

Parsing Algorithms	
Input	1. CFG 2. word (w)
Steps	Derivation for w: <ul style="list-style-type: none"> w is in the language No derivation <ul style="list-style-type: none"> w is not in the language
Output	Parse Tree (with leftmost derivation)

Augmented Grammars

Ensures S has only 1 production rule

Augmented Grammar	
S'	$S' \rightarrow \vdash S \dashv$

Predict Table

Leftmost does not eliminate uncertainty in **choice** of rules (only **order**)

Predict[A][a] : given a non-terminal A and a lookahead terminal a, will predict which rule to choose for A

- ① $S' \rightarrow \vdash S \dashv$
- ② $S \rightarrow b S d$
- ③ $S \rightarrow p S q$
- ④ $S \rightarrow c$
- ⑤ $c \rightarrow 1 c$
- ⑥ $c \rightarrow \varepsilon$

Nullable(A)	$A \Rightarrow^* \epsilon$	<table><tr><th>Iteration</th><th>0</th><th>1</th><th>2</th><th>3</th><th></th></tr><tr><td>S'</td><td>F</td><td>F</td><td>F</td><td>F</td><td>$S' \rightarrow \tau S \mid \rightarrow \tau C \mid \rightarrow \tau \epsilon \Rightarrow$</td></tr><tr><td>S</td><td>F</td><td>F</td><td>T</td><td>T</td><td>$S \rightarrow C \rightarrow \epsilon$</td></tr><tr><td>C</td><td>F</td><td>T</td><td>T</td><td>T</td><td>$C \rightarrow \epsilon$</td></tr><tr><td></td><td>↑ Start all false</td><td>↑ directly</td><td>↑ 2 rules</td><td>↑ 3 rules</td><td></td></tr></table>	Iteration	0	1	2	3		S'	F	F	F	F	$S' \rightarrow \tau S \mid \rightarrow \tau C \mid \rightarrow \tau \epsilon \Rightarrow$	S	F	F	T	T	$S \rightarrow C \rightarrow \epsilon$	C	F	T	T	T	$C \rightarrow \epsilon$		↑ Start all false	↑ directly	↑ 2 rules	↑ 3 rules			
Iteration	0	1	2	3																														
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	↑ Start all false	↑ directly	↑ 2 rules	↑ 3 rules																														
First(A)	<p>Look at the RHS of rule: $A \rightarrow BCD \dots$</p> <p>B is a terminal: add B</p> <p>Else: add First(B)</p> <p>If B is nullable: add First(C)</p>	<table><tr><th>Iteration</th><th>0</th><th>1</th><th>2</th></tr><tr><td>S'</td><td>{ }</td><td>{ t }</td><td>{ t }</td></tr><tr><td>S</td><td>{ }</td><td>{ b, p }</td><td>{ b, p, 1 }</td></tr><tr><td>C</td><td>{ }</td><td>{ 1 }</td><td>{ 1 }</td></tr><tr><td></td><td>↑ Start</td><td>↑ directly</td><td>↑ 2 rules</td></tr></table>	Iteration	0	1	2	S'	{ }	{ t }	{ t }	S	{ }	{ b, p }	{ b, p, 1 }	C	{ }	{ 1 }	{ 1 }		↑ Start	↑ directly	↑ 2 rules												
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C	{ }	{ 1 }	{ 1 }																															
	↑ Start	↑ directly	↑ 2 rules																															
Follow(A)	<p>Look at the RHS of rule: $C \rightarrow \dots A \dots$</p> <p>$C \rightarrow \dots A \dots$</p> <p>① $C \rightarrow \dots A a$ (add <i>a</i>)</p> <p>② $C \rightarrow \underbrace{AB_1B_2B_3}_\text{nullable} D \parallel AD$ (add First(<i>D</i>))</p> <p>③ $C \rightarrow \underbrace{AB_1B_2B_3}_\text{nullable} \parallel \dots A$ (add Follow(<i>C</i>))</p>	<table><tr><th>Iteration</th><th>0</th><th>1</th></tr><tr><td>S</td><td>{ }</td><td>{ t, d, t }</td></tr><tr><td>C</td><td>{ }</td><td>{ t, d, t }</td></tr><tr><td></td><td>↑ Start</td><td></td></tr></table>	Iteration	0	1	S	{ }	{ t, d, t }	C	{ }	{ t, d, t }		↑ Start																					
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	↑ Start																																	
Predict Table	<p>For Each rule: $A \Rightarrow B$</p> <p>1) add First(B): add the rule</p> <p>2) If Nullable(B)</p> <p> a) add Follow(A)</p>	<table><tr><th></th><th>τ</th><th>\neg</th><th>b</th><th>d</th><th>p</th><th>a</th><th>1</th></tr><tr><td>S'</td><td>1</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>S</td><td></td><td>4</td><td>2</td><td>4</td><td>3</td><td>4</td><td>4</td></tr><tr><td>C</td><td></td><td>6</td><td></td><td>6</td><td></td><td>6</td><td>5</td></tr></table>		τ	\neg	b	d	p	a	1	S'	1							S		4	2	4	3	4	4	C		6		6		6	5
	τ	\neg	b	d	p	a	1																											
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C		6		6		6	5																											

LL(1) Top-Down Parsing Algorithm

A grammar is LL(1) \Leftrightarrow each cell of the Predict table contains at most 1 rule

L Left-to-Right scan of input

L Leftmost Derivations

1 lookahead 1 symbol of input (for picking a rule)

```
push S'

for(each x in input)
{
  while(TOS is non-terminal)
  {
    rule # = predict[TOS][x]
    TOS.pop
    TOS.push rule

    ERROR if more than 1 rule
  }

  if(TOS != next input input) ERROR
  else
    pop
    pop (from input)
}
accept //stack is empty
```

2 ways to get **ERROR**

1. TOS is a terminal, does not match the next input symbol
2. Predict[A][a] = 0 or 2+ rules

Example 1

Predict Table:

- 1) $S' \rightarrow \vdash S \vdash$
- 2) $S \rightarrow AyB$
- 3) $A \rightarrow ab$
- 4) $A \rightarrow cd$
- 5) $B \rightarrow z$
- 6) $B \rightarrow wx$

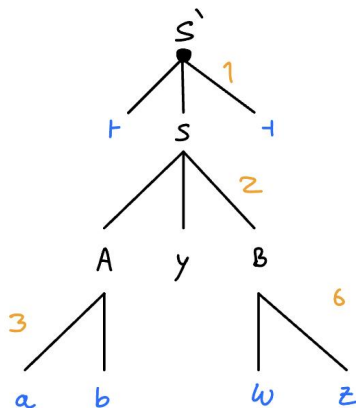
	\vdash	\vdash	y	a	b	c	d	w	x	z
S'	1									
S				2		2				
A				3		4				
B					5			6		

① push $S' \rightarrow \vdash S \vdash$

		TOS			
Read	Unread	Stack	non-terminal	terminal	Action
ϵ	$\vdash abywx \vdash$	$\vdash S \vdash$			
ϵ	$\vdash abywx \vdash$	$\vdash S \vdash$		\vdash matches \vdash	\rightarrow pop read
\vdash	$abywx \vdash$	$S \vdash$	$\text{predict}[S][a] = 2$		\rightarrow pop push(B,y,A)
\vdash	$abywx \vdash$	$AyB \vdash$	$\text{predict}[A][a] = 3$		\rightarrow pop push(b,a)
\vdash	$abywx \vdash$	$aByB \vdash$		a matches a	\rightarrow pop read
$\vdash a$	$bywx \vdash$	$bByB \vdash$		b matches b	\rightarrow pop read
$\vdash ab$	$ywx \vdash$	$yB \vdash$		y matches y	\rightarrow pop read
$\vdash aby$	$wx \vdash$	$B \vdash$	$\text{predict}[B][w] = 6$		\rightarrow pop push(x,w)
$\vdash aby$	$wx \vdash$	$wx \vdash$		w matches w	\rightarrow pop read
$\vdash abyw$	$x \vdash$	$x \vdash$		x matches x	\rightarrow pop read
$\vdash abywx$	\vdash	\vdash		\vdash matches \vdash	

Resulting Parse Tree:

Sequence of rules: 1, 2, 3, 6



Example 2:

Predict Table:

- 1) $S' \rightarrow TS \mid$
- 2) $S \rightarrow Tz$
- 3) $z \rightarrow +Tz$
- 4) $z \rightarrow \epsilon$
- 5) $T \rightarrow FT'$
- 6) $T' \rightarrow *FT'$
- 7) $T' \rightarrow \epsilon$
- 8) $F \rightarrow a|b|c$

Predict:

	T		+	*	a	b	c
S'	1						
S					2	2	2
z		4	3				
T					5	5	5
T'		7	7	6			
F					8	8	8

Nullable

S'	F
S'	F
S	F
z	T
T	F
T'	T
F	F

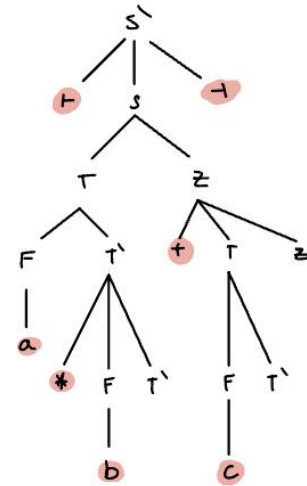
First

S'	T
S'	a, b, c
z	+
T	a, b, c
T'	*
F	a, b, c

Follow

S'	\emptyset
S'	
z	
T	+,
T'	+,
F	*, +,

Read	Stack	Action
ϵ	S'	
T	S'	predict[+][S'] = 1
T	$TS \mid$	match(T)
T	$S \mid$	predict[a][S] = 2
Ta	$Tz \mid$	predict[a][T] = 5
Ta	$FT'z \mid$	predict[a][F] = 8
Ta	$aT'z \mid$	match(a)
Taa	$T'z \mid$	predict[*][T'] = 6
Taa	$*FT'z \mid$	match(*)
Taab	$FT'z \mid$	predict[b][F] = 8
Taab	$bT'z \mid$	match(b)
Taab+	$T'z \mid$	predict[+][T'] = 7
Taab+	$z \mid$	predict[+][z] = 3
Taab+	$+Tz \mid$	match(+)
Taab+c	$Tz \mid$	predict[c][T] = 5
Taab+c	$FT'z \mid$	predict[c][F] = 8
Taab+c	$cT'z \mid$	match(c)
Taab+c	$T'z \mid$	predict[][T'] = 7
Taab+c	$z \mid$	predict[][z] = 4
Taab+c	\mid	match()
Taab+c	\emptyset	



Notes on LL1:

1. We push the symbols in reverse so that the leftmost symbol ends up on TOS. This ensures that the first non-terminal we encounter is the leftmost non-terminal.
2. At any point: current derived string = Read-Input + Stack

Limitations

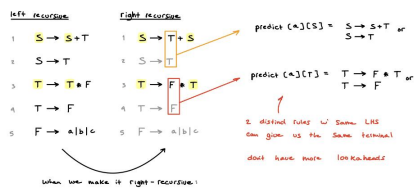
1. Only 1 rule in the predict table:
2. If 'A' is nullable, it can't have the same terminal in: $\text{first}(A)$ and $\text{follow}(A)$
3. Only 1 way to derive ϵ from a nullable symbol

Left-Associative Grammars are **never** LL(1)

Right-Associative Grammars **can be** LL(1)

Solution

1. We must **at least** make it right recursive
2. Factor **if needed**
3. Add precedence **if needed**



We could increase the lookahead: LL(k)

but the predict tables is far more complex

Left Factoring

$S \rightarrow \alpha T_1$
 $S \rightarrow \alpha T_2$
 $S \rightarrow \alpha T_3$

Factor $T \rightarrow T_1 | T_2 | T_3$

