Graphical Abstract

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Highlights

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- A physics-based irrigation environment is coupled with reinforcement learning control.
- Rule-based, learning-based, and hybrid neuro-physical strategies are systematically compared.
- Reinforcement learning improves water-use efficiency under stochastic climatic forcing.
- Neural ODE augmentation reduces extreme stress caused by model mismatch.
- Results clarify trade-offs between robustness, efficiency, and interpretability in irrigation control.

A physics-informed reinforcement learning framework for climate-robust irrigation control

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Abstract

Climate change increases hydro-climatic variability, challenging conventional irrigation strategies that rely on static rules and fixed thresholds. From a modelling perspective, irrigation management can be formulated as a dynamic environmental control problem operating under uncertainty. This study proposes a physics-informed reinforcement learning approach for irrigation control that integrates mechanistic soil–water dynamics with adaptive decision making. A physically inspired irrigation environment is constructed using a soil–water balance model subject to stochastic rainfall disturbances. Within this environment, three control strategies of increasing modelling complexity are analysed: (i) a rule-based controller derived from expert thresholds, (ii) a reinforcement learning controller trained using Proximal Policy Optimization (PPO) interacting directly with the physical model, and (iii) a hybrid neuro-physical formulation in which unmodelled residual dynamics are learned via a Neural Ordinary Differential Equation and embedded within the control loop. This progressive formulation enables a structured comparison between heuristic control, data-driven control, and physics-augmented learning. Simulation results show that reinforcement learning improves the trade-off between water use and crop water stress relative to rule-based control, particularly under variable climatic conditions. Incorporating learned residual dynamics further enhances robustness and stability without violating physical consistency. The proposed framework is transferable to other environmental systems requiring adaptive control under non-stationary conditions.

Keywords:
Intelligent irrigation, Environmental decision support, Physics-informed

reinforcement learning, Hybrid modelling, Soil–water dynamics, Climate uncertainty

1. Introduction

Climate change is increasing the frequency and intensity of hydro-climatic variability, placing growing pressure on agricultural water management systems. Irrigation, which accounts for a major share of global freshwater withdrawals, is particularly sensitive to rainfall uncertainty, seasonal variability, and soil–plant interactions. Conventional irrigation strategies, typically based on fixed schedules or expert-defined thresholds, often lack the flexibility required to adapt to non-stationary environmental conditions, leading to inefficient water use or increased crop water stress.

From a modelling perspective, irrigation management can be formulated as a dynamic environmental control problem in which decisions are made sequentially under uncertainty while respecting physical constraints. Process-based soil–water models have long been used to represent irrigation dynamics and to support decision making, offering interpretability and physical consistency through simplified representations such as bucket models or FAO-56-based approaches (Raes et al., 2009). However, these models rely on assumptions and parameterizations that may not fully capture unmodelled dynamics, context-dependent responses, or climatic variability.

Recent advances in reinforcement learning (RL) have enabled data-driven control policies capable of adapting to complex and stochastic environments (Sutton and Barto, 2018). RL-based approaches have been increasingly explored for irrigation and water management applications, showing potential improvements over static rule-based strategies. Nevertheless, purely data-driven control raises concerns regarding interpretability, physical plausibility, and robustness, particularly in safety-critical and resource-constrained environmental systems. These limitations have motivated growing interest in physics-informed and hybrid modelling approaches that integrate mechanistic knowledge with learning-based adaptability.

Physics-informed learning provides a principled means of constraining data-driven models using known system structure, thereby improving generalization and stability (Willard et al., 2022). In this context, Neural Ordinary Differential Equations (Neural ODEs) offer a flexible framework for learning continuous-time residual dynamics that augment existing physical models while preserving their interpretability (Rackauckas et al., 2020). When embedded within a control loop, such hybrid neuro-physical formulations have the potential to bridge process-based understanding and adaptive decision making in environmental systems.

Despite these advances, several open questions remain regarding the relative benefits of learning-based control compared to expert rule-based strategies, the role of learned residual dynamics in improving robustness, and the implications of increasing model–controller integration for interpretability and transferability. Addressing these questions requires controlled comparative studies that explicitly analyse modelling assumptions, control architectures, and performance trade-offs.

Within environmental modelling, increasing attention has been given to the integration of simulation models with adaptive decision-making and control frameworks, particularly in contexts characterised by uncertainty, nonlinearity, and competing objectives. Established contributions in Environmental Modelling & Software have emphasized the need for explicit uncertainty handling across the full modelling workflow (Refsgaard et al., 2007) and for disciplined, iterative model development and evaluation procedures to support robust decision-making (Jakeman et al., 2006).

This study addresses the following research questions:

- **RQ1:** To what extent does reinforcement learning improve irrigation control performance compared to expert rule-based strategies when interacting with a physics-based soil–water model?

- **RQ2:** Does embedding learned residual dynamics via a Neural ODE enhance the robustness and stability of learning-based irrigation control under variable climatic conditions?

- **RQ3:** How does increasing model–controller integration affect interpretability and transferability in environmental control systems?

The main contributions of this work are as follows:

1. A physics-informed reinforcement learning formulation that models irrigation as a sequential environmental control problem under climatic uncertainty.
2. A structured comparison of three control strategies—rule-based control, reinforcement learning, and hybrid neuro-physical reinforcement learning—within a unified physical simulation environment.

3. The integration of Neural ODEs as residual dynamic models to augment soil–water balance equations while preserving physical consistency.
4. A systematic evaluation under stochastic rainfall conditions, providing insights into performance trade-offs, robustness, and policy behavior.
5. A transferable modelling framework applicable to other environmental systems requiring adaptive control under non-stationary conditions.

The remainder of the paper is organized as follows. Section 2 reviews related work on irrigation modelling, rule-based control, reinforcement learning, and hybrid physics-informed approaches. Section 3 formulates irrigation control as a sequential decision-making problem. Section 4 describes the physics-based irrigation environment. Section 5 presents the three control scenarios investigated. Section 6 details the experimental design and evaluation protocol. Section 7 reports the results, which are discussed. Section 8 concludes the paper.

2. Related Work

This section reviews relevant literature on irrigation modelling and control, focusing on the transition from process-based and rule-based approaches toward learning-based and hybrid physics-informed methods. The aim is to position the present work within established environmental modelling paradigms and to identify methodological gaps addressed by the proposed framework.

2.1. Process-based modelling of irrigation systems

Process-based models remain a cornerstone of irrigation modelling, providing mechanistic representations of soil–water–plant interactions based on mass balance principles and evapotranspiration formulations. Classical approaches range from conceptual bucket models to FAO-56-based methods, as implemented in decision-support tools such as AquaCrop (Raes et al., 2009). These models offer interpretability, transparency, and relatively low data requirements, which are essential for scenario analysis and policy-oriented studies.

Nevertheless, process-based models rely on simplifying assumptions regarding soil heterogeneity, root water uptake, and boundary conditions. Their performance may degrade under highly variable or extreme climatic conditions, where unmodelled dynamics and parameter uncertainty become dominant. Recent reviews highlight that while physically based models are indispensable, their standalone use may be insufficient for adaptive irrigation management under climate change (Fatichi et al., 2016; Seneviratne et al., 2021).

2.2. Rule-based and heuristic irrigation control

Rule-based irrigation strategies, often derived from expert knowledge or agronomic guidelines, remain widely used in practice. Such approaches typically rely on fixed thresholds for soil moisture, crop water stress indices, or accumulated evapotranspiration deficits. Their appeal lies in their simplicity, interpretability, and ease of deployment, particularly in data-scarce environments.

However, heuristic controllers are inherently static and may struggle to adapt to non-stationary climatic conditions. Recent comparative studies emphasize that threshold-based rules can perform suboptimally when rainfall variability or delayed soil responses are significant, motivating the exploration of adaptive control strategies (Jones et al., 2022). As a result, rule-based control is increasingly regarded as a baseline rather than a long-term solution for climate-resilient irrigation.

2.3. Reinforcement learning for irrigation and water management

Reinforcement learning (RL) provides a flexible framework for sequential decision making in uncertain and dynamic environments (Sutton and Barto, 2018). In the context of water management, RL has been applied to reservoir operation, canal regulation, and irrigation scheduling, with several studies reporting improved performance compared to static policies (Yang et al., 2021; Giuliani et al., 2021).

Despite these advances, purely data-driven RL approaches face notable challenges in environmental applications. They often require extensive training data, may violate physical constraints, and can produce policies that are difficult to interpret or trust. Recent reviews stress that the lack of physical consistency and robustness remains a major barrier to the operational adoption of RL in environmental systems (Rolnick et al., 2022; Reichstein et al., 2019).

Within the environmental modelling community, adaptive control and policy search methods have long been investigated as means to support decision-making under deep uncertainty and competing objectives. In particular, policy search approaches have been shown to provide a principled framework for exploring trade-offs between robustness, efficiency, and risk in water management systems (Giuliani et al., 2016). Similarly, many-objective evolutionary algorithms, such as the Borg framework, have been widely adopted to identify diverse and non-dominated control strategies across complex environmental objectives (Hadka and Reed, 2013).

In this context, reinforcement learning can be viewed not as a replacement for existing environmental decision-support methodologies, but as a complementary adaptive control paradigm that shares conceptual foundations with policy search and many-objective optimisation, while offering enhanced capacity to exploit temporal structure and delayed system responses.

2.4. Physics-informed and hybrid learning approaches

To address these limitations, physics-informed machine learning has emerged as a promising paradigm that embeds mechanistic knowledge into data-driven models. Comprehensive surveys highlight that constraining learning with physical structure improves generalization, stability, and interpretability, particularly in data-limited regimes (Willard et al., 2022; Karniadakis et al., 2021).

Neural Ordinary Differential Equations (Neural ODEs) provide a natural framework for hybrid modelling by enabling continuous-time representations in which neural networks parameterize unknown or residual dynamics (Rackauckas et al., 2020). In environmental and Earth system sciences, Neural ODEs and related universal differential equation frameworks have been increasingly used to augment physical models rather than replace them, preserving interpretability while improving fidelity (Rackauckas et al., 2021; Beucler et al., 2021).

2.5. Hybrid modelling for control under uncertainty

While physics-informed learning has been extensively studied for system identification and forecasting, its integration with reinforcement learning for control remains comparatively underexplored in environmental modelling. Recent work in safe and model-based RL emphasizes the importance of incorporating physical structure to improve robustness and avoid unsafe or unrealistic control policies (Berkenkamp et al., 2017; Perkins et al., 2023).

Few studies have conducted systematic comparisons between rule-based control, reinforcement learning, and hybrid neuro-physical control within a unified environmental modelling framework. In particular, the potential of learning residual dynamics via Neural ODEs to enhance irrigation control robustness under climatic variability has received limited attention. This gap motivates the present work, which investigates control strategies of increasing modelling complexity within a physics-based irrigation environment.

3. Problem Statement

Irrigation management is formulated as a finite-horizon sequential decision-making problem for a dynamic environmental system subject to climatic uncertainty. Over a growing season, irrigation actions must be selected to regulate soil–water dynamics so as to balance crop water stress and water use efficiency while respecting physical and operational constraints.

3.1. System dynamics

We consider a single agricultural plot evolving over a season of length T, discretized into daily decision steps $t = 0, \dots, T-1$. Let

$$\mathbf{x}_t \in \mathbb{R}^n \tag{1}$$

denote the latent physical state of the system at time t, comprising soil moisture and related hydrological variables. The system is influenced by irrigation inputs and exogenous climatic forcing.

The evolution of the physical state is governed by a process-based soil–water balance model,

$$\mathbf{x}_{t+1} = f_{\text{phys}}(\mathbf{x}_t, u_t, d_t; \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t, \tag{2}$$

where u_t is the applied irrigation amount, d_t represents exogenous climatic drivers (e.g., rainfall and evapotranspiration), $\boldsymbol{\theta}$ denotes physical parameters, and $\boldsymbol{\varepsilon}_t$ captures modelling errors and unresolved processes. Such formulations are standard in irrigation and land-surface modelling (Raes et al., 2009; Fatichi et al., 2016).

3.2. Decision process formulation

At each decision step, the controller selects an action

$$u_t \in \mathcal{U}, \tag{3}$$

where $\mathcal{U}$ is a bounded set of feasible irrigation inputs reflecting operational constraints (e.g., maximum daily application).

The controller does not necessarily observe the full physical state $\mathbf{x}_t$. Instead, it receives an observation vector

$$\mathbf{s}_t = h(\mathbf{x}_t, d_t), \tag{4}$$

where $h(\cdot)$ denotes an observation mapping that may include partial state information, climatic variables, and normalized features. This formulation allows both fully observed and partially observed settings while remaining compatible with practical implementations.

The resulting control problem can be cast as a Markov Decision Process (MDP),

$$\mathcal{M} = \langle \mathcal{S}, \mathcal{U}, p, r, \gamma \rangle, \tag{5}$$

where $\mathcal{S}$ is the observation space, $p(\mathbf{s}_{t+1} \mid \mathbf{s}_t, u_t)$ is the transition kernel induced by Eq. (2), r is the reward function, and $\gamma \in (0, 1]$ is a discount factor (Sutton and Barto, 2018).

3.3. Control objective

The irrigation objective is to limit crop water stress while minimizing irrigation water use over the season. This trade-off is expressed through a stage-wise reward function

$$r_t = r(\mathbf{s}_t, u_t), \tag{6}$$

which penalizes both stress-related deviations from desirable soil moisture conditions and excessive irrigation.

A generic formulation is

$$r_t = -\Big(\alpha\, \mathcal{S}(\mathbf{x}_t) + \beta\, \mathcal{C}(u_t)\Big), \tag{7}$$

where $\mathcal{S}(\mathbf{x}_t)$ is a scalar indicator of crop water stress, $\mathcal{C}(u_t)$ denotes the irrigation cost, and $\alpha, \beta > 0$ are weighting coefficients. Additional penalty terms may be introduced to enforce soft constraints, without loss of generality.

The performance of a control policy π is evaluated through the expected discounted return

$$J(\pi) = \mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \gamma^t r_t \right]. \tag{8}$$

where $\gamma \in (0, 1]$ is a discount factor, and the expectation is taken over stochastic climatic disturbances and system noise. This formulation aligns with standard Markov Decision Process (MDP) representations used in reinforcement learning (Sutton and Barto, 2018).

3.4. Climatic uncertainty

Climatic inputs, particularly rainfall, are treated as stochastic disturbances,

$$d_t \sim \mathcal{P}_d, \tag{9}$$

where $\mathcal{P}_d$ denotes an unknown and potentially non-stationary distribution. This formulation explicitly accounts for hydro-climatic variability and supports robustness analysis under changing climatic conditions (Seneviratne et al., 2021).

3.5. Learning-based and hybrid control formulations

Within this general framework, different control strategies can be defined according to the structure of the policy π and the assumed system dynamics.

Rule-based controllers specify π using fixed thresholds derived from expert knowledge. Learning-based controllers parameterize π and optimize it through interaction with the simulated environment, as in reinforcement learning.

To improve robustness and physical consistency, the process-based dynamics in Eq. (2) can be augmented by a learned residual component,

$$\mathbf{x}_{t+1} = f_{\text{phys}}(\mathbf{x}_t, u_t, d_t) + f_{\text{res}}(\mathbf{x}_t, u_t, d_t), \tag{10}$$

where $f_{\text{res}}(\cdot)$ captures unmodelled or poorly represented processes. This residual may be parameterized using a continuous-time formulation, such as a Neural Ordinary Differential Equation, allowing the hybrid model to preserve the interpretability of the physical core while improving adaptability (Rackauckas et al., 2020; Willard et al., 2022).

This progressive formulation provides a unified theoretical basis for comparing rule-based control, reinforcement learning, and hybrid neuro-physical reinforcement learning within a consistent environmental modelling framework.

4. Physics-based irrigation environment

This section describes the physics-based irrigation environment used to simulate soil–water dynamics and to evaluate the different control strategies. The environment provides a mechanistic yet computationally tractable representation of irrigation processes, enabling controlled experimentation under variable climatic conditions while maintaining physical interpretability.

4.1. Environment overview

The irrigation environment represents a single agricultural plot over a finite growing season of length T. Time is discretized into daily steps, consistent with common irrigation scheduling practices. At each step, the environment updates the soil–water state in response to irrigation inputs and exogenous climatic forcing, and returns observations and rewards to the controller.

The environment is designed to serve two complementary purposes: (i) to encode prior physical knowledge of soil–water processes, and (ii) to provide a stable and reproducible interface for learning-based control algorithms.

4.2. Soil–water balance model

The core of the environment is a process-based soil–water balance model governing the evolution of root-zone soil moisture. Let w_t denote the effective soil water content at time t. Its dynamics are described by

$$w_{t+1} = w_t + I_t + R_t - ET_t - D_t, \tag{11}$$

where I_t is the irrigation input, R_t is rainfall, ET_t is actual evapotranspiration, and D_t represents drainage or percolation losses. Each term is constrained to ensure physical plausibility (e.g., non-negativity and capacity limits).

Evapotranspiration and drainage terms depend on soil moisture and climatic conditions, introducing nonlinearities and delayed effects that are characteristic of real irrigation systems. While simplified, this formulation captures the dominant processes influencing short-term irrigation decisions and is consistent with commonly used conceptual models (Raes et al., 2009; Fatichi et al., 2016).

4.3. Physical constraints and saturation effects

Soil moisture is bounded by physical limits corresponding to wilting point and field capacity. These bounds are enforced within the environment:

$$w_{\min} \leq w_t \leq w_{\max}. \tag{12}$$

When soil moisture exceeds field capacity, excess water contributes to drainage losses. Conversely, when soil moisture approaches the lower bound, evapotranspiration is reduced, leading to increased crop water stress.

Such saturation and threshold effects introduce nonlinear responses that challenge static control rules and motivate adaptive control strategies.

4.4. Climatic forcing and stochasticity

Climatic variables, in particular rainfall, are treated as exogenous inputs to the environment. Rainfall may be specified deterministically for scenario analysis or sampled from a stochastic process to emulate hydro-climatic variability:

$$R_t \sim \mathcal{P}_R. \tag{13}$$

This design allows the same environment to be used for both controlled experiments and robustness evaluations under uncertain climatic conditions, which is essential for assessing climate-resilient irrigation strategies.

4.5. Observation generation

At each time step, the environment provides the controller with an observation vector derived from the underlying physical state and climatic inputs:

$$\mathbf{s}_t = h(w_t, R_t, ET_t, t), \tag{14}$$

where $h(\cdot)$ denotes an observation mapping that may include normalized soil moisture indicators, recent climatic variables, and temporal information. This abstraction supports partial observability and reflects realistic sensing conditions.

4.6. Environment interface for control

The environment follows a standard step-based interaction pattern commonly used in control and reinforcement learning frameworks. Given an irrigation action I_t, the environment:

1. updates the soil–water state using Eq. (11),

2. computes the resulting reward based on stress and water use,
3. generates the next observation $\mathbf{s}_{t+1}$,
4. advances the simulation time.

Episodes terminate after T steps, corresponding to the end of the growing season. This episodic structure enables consistent evaluation of seasonal irrigation performance.

4.7. Role of the environment in hybrid modelling

The physics-based environment serves as the reference model for all control scenarios considered in this study. In learning-based settings, it acts as the simulator with which the controller interacts. In hybrid formulations, its dynamics may be augmented by learned residual components, while the physical core remains intact.

By separating physical modelling from control logic, the environment provides a transparent and extensible foundation for comparing rule-based, reinforcement learning, and hybrid neuro-physical irrigation strategies within a unified framework.

5. Control scenarios

This section describes the three irrigation control scenarios investigated in this study. All scenarios interact with the same physics-based irrigation environment described in Section 4, ensuring that observed differences in performance arise solely from the control strategy and not from changes in the underlying system dynamics. The scenarios are designed to represent increasing levels of modelling and control sophistication, enabling a structured comparison between heuristic, learning-based, and hybrid neuro-physical approaches.

5.1. Scenario 1: Physics-based model with rule-based control

The first scenario relies on a rule-based controller operating on top of the physics-based irrigation environment. The control policy is defined by a set of fixed decision rules derived from expert knowledge of soil–water processes. At each time step, the irrigation input is determined as a function of the current observation vector,

$$u_t = \pi_{\text{rule}}(\mathbf{s}_t), \tag{15}$$

where π_{rule} encodes threshold-based logic, such as triggering irrigation when soil moisture falls below a prescribed level.

Rule-based control serves as a baseline reflecting common irrigation practices. Its main advantages are simplicity, interpretability, and negligible computational cost. However, because decision thresholds are fixed a priori, the controller does not adapt to changing climatic conditions or evolving system dynamics. This scenario provides a reference against which the benefits of learning-based approaches can be assessed.

5.2. Scenario 2: Physics-based model with reinforcement learning control

In the second scenario, irrigation decisions are determined by a learning-based controller trained using reinforcement learning, while the environment dynamics remain purely physics-based. The control policy is parameterized by a stochastic function approximator,

$$u_t \sim \pi_\theta(\cdot \mid \mathbf{s}_t), \tag{16}$$

where θ denotes the policy parameters.

Policy optimization is performed through repeated interaction with the environment, with the objective of maximizing the expected cumulative reward defined in Section 3. A policy-gradient method is employed to update θ, enabling the controller to adapt its decisions to delayed effects, stochastic rainfall, and nonlinear soil–water responses.

Compared to rule-based control, reinforcement learning allows the controller to exploit temporal patterns and trade-offs that are difficult to encode explicitly. However, because the learning process relies on the accuracy of the underlying physical model, modelling errors and unrepresented dynamics may limit robustness, particularly under conditions not encountered during training.

5.3. Scenario 3: Physics-based model augmented with NeuroODE and reinforcement learning

The third scenario extends the learning-based control formulation by augmenting the physics-based environment with a learned residual dynamics component. In this hybrid neuro-physical setting, the soil–water dynamics are expressed as

$$\mathbf{x}_{t+1} = f_{\mathrm{phys}}(\mathbf{x}_t, u_t, d_t) + f_{\mathrm{res}}(\mathbf{x}_t, u_t, d_t), \tag{17}$$

where f_{phys} denotes the process-based model and f_{res} captures residual dynamics that are not explicitly represented in the physical formulation.

The residual component is parameterized using a Neural Ordinary Differential Equation, allowing it to model continuous-time corrections to the physical dynamics while preserving the interpretability of the physical core. The reinforcement learning controller interacts with this hybrid environment and is trained using the same objective as in Scenario 2.

This scenario is designed to assess whether incorporating learned residual dynamics improves control robustness and stability under climatic variability. By embedding learning within the system dynamics rather than solely within the control policy, the hybrid approach aims to reduce model mismatch effects while maintaining physical consistency.

5.4. Comparative perspective

Across the three scenarios, the action space, reward structure, and climatic forcing remain identical. The scenarios differ only in the degree to which learning is introduced into the control loop and the system dynamics. This progressive design enables a systematic evaluation of:

- the performance gap between heuristic and adaptive control,
- the benefits and limitations of reinforcement learning when coupled with a fixed physical model,
- the added value of hybrid neuro-physical modelling for irrigation control under uncertainty.

Together, these scenarios provide a coherent framework for analysing how increasing levels of learning and model integration affect irrigation performance, robustness, and interpretability.

6. Experimental design

This section describes the experimental design used to evaluate the three control scenarios. To ensure reproducibility and consistency across scenarios, the environment physics, weather forcing, training budget, and (when applicable) rule-based controller parameters are centralized in a configuration module. This configuration-driven design separates modelling assumptions from algorithmic components and supports controlled sensitivity analyses.

6.1. Configuration-driven experimental setup

The experimental setup is defined by four parameter groups:

- **Environment parameters** controlling the episode horizon and operational constraints: season length T (in days), maximum admissible daily irrigation $u_{\max}$, and a random seed for reproducibility.
- **Soil parameters** controlling storage capacity, soil tension thresholds, drainage, and irrigation efficiency.
- **Weather parameters** defining the reference evapotranspiration (ET0) seasonal pattern and the stochastic rainfall generator.
- **Training parameters** specifying the training budget for reinforcement learning (total number of environment steps).

All learning-based controllers are trained and evaluated under the same physical parameterization and forcing protocol unless explicitly stated otherwise.

6.2. Soil parameterization

The soil configuration specifies (i) the effective root-zone depth Z_r, (ii) volumetric water content limits, and (iii) a soil water tension representation used to model crop stress and evapotranspiration reduction. The main parameters include:

- **Root zone and water contents:** Z_r, θ_s (saturation), θ_{fc} (field capacity), θ_{wp} (wilting point);
- **Soil water tension thresholds (cbar):** ψ_{sat}, ψ_{fc}, ψ_{wp};
- **Drainage and irrigation efficiency:** drainage coefficient k_d, irrigation efficiency η_I;
- **Evapotranspiration stress threshold:** ψ_{ET}^{crit} controlling stress-induced ET reduction.

These parameters define the admissible soil moisture range and the nonlinear response of drainage and evapotranspiration to soil water status, which are central to irrigation decision making.

6.3. Weather forcing configuration

Weather forcing is parameterized through (i) a seasonal reference evapotranspiration (ET0) process and (ii) a stochastic rainfall generator. ET0 is defined through a baseline level and seasonal modulation with additive noise:

$$ET0_t = \mathrm{ET0}_{base} + \mathrm{ET0}_{amp}\, g(t) + \xi_t, \tag{18}$$

where $g(t)$ is a periodic seasonal signal and ξ_t is a zero-mean noise term governed by an amplitude parameter.

Rainfall occurrence is modeled through season-dependent probabilities and bounded rainfall amounts:

- **Occurrence probabilities:** $p_{\text{rain,early}}$, $p_{\text{rain,mid}}$, $p_{\text{rain,late}}$;
- **Amount bounds:** $R_t \in [R_{\min}, R_{\max}]$ with parameters `rain_min` and `rain_max`.

This forcing design enables robustness assessment under hydro-climatic variability while preserving experimental control.

6.4. Scenario 1: Rule-based controller configurations

For Scenario 1, rule-based irrigation policies are parameterized to enable controlled comparison and reproducible baselines. Three rule families are considered:

- **Single-threshold rule** ("seuil unique"): an irrigation dose is applied when soil water tension exceeds a threshold, with optional reduction based on forecasted rainfall. Parameters include the tension threshold, irrigation dose, rainfall threshold, and a reduction factor.
- **Comfort-band rule** ("bande de confort"): irrigation is triggered to maintain soil tension within a lower/upper band, with parameters $(\psi_{low}, \psi_{high})$ and an irrigation dose.
- **Proportional rule**: irrigation is proportional to the deviation from a target soil tension ψ_{target} with proportional gain k_I.

These parameterized baselines support fair comparison with learning-based controllers while preserving interpretability.

6.5. Scenario 2: PPO training configuration (Physics + PPO)

In Scenario 2, the irrigation policy is learned using reinforcement learning while interacting with the physics-based environment (Section 4). Training is performed with Proximal Policy Optimization (PPO), using a stochastic policy $\pi_\theta(u_t \mid \mathbf{s}_t)$ and a value function $V_\psi(\mathbf{s}_t)$ optimized from simulated rollouts.

Training budget and reproducibility. The training budget is defined by the total number of environment interaction steps (`total_timesteps`). Randomness due to climatic forcing and learning initialization is controlled through an explicit seed, ensuring reproducibility across runs.

Rollout collection and advantage estimation. Training data are collected in batches of trajectories generated by the current policy. Advantage estimates are computed using temporal-difference residuals and (when enabled) Generalized Advantage Estimation (GAE), characterized by a discount factor γ and a trace-decay parameter λ:

$$\hat{A}_t = \sum_{l=0}^{\infty} (\gamma\lambda)^l \delta_{t+l}, \qquad \delta_t = r_t + \gamma V_\psi(\mathbf{s}_{t+1}) - V_\psi(\mathbf{s}_t). \tag{19}$$

PPO objective and stabilization mechanisms.. Policy updates maximize a clipped surrogate objective to prevent destructive updates:

$$\mathcal{L}^{\mathrm{PPO}}(\theta) = \mathbb{E}_t \left[\min \left(r_t(\theta)\, \hat{A}_t,\ \mathrm{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\, \hat{A}_t \right) \right], \tag{20}$$

with likelihood ratio $r_t(\theta) = \pi_\theta(u_t \mid \mathbf{s}_t)/\pi_{\theta_{\mathrm{old}}}(u_t \mid \mathbf{s}_t)$. The full training loss typically includes (i) a value-function regression term and (ii) an entropy regularization term that encourages exploration. Additional stabilization mechanisms may include gradient clipping, reward scaling, and observation normalization, all implemented consistently within the training utility.

Evaluation protocol. To ensure fair comparison with Scenario 1, the learned policy is evaluated on fixed seasonal episodes under identical physics and weather settings, using deterministic action selection where relevant (e.g., mean action for continuous policies). Performance metrics are computed from seasonal trajectories (water use, stress indicators, and robustness statistics), as described in Section 6.

6.5.1. Scenario 3: Neural ODE residual model (architecture and inputs/outputs)

To improve reproducibility of Scenario 3, we specify the architecture of the residual dynamics component used in the Neural ODE augmentation. The residual function is parameterised by a multilayer perceptron (MLP), denoted f_θ, which predicts a correction term to the nominal physics-driven update. In our implementation, the MLP takes a four-dimensional input vector,

$$\mathbf{x}_t = \left[\psi_t,\ I_t,\ R_t,\ ET0_t\right]^\top, \tag{21}$$

where ψ_t is the soil water tension (cbar), I_t is the applied irrigation (mm), R_t is rainfall (mm), and $ET0_t$ is reference evapotranspiration (mm/day). The network outputs a scalar residual correction $\Delta\psi_t$:

$$\Delta\psi_t = f_\theta(\mathbf{x}_t). \tag{22}$$

The MLP has two hidden layers of 64 units each with tanh activations:

$$\mathbf{h}_1 = \tanh(\mathbf{W}_1\mathbf{x}_t + \mathbf{b}_1), \tag{23}$$

$$\mathbf{h}_2 = \tanh(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2), \tag{24}$$

$$\Delta\psi_t = \mathbf{W}_3\mathbf{h}_2 + b_3, \tag{25}$$

where $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3$ and $\mathbf{b}_1, \mathbf{b}_2, b_3$ denote the trainable weights and biases. This residual correction is integrated within the hybrid update used by the Neural ODE formulation, enabling the model to compensate for systematic discrepancies between simplified soil–water physics and observed trajectories under stochastic forcing and control actions.

6.6. Training and evaluation protocol

Learning-based controllers are trained using a fixed training budget (total environment interaction steps). Training and evaluation are separated: policies are learned under the configured stochastic forcing, then evaluated under predefined forcing scenarios (deterministic and/or stochastic) using identical physical parameters.

Experiments are repeated across independent random seeds affecting stochastic forcing and learning initialization. Reported results are aggregated across runs, and variability indicators are provided where relevant.

6.7. Sensitivity ranges and reproducibility

To facilitate sensitivity analysis and interactive exploration, the configuration specifies explicit ranges (min, max, step) for key environment, soil, weather, and rule parameters. These ranges define admissible perturbations used in experiments and user interfaces, and provide a transparent envelope for robustness analysis.

7. Results and discussion

This section presents and discusses the results obtained for the three irrigation control scenarios described in Section 5: (i) rule-based control, (ii) reinforcement learning using a physics-based environment, and (iii) hybrid reinforcement learning with Neural ODE–augmented dynamics. All simulations are conducted over a full growing season under identical physical configurations and stochastic climatic forcing.

The experimental design and evaluation protocol follow established best practices in Environmental Modelling & Software for transparent, reproducible, and interpretable model assessment. In particular, model performance is characterised using complementary indicators that capture both system-level behaviour and decision-relevant outcomes, in line with EMS recommendations for the evaluation of environmental models (Bennett et al., 2013). The use of fixed physical configurations, controlled stochastic forcing, and consistent training and evaluation settings across scenarios ensures that observed performance differences can be attributed to the control strategy rather than experimental artefacts.

7.1. Scenario 1: Rule-based control — conservative stability

Figure 1 illustrates the seasonal dynamics obtained with rule-based irrigation strategies.

The results show that rule-based control maintains soil water tension predominantly within or close to the agronomically optimal range. This conservative behaviour results in a high proportion of days spent in the comfort zone, reflecting the explicit enforcement of predefined thresholds or bands. However, irrigation actions are triggered reactively and often abruptly, leading to frequent water applications.

Soil water storage remains close to field capacity for most of the season, but this stability is achieved at the expense of substantial drainage losses, particularly when irrigation coincides with rainfall. These results indicate

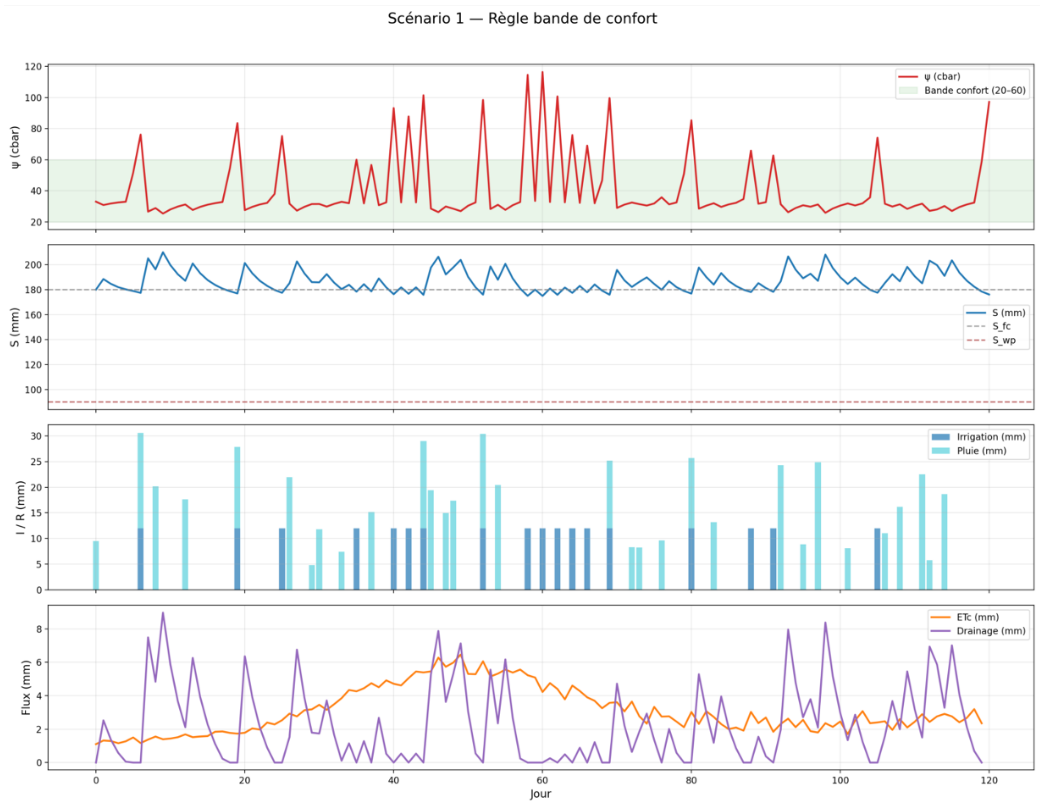

Figure 1: Seasonal dynamics of rule-based irrigation control (Scenario 1): soil water tension, soil water storage, irrigation and rainfall inputs, and hydrological fluxes.

that while rule-based control is robust in terms of stress avoidance and interpretability, it suffers from inefficient water use under stochastic climatic conditions.

7.2. Scenario 2: Physics-based reinforcement learning — efficiency with risk

Figure 2 presents the seasonal trajectories obtained with the PPO controller interacting with the physics-based environment.

Compared to Scenario 1, irrigation actions are smoother and significantly reduced in magnitude, leading to the lowest cumulative irrigation volume among the three scenarios. This results in the highest water-use efficiency. However, the soil water tension time series exhibits pronounced and persistent peaks during dry periods, indicating severe stress episodes.

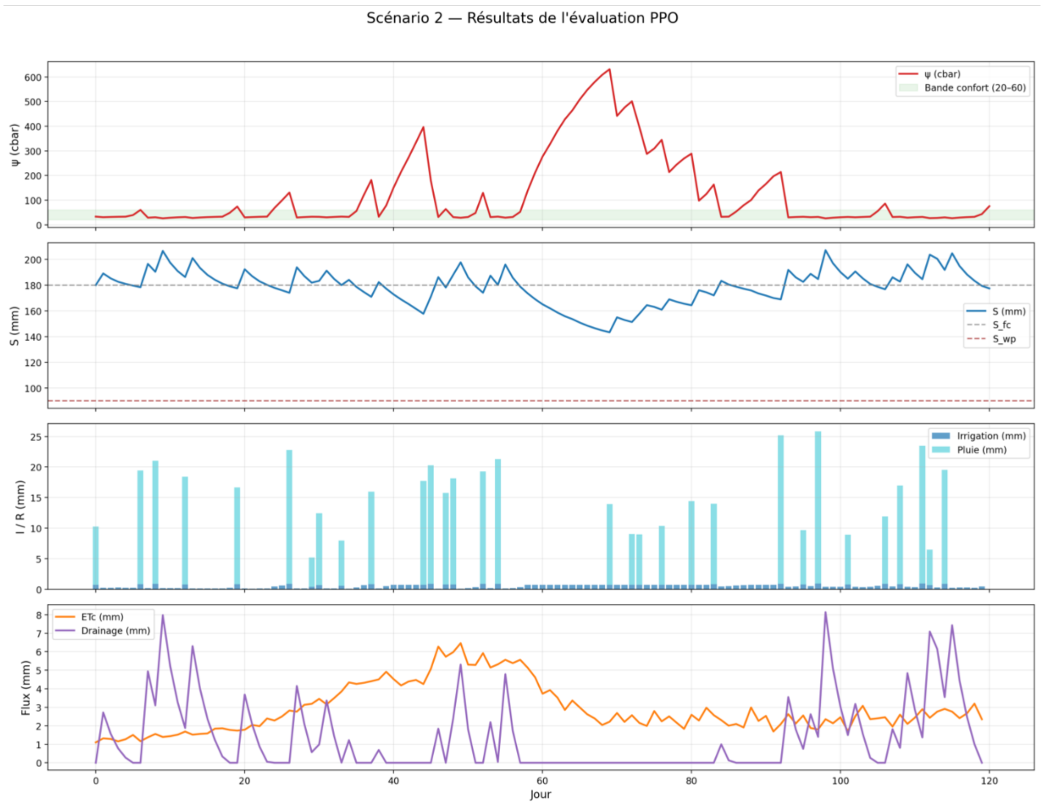

Figure 2: Seasonal dynamics of PPO-based irrigation control using the physics-based environment (Scenario 2).

These extreme tension values reveal limitations in the controller's anticipation of cumulative water deficits when relying solely on a simplified physical model. While drainage losses are reduced relative to the rule-based baseline, the occurrence of extreme stress raises concerns regarding agronomic reliability.

7.3. Scenario 3: Hybrid reinforcement learning with Neural ODE — moderated trade-offs

Figure 3 reports the results obtained with the hybrid neuro-physical control strategy.

The hybrid approach significantly reduces the severity and duration of extreme soil water tension peaks observed in Scenario 2. Soil water storage trajectories are smoother and avoid deep depletion during prolonged dry

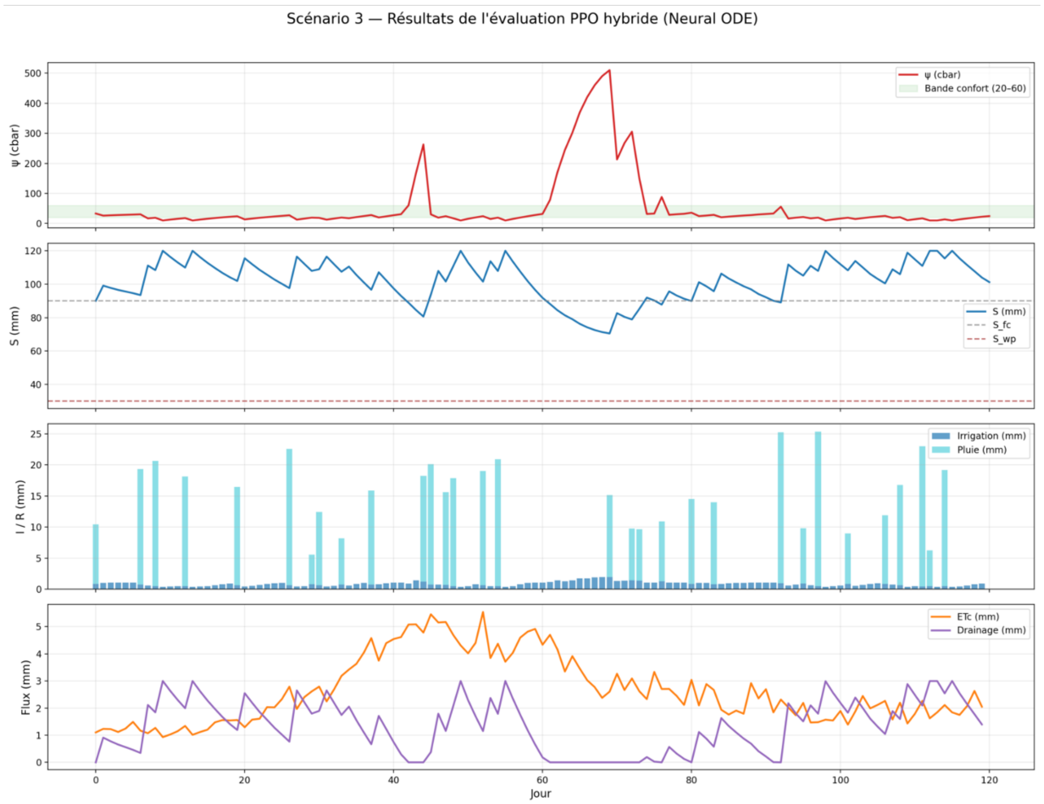

Figure 3: Seasonal dynamics of hybrid PPO control with Neural ODE–augmented dynamics (Scenario 3).

periods. Irrigation actions remain moderate and better coordinated with rainfall events, resulting in reduced drainage losses.

Nevertheless, Scenario 3 does not maximize the time spent in the optimal tension range, which remains lower than that achieved by the rule-based strategy. Instead, the hybrid controller achieves a compromise between stress mitigation and water-use efficiency.

7.4. Comparative analysis across scenarios

To consolidate the scenario-wise analysis, we now examine comparative indicators derived from the seasonal simulations.

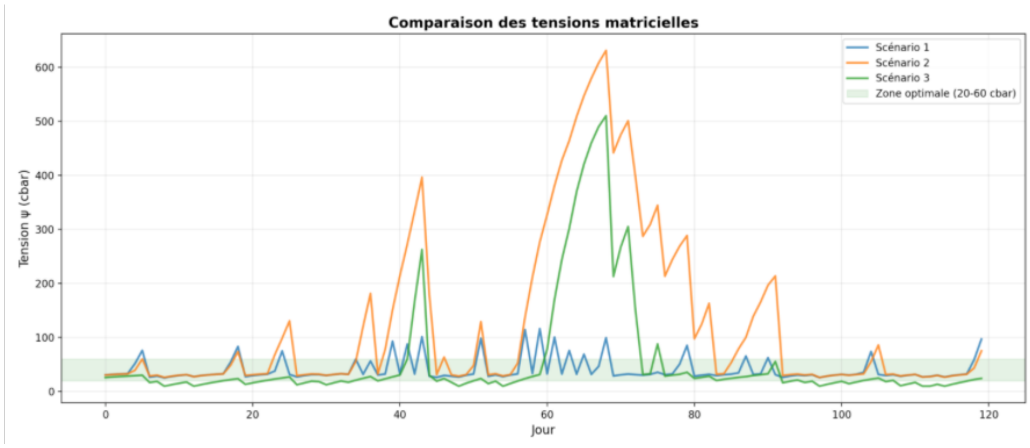

Figure 4: Comparison of soil water tension dynamics across the three scenarios. The shaded area indicates the agronomically optimal tension range.

7.4.1. Comparison of soil water tension dynamics

Figure 4 compares soil water tension trajectories across the three scenarios.

Scenario 1 maintains tension values predominantly within the optimal range. Scenario 2 exhibits extreme stress peaks, confirming the potentially risk-prone nature of efficiency-driven control under simplified physical modelling assumptions. Scenario 3 attenuates these extremes relative to Scenario 2 but does not fully recover the conservative behaviour of Scenario 1.

Comparison of soil water storage. Figure 5 compares soil water reserves over the growing season.

Rule-based control maintains storage close to field capacity through frequent irrigation. Physics-based reinforcement learning allows deeper depletion during dry periods, explaining the observed stress peaks. The hybrid approach moderates these dynamics, avoiding both excessive depletion and excessive irrigation.

7.4.2. Comparison of cumulative water volumes

Figure 6 reports cumulative irrigation and rainfall volumes.

Rainfall contributions are identical across scenarios, while irrigation volumes differ substantially. Scenario 1 applies the highest irrigation depth, Scenario 2 the lowest, and Scenario 3 an intermediate amount, confirming the efficiency–robustness trade-off.

Figure 5: Comparison of soil water storage trajectories across the three scenarios, relative to field capacity and wilting point.

7.4.3. Comparison of aggregated performance indicators

Figure 7 summarizes drainage losses, time spent in the optimal tension range, mean soil water tension, and water-use efficiency.

Rule-based control maximizes time spent in the optimal zone but suffers from high drainage losses. Physics-based reinforcement learning achieves the highest water-use efficiency but exhibits the worst stress indicators. The hybrid approach reduces drainage and extreme stress while maintaining acceptable efficiency, illustrating its role as a balanced compromise.

7.4.4. Comparative synthesis

Across all figures, a consistent pattern emerges. Rule-based control prioritizes stress avoidance at the expense of water efficiency. Physics-based reinforcement learning prioritizes efficiency at the expense of robustness. Hybrid neuro-physical reinforcement learning reshapes this compromise, mitigating the most severe stress events while preserving substantial water savings relative to heuristic control.

Learning-based controllers outperform rule-based strategies when irrigation objectives extend beyond strict stress avoidance and require anticipation of delayed hydrological responses under stochastic climatic forcing. In particular, when rainfall variability, cumulative deficits, and water efficiency constraints dominate system behaviour, fixed thresholds become insufficient, whereas learning-based policies can exploit temporal structure to improve

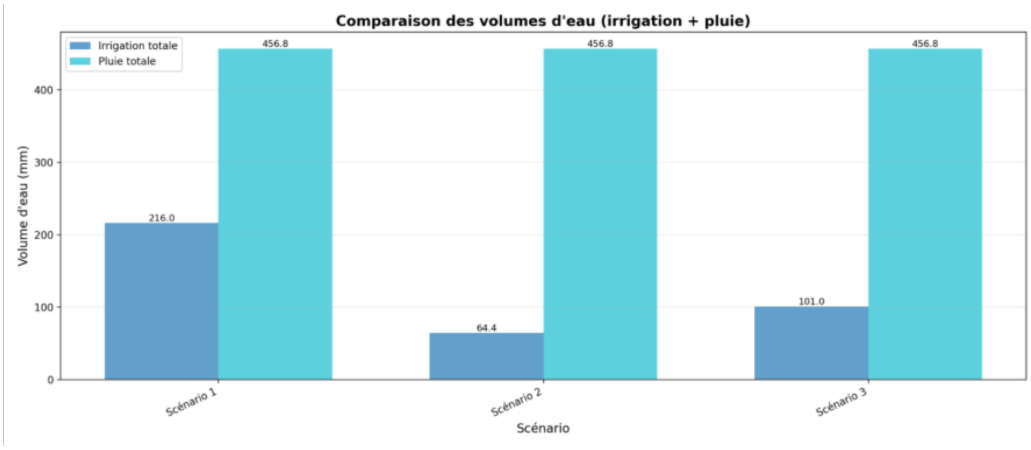

Figure 6: Comparison of cumulative irrigation and rainfall volumes across scenarios.

performance.

The NeuroODE does not aim to maximize time in agronomic comfort zones but to correct structural inaccuracies in the physical model that lead to delayed or extreme stress responses, thereby reshaping—rather than eliminating—the trade-off between efficiency and robustness.

These results confirm that physics-informed learning is not a replacement for physical modelling or agronomic expertise, but a powerful mechanism for navigating trade-offs in climate-uncertain irrigation systems.

From a deployment perspective, the results suggest that hybrid neuro-physical controllers are best suited for operational contexts where basic physical knowledge is available but system behaviour remains partially uncertain. Rule-based strategies remain attractive for low-risk, low-complexity settings, whereas learning-based controllers become advantageous when irrigation must adapt to evolving climatic patterns, soil heterogeneity, or policy-driven water constraints. Training costs and data availability remain limiting factors, motivating incremental and transfer-based learning approaches.

7.4.5. Limitations

Several limitations remain and define clear directions for future work. First, the current controllers rely on Markovian state representations that incompletely capture long-term dependencies and irregular temporal sampling, motivating the integration of Neural Controlled Differential Equations to better model continuous-time dynamics driven by sparse observations. Second,

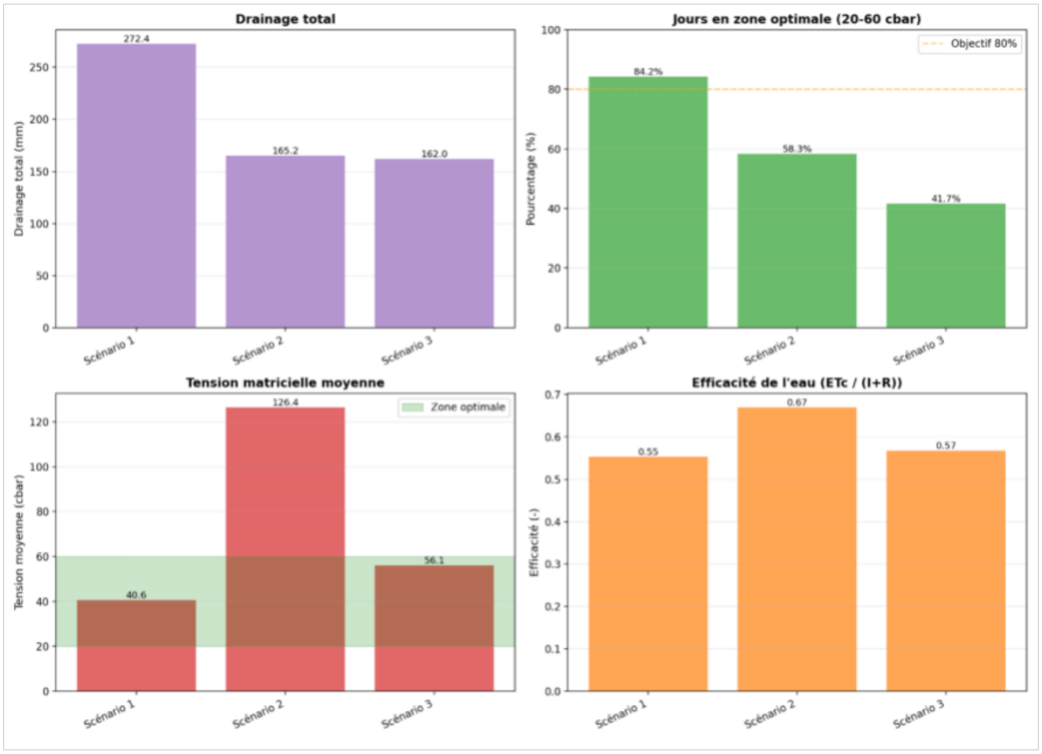

Figure 7: Comparison of aggregated performance indicators: total drainage, percentage of days in the optimal tension range, mean soil water tension, and water-use efficiency.

the use of simplified stochastic weather generators limits anticipatory capabilities; integrating time-series foundation models such as PatchTST would enable more realistic climate-aware decision-making. Finally, the absence of an explicit world model constrains planning and counterfactual analysis, suggesting future extensions toward latent world-model architectures that jointly learn dynamics, uncertainty, and control.

Effect of increased physical complexity. The soil–water balance model used in this study is intentionally simplified to enable a controlled, reproducible comparison of control paradigms under identical forcing. Incorporating additional processes—such as soil heterogeneity, layered profiles, and dynamic crop growth influencing root water uptake and evapotranspiration—would increase state uncertainty, introduce longer delays, and amplify nonlinear interactions. In such settings, rule-based strategies (Scenario 1) would likely require repeated re-tuning across soils and seasons and may exhibit larger oscillations and drainage losses due to spatial variability and deep percolation pathways. Physics-based reinforcement learning (Scenario 2) may become more sensitive to structural model mismatch when key nonlinearities are omitted (e.g., layer-wise redistribution and growth-driven ET changes), potentially increasing stress episodes unless observations are enriched and training explicitly accounts for parameter variability (e.g., via domain randomization). The hybrid approach (Scenario 3) is expected to remain advantageous primarily by correcting systematic residual errors arising from simplified physics, but its benefit would likely manifest as improved robustness and safer behaviour under mismatch rather than universal dominance across all metrics. These considerations motivate a progressive validation protocol in future work, where process fidelity is increased stepwise (layered soil, heterogeneous fields, crop modules) and policies are evaluated for transfer performance and robustness to physically meaningful perturbations.

Climatic forcing and non-stationarity.. The stochastic weather generator employed in this study introduces variability through seasonal patterns and noise in rainfall occurrence and reference evapotranspiration, but it does not explicitly represent regime shifts or long-term non-stationarity. This design choice was made to support a controlled comparison of control strategies under identical forcing, ensuring that observed differences are attributable to decision-making logic rather than to climatic confounders.

Nevertheless, a more stringent robustness assessment would involve evaluating controllers under distinct climatic regimes (e.g., arid, temperate, tropical) using historical weather records or synthetic trajectories derived from climate model outputs. Under such conditions, fixed rule-based strategies would likely require repeated re-calibration, while learning-based controllers would face distribution shift relative to their training data. The hybrid neuro-physical approach is expected to be particularly relevant in these settings, as residual learning can correct persistent biases induced by unmodelled climatic effects, provided that training or adaptation mechanisms explicitly account for non-stationarity. Future work will therefore focus on cross-climate evaluation and transfer, aligning with EMS best practices for robustness analysis under climatic uncertainty.

Neural ODE residual dynamics. The Neural ODE component is used to model residual dynamics that are not captured by the simplified soil–water balance equations. Importantly, the residual model is not pre-trained on data from a higher-fidelity simulator. Instead, it is trained concurrently with the reinforcement learning agent using trajectories generated online during interaction with the environment. At each time step, the physical model provides a nominal state derivative, while the Neural ODE learns an additive correction term that accounts for systematic model mismatch induced by stochastic climate forcing and control actions.

The residual dynamics function f_{res} is parameterized as a lightweight multilayer perceptron embedded within a continuous-time ODE solver. Its inputs consist of the current state variables (e.g., soil water storage or tension), control actions, and relevant exogenous drivers (e.g., reference evapotranspiration), while its outputs correspond to residual corrections to the state derivatives. This formulation allows the hybrid model to preserve physical interpretability while flexibly compensating for unmodelled nonlinearities during learning.

Transferability to other environmental systems. The proposed framework is transferable in the sense of its modelling and control structure, rather than through direct reuse of a trained policy. Applying it to another environmental system requires identifying four analogous components. First, a process-based model f_{phys} describing the dominant system dynamics (e.g., mass balance equations for reservoir storage, or atmospheric transport models for air quality). Second, control actions u_t corresponding to management decisions (e.g., release policies in reservoir operation or emission control measures in air quality management). Third, stochastic drivers d_t representing exogenous forcing, such as inflow uncertainty, meteorological variability, or emission fluctuations. Finally, an objective function encoding system-specific trade-offs, such as reliability versus flood risk in reservoir management, or exposure reduction versus economic cost in air quality control.

Within this structure, learning-based or hybrid controllers can be used to reshape trade-offs under uncertainty by complementing simplified physics with data-driven adaptation. Transferability therefore lies in the general decision-support paradigm—combining process knowledge, stochastic forcing, and adaptive control—rather than in assuming that a specific irrigation policy or neural architecture can be directly deployed across domains.

8. Conclusion and perspectives

This study investigated the potential of physics-informed reinforcement learning for intelligent irrigation control under stochastic climatic forcing.

Three control paradigms were systematically compared: a rule-based heuristic strategy, reinforcement learning interacting with a physics-based soil–water model, and a hybrid neuro-physical approach in which the physical model is augmented by Neural Ordinary Differential Equation (Neural ODE) residual dynamics. The results highlight that no single strategy dominates across all performance criteria. Rule-based control provides strong guarantees in terms of stress avoidance and interpretability, but suffers from excessive irrigation and drainage losses due to its reactive nature. Physics-based reinforcement learning substantially improves water-use efficiency by exploiting temporal structure and delayed system responses, yet remains vulnerable to severe stress episodes caused by model mismatch and limited anticipation. The hybrid neuro-physical approach reshapes this trade-off by attenuating extreme stress events and reducing drainage while preserving much of the efficiency gain achieved by learning-based control.

Beyond quantitative performance, an important contribution of this work lies in clarifying *when* and *why* learning-based controllers outperform heuristic rules. Learning becomes advantageous when irrigation objectives extend beyond strict constraint satisfaction and require anticipation under climatic variability, cumulative deficits, and competing agronomic and hydrological objectives. In such contexts, fixed thresholds become insufficient, whereas learning-based policies can adapt decisions to evolving system trajectories. The role of Neural ODEs is shown to be corrective rather than substitutive. By learning residual dynamics on top of a physically interpretable core, the hybrid controller improves robustness to structural simplifications without sacrificing transparency. This positioning is critical for real-world deployment, where trust, explainability, and physical consistency remain essential.

From an operational perspective, the results suggest that different control paradigms may be appropriate depending on context. Rule-based strategies remain attractive for low-risk, low-complexity settings with limited data availability. Learning-based and hybrid approaches become increasingly relevant in environments characterized by climatic uncertainty, heterogeneous soils, or policy-driven water constraints, provided that training costs and data requirements can be managed through incremental or transfer learning.

Several limitations of the present study define clear directions for future work. First, the current Markovian state representations incompletely capture long-term dependencies and irregular temporal sampling inherent to real irrigation systems. Integrating Neural Controlled Differential Equations offers a principled pathway to model continuous-time dynamics driven by sparse and asynchronous observations. Second, reliance on simplified stochastic weather generators limits anticipatory capabilities; coupling the control framework with time-series foundation models such as PatchTST could enable climate-aware decision-making over longer horizons. Finally, the absence of an explicit world model restricts planning and counterfactual reasoning, motivating future extensions toward latent world-model architectures that jointly learn dynamics, uncertainty, and control.

In conclusion, this work demonstrates that physics-informed learning provides a powerful framework for navigating the inherent trade-offs of intelligent irrigation under uncertainty. Rather than replacing physical modelling or agronomic expertise, hybrid learning approaches offer a complementary pathway toward adaptive, robust, and interpretable decision support systems for climate-resilient water management.

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