**■ HullRobert-HW-1.md** 

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HW 1, ECE 523

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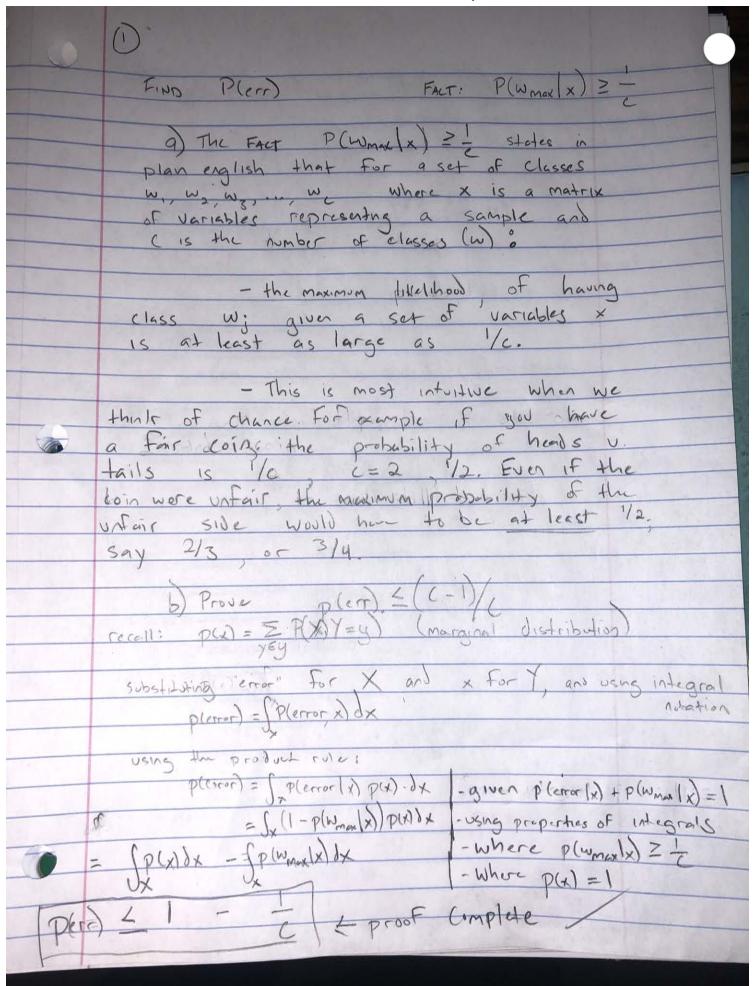
# 1 Probability and Discriminant Classifiers

## Part 1: Maximum and Posterior v Probability of Chance

PART I: Maximum Posterior vs Probability of Chance Show/explain that  $P(\omega_{\text{max}}|\mathbf{x}) \geq \frac{1}{c}$  when we are using the Bayes decision rule, where c is the number of classes. Derive an expression for p(err). Let  $\omega_{\text{max}}$  be the state of nature for which  $P(\omega_{\text{max}}|\mathbf{x}) \geq P(\omega_i|\mathbf{x})$  for  $i = 1, \ldots, c$ . Show that  $p(\text{err}) \leq (c-1)/c$  when we use the Bayes rule to make a decision. Hint, use the results from the previous questions.

Answer:

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### Part 2: Bayes Decision Rule Classifier



**PART II: Bayes Decision Rule Classifier** Let the elements of a vector  $\mathbf{x} = [x_1, \dots, x_d]^\mathsf{T}$  be binary valued. Let  $P(\omega_j)$  be the prior probability of the class  $\omega_j$   $(j \in [c])$ , and let

$$p_{ij} = P(x_i = 1 | \omega_j)$$

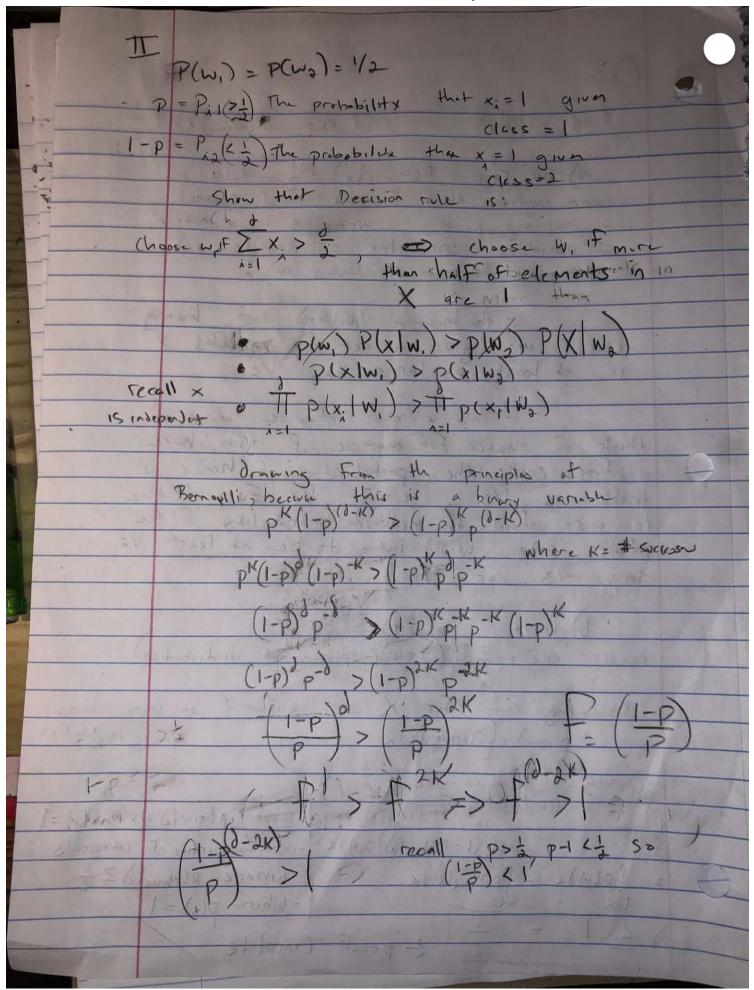
with all elements in **x** being independent. If  $P(\omega_1) = P(\omega_2) = \frac{1}{2}$ , and  $p_{i1} = p > \frac{1}{2}$  and  $p_{i2} = 1 - p$ , show that the minimum error decision rule is

Choose 
$$\omega_1$$
 if  $\sum_{i=1}^d x_i > \frac{d}{2}$ 

Hint: Think back to ECE503 and types of random variables then start out with

Choose 
$$\omega_1$$
 if  $P(\omega_1)P(\mathbf{x}|\omega_1) > P(\omega_2)P(\mathbf{x}|\omega_2)$ 

#### **Answer**



	Proof by trial of error choose will  o if condition satisfied choose will  o if condition had satisfied choose will  I-D d-dk  Whom I-D ( )  Proof by trial of error  whom I-D ( )  Result: becase is a positive  they left had side distance is a positive  Result: becase is a pos
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19	
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8	2 Constitute
	Rosult: deponent is negative
-	(FLI) > 1 SO L.H.S. 7 R.H.S. Choose W,
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-	(3) Condition K= =
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1	INI MUST Choose W In plain words if more than
0	uni must choose W, In plain words if more than half of elevents in x = 1 then choose W,
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9	

## Part 3: The Ditzler Household Growing Up

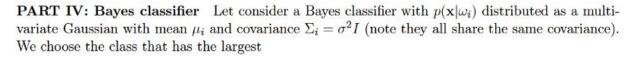
**PART III: The Ditzler Household Growing Up** My parents have two kids now grown into adults. Obviously there is me, Greg. I was born on a Wednesday. What is the probability that I have a brother? You can assume that  $\mathbb{P}(\text{boy}) = \mathbb{P}(\text{girl}) = \frac{1}{2}$ .

**Answer** 

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BTH XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	6
F × 13/99 F	E
5 × 15 11 12 13	e
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F X F	
51 X 5	9
50 X S	
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P(6B)=1/4 P(66)=1/4	
We want: P(BB B) = P(B BB) · P(BB)	
We want: P(BBB) = P(B BB) · P(BB)	
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6. D(BAIB) - ERR 13 - 13 1101	
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## Part 4: Bayes Classifier

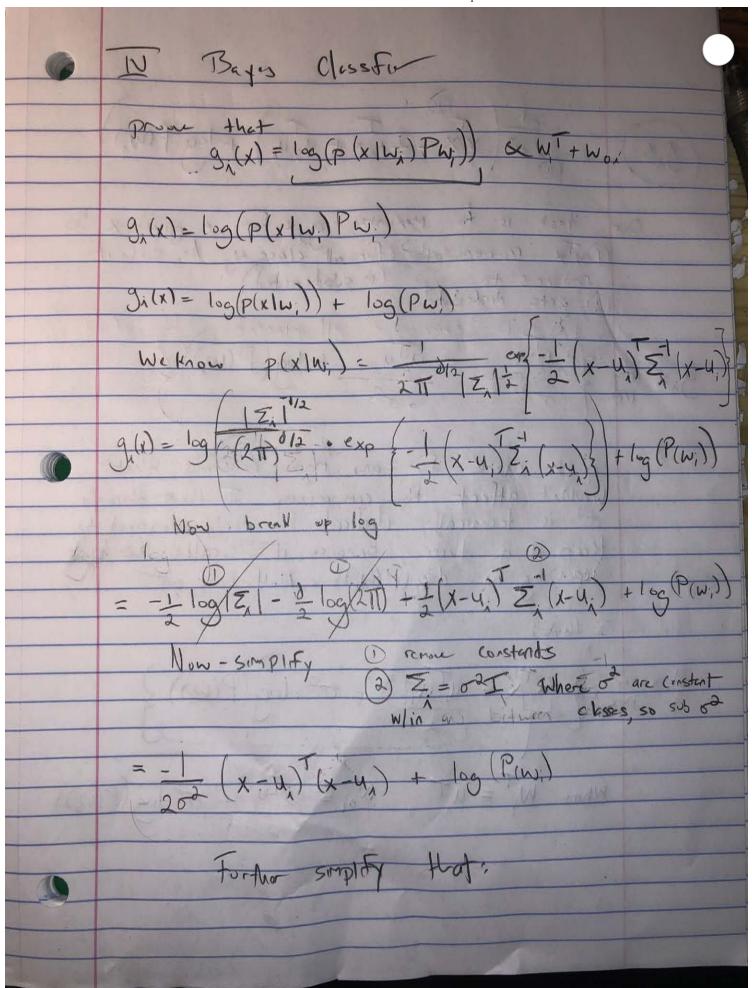


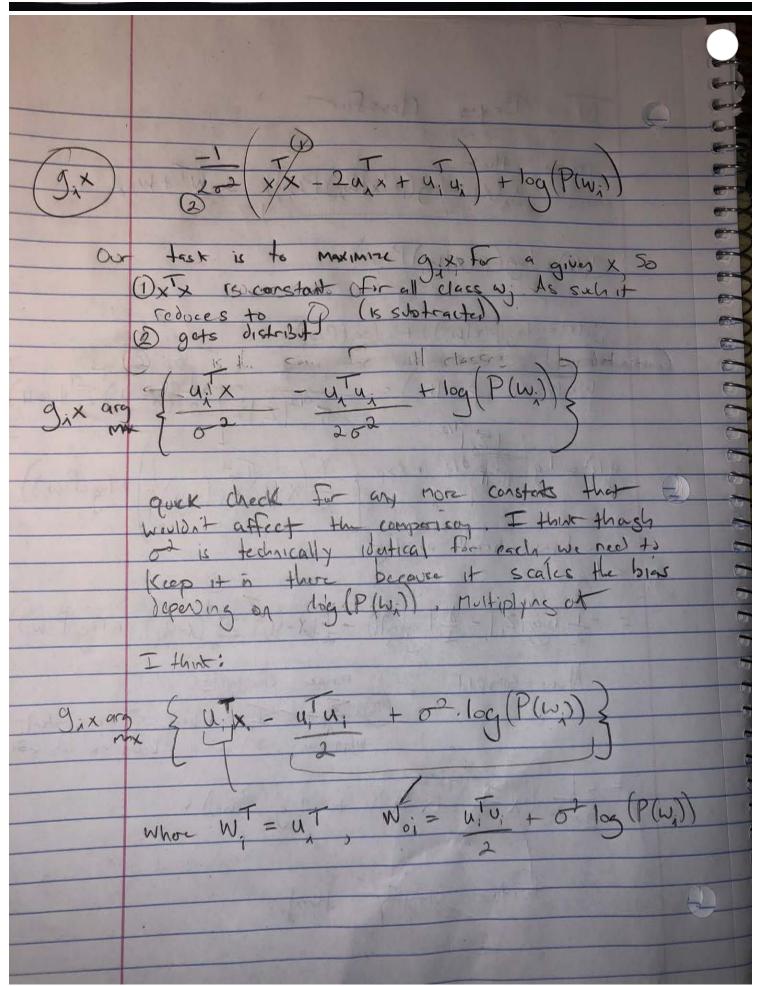
$$g_i(\mathbf{x}) = \log(p(\mathbf{x}|\omega_i)P(\omega_i)) \propto \mathbf{w}_i^\mathsf{T} \mathbf{x} + w_{0i}$$

Find  $\mathbf{w}_i$  and  $w_{0i}$ . Fact:

$$p(\mathbf{x}|\omega_i) = \frac{1}{(2\pi)^{\frac{d}{2}}|\Sigma_i|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2} (\mathbf{x} - \mu_i)^\mathsf{T} \Sigma_i^{-1} (\mathbf{x} - \mu_i)\right\}$$

#### Answer?





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