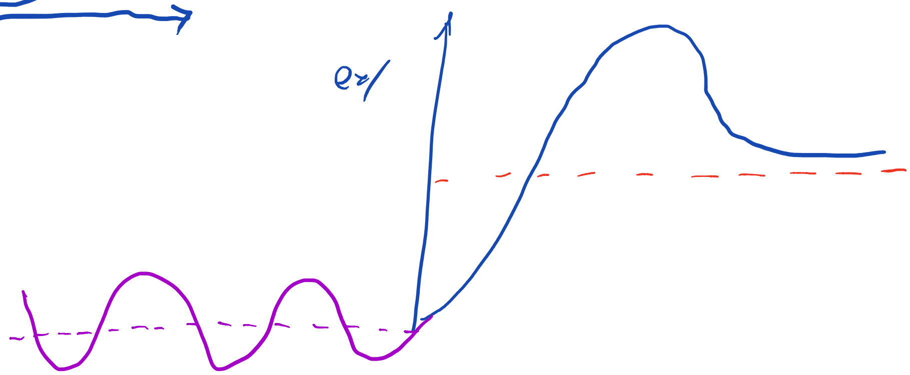
Horizontal Assymptotes



* No more than two horizontal asymptotes

* A fcn can pass its horizontal asymptote

ex/



Algebra of F^{cns}

ex/ Suppose $f(x) = x^2$ & $g(x) = x^2 - 1$, define a new f^{cn} by the rule:

$$y = f(x) + g(x) = x^2 + (x^2 - 1) = 2x^2 - 1$$

Name this f^{cn} "f+g" $(f+g)(x) = 2x^2 - 1$

$$(f-g)(x) = f(x) - g(x) = x^2 - (x^2 - 1) = 1$$

$$(f \cdot g)(x) = f(x) \cdot g(x) = x^2(x^2 - 1) = x^4 - x^2$$

$$(7g)(x) = 7g(x) = 7(x^2 - 1)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{x^2 - 1}$$

Composition of f^{cns}

we can also define a new f^{cn} by "composing" f & g & we name this new f^{cn} "$f \circ g$"

$(f \circ g)(x) = f(g(x)) = f(x^2-1) = (x^2-1)^2$

outside fun　　inside f^{cn}

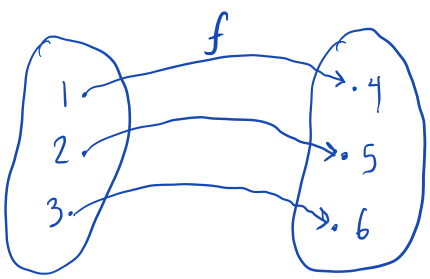
$\boxed{(f \circ g)(x) \neq (g \circ f)(x)}$

$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)^2 - 1 = x^4 - 1$

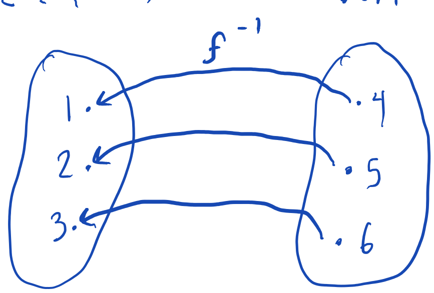
INVESE OF F^{cns}

ex/

DOM = $\{1,2,3\}$　　RANGE = $\{4,5,6\}$



RANGE = $\{1,2,3\}$　　DOM = $\{4,5,6\}$



$\boxed{f^{-1}(f(x)) = x \text{ for all } x \text{ in Dom } f}$

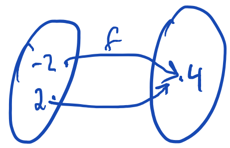
$\boxed{f(f^{-1}(x)) = x \text{ for all } x \text{ in Dom } f^{-1}}$

ex/ $f^{-1}(f(1)) = f^{-1}(4) = 1$

ex/ $f(f^{-1}(4)) = f(1) = 4$

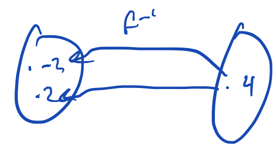
ex/ $g(x) = x^2$

DOM = $\mathbb{R}$ Range = $[0, \infty)$



$\Rightarrow$

If we "reverse arrows" we get



Not fcn

Only f^{ns} that have a <u>one-to-one</u> can have an inverse fcn.

ex/ consider $f(x) = \dfrac{x-2}{x+1}$ find f^{-1}

Let $y = \dfrac{x-2}{x+1}$ $(x \neq -1)$

$x = \dfrac{y-2}{y+1}$ $(y \neq -1)$

$x(y+1) = y-2$

$xy + x = y-2$

$xy - y = -x-2$

$y(x-1) = -x-2$

$y = \dfrac{-x-2}{x-1}$ $(y \neq -1)$

So $f^{-1}(x) = \dfrac{-x-2}{x-1}$

ex/ Consider $y = \sqrt{2-x}$, find the inverse f^{n}

$y = \sqrt{2-x}$ DOM: $x \leq 2$

RANGE: $y \geq 0$

$x = \sqrt{2-y}$ RANGE: $y \leq 2$

DOM: $x \geq 0$

$y = 2 - x^2$ $(x \geq 0)$ ← Inverse f^{cn}