CS159 - Assignment 2a

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February 2, 2023

1.

$$P(a) = \frac{6}{12} = 0.5$$
$$P(b) = \frac{5}{12} = 0.417$$
$$P(c) = \frac{1}{12} = 0.083$$

2.

$$P(a|a) = \frac{1}{5} = 0.2$$
$$P(b|a) = \frac{3}{5} = 0.6$$
$$P(c|a) = \frac{1}{5} = 0.2$$
$$P(a|b) = \frac{3}{5} = 0.6 \quad 2/3$$
$$P(b|b) = \frac{2}{5} = 0.4 \quad 1/3$$
$$P(c|b) = \frac{0}{5} = 0$$
$$P(a|c) = \frac{1}{1} = 1$$
$$P(b|c) = \frac{0}{1} = 0$$
$$P(c|c) = \frac{0}{1} = 0$$

3.

$$P(a|a) = \frac{2}{8} = 0.25$$
$$P(b|a) = \frac{4}{8} = 0.5$$
$$P(c|a) = \frac{2}{8} = 0.25$$
$$P(a|b) = \frac{4}{8} = 0.5 \quad 3/6$$
$$P(b|b) = \frac{3}{8} = 0.375 \quad 2/6$$
$$P(c|b) = \frac{1}{8} = 0.125 \quad 1/6$$
$$P(a|c) = \frac{2}{4} = 0.5$$
$$P(b|c) = \frac{1}{4} = 0.25$$
$$P(c|c) = \frac{1}{4} = 0.25$$

4.

<UNK>	a	<UNK>	b
a	<UNK>	a	b
b	a	b	a

5. Unigrams:

$$P(a) = \frac{5}{12} = 0.417$$
$$P(b) = \frac{4}{12} = 0.333$$
$$P(\langle \text{UNK} \rangle) = \frac{3}{12} = 0.25$$

Bigrams:

$$P(a|a) = \frac{0}{4} = 0$$
$$P(b|a) = \frac{2}{4} = 0.5$$
$$P(\langle \text{UNK} \rangle|a) = \frac{2}{4} = 0.5$$
$$P(a|b) = \frac{3}{4} = 0.75$$
$$P(b|b) = \frac{1}{4} = 0.25$$
$$P(\langle \text{UNK} \rangle|b) = \frac{0}{4} = 0$$
$$P(a|\langle \text{UNK} \rangle) = \frac{2}{3} = 0.667$$
$$P(b|\langle \text{UNK} \rangle) = \frac{1}{3} = 0.333$$
$$P(\langle \text{UNK} \rangle|\langle \text{UNK} \rangle) = \frac{0}{3} = 0.333$$

6. α calculations:

$$reserved\_mass(a) = \frac{2 \times 0.5}{4} = \frac{1}{4}$$
$$1 - \sum_{X:C(AX)>0} p(X) = 1 - \left(\frac{4}{12} + \frac{3}{12}\right) = \frac{5}{12}$$
$$\alpha(a) = \frac{1/4}{5/12} = \frac{3}{5}$$

$$reserved\_mass(b) = \frac{2 \times 0.5}{4} = \frac{1}{4}$$
$$1 - \sum_{X:C(BX)>0} p(X) = 1 - \left(\frac{5}{12} + \frac{4}{12}\right) = \frac{1}{4}$$
$$\alpha(b) = \frac{1/4}{1/4} = 1$$

$$reserved\_mass(\langle \text{UNK} \rangle) = \frac{2 \times 0.5}{3} = \frac{1}{3}$$
$$1 - \sum_{X:C(\langle \text{UNK} \rangle X)>0} p(X) = 1 - \left(\frac{5}{12} + \frac{4}{12}\right) = \frac{1}{4}$$
$$\alpha(\langle \text{UNK} \rangle) = \frac{1/3}{1/4} = \frac{4}{3}$$

Smoothed bigram calculations:

$$
\begin{aligned}
P(a|a) &= \frac{3}{5} \times \frac{5}{12} = \frac{1}{4} \\
P(b|a) &= \frac{2-0.5}{4} = \frac{3}{8} \\
P(\langle \text{UNK} \rangle | a) &= \frac{2-0.5}{4} = \frac{3}{8} \\
P(a|b) &= \frac{3-0.5}{4} = \frac{5}{8} \\
P(b|b) &= \frac{1-0.5}{4} = \frac{1}{8} \\
P(\langle \text{UNK} \rangle | b) &= 1 \times \frac{1}{4} = \frac{1}{4} \\
P(a|\langle \text{UNK} \rangle) &= \frac{2-0.5}{3} = \frac{1}{2} \\
P(b|\langle \text{UNK} \rangle) &= \frac{1-0.5}{3} = \frac{1}{6} \\
P(\langle \text{UNK} \rangle | \langle \text{UNK} \rangle) &= \frac{4}{3} \times \frac{1}{4} = \frac{1}{3}
\end{aligned}
$$