A Taste of Computational Mathematics

Introduction to Python and Digital Image Processing

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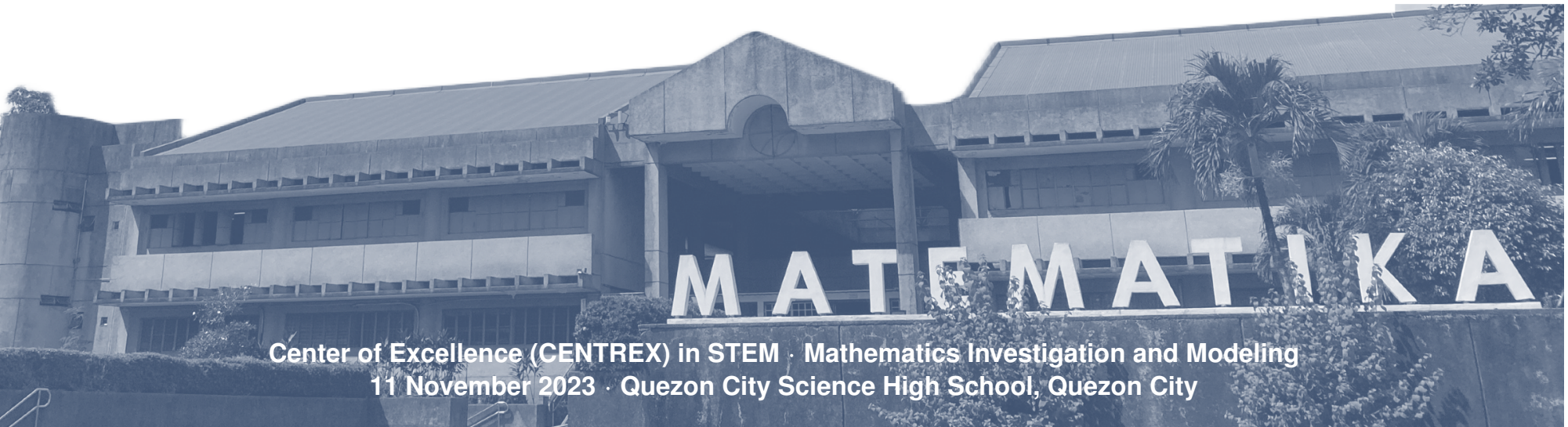

Center of Excellence (CENTREX) in STEM · Mathematics Investigation and Modeling
11 November 2023 · Quezon City Science High School, Quezon City

Mathematical Problem

Find x.

Mathematical Problem

Find x such that

$$\varphi(x; d) = 0$$

d, set of data which the solution depends on

φ, functional relation between x and d

Mathematical Problem

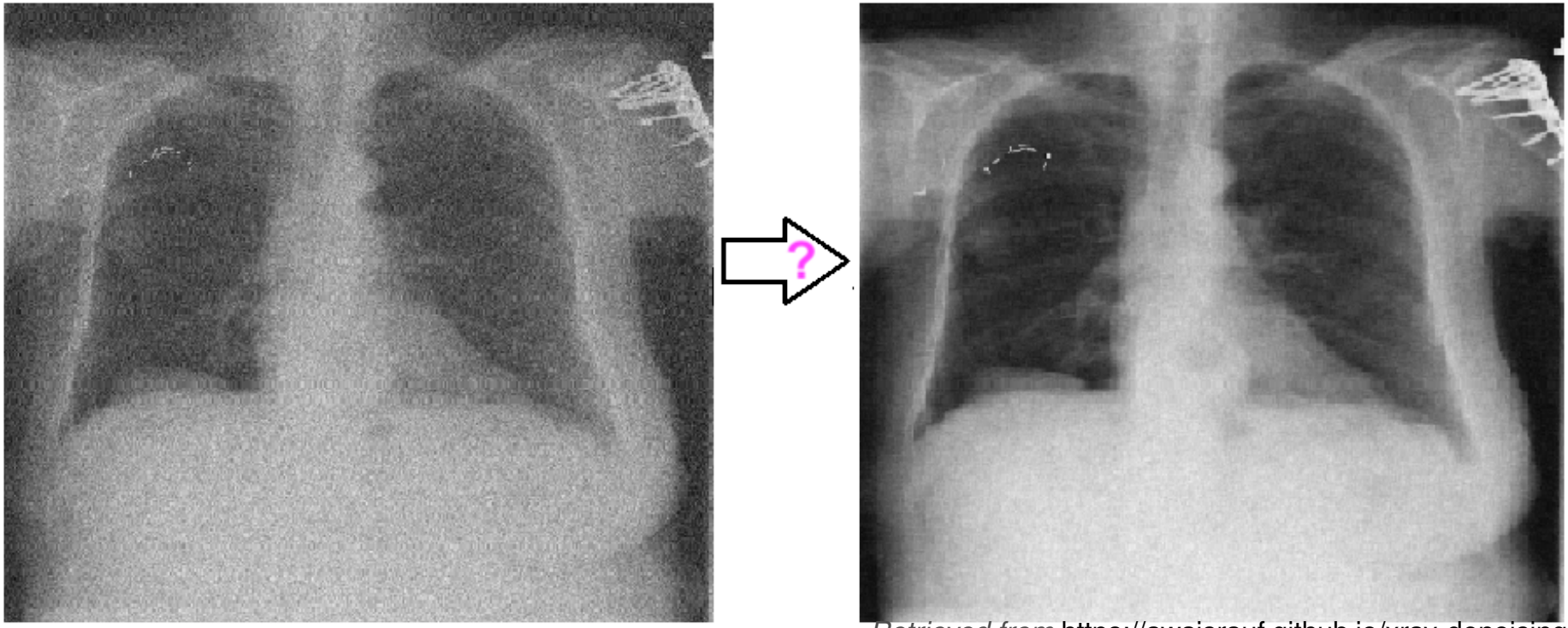

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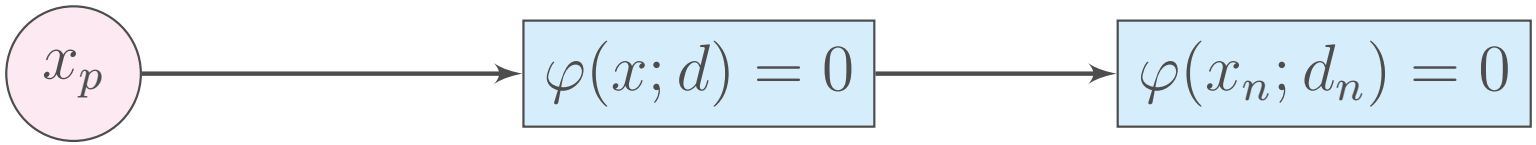
image input u_0 $\rightarrow$ image processor T $\rightarrow$ desired output F

Find T such that

$$T(u_0) = F$$

Computational Model

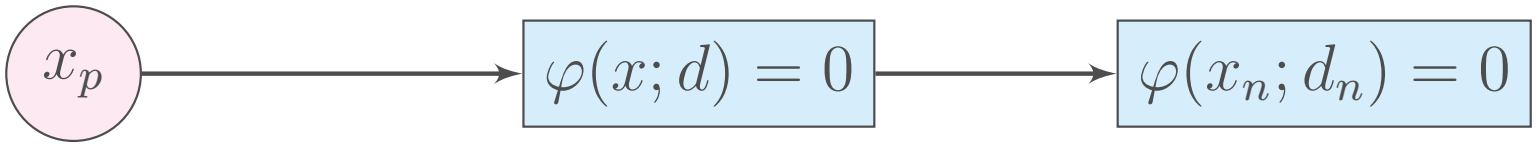
Physical Problem $\rightarrow$ Mathematical Problem $\rightarrow$ Computational Model



Mathematical Investigation *Modeling*

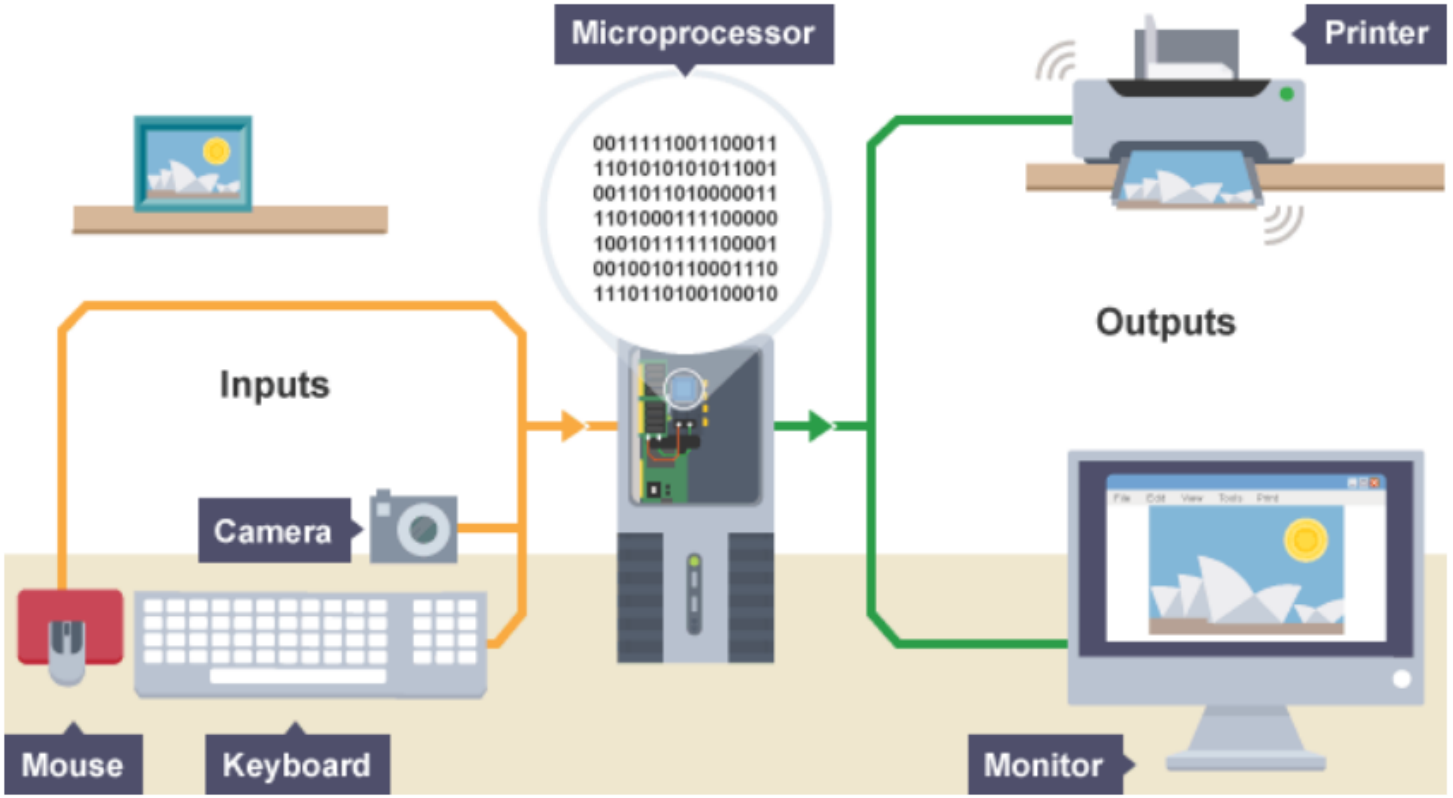
Computational Model

Physical Problem $\rightarrow$ Mathematical Problem $\rightarrow$ Computational Model



- Numerical analysis - a branch of Mathematics that is concerned with the development and investigation of constructive methods for the numerical solution of mathematical problems

Binary Machine Numbers



- circuits in a computer's processor are made up of billions of transistors – tiny switches activated by electronic signals
- on and off states of a transistor are reflected by the binary digits (bits): 1 and 0 – the smallest unit of data in computing

Binary Machine Numbers

- Numbers are stored in memory in bits – long strings of 0s and 1

$$9 \div 2 = 4 \text{ remainder } 1$$
$$4 \div 2 = 2 \text{ remainder } 0$$
$$2 \div 2 = 1 \text{ remainder } 0$$
$$1 \div 2 = 0 \text{ remainder } 1$$

$$9_{10} = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1001_2$$

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$$156_{10} = 10011100_2$$

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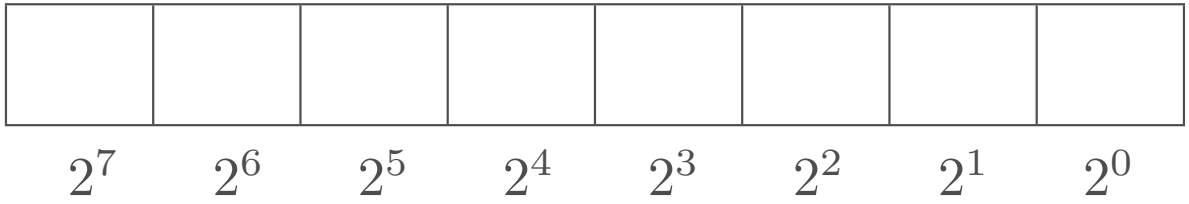

Binary Machine Numbers

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1	1	1	1	1	1	1	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

that is, the largest value it can take will be 255.

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$$\begin{array}{rcl} 255 & \Longrightarrow & 11111111 \\ +\ \ 1 & \Longrightarrow & +\ \ \ \ \ \ \ 1 \\ \hline & & 100000000 \end{array} \quad \Longrightarrow \quad 256$$

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1	0	0	0	0	0	0	0	0

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1	0	0	0	0	0	0	0	0

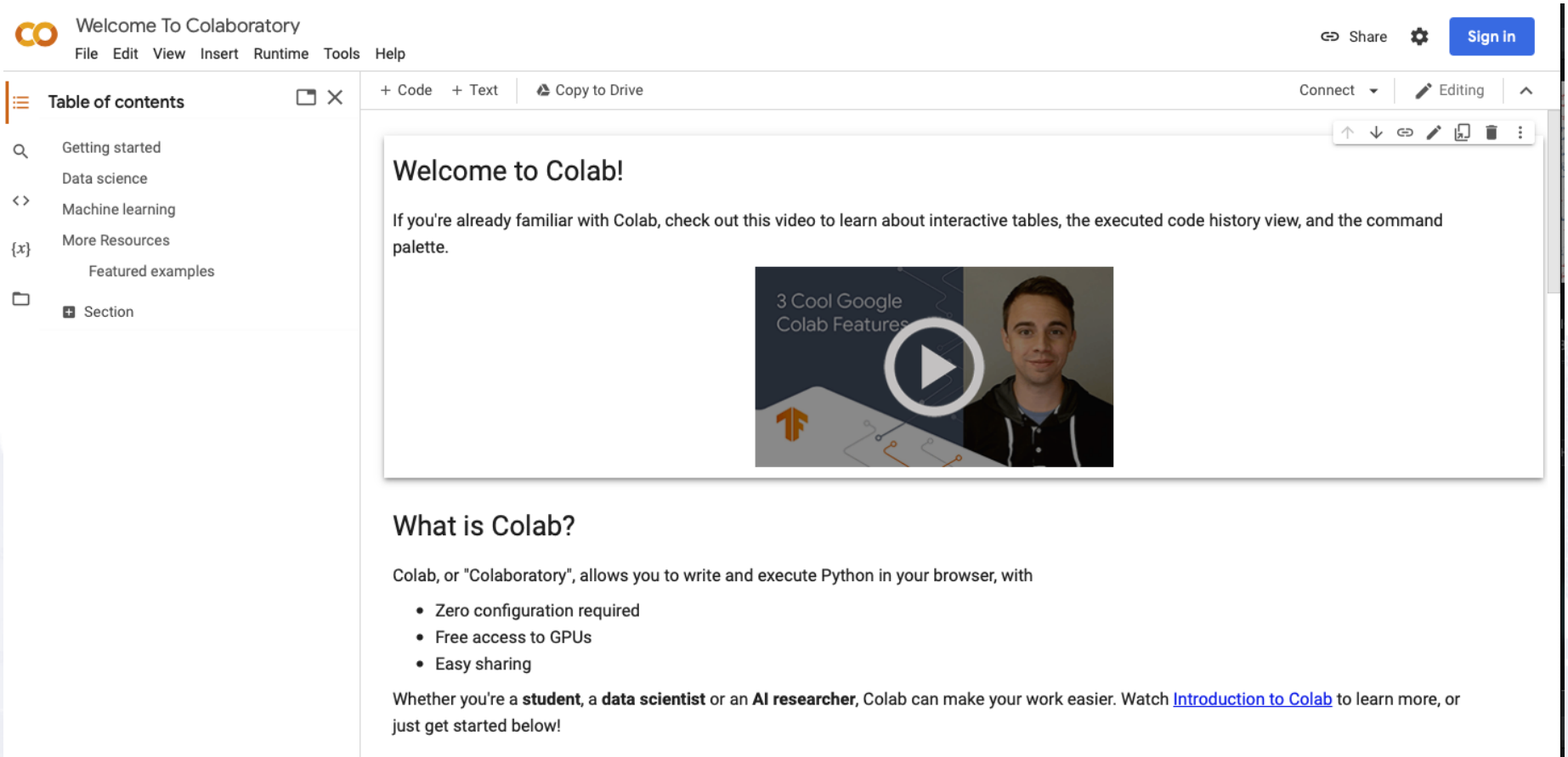
- CPU drops the overflow digit since the computer cannot store it.

Python Basics

- There are two types:
  - minimize human coding time (e.g. Python, Julia, R, MATLAB)
  - minimize computing time (e.g. C, C++, Fortran)

- Open and activate your DepEd Gmail account on

  `https://colab.research.google.com`



Python Basics

Open the following link

`https://github.com/rhudaina/CENTREX-for-STEM-DepEd-QCSHS`

in your browser.

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Matrices and NumPy

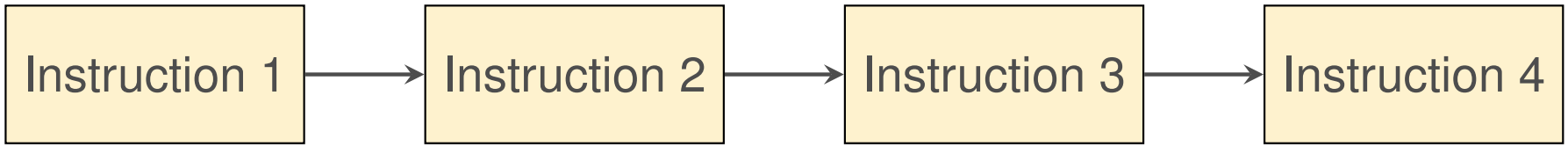
- A matrix is a 2-dimensional array of numbers arranged in rows and columns.

$$A = \begin{bmatrix} 7 & -2 & 1 & 1 \\ 3 & 0 & 0 & -1 \\ -1 & 9 & 0 & 0 \\ 1 & 5 & -1 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & 0 & 5 \end{bmatrix}$$

  Note that matrix A has 6 rows and 5 columns.

Sequence

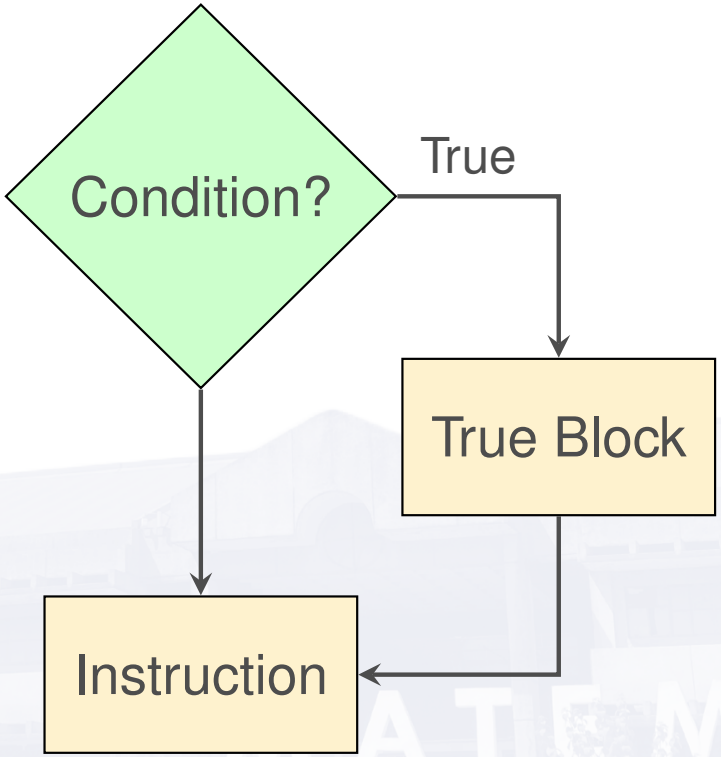
- Sequence expresses the trivial idea that unless you direct it otherwise, the computer code is to be implemented one instruction at a time



Selection

- Single-alternative decision

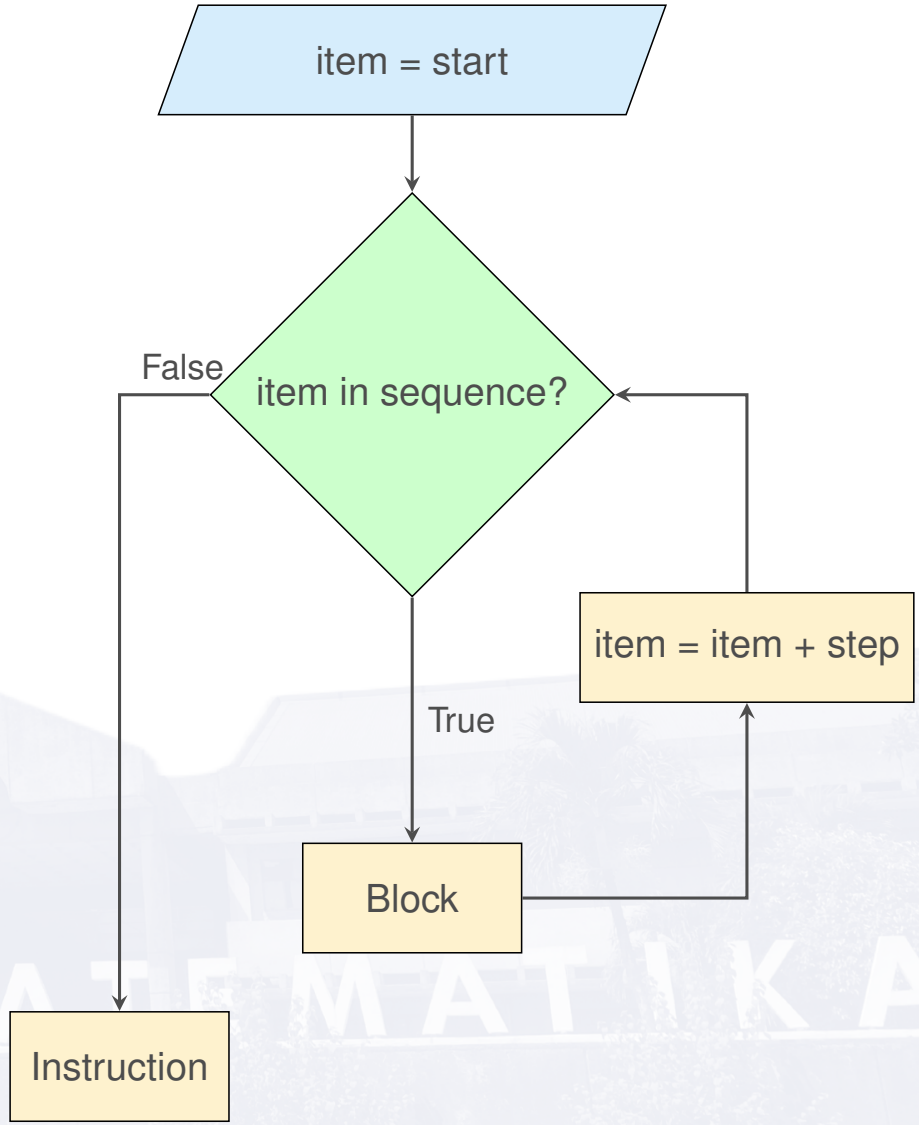
```
1 if condition :
2     TRUE block
```



Repetition

- Count-controlled loop performs a specified number of repetitions or iterations

```
1 for item in sequence:
2     Block
```



Introduction to Image Processing

Often, images may have been degraded due to either poor imaging conditions or problems during storage and communication

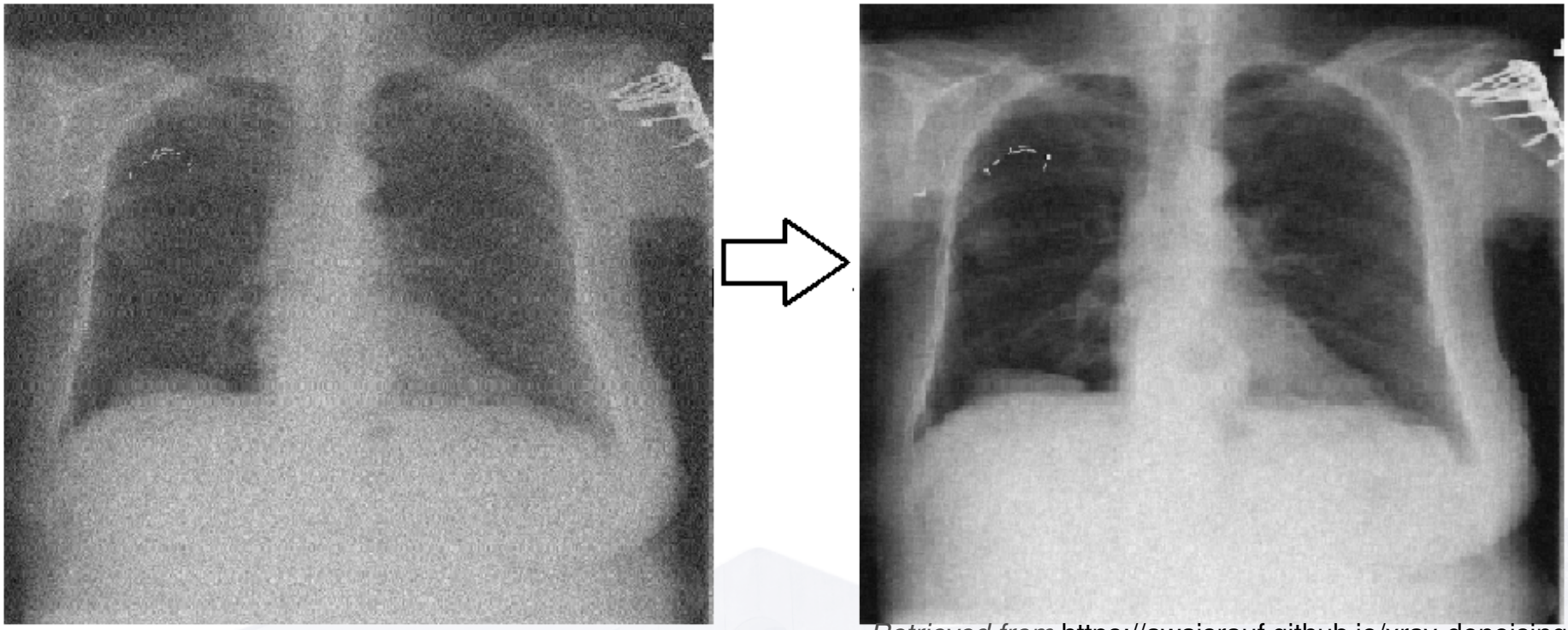


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image input u_0 $\rightarrow$ image processor T $\rightarrow$ output $F = T[u_0]$.

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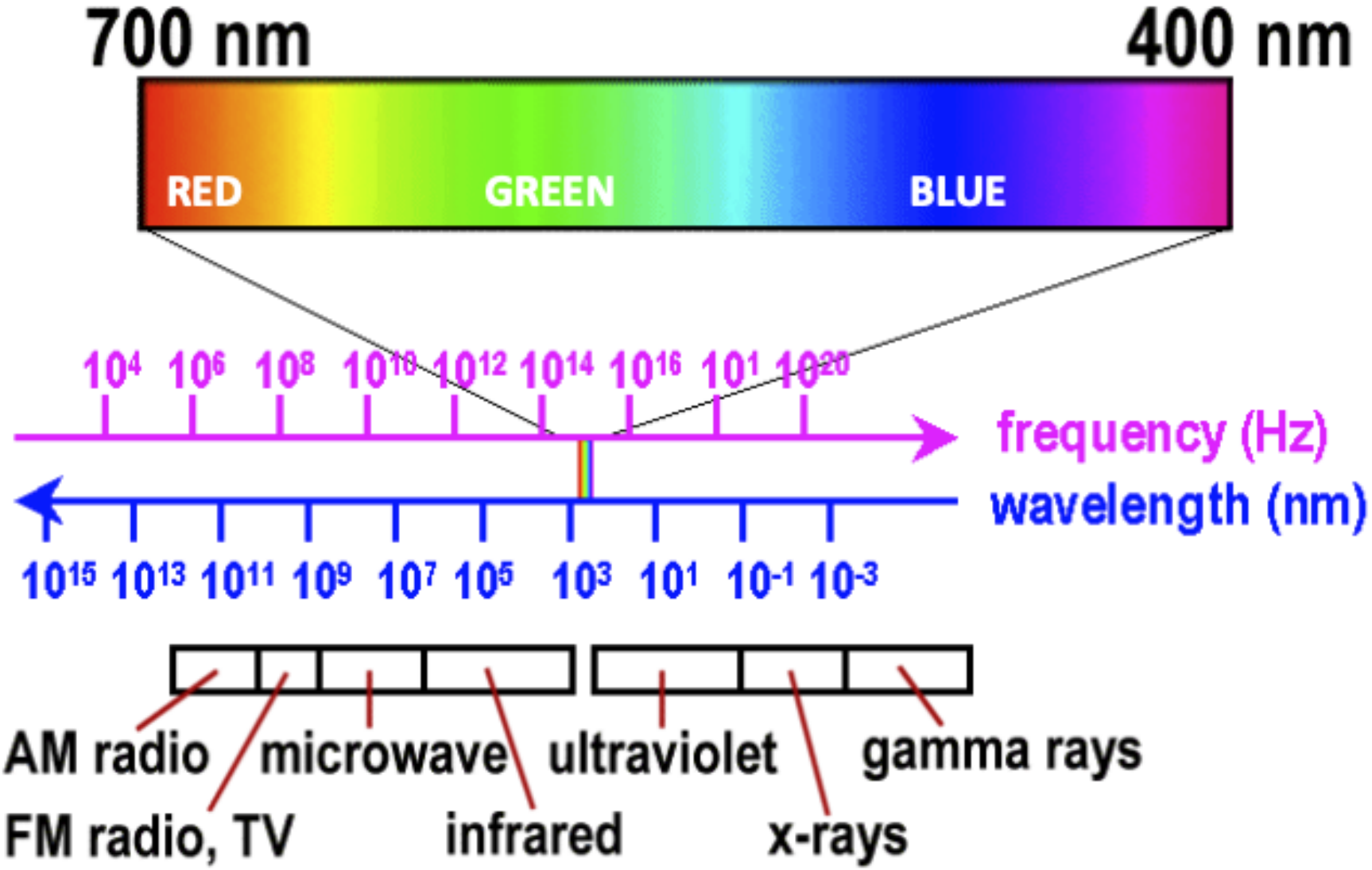
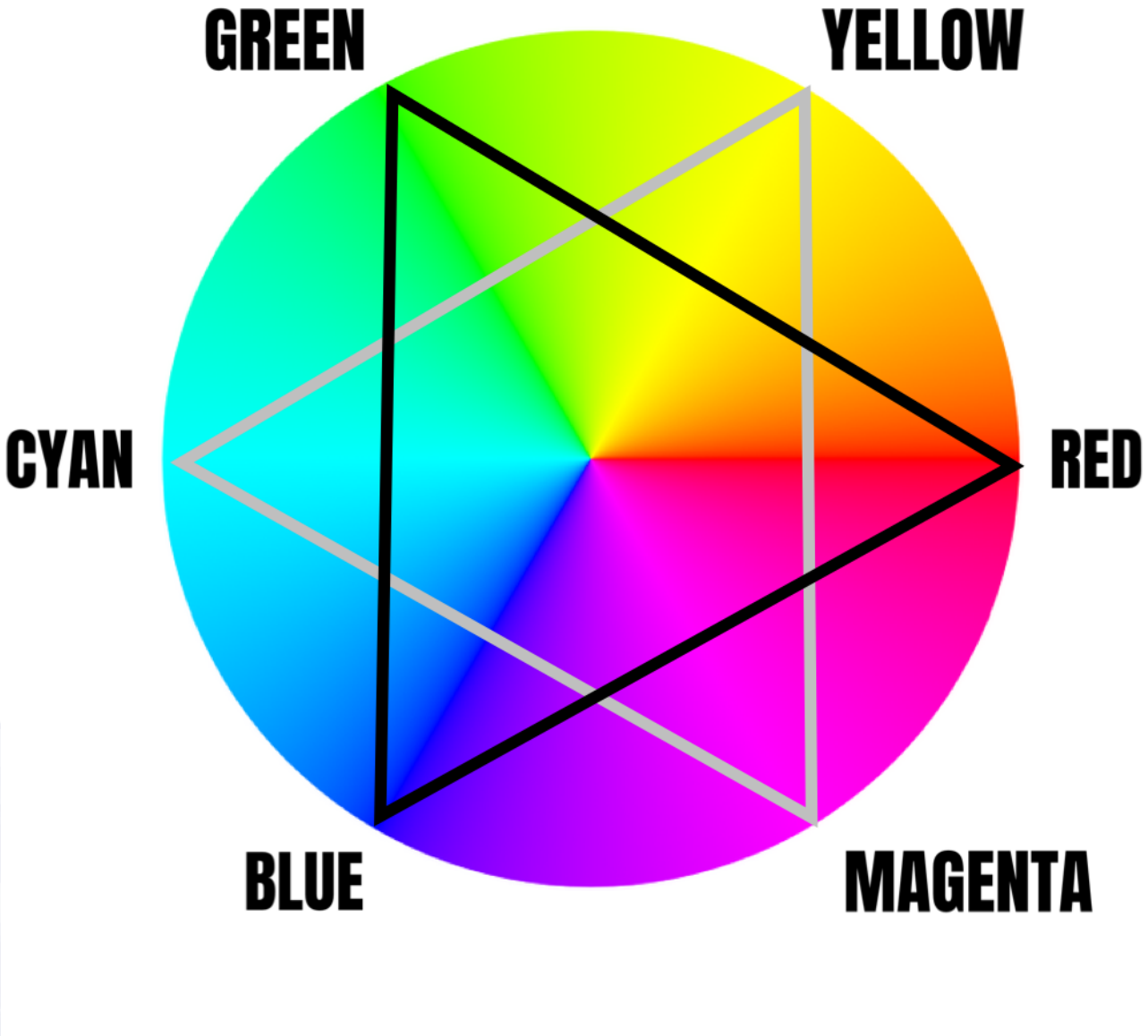
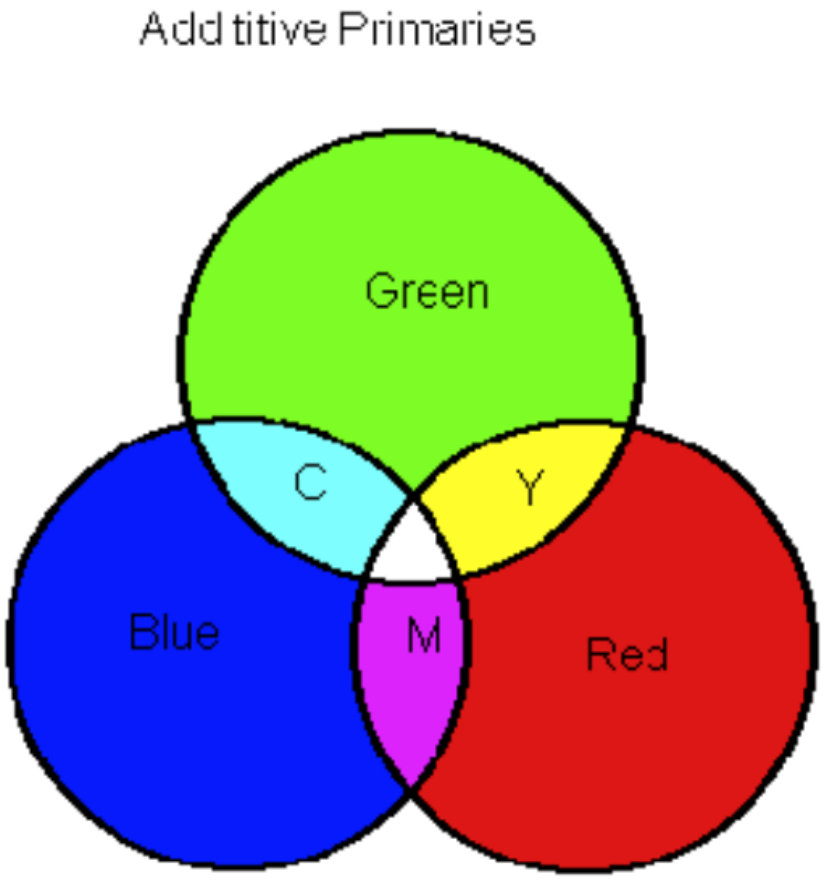
Visible Spectrum



Color Wheel



Primary Colors



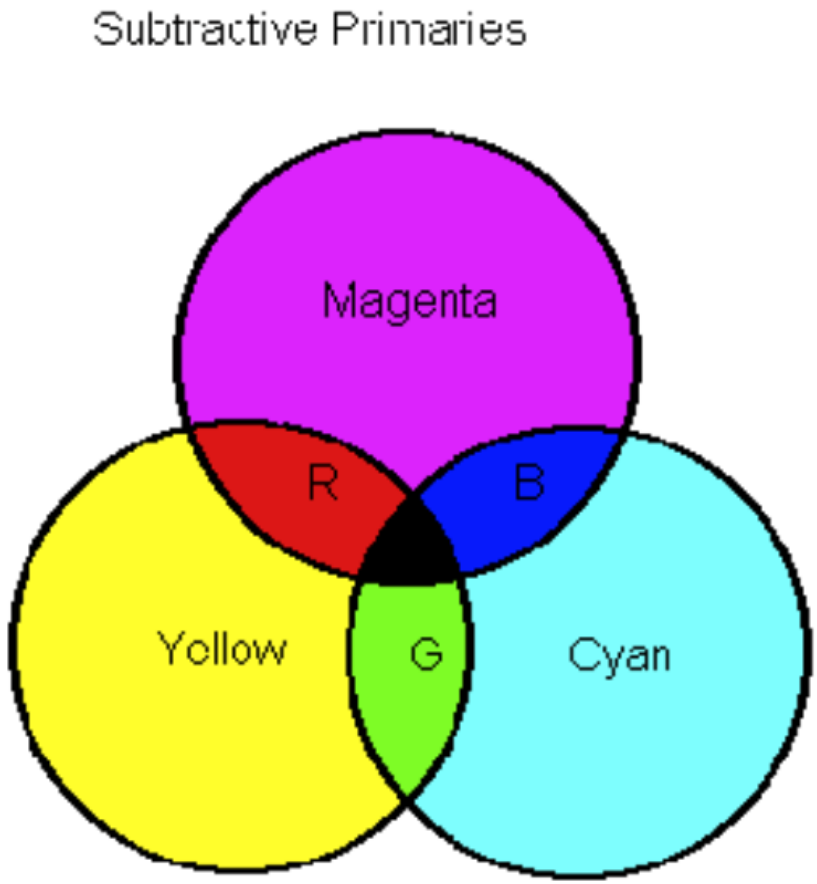


Image Denoising and Heat Equation

Heat flows from the warmer body to the cooler body until they reach the same temperature.

Image Denoising and Heat Equation

Heat flows from the warmer body to the cooler body until they reach the same temperature.

- Let u be temperature of a system at point (x, y) at time t

$$u = u(t, x, y)$$

- Heat (or diffusion) equation models how heat spreads in a body/system

$$\underbrace{u_t}_{\text{how } u \text{ changes in time}} = \underbrace{u_{xx} + u_{yy}}_{\text{how } u \text{ changes in space}}$$

  - u_t, first partial derivative
  - u_{xx} and u_{yy}, second partial derivatives

All routes to

STEM

Science, Technology, Engineering, and Mathematics

involves

CALCULUS

study of change (motion)

Thank you for your attention!