APMA 1650 – Problem Session 2

Wednesday, July 20, 2016

There are eleven problems on this sheet. There is no particular order to them, so I recommend that you work on the ones you find the most interesting.

1. For a certain section of a pine forest, the number of diseased trees per acre, Y , has a Poisson distribution with mean $\lambda = 10$. The diseased trees are sprayed with an insecticide at a cost of \$3 per tree, plus a fixed overhead cost for equipment rental of \$50. Letting C denote the total spraying cost for a randomly selected acre, find the expected value and variance for C. Within what interval would you expect C to lie with probability of at least 0.75?

2. You are a barista at a local coffee shop. The average number of customers per hour who enter your shop is 10. Assume customers arrive one-at-a-time and their arrivals are independent from each other.
   (a) What is the average time between the arrival of two customers?
   (b) What is the probabiltity that there will be an interval of 10 minutes or more between the arrival of one customer and the next?

3. Lifetimes of automotive tires are given in miles; higher performance tires are rated to last more miles. A manufacturer of tires wants to advertise a mileage interval that includes 90% of the mileage on tires they manufacture. All they know is that, for a large number of tires tested, the mean mileage was 25,000 miles, and the standard deviation was 4000 miles. What interval would you suggest?

4. A machine used to fill cereal boxes dispenses, on average, μ ounces per box. Let Y be the amount of ounces of cereal dispensed by the machine. The manufacturer wants Y to be within 1 ounce of μ at least 75% of the time. What is the largest value of the standard deviation σ of Y that can be tolerated if the manufacturer's objectives are met.

5. Beginning at 12:00 midnight, a call center is up for one hours and then down for two hours on a regular cycle. A person who is unaware of this schedule dials the center at a random time between 12:00 midnight at 5:00 am. What is the probability that the call center is up when the person's call comes in?

6. Weekly CPU time used by a hedge fund can be modeled by a random variable Y which has probability density function (measured in hours) given by:

$$f(y) = \begin{cases} cy^2(4-y) & 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of c such that this is a valid density function.

(b) Find the expected value and variance of Y.

(c) The CPU time costs the firm \$200 per hour. Find the expected value and variance of the weekly cost for CPU time.

7. The magnitude of earthquakes can be modeled as an exponential distribution with mean 2.4 (as measured on the Richter scale).

   (a) Find the probability that an earthquake will exceed 3.0 on the Richter scale.

   (b) Find the probability that an earthquake will fall between 2.0 and 3.0 on the Richter scale.

8. Let X and Y have a joint density function given by

$$f(x,y) = \begin{cases} cx & 0 \le y \le x \le 3 \\ 0 & otherwise \end{cases}$$

   (a) Find the value of c such that this is valid joint density function.

   (b) Find $\mathbb{P}(X \le 2, Y \ge 1)$.

   (c) Find the marginal densities of X and Y.

   (d) Find the expected values of X and Y.

   (e) Find the conditional density of Y given $X = x$.

   (f) Find $\mathbb{P}(X \le 2 | Y = 1)$

   (g) Find the conditional expected value $\mathbb{E}[Y|X = x]$.

9. Let X and Y be random variables with joint density given by

$$f(x,y) = \begin{cases} 6(1-y) & 0 \le x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find the covariance of X and Y. Are X and Y independent?

10. Scores on an examination are (roughly) normally distributed with mean 78 and variance 36.

    (a) What is the probability that a student scores higher than 72?

    (b) Suppose that students in the top 20% will receive an A on the exam? (This is now how I grade my exams.) What is the minimum score needed to receive an A?

11. A forester studying the effects of fertilization on pine forests is interested in estimating the average basal area of pine trees (basal area is the area of a given section of land that is occupied by the cross-section of tree trunks at their base). She has discovered that these measurements (in square inches) are normally distributed with standard deviation of 4 square inches.

(a) If she samples $n = 9$ trees, what is the probability that the sample mean will be within 2 square inches of the population mean.

(b) If she would like the sample mean to be within 1 square inch of the population mean with probability 0.90, how many trees must she measure in order to ensure this degree of accuracy?