Fast Fourier Transform

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What Is An FFT?

- FFT stands for Fast Fourier Transform
- FFT is an algorithm to compute the DFT of a sequnce.
- Fast Fourier Transforms are widely used in engineering, science, and mathematics

DFT

Fourier Series

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{2\pi ikn/N} \quad n \in \mathbb{Z}$$

Discrete Fourier Transform

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi ikn/N} \quad k \in \mathbb{Z}$$

Matrix Representation

An N-point DFT is expressed as the multiplication $X = Wx$, where x is the original input signal, W is the $N \times N$ square DFT matrix, and X is the DFT of the signal.

$$W = \left(\frac{\omega^{jk}}{\sqrt{N}}\right)_{j,k=0,1,\ldots,N-1} = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{(N-1)} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

where $\omega = e^{-2\pi i/N}$.

How Does FFT Work?

Wx is of order $O(N^2)$ and FFT utilizes the symmetry of this matrix to reduce the time of multiplication to $O(NlogN)$

History and Importance

James Cooley and John Tuckey are credited for the invention of the FFT algorithm.

James Cooley

John Tukey

FFT Libraries

- In python one can use numpy.fft.fft().
- In MATLAB just need to use fft() or fftw().
- There are various libraries available in C but probably the most popular one is fftw. The fftw package was developed at MIT by Matteo Frigo and Steven G. Johnson.

FFTW in C

Example

```
#include <fftw3.h>
...
{
fftw_complex *in, *out;
fftw_plan p;
...
in = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
out = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
p = fftw_plan_dft_1d(N,in,out,FFTW_FORWARD,FFTW_ESTIMATE);
...
fftw_execute(p);
...
fftw_destroy_plan(p);
fftw_free(in); fftw_free(out);
```

References

1. https://en.wikipedia.org/wiki/Fast_Fourier_transform
2. https://en.wikipedia.org/wiki/Discrete_Fourier_transform
3. http://www.fftw.org
4. https://www.mathworks.com/help/matlab/ref/fft.html
5. https://docs.scipy.org/doc/numpy/reference/generated/numpy.fft.fft.html

The End