

*An Information Flow Model for Conflict and Fission in Small Groups*¹

WAYNE W. ZACHARY

Data from a voluntary association are used to construct a new formal model for a traditional anthropological problem, fission in small groups. The process leading to fission is viewed as an unequal flow of sentiments and information across the ties in a social network. This flow is unequal because it is uniquely constrained by the contextual range and sensitivity of each relationship in the network. The subsequent differential sharing of sentiments leads to the formation of subgroups with more internal stability than the group as a whole, and results in fission. The Ford-Fulkerson labeling algorithm allows an accurate prediction of membership in the subgroups and of the locus of the fission to be made from measurements of the potential for information flow across each edge in the network. Methods for measurement of potential information flow are discussed, and it is shown that all appropriate techniques will generate the same predictions.

THE PROBLEM OF HOW and why fission takes place in small bounded groups has long been a central issue in social anthropology, even though the small groups studied have often been described under some other rubric, such as kinship. Fission in kinship groups has been studied from a variety of perspectives, especially those of descent theory (e.g. Evans-Pritchard 1940; Middleton and Tait 1958; Peters 1960; Forde 1964), and ecological adaptation (e.g. Reay 1967; Kelly 1968; Rappaport 1969; Nelson 1971). Another type of small bounded group frequently studied by anthropologists is the voluntary association (e.g. Mangin 1965, 1970; Doughty 1970, Goode 1970), although seldom with regard to fission or faction formation. In this paper I present data from such a group, a university-based karate club, in which a factional division led to a formal separation of the club into two organizations. The process leading to this fission is analyzed using a new model of fission, based on a social network approach. This model is a formal one, taken from the family of mathematical structures known as capacitated networks, and was developed directly from the ethnographic material outlined in the following section. The model allows the locus of fission within the group to be accurately predicted (greater than 97% accuracy for the data reported here). Moreover, this result is not limited to voluntary associations or to American culture, but rather is applicable

¹ I would like to thank Henry Selby, Lucy Garretson, and David Smith who read and commented on this and several earlier versions of this paper, as well as Russell Bernard, Peter Killworth, Lori Corwin, and Arthur Murphy who commented on earlier versions only. I would also like to thank George Benko and Robert Leskovec who helped me collect the original data. Still, any errors and misrepresentations are my own doing. Finally, I would like to thank the National Science Foundation which supported me as a Graduate Fellow during the analysis of the data.

to bounded social groups of all types in all settings. Also, the data required can be collected by a reliable method currently familiar to anthropologists, the use of nominal scales.

THE ETHNOGRAPHIC RATIONALE

The karate club was observed for a period of three years, from 1970 to 1972. In addition to direct observation, the history of the club prior to the period of the study was reconstructed through informants and club records in the university archives. During the period of observation, the club maintained between 50 and 100 members, and its activities included social affairs (parties, dances, banquets, etc.) as well as regularly scheduled karate lessons. The political organization of the club was informal, and while there was a constitution and four officers, most decisions were made by consensus at club meetings. For its classes, the club employed a part-time karate instructor, who will be referred to as Mr. Hi.²

At the beginning of the study there was an incipient conflict between the club president, John A., and Mr. Hi over the price of karate lessons. Mr. Hi, who wished to raise prices, claimed the authority to set his own lesson fees, since he was the instructor. John A., who wished to stabilize prices, claimed the authority to set the lesson fees since he was the club's chief administrator.

As time passed the entire club became divided over this issue, and the conflict became translated into ideological terms by most club members. The supporters of Mr. Hi saw him as a fatherly figure who was their spiritual and physical mentor, and who was only trying to meet his own physical needs after seeing to theirs. The supporters of John A. and the other officers saw Mr. Hi as a paid employee who was trying to coerce his way into a higher salary. After a series of increasingly sharp factional confrontations over the price of lessons, the officers, led by John A., fired Mr. Hi for attempting to raise lesson prices unilaterally. The supporters of Mr. Hi retaliated by resigning and forming a new organization headed by Mr. Hi, thus completing the fission of the club.

During the factional confrontations which preceded the fission, the club meeting remained the setting for decision making. If, at a given meeting, one faction held a majority, it would attempt to pass resolutions and decisions favorable to its ideological position. The other faction would then retaliate at a future meeting when it held the majority, by repealing the unfavorable decisions and substituting ones

² All names given are pseudonyms in order to protect the informants' anonymity. For similar reasons, the exact location of the study is not given.

favorable to itself. Thus, the outcome of any crisis was determined by which faction was able to "stack" the meetings most successfully.

The factions were merely ideological groupings, however, and were never organizationally crystallized. There was an overt sentiment in the club that there was no political division, and the factions were not named or even recognized to exist by club members. Rather, they were merely groups which emerged from the existing network of friendship among club members at times of political crisis because of ideological differences. There was no attempt by anyone to organize or direct political strategies of the groups, and, in general, there was no barrier to interaction between members of opposing factions. Only at times of direct political conflict did individuals selectively interact with others who shared the same ideological position, to the exclusion of those holding other positions. This selective association during confrontations is what brought the factions together only at crisis moments.

Political crisis, then, also had the effect of strengthening the friendship bonds within these ideological groups, and weakening the bonds between them, by the pattern of selective reinforcement. A series of political crises, like that which preceded the fission had the effect of "pulling" apart the network of friendship ties which held the club together, until the group completely and formally separated.

There are several reasons for formalizing this ethnographic description of the fission process into a mathematical model. First, the description is clarified. The formalization of the data necessary for the construction of a mathematical model requires that the intuitively based description be made precise and that all the relationships among descriptive categories be made clear. Second, the intuitive conclusions drawn can be formally stated and included in the model. The case study can then be generalized on the basis of the formal properties of these conclusions. In particular, the conclusions drawn about this group force a rejection of a standard anthropological social network model and require the construction of a more powerful network model—a network flow model. This model is new to anthropology and suggests several important new avenues of investigation in small-group studies. Third, hypotheses about the fission process, intuited from observation of the club, may be rigorously stated within the terms of the model and mathematically tested. Alternatively, assumptions about the basic conditions of the political process could be stated and then simulated within the model to assess their validity. Only the former approach will be used here, however.

The model is constructed by formalizing those relationships, or those components of the system, believed to be the most significant or the most explanatory. The feature of the karate club that appeared most

important in the ethnographic data was the network of friendship relationships among club members. While only (affective) friendship is considered here, any other dyadic relationship, such as effective friendship, patron-client relationships, or kinship ties, could have been used to specify the model. This flexibility gives the model a great generality.

A formal model of the friendship network within the club, one that contains sufficient complexity and precision to allow the testing of propositions about the political activities and the fission process, can be constructed. The model-building process will begin from the point of the social network model, an intuitive, nonformal model which can easily be transformed into a mathematical one.

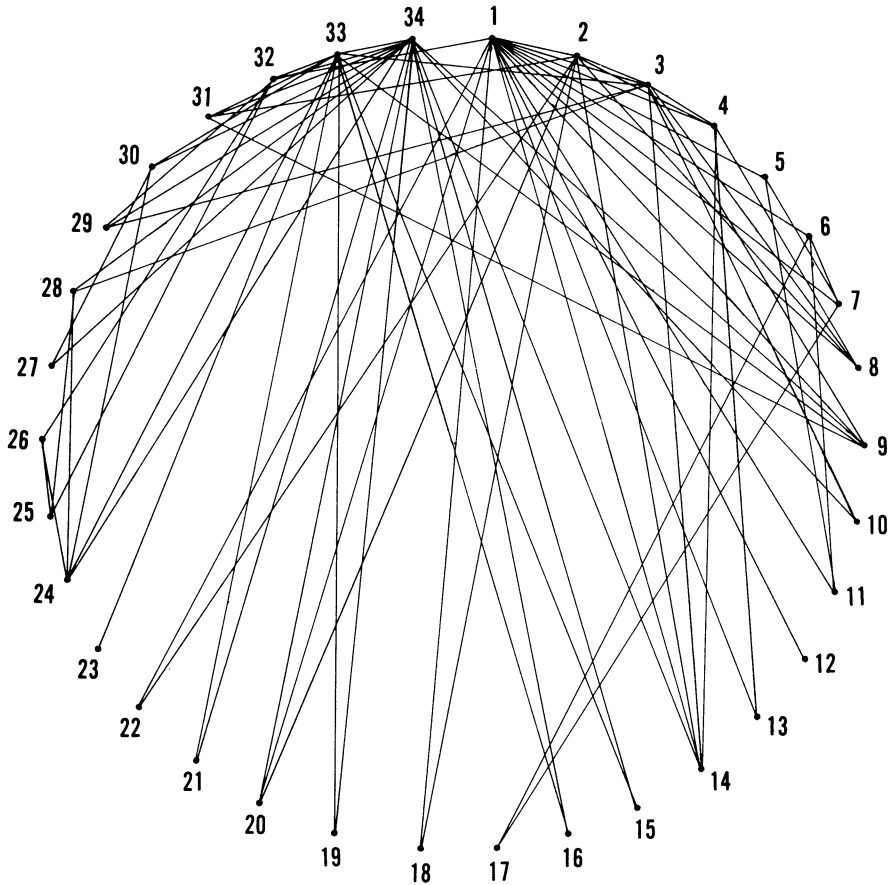
SOCIAL NETWORKS

The idea of the social network model is now well-established in the social science literature (Barnes 1968; Wolfe and Whitten 1974; Bott 1971; and Mitchell 1969), and is thus familiar to most anthropologists. In a social network model, the social relationships among the individuals in some bounded group are represented by a graph. Each individual in the group is represented by a point on the graph, with a line (called an edge) being drawn between any two points on the graph if, and only if, the relationship under consideration exists between the corresponding individuals. The analysis of patterns of social relationship in the group is then conducted on the graph, which is merely a shorthand representation of the ethnographic data. Barnes (1969), for example, has programmatically suggested the investigation of such features as the density (the ratios of actual to possible edges in the graph) or the average length of closed circuits in the graph.

An alternative, more formal, representation of the social network is as a square matrix of ones and zeroes. In this form, each individual is represented by a row and column in the matrix; for example, the i th individual being represented by the i th row and the i th column. For each cell in the matrix, a one is assigned to it if, and only if, an edge was drawn between the points corresponding to the row and column designating that cell. A zero is assigned otherwise. The matrix can then be manipulated by matrix algebraic procedures to uncover formal relationships which are not superficially evident in the data. Examples of techniques for using the matrix form to analyze the structure of social relationships are presented in Flament (1963), Holland and Leinhardt (1970), and Lorraine and White (1971).

Both the graph and the matrix representations can be constructed for the karate club. The graph representation of the relationships in the club (shortly before the fission) is given in Figure 1. A edge is drawn if

FIGURE 1
Social Network Model of Relationships in the Karate Club



This is the graphic representation of the social relationships among the 34 individuals in the karate club. A line is drawn between two points when the two individuals being represented consistently interacted in contexts outside those of karate classes, workouts, and club meetings. Each such line drawn is referred to as an edge.

two individuals consistently were observed to interact outside the normal activities of the club (karate classes and club meetings). That is, an edge is drawn if the individuals could be said to be friends outside the club activities. This graph is represented as a matrix in Figure 2. All the edges in Figure 1 are *nondirectional* (they represent interaction in both directions), and the graph is said to be *symmetrical*. It is also possible to draw edges that are *directed* (representing one-way relationships); such

FIGURE 2

MATRIX OF RELATIONSHIPS IN THE CLUB: THE MATRIX *E*

	Individual Number																																			
	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4		
1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
2	1	0	1	1	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0	
3	1	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	
4	1	1	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
7	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
8	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
9	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
11	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
13	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
14	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
17	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
20	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
22	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
28	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
29	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
31	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
33	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0
34	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1	0	1	1	0	1	1	0	0	1	1	1	1	1	1	0

This is the matrix representation of the graph shown in Figure 1. The rows and columns represent individuals in the club. An entry, determined by a row/column pair, is valued at 1 if an edge was drawn in Figure 1 between the two individuals represented by the row and column. The entry is valued at 0 otherwise. For example, the entry in the fifth row, seventh column, is a 1, indicating there is an edge existing between individuals 5 and 7. Notice that the matrix is symmetrical—i.e., the seventh row fifth column is also valued at 1, since it identifies the same edge. Because, by definition, no individual can interact with himself, zeroes occupy the main diagonal (from row/column 1 to row/column 34). Later, this matrix will be referred to by the symbol *E*.

graphs are called *asymmetrical*. However, only symmetrical networks will be considered in this paper.

Only 34 individuals are presented in Figures 1 and 2. Although club membership was near 60 at that time, none of the 26 members not represented interacted with other club members outside the context of meetings and classes. Because these individuals would only be unconnected points in the graph, and rows and columns of zeroes in the matrix, they are not included. These individuals belonged to neither faction, and did not participate in the politics of the club. Most joined neither club at the time of the fission, preferring to quit the study of karate altogether because of the political conflict.

Density analysis, or the various forms of matrix analysis, are not conducted here because they, like all social network analyses, are static. Lorraine and White (1971) have pointed out that an analysis of the relationships between edges in the network is the only kind of analysis possible on graph and matrix representations. Such an analysis is purely structural (by the classic Jacobsonian definition) and must ignore all processual aspects of the social activity represented by the network. Since the political process leading to the fission rather than the political structure of the group is the focus of this paper, neither the graph nor the matrix representation is adequate. However, further information can be included along with the matrix form to make a processual analysis possible.

CAPICITATED NETWORKS

In constructing a processual model, the network shown in Figures 1 and 2 must be considered only as it represents the social constraints of the system within which the political process operates. The model built here is based on the friendship structure within the club, and the feature of the political process which operated through these friendship relations was the transmission of political information (both tactical, e.g., when a meeting would occur, and ideological) during times of political crisis. This process brought the factions into existence during crisis periods, and was the mechanism for the positive feedback relationship which existed between crisis and factional organization. In this relationship, crises caused the reinforcement of only those relationships within factions, thus making the factions more defined, and also making another confrontation more likely.

From the point of view of information flow, the ties in the network can be thought of as channels across which information may flow. The absence of an edge precludes any direct passage of political information between the two individuals represented by the potential edge. It is obvious that the flow, or potential for flow, is not identical across all

edges in the network. Some individuals are much closer friends than others, some see each other only in the presence of members of the other factions. All these differing relationships are represented in the same way in Figures 1 and 2. In order to represent the network accurately as a net of channels for information flow, the different potentials for information flow of the different edges in the network must be quantified and included in the model.

By quantifying the strengths/weaknesses of the edges in the network, a mathematical model can be specified that is formally adequate for the investigation of information flow in the karate club, and its effect on the fission process. The matrix representation of the social network shown in Figures 1 and 2 will be retained as one component of this model. The matrix form is chosen because of its greater ease of algebraic manipulation. The matrix in Figure 2 will be referred to as the *existence matrix*, and all entries of value one will be referred to as *existing edges* in the network. A second matrix must be created which quantifies the relative strengths/weaknesses of the existing edges in the network. This second matrix is called the *capacity matrix*.

The complete mathematical model of the karate club can now be specified as an ordered triplet (V, E, C) , where V is the set of individuals included in the network, E is an existence matrix, and C is a capacity matrix. The elements of set V are called *nodes* in the network, and the triplet (V, E, C) is called a *capacitated network*. The mathematical development of capacitated networks is presented in Ford and Fulkerson (1962), Hu (1969), Maki and Thompson (1974), and Zachary (1977).

Each value of the matrix C can be interpreted as representing a "capacity" or "value" of maximum possible flow for the corresponding edge in the existence matrix E . It can be shown mathematically that most features of flow in networks, (in this case flow of political information) are functions of the edge capacities in the network (see Ford and Fulkerson 1962, or Zachary 1977). In particular, the maximum flow in the network is completely determined by the values in the matrix of edge capacities.

Unfortunately, there is no way known to quantify what has been referred to as "political information," and by extension, there is no sure way to specify actual capacities of information flow for the edges in the network. However, by making certain assumptions about the social settings in which information was communicated, this problem can be circumvented.

Political information was communicated in contexts outside the regular activities of the club, and club members interacted in a number of such contexts. Since political activity was not overtly recognized by

club members, the transmission of political information was then incidental to normal social interaction. It can be assumed that the amount of information transmitted was related to the number of contexts in which the information might have been communicated. Mathematically, this can be stated as assuming that the amount of information communicated is a function of the number of contexts in which communication could take place. If, from a knowledge of the karate club, some specific relationship between the amount of information and the number of contexts can be determined, a procedure for assigning values to C based on the number of contexts can be devised. The functional relationship is at least *monotone increasing*. That is, the greater the number of contexts in which a pair interacted, the more the information that could be passed between them. The data indicate that the relationship is roughly linear. For instance, twice as much information could be passed over time between individuals who interacted in four contexts as between individuals who interacted in only two contexts.

Some ethnographic description of this proposed linearity can be provided. A typical message unit transmitted through the network was the news of a club meeting. The "goal" of any faction member was to pass along such information to as many members as possible of the same faction and to as few as possible of the opposing faction. However, as previously stated, the communication of such items of information was incidental to normal interaction. The assumption of linearity, then, claims that the likelihood of this item of information being communicated to any individual increases linearly with the number of contexts in which interaction with that individual takes place.

The assumption of linearity can also be justified mathematically. Given a monotone increasing function, it can be reasonably approximated by a single linear function if there is no severe change in the rate of increase of the function in the interval under consideration. The data provide no indication that such a change in rate of increase in information flow occurs, and thus the assumption of linearity can be accepted.

The linearity of the relationship between information flow and the number of contexts of interaction suggests a method for generating values for the matrix C . Since information cannot be quantified, but has been shown to be a linear function of a variable that can be, the quantifiable variable may be used in its place, and the analysis to follow will not be effected, as proved in Zachary (1977, see Theorem 4).³ A

3 In particular, Zachary (1977: Theorem 4) shows that any capacity matrix which is a linear multiple of the unknown but "true" capacity matrix will produce the same minimum cut as the "true" matrix when the maximum flow–minimum cut labeling procedure (used throughout the

specific procedure can now be outlined for generating the values of C . A finite set of possible contexts, chosen on the basis of observation of the group, will be used as the domain of a scale variable. Then, the relationship between each pair of individuals in the network is examined against this (nominal) scale. A value, equal to the total number of contexts from the scale in which the two individuals interacted, is then assigned to the corresponding entry in C . Eight contexts are included in the domain of the scale applied to the edges in the karate club network. They are:

- (1) Association in and between academic classes at the university.
- (2) Membership in Mr. Hi's private karate studio on the east side of the city where Mr. Hi taught nights as a part-time instructor.
- (3) Membership in Mr. Hi's private karate studio on the east side of the city, where many of his supporters worked out on weekends.
- (4) Student teaching at the east-side karate studio referred to in (2). This is different from (2) in that student teachers interacted with each other, but were prohibited from interacting with their students.
- (5) Interaction at the university rathskeller, located in the same basement as the karate club's workout area.
- (6) Interaction at a student-oriented bar located across the street from the university campus.
- (7) Attendance at open karate tournaments held through the area at private karate studios.
- (8) Attendance at intercollegiate karate tournaments held at local universities. Since both open and intercollegiate tournaments were held on Saturdays, attendance at both was impossible.

This scale was applied to the relationships between all pairs of individuals in the karate club, using data compiled over the three years of direct observation of interactions in the club. For each existing edge in E (Figure 2), the pair of individuals involved interacted in at least one of the above eight contexts. The quantified matrix of contexts is given in Figure 3, and is the third component in the capacitated network model (V, E, C) .

NETWORK FLOWS

The flow of information in the club can now be analyzed with the fully specified model (V, E, C) , by using the family of mathematical problems known as *network flows*, or Ford-Fulkerson problems, after the

remainder of the paper) is applied. A linear multiple is a linear function in which the zero in the domain (here, information flow) is mapped into the zero in the range (here, number of contexts of interaction). Since zero potential for information flow clearly maps into zero contexts of interaction, the linear function used is really a linear multiple.

FIGURE 3

**QUANTIFIED MATRIX OF RELATIVE STRENGTHS OF THE RELATIONSHIPS
IN THE KARATE CLUB: THE MATRIX C**

[illegible]

The scale given in the text (p. 461) was used to assign values of relative strength/weakness of the relationships in the club. This matrix gives the values assigned to each edge specified in matrix E (Figure 2). The value represents the number of contexts in which interaction took place between the two individuals involved. The row/column ordering is the same as in Figure 2.

two mathematicians who first extensively studied flows in networks (Ford and Fulkerson 1962).

An item of political information (for example, a decision to call a meeting) originated with one or several individuals and was spread through the club. This can be interpreted in the model as a flow from one node to all other nodes in the network. The effect of the basic

structure of *E* and *C* (that is, the paths and strengths of relationships in the club) on the passage of this item of information through the network can be observed by noting the paths that the item will take in a given communication. The information (the calling of a meeting) usually began with one of the factional leaders, Mr. Hi or John A. This individual will be called the "source of the information," or simply, the *source*. Given the course of political conflict in the club, it would be advantageous to the source if individuals in the opposing faction did not receive this information. Further, the source would benefit more if the information would be deprived to individuals proportionately to their affiliation with the other faction—the more closely a person was aligned with the other side, the less the source would benefit from his or her knowing about the meeting. By this reasoning, the leader of the other faction should be the last to receive the information. He will therefore be called the "sink of the information," or simply, the *sink*—the terms "source" and "sink" being standard terms from network flow theory.

A hypothesis to be investigated is that no overt attempt may have had to be made to avoid mentioning the news in the presence of members of the other faction because the network tended to prevent such meetings from occurring. The hypothesis suggests that the sink's faction generally would not receive the information, since members of opposing factions were less likely to associate, or did so in fewer different contexts, than members of the same faction.

Not all individuals in the network were solidly members of one faction or the other. Some vacillated between the two ideological positions, and others were simply satisfied not to take sides. These individuals are key nodes in the network, in that it was through them that information was likely to pass from one faction to the other. The feedback principle can be used to state that as the frequency of crisis increases, these individuals are likely to be included in the network ties of the members of one faction to the exclusion of those of the other faction, regardless of whether they are politically part of that faction. It would be expected that these individuals would join the club formed by the faction with which they were associated when the split in the club occurred. A second, more general, hypothesis can be framed from this discussion: that a *bottleneck* in the network, representing a structural limitation on information flow from the source to the sink, will predict the break that occurred in the club at the time of the fission. The term "bottleneck" refers to the first hypothesis, which claims the existence of some structural feature in the network inhibiting information flow between factions.

The validity of both hypotheses can be evaluated by a single mathematical technique called the *maximum flow–minimum cut labeling*

procedure, developed by Ford and Fulkerson (1962) when they established that the value of the maximum flow in a capacitated network is strictly determined by the values assigned to the capacities of the edges in the network. Intuitively stated, they proved that the maximum flow is equal to the capacity of the smallest possible break in the network separating the source from the sink.

A break in the network is conceptualized as a division of the set of nodes V into two subsets, one containing the source and the other, the sink. The edges connecting elements of one subset to elements of the other are called the *cut*. The sum of the capacities of all edges in the cut is its capacity. From Ford and Fulkerson, the maximum flow is equal to the capacity of the minimum cut. The minimum cut, then, represents a bottleneck in the network and possesses the properties described in the two hypotheses.

The minimum cut is located by means of an algorithm which begins by starting a flow moving in the network from the source to the sink. This flow is then systematically incremented until no more flow can be added because it is equal to the capacity of some edge on each possible path from the source to the sink. The minimum cut is comprised of those edges on each path from the source to the sink for which the flow equals the capacity. If the flow is equal to the capacity of more than one edge on the path, only the edge closest to the source is in the cut. Mathematically, the first hypothesis states that the members of one faction will all be on one side of the minimum cut and that the members of the other faction will all be on the other side. The second hypothesis states that the minimum cut will separate the club members who joined Mr. Hi's club after the split from those who joined the officers' club.

NETFLOW (Smillie 1969) is an APL language computer program which carries out the maximum flow–minimum cut labeling procedure of Ford and Fulkerson. NETFLOW was run on the model of the karate club, (V, E, C) . The program requires that the first row/column of matrices E and C represent the source and that the last row/column (here, row/column 34) represent the sink. Hence, row/column 1 in these matrices designates Mr. Hi, and row/column 34 designates John A. The results of NETFLOW support both hypotheses, and are summarized in Table 1. In all cases, except individual number 9, individuals on the source side of the cut belonged to Mr. Hi's faction (or no faction), and joined the club formed by his supporters after the split. Individuals on the sink side of the cut belonged to John A.'s faction (or no faction) and joined the club founded by the officers. Person number 9 was a weak supporter of John but joined Mr. Hi's club after the split. This can be explained by noting that he was only three weeks away from a test for black belt (master status) when the split in the club occurred. Had he

TABLE 1
RESULTS OF INITIAL *NETFLOW* RUN

INDIVIDUAL NUMBER	SIDE OF CUT	FACTION	CLUB AFTER FISSION
1	Source	Mr. Hi - Strong	Mr. Hi's
2	Source	Mr. Hi - Strong	Mr. Hi's
3	Source	Mr. Hi - Strong	Mr. Hi's
4	Source	Mr. Hi - Strong	Mr. Hi's
5	Source	Mr. Hi - Strong	Mr. Hi's
6	Source	Mr. Hi - Strong	Mr. Hi's
7	Source	Mr. Hi - Strong	Mr. Hi's
8	Source	Mr. Hi - Strong	Mr. Hi's
9	Sink	John - Weak	Mr. Hi's
10	Sink	None	Officers'
11	Source	Mr. Hi - Strong	Mr. Hi's
12	Source	Mr. Hi - Strong	Mr. Hi's
13	Source	Mr. Hi - Weak	Mr. Hi's
14	Source	Mr. Hi - Weak	Mr. Hi's
15	Sink	John - Strong	Officers'
16	Sink	John - Weak	Officers'
17	Source	None	Mr. Hi's
18	Source	Mr. Hi - Weak	Mr. Hi's
19	Sink	None	Officers'
20	Source	Mr. Hi - Weak	Mr. Hi's
21	Sink	John - Strong	Officers'
22	Source	Mr. Hi - Weak	Mr. Hi's
23	Sink	John - Strong	Officers'
24	Sink	John - Weak	Officers'
25	Sink	John - Weak	Officers'
26	Sink	John - Strong	Officers'
27	Sink	John - Strong	Officers'
28	Sink	John - Strong	Officers'
29	Sink	John - Strong	Officers'
30	Sink	John - Strong	Officers'
31	Sink	John - Strong	Officers'
32	Sink	John - Strong	Officers'
33	Sink	John - Strong	Officers'
34	Sink	John - Strong	Officers'

This table summarizes the results of the first run of *NETFLOW*, using matrices *E* and *C* as input. "Individual Number" identifies the individual with the corresponding row/column in the matrices. "Side of Cut" refers to the subset of *V* to which the individual was assigned by *NETFLOW*, either the source side or the sink side. "Faction" gives the factional affiliation of the individual, either with that of John A., that of Mr. Hi, or none. The strong/weak designations in this column indicate whether the individual was a strong or a weak supporter of the faction's ideological position. Finally, "club after fission" indicates which club was joined after the fission, either that formed by Mr. Hi, or that formed by the officers of the original club.

joined the officers' club he would have had to give up his rank and begin again in a new style of karate with a white (beginner's) belt, since the officers had decided to change the style of karate practiced in their new club. Having four years of study invested in the style of Mr. Hi, the individual could not bring himself to repudiate his rank and start again.

In fact, the inclusion of individual 9 in the sink side of the cut is supportive of both hypotheses. He was a weak political supporter of John A., and during the crises preceding the fission, he became increasingly associated with the members of John's faction to the exclusion of those of Mr. Hi's. He was clearly a structural part of John's faction and should have been assigned to the sink side of the cut by the labeling procedure. That he did not follow the factional affiliation and join the officers' club resulted from an overriding interest, not shared by other club members, in remaining with Mr. Hi—his black belt. It is important that all individuals with no factional alliance joined the club founded by the faction to which they were assigned by the labeling procedure. This is the faction to which they structurally, though not necessarily ideologically, belonged.

The results of NETFLOW verify the first hypothesis by demonstrating that the factions in the club could be represented by the minimal cut in the network model. The second hypothesis is validated by demonstrating that the membership of the clubs could be predicted by the minimal cut in the network model. These results are not mathematically conclusive, however, unless the uniqueness of the minimal cut can be established. There could be several possible cuts in the network having the same minimal capacity. It must be proven that the cut chosen by NETFLOW is the only minimal cut in the network. Fortunately, Ford and Fulkerson (1962:ch. 2) also established a procedure by which the uniqueness of a minimum cut can be proven.

In this procedure, the entire original network is reversed, so that the source becomes the sink, and the sink, the source. The labeling procedure is then applied to the reversed network; if the same minimum cut is found, then it is unique. This procedure can be accomplished by reversing the row/column ordering of matrices *E* and *C*, (so that row/column 1 becomes row/column 34, row/column 2 becomes row/column 33, etc.), and then running the NETFLOW program on these reversed matrices.⁴ The results of this run are summarized in Table 2. As can be seen from Table 2, the same cut is found in the reversed network as in the original network. The minimum cut is then unique, in that there is no other cut in the network with the minimum capacity. The two hypotheses can be formally accepted on the basis of this uniqueness. The test of the two hypotheses is summarized in Table 3.

4 Note that the matrix reversal operation used here bears no similarity or relation to either the matrix inversion or the matrix transposition operations frequently used in matrix algebra.

TABLE 2
RESULTS OF SECOND *NETFLOW* RUN

INDIVIDUAL NUMBER IN ORIGINAL MATRIX C	INDIVIDUAL NUMBER IN REVERSED MATRIX C*	SIDE OF CUT IN ORIGINAL MATRIX C	SIDE OF CUT IN ORIGINAL MATRIX C*
1	34	Sink	Source
2	33	Sink	Source
3	32	Sink	Source
4	31	Sink	Source
5	30	Sink	Source
6	29	Sink	Source
7	28	Sink	Source
8	27	Sink	Source
9	26	Sink	Source
10	25	Sink	Source
11	24	Sink	Source
12	23	Sink	Source
13	22	Source	Sink
14	21	Sink	Source
15	20	Source	Sink
16	19	Sink	Source
17	18	Source	Sink
18	17	Source	Sink
19	16	Sink	Source
20	15	Sink	Source
21	14	Source	Sink
22	13	Source	Sink
23	12	Source	Sink
24	11	Source	Sink
25	10	Sink	Source
26	9	Sink	Source
27	8	Source	Sink
28	7	Source	Sink
29	6	Source	Sink
30	5	Source	Sink
31	4	Source	Sink
32	3	Source	Sink
33	2	Source	Sink
34	1	Source	Sink

This table summarizes the results of the second run of the program *NETFLOW*, this time using the reversed matrices as input. Column 1 of the table gives the order of the individuals as used in Table 1 and in matrices *E* and *C*. Column 2 gives the individual numbers in the reversed matrices. Column 3 indicates the side of the cut to which the individual was assigned by the first run of *NETFLOW* (from Table 1), and column 4 lists the side of the cut to which the individual was assigned by the second run, using the reversed matrices as input. In each case, the entry in column 3 is the opposite the entry in column 4. Thus, the same cut is defined by both runs, and the minimum cut is unique.

CONCLUSIONS

It has been shown how the flow of political information through the club interacts with the political strategy of the factions to pull apart, or bottleneck, the network at the factional boundary. This boundary corresponds to an ideological as well as an organizational division in the

TABLE 3
EVALUATION OF THE HYPOTHESES

INDIVIDUAL NUMBER IN MATRIX C	FACTION MEMBERSHIP FROM DATA	FACTION MEMBERSHIP AS MODELED	HIT/ MISS	CLUB AFTER SPLIT FROM DATA	CLUB AFTER SPLIT AS MODELED	HIT/ MISS
1	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
2	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
3	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
4	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
5	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
6	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
7	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
8	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
9	John	John	Hit	Mr. Hi's	Officers'	Miss
10	John	John	Hit	Officers'	Officers'	Hit
11	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
12	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
13	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
14	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
15	John	John	Hit	Officers'	Officers'	Hit
16	John	John	Hit	Officers'	Officers'	Hit
17	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
18	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
19	John	John	Hit	Officers'	Officers'	Hit
20	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
21	John	John	Hit	Officers'	Officers'	Hit
22	Mr. Hi	Mr. Hi	Hit	Mr. Hi's	Mr. Hi's	Hit
23	John	John	Hit	Officers'	Officers'	Hit
24	John	John	Hit	Officers'	Officers'	Hit
25	John	John	Hit	Officers'	Officers'	Hit
26	John	John	Hit	Officers'	Officers'	Hit
27	John	John	Hit	Officers'	Officers'	Hit
28	John	John	Hit	Officers'	Officers'	Hit
29	John	John	Hit	Officers'	Officers'	Hit
30	John	John	Hit	Officers'	Officers'	Hit
31	John	John	Hit	Officers'	Officers'	Hit
32	John	John	Hit	Officers'	Officers'	Hit
33	John	John	Hit	Officers'	Officers'	Hit
34	John	John	Hit	Officers'	Officers'	Hit
TOTALS	34 hits, 0 misses 100% hits, 0% misses			33 hits, 1 miss 97% hits, 3% misses		

This table gives the results of the NETFLOW runs used to test the two hypotheses (see pp. 462 ff.). The faction membership (column 2) and the club joined after the fission (column 5) entries were taken from the ethnographic data. These columns merely state what the individuals actually did. Column 3 gives the faction membership as predicted by the model (based on which side of the minimum cut the individual was placed). Column 4 gives the accuracy of each of these predictions. The model was 100% accurate in predicting faction membership. Column 6 gives the membership in the two clubs formed after the fission, again as predicted by the model (based on which side of the minimum cut the individual was placed). Column 7 gives the results of these predictions. The model was 97% accurate in predicting club membership after the split. Thus, both hypotheses can be accepted.

club. It is possible to extend the argument used in the description of the fission process to include the ideological division in the same feedback framework used to describe the development of the organizational division.

Conflict in any setting must be bounded by some sharing of rules which allows the combatants or contestants to know what and how they are contesting. This knowledge can be at any level, from the rules of a game, to the shared meanings of ritual items for the *gumsa* and *gumlao* Kachin (Leach 1954). Whatever the level, these rules must continue to be understood by both players of the game, or the conflict ceases as the activity becomes undefined for the players.

In the karate club, both factions shared an ideological position toward the club in the early stages of the conflict, but followed increasingly divergent positions. Their divergent positions made the basis for conflict increasingly unclear to the participants. It also became more and more difficult for members of the two factions to understand how the opposing side could even consider themselves as part of the same club. Just as a faction tended to know less and less about the political activity of its opposition, it tended to understand less and less its common ground with the opposition, until even the existence of the club ceased to serve as a basis for unity.

It can be concluded, then, that not just political information about club meetings was communicated in the friendship network outside the club. In these contexts, club members communicated their perceptions and understandings of the nature of the club in various conscious and unconscious ways, as well. If the bottlenecking of the network prevented the flow of political information between factions, it also prevented the sharing of perceptions of the club held by various club members. In particular, it tended to allow sharing of these items to a greater extent within the factions than across the factional boundary.

A feedback relationship, then operated in two directions, one organizational and one ideological. The strategy of the factions (whose membership was based on ideology) acted to strengthen the cut in the network, which itself was the organizational basis for the factions' existence. The cut in the network acted to intensify the ideological divisions on which club members based their factional affiliation. These feedback relationships constituted a vicious cycle, acting as a mechanism which virtually assured the fission in the club. While the content of this mechanism was specific to the karate club, its form is a general feature of communication patterns in small groups, and corresponds to a feature inherent in the type of model used.

This conclusion has broad implications for the anthropological study of small groups. The positive feedback process can be formally

represented as a feature of a model of the group, the minimum cut in a capacitated network. This type of model can be constructed for any group, and the existence and the significance of the minimum cut, or bottleneck, can be tested. The results of this study certainly suggest that whenever a unique minimum cut exists in a capacitated network representation of a small group, it will act as a barrier to group unity, and under certain conditions, will act as a selective factor for group fission.

As suggested above, the model can be generalized to include other types of social relationships than that used here, effective friendship. Moreover, the method used to generate values for the edge capacities can also easily be described generally as constructing a nominal scale, choosing subsets of elements from the nominal scale, and assigning the number denoting the size of the subset to the edge in question. This method places minimal restrictions on both ethnographers and informants. Other equally simple procedures could also be devised.

The capacitated network model constructed here is mathematically much more powerful than the usual social network model. There are two reasons for this increased power, one mathematical and one anthropological. Mathematically, more information is included in the capacitated network model, in the form of the strengths/weaknesses of the edges given in matrix *C*. Models containing more information are always more powerful than those containing less, the example of the increased power of known-distribution statistics as compared to distribution-free statistics being perhaps the best-known to anthropologists.

Anthropologically, the approach used to build the model is processual rather than static. The network model is constructed from a view of the social system as a processor of information—a cybernetic machine—rather than as a static set of relationships. While the structural features of the network are investigated, it is the structure of a cybernetic process that is really being studied. It is from this information processing approach that such things as feedback and information flow can be discussed and included in the model.

Further refinement of the mathematical assumptions built into the model is possible. The most plausible of the assumptions made is that the relationship between the amount of information and the number of contexts is monotone increasing. While any monotonic function which preserves addition may be used, the present assumption of linearity can be justified both ethnographically and as a standard computational device. While this approximation has proven sufficient for this study, further ethnographic methods must be developed for specifying the relationship accurately in other cases.

Finally, the method used to build the network flow model differs from that used in much of the current model-building work in network theory (e.g. Hunter 1974; Killworth and Bernard 1975) in that it began with an existing set of data rather than a set of purely theoretical propositions. The model built here is a formalization of ethnographic description of the club, and all assumptions made are ethnographically justified. Only after it had been fully constructed was the model generalized, and the accuracy of the predictions made by the model are certainly dependent on this data-based aspect. Many of the mathematically more sophisticated models (Killworth and Bernard 1975, for example) are constructed solely from theory and intuition, and it is openly admitted that the results may not be close approximations of social reality (Killworth and Bernard 1975: ch.1). Because of its data orientation, the network flow model has the advantages of simplicity and accuracy of representation.

BIBLIOGRAPHY

- BARNES, J. A.
1968 Networks and Political Processes. Pp. 107-30 in *Local-Level Politics* (ed. by Marc Swartz). Chicago: Aldine Publishing Company.
- BOTT, ELIZABETH
1971 *Family and Social Networks*. 2d ed. London: Tavistock.
- DOUGHTY, PAUL
1970 Behind the Back of the City. Pp. 30-46 in *Peasants in Cities* (ed. by W. Mangin). Boston: Houghton-Mifflin Co.
- EVANS-PRITCHARD, E. E.
1940 *The Nuer*. London: Oxford University Press.
- FLAMENT, CLAUDE
1963 *Applications of Graph Theory to Group Structure*. Englewood-Cliffs, N. J.: Prentice-Hall.
- FORD, L. AND D. FULKERSON
1962 *Flows in Networks*. Princeton: Princeton University Press.
- FORDE, DARYLL
1964 Fission and Accretion in the Patriline. Pp. 49-84 in *Yako Studies*, (ed. by Daryll Forde). London: Oxford University Press.
- GOODE, JUDITH
1970 Latin American Urbanism and Corporate Groups. *Anthropological Quarterly* 43:146-67.
- HOLLAND, PAUL AND SAMUEL LEINHARDT
1970 A Method for Detecting Structure in Sociometric Data. *American Journal of Sociology* 75:492-513.
- HU, T. C.
1969 *Interger Programming and Network Flows*. Reading, Mass.: Addison-Wesley.

HUNTER, JACK

- 1974 Dynamic Sociometry. Paper presented at Mathematical Social Science Board Conference on Social Networks at Cheat Lake, W. Va., May 1974.

KELLY, RAYMOND

- 1968 Demographic Pressure and Descent Group Structure in the New Guinea Highlands. *Oceania* 39:36-63.

KILLWORTH, PETER AND RUSSELL BERNARD

- 1975 A Model of Human Group Dynamics. Office of Naval Research, Interim Report number BK-108-75.

LEACH, EDMUND

- 1954 Political Systems of Highland Burma. Boston: Beacon Press.

LORRAINE, FRANCOIS AND HARRISON WHITE

- 1971 Structural Equivalence of Individuals in Social Networks. *Journal of Mathematical Sociology* 1:49-80.

MAKI, DANIEL AND MAYNARD THOMPSON

- 1974 Mathematical Models and Applications. Englewood-Cliffs, N.J.: Prentice-Hall.

MANGIN, WILLIAM

- 1965 The Role of Regional Associations in the Adaptation of Rural Migrants to Cities in Peru. Pp. 311-23 in *Contemporary Cultures and Societies of Latin America* (ed. by D. B. Heath and R. N. Adams). New York: Random House.

- 1970 Peasants in Cities. Boston: Houghton-Mifflin Co.

MIDDLETON, JOHN AND DAVID TAIT

- 1958 Tribes Without Rulers. New York: Humanities Press.

MITCHELL, CLYDE

- 1969 Social Networks in Urban Situations. Manchester: Manchester University Press.

NELSON, H. E.

- 1971 Disease Demography and Evolution of the Social Structure in Highland New Guinea. *Journal of the Polynesia Society* 80:204-16.

PETERS, EMERY

- 1960 The Proliferations of Segments in the Lineages of the Bedouin of Cyrenaica. *Journal of the Royal Anthropological Institute* 90:29-53.

RAPPAPORT, ROY

- 1969 Population Dispersal and Land Redistribution Among the Maring of New Guinea. Pp. 113-27 in *Ecological Essays*, National Museum of Canada Bulletin 230 (ed. by D. Damas) Ottawa: National Museum of Canada.

REAY, MARIE

- 1967 Structural Co-variance of Land Shortages Among Patrilineal Peoples. *Anthropological Forum* 2:4-19.

SMILLIE, K. W.

- 1969 STATPACK 2: An APL Statistical Package. University of Alberta Dept. of Computing Sciences publication 17, Edmonton, Alberta.

WOLFE, ALVIN AND NORMAN WHITTEN

1974 Network Analysis. Pp. 717–46 in *Handbook of Social and Cultural Anthropology* (ed. by J. Honnigman). New York: Rand McNally.

ZACHARY, WAYNE

1977 Elements of Network Flow Theory: A Review. In *Mathematics of the Cultural Sciences* (ed. by Paul Ballonoff). In press.

DEPARTMENT OF ANTHROPOLOGY

TEMPLE UNIVERSITY

PHILADELPHIA, PA. 19122