MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1. (a) Printing money will dramatically increase the amount of money in circulation in a country, and this will result in an undesirable inflation. On the other hand, when government issue bonds, they are able to maintain the amount of money in circulation. Thus, will not affect the value of currency.

   (b) A flatten yield curve may occur when investors worry about the growth prospect of a country's economy. As a result, they anticipate the interest rate to stay the same in the long-term.

   (c) Quantitative Easing is a monetary policy that central banks use to increase the money supply in an economy by purchasing long-term securities. Since the start of COVID-19, the Fed has purchased 1.45 trillion dollar worth of T-bill. [1]

2. Bonds included in the construction of yield curve are: "CAN 1.75 May 21", "CAN 1.5 Aug 21", "CAN 0.5 Mar 22", "CAN 0.25 Aug 22", "CAN 1.75 Mar 23", "CAN 8 Jun 23", "CAN 2.25 Mar 24", "CAN 1.5 Sep 24", "CAN 1.25 Mar 25" , "CAN 0.5 Sep 25", " CAN 0.25 Mar 26" and "CAN CAN 0.25 Mar 26".

   The bonds chosen in the list have maturity approximately 6 months apart with one another. This selection give us an evenly space maturity time and accurate "yield to maturity" to bootstrap for interpolation as we are plotting our yield curve.

   One exception is between now to 1 year from now. The bonds chosen within this period are 3 months from now, 6 months from now then 1 year from now. ("CAN 1.75 May 21", "CAN 1.5 Aug 21", "CAN 0.5 Mar 22"). The reason for this choice is because we want to have bonds that have no coupon payment left so we can treat them as zero-coupon bond. This would make our bootstrapping process later a lot easier.

3. According to Principle Component Analysis(PCA), the eigenvalues and eigenvectors associated with the covariance matrix of the stochastic processes give us the relative responsibility for the change in value of each process comparing with others. For example, assume we are tracking the S and P 500 index, and notice that as market opens the index fell by 2 percent. By looking into each individual stock and find the covariance matrix of the index, we can rank the eigenvalues of each stock by magnitude. The stock with the largest eigenvalue is the one that is most responsible for the drop in index points than the other 499 stocks. It is a powerful technique to find the most significant variable out of a collection of all variables. It is also a good way to approximate the direction of movement of an index such as S and P 500. [2]

Empirical Questions - 75 points

4.

(a) Start with a data set that contains all the bond information. group the data set by date of collection Then use bond.TCF function from frvFinance package to calculate the yield.

Since the time is not exactly 0.5 year apart from each other, we want to interpolate the time period so that they are at the appropriate time range.

Let y be current ytm and let x be the current time (still not a multiple of 0.5). Define an empty vector call interpolated ytm.

Run for loop from 2 to (length of the time interval - 1) in total 10 times since the first time chosen was only 3 months to maturity so we can safely ignore. If the time[i] is not a multiple of 0.5. Define interpolated ytm[i] to be the linear interpolatation of ytm from time[i-1] and time[i]. Else, keep the original ytm if the ytm is coincidentally from time period that's a multiple of 0.5.

At the end, return interpolated ytm with its associated time period and date extracted. Plot them superimpose on each other then we get the yield to maturity plot in the reference page.

(b) To calculate the spot rate, I design helper functions that help me calculate dirty price and present value of all payments except the last coupon payment plus principle. Obtaining these, we are able to calculate their rates by bootstrapping. Calculating the spot rate one by one starting with the bonds that's closest to maturity.

Obtain the spot rates by dates of extraction then plot them superimpose on each other on the same graph. Then, we get the Spot Rate plot in the reference page.

(c) In order to calculate the 1 year forward rate, first we pick out the data points with time to maturity that are exactly 1, 2, 3, 4 and 5 years from now.

Using the spot rates we obtained from the previous section, we discount them back semi-annually to recover the 1 year future spot rate.

Plot all the future rates on the same graph and colour code them by dates, we eventually end up with the One Year Forward Rate plot in the reference page.

5. The covariance matrix for the 5 years yields is:

$$\begin{bmatrix} 0.007937 & 0.003781 & 0.002879 & 0.000851 & 0.000739 \\ 0.003781 & 0.002502 & 0.001425 & 0.000326 & 0.000265 \\ 0.002879 & 0.001425 & 0.001920 & 0.000427 & 0.000406 \\ 0.000851 & 0.000326 & 0.000427 & 0.001363 & 0.001150 \\ 0.000739 & 0.000265 & 0.000406 & 0.001150 & 0.000989 \end{bmatrix}$$

The covariance matrix of the forward rates (f1.1, f1.2, f1.3, f1.4) is:

$$\begin{bmatrix} 0.002264 & 0.000746 & -0.000302 & -0.000244 \\ 0.000746 & 0.002274 & -0.000088 & -0.000003 \\ -0.000302 & -0.000088 & 0.001466 & 0.001226 \\ -0.000244 & -0.000003 & 0.001226 & 0.001056 \end{bmatrix}$$

6. The eigenvalues of the covariance matrix of yields are: (0.011210, 0.002174, 0.000772, 0.000548, 0.000010), with the associated eigenvectors by columns:

$$\begin{bmatrix} 0.830628 & 0.117182 & -0.222525 & 0.496797 & -0.000875 \\ 0.422162 & 0.157371 & -0.252416 & -0.855978 & 0.024523 \\ 0.331678 & -0.057768 & 0.932854 & -0.123147 & -0.035661 \\ 0.111584 & -0.745038 & -0.119027 & -0.065277 & -0.643459 \\ 0.096829 & -0.634889 & -0.048841 & -0.032670 & 0.764256 \end{bmatrix} \tag{1}$$

Similarly, we obtained the eigenvalues of the covariance matrix of the future rates: (0.003177, 0.002395, 0.001471, 0.000017), and their associated eigenvectors by columns:

$$\begin{bmatrix} -0.671857 & -0.156769 & 0.723887 & 0.004453 \\ -0.589798 & -0.477588 & -0.650676 & -0.025830 \\ 0.349777 & -0.652275 & 0.187348 & -0.645828 \\ 0.280005 & -0.567336 & 0.132320 & 0.763033 \end{bmatrix} \tag{2}$$

According to Principle Component Analysis, the combination with the largest eigenvalue is the one that give us the most information in both our yield curve calculation and 1 year forward rate calculations.

References and GitHub Link to Code

Citations

1. Cheng, J., Wessel, D., amp; Younger, J. (2020, May 01). How did COVID-19 disrupt the market for U.S. Treasury debt? Retrieved February 08, 2021, from https://www.brookings.edu/blog/up-front/2020/05/01/how-did-covid-19-disrupt-the-market-for-u-s-treasury-debt/

2. Jaadi, Z. (n.d.). A step-by-step explanation of principal component analysis. Retrieved February 08, 2021, from https://builtin.com/data-science/step-step-explanation-principal-component-analysis

Data Visualization Plots