

Semantic Relations Among Propositions:

- $\alpha \models \beta$
- 1) $\forall S : \overline{S}(\alpha) = T \rightarrow \overline{S}(\beta) = T$
 - 2) $\forall S : \overline{S}(\alpha \rightarrow \beta) = T$
 - 3) $\alpha \rightarrow \beta$ is a tautology

$\alpha \equiv \beta$ $\alpha \models \beta \wedge \beta \models \alpha$

Satisfiable $\iff \exists \alpha : \Sigma \not\models \alpha$

\neg Satisfiable $\iff \forall \alpha : \Sigma \models \alpha$

Complete $\forall \alpha : [\Sigma \models \alpha] \vee [\Sigma \models \neg \alpha]$

\neg Consistent $\iff \Gamma \vdash \text{contradiction}$

$\Gamma \vdash \alpha \not\Rightarrow \Gamma \cup \{\alpha\}$ Consistent

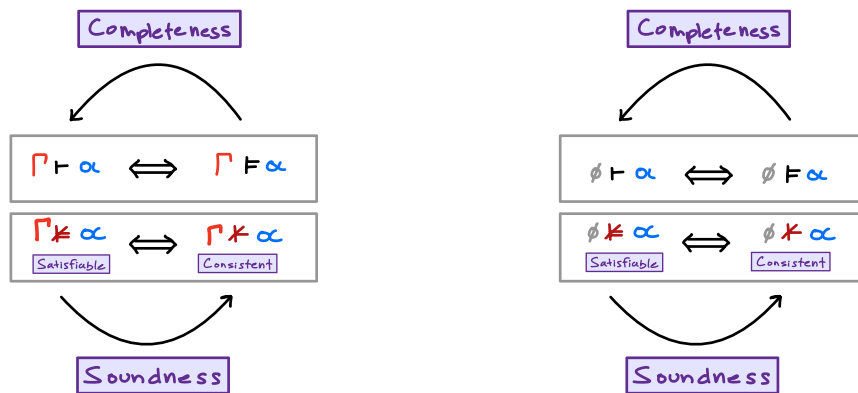
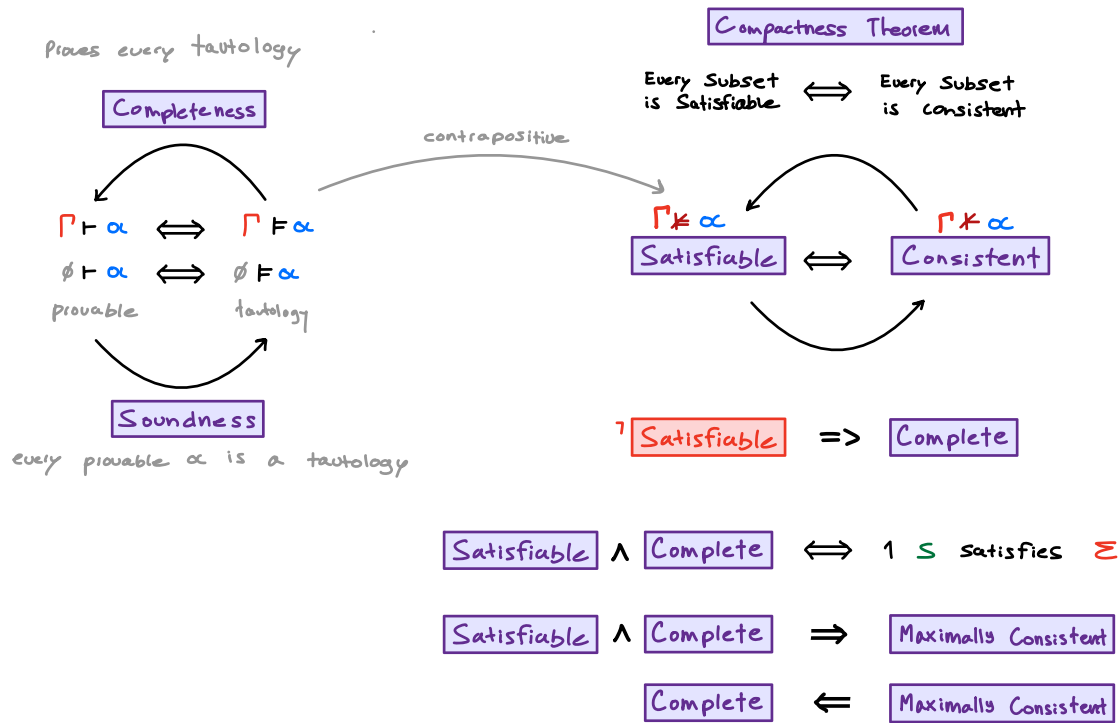
$\Gamma \not\vdash \alpha \Rightarrow \Gamma \cup \{\alpha\} \neg$ Consistent

- Consistent
- $\nexists \alpha$ such that: $[\Gamma \vdash \alpha] \wedge [\Gamma \vdash \neg \alpha]$
 - $\exists \beta : \Gamma \not\vdash \beta$
 - consistent \Rightarrow every subset is consistent

Maximally Consistent proves everything possible (except contradictions)

- 1) $\Sigma \not\vdash \alpha \Rightarrow \Sigma \cup \{\alpha\}$ is inconsistent
 $\Rightarrow \Sigma \cup \{\neg \alpha\}$ is inconsistent
- 2) $\Sigma \vdash \alpha \vee \Sigma \vdash \neg \alpha$ (not both)

Connecting Syntax & Semantics



Monotonicity (More Assumptions = more conclusions)

1) $\Gamma_1 \subseteq \Gamma_2 : \Gamma_1 \vdash \alpha \Rightarrow \Gamma_2 \vdash \alpha$

2) $\Gamma \vdash \alpha \ \forall \alpha \in \Gamma \Rightarrow \{\alpha : \Gamma \vdash \alpha\} = \{\alpha : \Gamma \vdash \alpha\}$

If a set
can prove
everything in
another set

Then the set of things
this set can prove
is greater.