Lecture 7

HASH FUNCTIONS

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CSSY2201 : Introduction to Cryptography

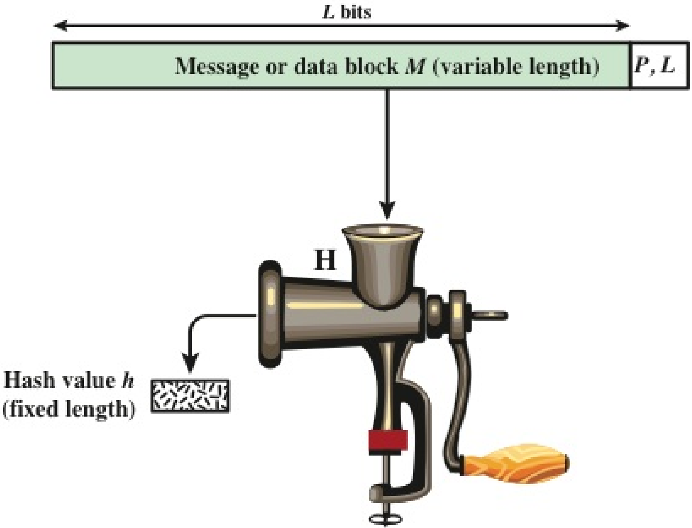
Plan

1. Hash functions
2. Secure hash Algorithm : SHA

Hash functions

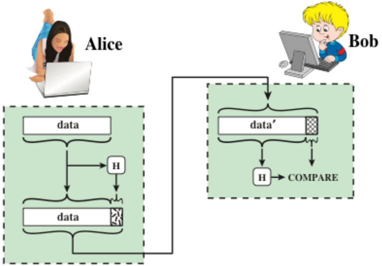
- Hash function accepts variable-length input and outputs a digest or hash of fixed length
- $h = H(M)$
- Its main purpose is integrity checking
- A cryptographic hash function is an algorithm that is mathematically difficult to :
  - Find an entry that gives a well-specified digest (Property : one-way function)
  - find two entries that give the same digest (Property : Collision-free)

Cryptographic Hash function h=H(M)

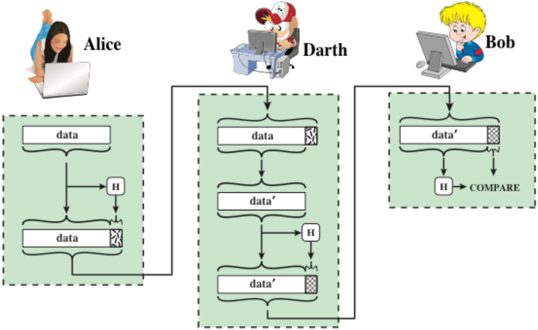


the output of a hash function from a message is named digest

- The digest of the message is its fingerprint
- Much smaller than original post
- easy to calculate
- cannot find message from digest
- Changing the message automatically changes the digest



(a) Use of hash function to check data integrity



(b) Man-in-the-middle attack

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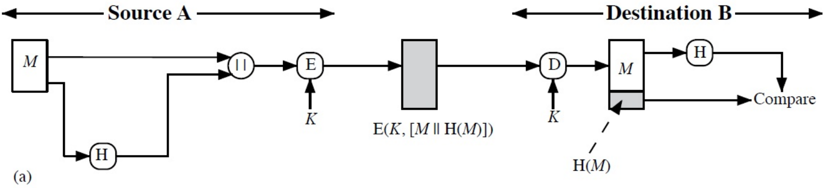
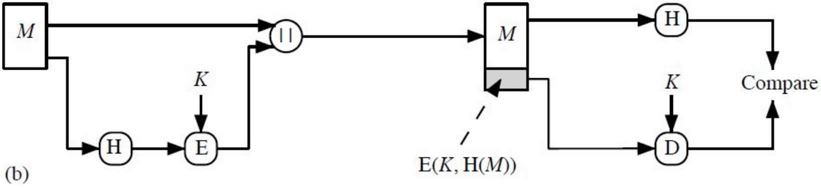
Message Authentication

- Verify message integrity
  - Ensure received data is exactly as sent
  - Ensure sender identity is valid
- Example 1 : Encrypt the message and its digest with a symmetric cryptosystem



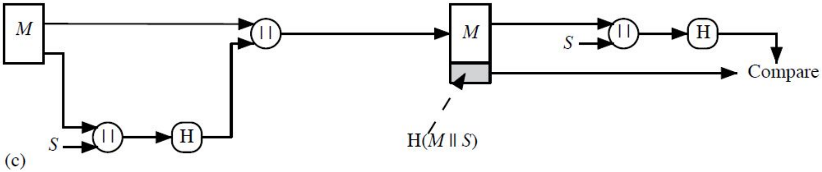
Message Authentication

- Example 2 : Encrypt only the message digest
- it reduces the complexity of calculation if confidentiality is not requested



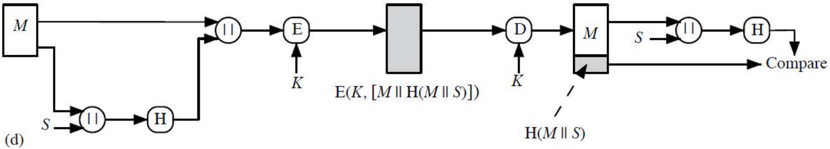
Message Authentication

- Example 3 : A shared secret is hashed
- No need for encryption



Message Authentication

- Example 4 : A shared secret combined with confidentiality



Other Uses of Hash Functions

- Used to create password files
  - When a user types a password, the hash of the password is compared to the saved hash for verification
  - This approach is used by the majority of operating systems
- used to detect intrusions and viruses
  - save the H(f) of each file to disk
  - the antivirus can later check if the file has been altered or not by recalculating its digest H(f)
  - An intruder will try to change F without changing H(f) : very difficult !
- can be used to build PRNG pseudo-random sequence generators
  - generate keystreams, secret keys

Requirements of a hash function

- Variable length entry
- Fixed length output
- Efficiency : given x, it is easy to generate the digest H(x) in s/w or h/w
- One-way function (Pre-image resistant) : For a given digest h, it is impossible to find y such that $H(y) = h$
- No broad sense collision (Second pre-image resistant : weak collision resistant) : For any given x, it is impossible to find $y \neq x$ such that $H(y) = H(x)$
- No collision in the strict sense (collision resistant : Strong collision resistant) : it is impossible to find a pair (x, y) such that $H(x) = H(y)$
- Random criterion : The output of H must be random according to the standard tests (NIST : 16 tests of the random criterion)

NB : "impossible" = "mathematically or by calculation difficult"

Requirements of a hash function

	Preimage Resistant	Second Preimage Resistant	Collision Resistant
Hash + digital signature	yes	yes	yes*
Intrusion detection and virus detection		yes	
Hash + symmetric encryption			
One-way password file	yes		
MAC	yes	yes	yes*

* Resistance required if attacker is able to mount a chosen message attack

Birthday Paradox

- In a class, what is the probability that 2 students celebrate their birthdays on the same day ?
- With 365 days a year, about thirty students in the class, we say to ourselves that it must be weak...
- We will calculate the probability that, in a group of k people, these people all have a different birthday :
  - If there are 2 people, the first can have a birthday anytime, the second any other day. we therefore have : $p_2 = \frac{364}{365} = 1 - \frac{1}{365}$
  - if mnt we have k people : $p_3 = (1 - \frac{1}{365})(1 - \frac{2}{365})$
  - in a group of k people, $p_k = (1 - \frac{1}{365})(1 - \frac{2}{365}) \ldots (1 - \frac{k-1}{365})$

Student Numbers	Prob. that their **anniv. dont match**
1	1
2	0.99
5	0.97
10	0.88
20	0.58
22	0.52
23	0.49
30	0.29
50	0.03

$\Longrightarrow$ It therefore only takes 23 people for there to be more than one chance in 2 (chance > 0.5) for 2 people to have their birthday on the same day.

Attacks based on the Birthday Paradox

- In an attack that looks for collisions, the adversary wants to find 2 messages that give the same digest.
- In a class of 23 students the probability of finding 2 students with the same birthday is > 0.5
- If the digest is encoded on b bits, there are 2^b possible hashes.
- if we take k different msgs, the probability of finding 2 msgs with the same digest is :

$$p = 1 - (1 - \frac{1}{2^b})(1 - \frac{2}{2^b})\ldots(1 - \frac{k-1}{2^b})$$

- for $p \geq \frac{1}{2}$, it suffices that $\overline{p} = (1 - \frac{1}{2^b})(1 - \frac{2}{2^b})\ldots(1 - \frac{k-1}{2^b}) \leq \frac{1}{2}$
- we have $(1 - \frac{j}{2^b}) \sim e^{(-\frac{j}{2^b})}$
- we then have $(1 - \frac{1}{2^b})(1 - \frac{2}{2^b})\ldots(1 - \frac{k-1}{2^b}) \sim e^{(-\frac{k(k-1)}{2^{b+1}})}$
- it is therefore necessary that $e^{(-\frac{k(k-1)}{2^{b+1}})} \leq \frac{1}{2}$

Digest Size	Total nb of trials to find a collision
8	20
16	302
32	77169
64	2 x 10^{10}
128	9 x 10^{19}
160	8 x 10^{25}
256	3 x 10^{40}

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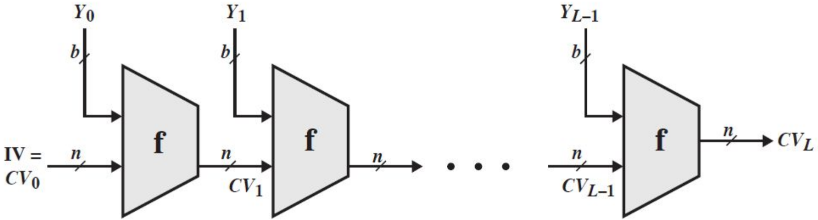
General structure of a hash function



IV	=	Initial value	L	=	number of input blocks
CV_i	=	chaining variable	n	=	length of hash code
Y_i	=	ith input block	b	=	length of input block
f	=	compression algorithm			

Secure Hash Algorithm (SHA)

- SHA was designed by "National Institute of Standards and Technology (NIST)" and published as "federal information processing standard" (FIPS 180) in 1993
- was revised in 1995 as SHA-1
- Based on MD4 hash function
- Produces a 160-bit size digest
- In 2002 NIST produced a revised version of the standard to define 3 more SHAs with lengths 256, 384, and 512 Known as SHA-2

Comparaison of SHA versions

	SHA-1	SHA-224	SHA-256	SHA-384	SHA-512
Message Digest Size	160	224	256	384	512
Message Size	$< 2^{64}$	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block Size	512	512	512	1024	1024
Word Size	32	32	32	64	64
Number of Steps	80	64	64	80	80

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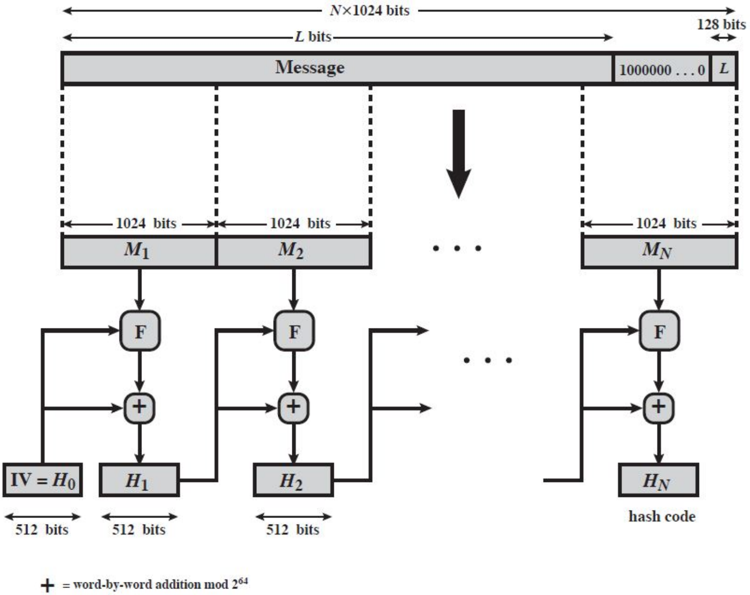
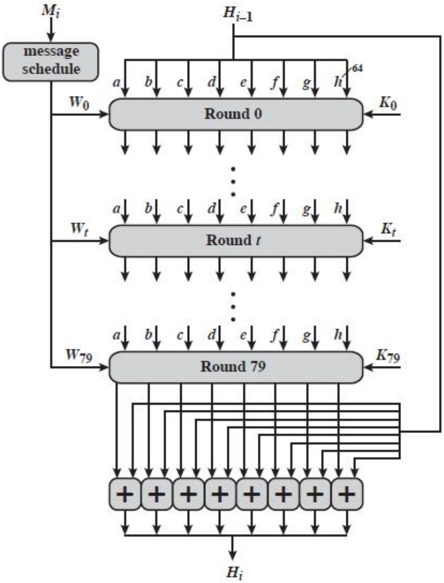
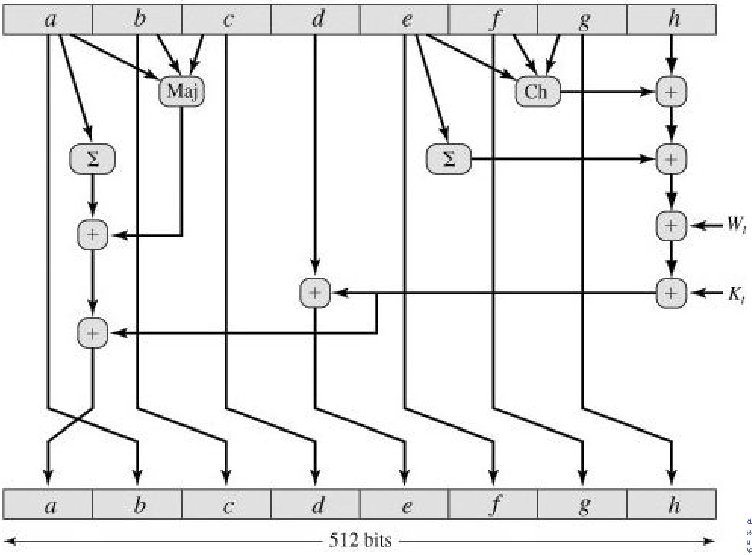
SHA-512


N×1024 bits
L bits
128 bits
Message
1000000 . . . 0
L
1024 bits
1024 bits
1024 bits
M1
M2
MN
F
F
F
IV = H0
H1
H2
HN
512 bits
512 bits
512 bits
hash code
+ = word-by-word addition mod 2^64

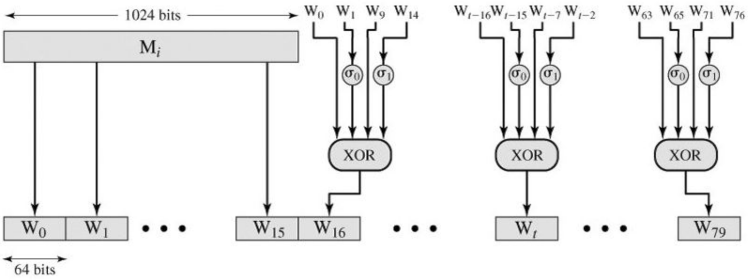
SHA-512 : Processing of a 1024-Bit block



SHA-512 : buffers update



SHA-512 : Processing of the message M_i



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