Semantic Relations Among Propositions:

$\boxed{\alpha \models \beta}$

1) $\forall s : \bar{s}(\alpha) = T \rightarrow \bar{s}(\beta) = T$

2) $\forall s : \bar{s}(\alpha \rightarrow \beta) = T$

3) $\alpha \rightarrow \beta$ is a tautology

$\boxed{\alpha \equiv \beta}$ $\alpha \models \beta \wedge \beta \models \alpha$

$\boxed{\text{Satisfiable}} \iff \exists \alpha : \boxed{\Sigma \not\models \alpha}$

$\neg \boxed{\text{Satisfiable}} \iff \forall \alpha : \boxed{\Sigma \models \alpha}$

$\boxed{\text{Complete}}$ $\forall \alpha : [\Sigma \models \alpha] \vee [\Sigma \models \neg\alpha]$

$\neg \boxed{\text{Consistent}} \iff \Gamma \vdash$ contradiction

$\Gamma \vdash \alpha \not\Rightarrow \Gamma \cup \{\alpha\}$ $\boxed{\text{Consistent}}$

$\Gamma \nvdash \alpha \Rightarrow \Gamma \cup \{\alpha\}$ $\neg \boxed{\text{Consistent}}$

$\boxed{\text{Consistent}}$

- $\nexists \alpha$ such that: $[\Gamma \vdash \alpha] \wedge [\Gamma \vdash (\neg\alpha)]$
- $\exists \beta : \Gamma \nvdash \beta$
- consistent $\Rightarrow$ every subset is consistent

$\boxed{\text{Maximally Consistent}}$ proves everything possible (accept contradictions)

1) $\Sigma \nvdash \alpha$ $\Rightarrow \Sigma \cup \{\alpha\}$ is inconsistent
   $\Rightarrow \Sigma \cup \{\neg\alpha\}$ is inconsistent

2) $\Sigma \vdash \alpha \quad \vee \quad \Sigma \vdash (\neg\alpha)$ (not both)

Connecting Syntax & Semantics

Proves every tautology

Completeness

$\Gamma \vdash \alpha \iff \Gamma \vDash \alpha$

$\emptyset \vdash \alpha \iff \emptyset \vDash \alpha$

provable tautology

Soundness

every provable α is a tautology

contrapositive

Compactness Theorem

Every Subset is Satisfiable $\iff$ Every Subset is consistent

$\Gamma \nvDash \alpha$ **Satisfiable** $\iff$ $\Gamma \nvdash \alpha$ **Consistent**

$\neg$ **Satisfiable** $\Rightarrow$ **Complete**

Satisfiable $\wedge$ **Complete** $\iff$ 1 S satisfies Σ

Satisfiable $\wedge$ **Complete** $\Rightarrow$ **Maximally Consistent**

Complete $\Leftarrow$ **Maximally Consistent**

Completeness

$\Gamma \vdash \alpha \iff \Gamma \vDash \alpha$	$\emptyset \vdash \alpha \iff \emptyset \vDash \alpha$
$\Gamma \nvDash \alpha$ (Satisfiable) $\iff$ $\Gamma \nvdash \alpha$ (Consistent)	$\emptyset \nvDash \alpha$ (Satisfiable) $\iff$ $\emptyset \nvdash \alpha$ (Consistent)

Soundness

Monotonicity (More Assumptions = more conclusions)

1) $\Gamma_1 \subseteq \Gamma_2$: $\Gamma_1 \vdash \alpha \Rightarrow \Gamma_2 \vdash \alpha$

2) $\underbrace{\Gamma \vdash \alpha \quad \forall \alpha \in \Gamma}_{\text{If a set can prove everything in another set}} \Rightarrow \underbrace{\{\alpha : \Gamma \vdash \alpha\} \subseteq \{\alpha : \Gamma \vdash \alpha\}}_{\text{Then the set of things this set can prove is greater.}}$