Characterizing Analog Noise Generators

Master's Thesis

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Abstract

Stochastic differential equations (SDEs) are differential equations that involve both deterministic and stochastic components. The key challenges in SDEs lies in their inherent complexity due to the inclusion of stochastic or random terms. In context, these random terms can represent uncertain factors or external influences in various fields including physics, engineering and finance, and can help to reach better accuracy if the randomness within the sequence used in the SDEs is true. Since 1990s, various statistical tests were developed to characterize the randomness like Square test, cube test, average value, etc. to test how good are the random numbers for specific applications. Followed by such test, a test suite which was a combination of multiple test was also developed by George Marsaglia, named as Diehard Tests.

Such application of random numbers influenced application of physical noise generators to enhance the results of SDEs. The noise generators we are using in this project are 3722 A - manufactured by HP, RG-1 by Wandel and Goltermann, Bipolar Junction Transistor (BJT) based noise generators and Zener diode based noise generators. The output of such noise generators are time-continuous random signals, which are time dependent. The behaviour of such generators depends on power supply voltage, temperature, age of the device and also differs on which model we are using. Hence, in this work, we are performing statistical tests on such physical noise generators to ensure the quality and goodness of signals they are generating. The results of these statistical tests help us conclude which type of noise the generator is creating, mainly white noise, pink noise or brown noise.

Our approach involved transforming the signal into datasets for various time intervals and applying statistical tests on the datasets, with specific focus on tests like autocorrelation, Fourier transformation and wavelet analysis.

This project implied to the conclusion that the noise generator RG-1 by Wandel and Goltermann and HP 3722-A generates pure white noise. Specifically at certain frequency. This noise signals can be used for further applications like cryptography. The noise generated by other generators is mostly pink noise, and can be used for the applications where pink noise will be a valuable input for a random variable.

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Chapter 1

Introduction

In the field of electronic design, noise is a big challenge that can influence the performance and reliability of devices. Analog noise generators are pivotal in simulating and characterizing the unpredictable signals and play a crucial role in testing and calibration processes across various electronic applications. Despite their importance, accurately characterizing the outputs of these generators using traditional methods poses significant challenges due to the stochastic nature of noise. This thesis project explores the use of sophisticated statistical tools—namely Fast Fourier Transform (FFT), Autocorrelation, and Wavelet analysis — to offer a more deeper understanding of noise characteristics exhibited by these noise generators. By showcasing the effects and insights provided by each method, this research aims to contribute to the broader field of electronic testing and signal analysis. Through this work, we seek to establish a methodological framework that improves the reliability and analytical depth in studying analog noise generators, thus paving the way for more robust electronic design and diagnostics.

1.1 Overview of Noise in Electronics

Noise in electronic circuits is an inherent phenomenon that impacts the performance and reliability of electronic devices. Generally regarded as any unwanted or random spurious signals that obscure or interfere with the desired signal, noise can originate from various sources, both internal and external to the circuit [1].

In this project, we are going to characterise the noise signals into three majorly differentiated noise types - white noise, pink noise and brown noise.

1.1.1 White Noise

White noise has a constant power spectral density across a wide frequency range. It is called white noise because it contains all frequencies much like white light contains all colors. This type of noise is generated primarily due to the thermal agitation of electrons in a conductor and is thus unavoidable at non-zero temperatures [2].

1.1.2 Pink Noise

Pink noise, or "1/f noise" shows a power spectral density that is inversely proportional to the frequency. This means it has more power at lower frequencies and less at higher frequencies. It is commonly found in biological systems and is characteristic of semiconductor and carbon composition resistors, as well as flicker noise observed in electronic devices [3].

1.1.3 Brown Noise

Brown noise is also known as "Brownian noise". This type of noise has a power density that decreases with the square of the frequency. It sounds deeper compared to white and pink noise and resembles the random walk pattern or Brownian motion of particles. It is characterized by strong energy at low frequencies, gradually tapering off as the frequency increases [4].

Though this project primarily focuses on the the noise, the analysis of noise is import in crucial in electronics. For high-precision applications such as audio electronics, medical instruments, and communication systems, understanding noise levels and characteristics is essential to ensure accurate readings and high-quality signal transmission.

1.2 Analog Noise Generators

Analog noise generators are specialized electronic devices designed to produce different types of noise signals, such as white, pink, and brown noise. These generators play a crucial role in various applications by simulating noise under controlled conditions to test and analyze the performance of electronic circuits and systems. We will be using the following noise generators for this projects in table 1.2.

In this project, we are considering the application of noise generators specifically for the production of keys in cryptography. The unknown, unpredictable, non-deterministic sequence of key will be inspired from the random signal generated by analog noise generators.

Table 1.1: List of Noise Generators

Sr. No.	Noise Generator	Model
1	Bipolar Junction Transistor based noise generator 1	-
2	Bipolar Junction Transistor based noise generator 2	-
3	Noise generator by Hewlett Packard	3722-A
4	Noise generator by Wandel and Goltermann	RG-1
5	Zener diode based noise generator 1	-
6	Zener diode based noise generator 2	-

1.2.1 Challenges in characterizing noise from noise generators

When undertaking the task of characterizing noise from analog generators, we are faced with several intricate challenges that can profoundly impact the accuracy and reliability of their results.

- **Accuracy of Spectral Density in Fourier Transform**: Accurately characterizing the spectral density in Fourier Transform of generated noise can be challenging due to the stochastic nature of noise and the limitations of measuring equipment.
- **Consistency and Stability**: This is an assumption that the noise output is consistent over time and stable under varying environmental conditions can be difficult and crucial for reliable testing.
- **Interference**: External electromagnetic interference can affect the accuracy of noise generation and measurement, necessitating careful isolation and shielding techniques.
- **Complexity of analysis**: The analysis of noise, especially for non-linear and dynamic characteristics like in pink and brown noise, requires sophisticated statistical tools and methodologies that can interpret data effectively across different conditions and applications. For this, we have used 3 different statistical methods for analysis which are discussed briefly in the next section.

1.3 Statistical Methods in Noise Analysis

In the field of noise analysis, statistical methods play a pivotal role in dissecting the complex nature of noise and extracting meaningful information from seemingly random signals. The application of these methods is both an art and a science, requiring a blend of mathematical rigor and practical insight. Among the most powerful tools in this domain are the Fast Fourier Transform (FFT), Autocorrelation, and wavelet analysis, each serving a unique purpose in the characterization of noise. We used Python programming language for these analysis.

- FFT is a computational method that converts signals from the time domain into the frequency domain, allowing analysts to inspect the signal's spectral components. Its efficiency and speed make it indispensable for real-time noise analysis, providing clarity on the frequency distribution and energy of the noise across a spectrum [5].
- Autocorrelation, on the other hand, offers a lens into the time domain characteristics of noise, showing the dependencies and repeating patterns within a signal. By measuring the correlation of a signal with a delayed version of itself, it assists in identifying periodicity and the presence of underlying structures in what might initially appear as random noise [6].
- Wavelet analysis introduces a further layer of sophistication, enabling the examination of noise signals at multiple scales. It excels where FFT falls short — in analyzing non-stationary signals whose frequency components change over time. Through wavelet analysis, researchers can delve into the transient features of noise, capturing the nuances of its behavior with a fine resolution in both time and frequency domains [7].

Collectively, these statistical methods form a robust toolkit for noise analysts, each offering different insights that, when combined, provide a comprehensive understanding of noise in electronic systems. The application of FFT, Autocorrelation, and wavelet analysis in noise analysis represents a convergence of mathematical concepts with practical engineering, underpinning advances in the field of electronics and signal processing.

1.4 Objective of the study

The primary objectives of this study on the characterization of noise signals using statistical methods are centered around a comprehensive analysis of noise within electronic systems. The research is meticulously designed to advance the understanding of noise behavior and its implications on electronic devices. The specific goals of the study are:

1. **Characterization of Spectral Components**: Utilize the Fast Fourier Transform (FFT) to dissect noise signals into their constituent frequency components. The aim is to map out the spectral density and identify predominant frequencies within the noise for different types of noise generators.

2. **Temporal Analysis and Periodicity Detection**: Employ Autocorrelation techniques to examine the time-dependency characteristics of noise signals. This will assist in detecting any periodicity or time-based patterns that could be critical in understanding the behavior of noise over time.

3. **Detailed Time-Frequency Analysis**: Apply wavelet analysis to achieve a granular breakdown of noise signals into their time-varying frequency components. This goal is directed towards capturing the transient and non-stationary aspects of noise, which are often missed by traditional Fourier methods.

4. **Comparative Assessment of Noise Types**: Through the analysis of white, pink, and brown noise signals generated by analog noise generators, the research aims to delineate the unique properties and challenges each type of noise presents.

5. **Visualization of Noise Characteristics**: Develop a series of plots and visual representations that make the properties and nuances of noise signals accessible and understandable. These plots will serve as a tool to visually compare and contrast the effects of different noise types.

6. **Methodological Advancement**: Evaluate the effectiveness and limitations of the statistical methods used, contributing to the refinement of noise analysis techniques. This study aims to provide a methodological benchmark for future research in the field.

1.5 Thesis outline

The blueprint of this thesis report is made to study and understand the intricate landscape of noise characterization in electronic systems. With each chapter explaining the foundations, the structure progresses systematically from introducing the fundamental concepts to exploring the intricate details of noise and its analysis.

1. **Noise Generators**: : Chapter 2 delves into the specifics of various noise generators. Detailed descriptions of their design, operational principles, and the types of noise they produce will be provided, giving a firm grounding in the tools used to create the noise signals analyzed throughout the study.

2. **Methods Used for Analysis**: Chapter 3 introduces the reader to the statistical methods at the core of this research: FFT, Autocorrelation, and wavelet analysis. Each method will be discussed in detail, explaining how they work, their relevance to noise analysis, and the reasons for their selection.

3. **Noise Spectrums**: In Chapter 4, the focus shifts to the noise itself. The characteristics of white, pink, and brown noise are introduced and explored. This chapter will unpack the theoretical underpinnings of these noise types and their significance in the context of noise analysis.

4. **Analysis of the Signals**: The penultimate chapter is the heart of the thesis, where the analysis takes place. It will detail the process of applying FFT, Autocorrelation, and wavelet analysis to the signals generated by the noise generators. Through plots and visual data, the noise will be characterized, and the performance of each analysis method will be evaluated.

5. **Future Scope of Research**: Chapter 5 will reflect on the study's findings and discuss the potential avenues for future research. It will lay out the implications of the current research for the field and suggest how these methods and findings can be extended or applied to new challenges in noise analysis.

1.6 Contribution to the Field

This thesis project, through its analytical approach, stands to make several contributions to the existing body of knowledge in the fields of electronics and noise analysis. By applying statistical tests to physical noise generators, this thesis provides insights into the quality and type of randomness that SDEs can utilize, particularly in electronic systems where noise is a fundamental aspect. The work delineates a methodology to assess the quality of signals from noise generators, which is critical for applications where signal purity is paramount. The identification of pure white noise generators could lead to improved models in simulations and more accurate results in systems relying on stochastic inputs.

With the classification of noise types produced by various generators, the thesis aids in the selection process for applications requiring specific noise profiles, such as white noise in cryptography and pink noise in other domains. By performing and documenting statistical tests for noise characterization, the thesis provides insights that could inform future standards and best practices in noise generator testing, improving the reliability of these devices across the industry and also holds a potential to encourage deeper analysis of signals.

Chapter 2

Noise Generators

The signals that are being characterized in this project are generated through various physical noise generators that are mentioned in chapter 1. For comparison purpose, we have also included an algorithm based random number generator that produces pseudo-random numbers.

It is important for the reader to understand the functionality and application of the various noise generators that are used in this project. We have used the analog noise generators shown in the figure 2.1.

Figure 2.1: Noise Generators

After the acquisition of data from these noise generators, followed by post processing, the data does not remain continuous time signal. Since we observe the value of amplitude at each time stamp, the continuous-time signal gets converted into discrete-time signal.

2.1 Pseudo-Random Number Generators

Pseudo-Random Number Generators (PRNGs) are the algorithmic approach to generate sequences of numbers that appear to be random but are, in fact, deterministic [2]. Unlike true random number generators that use physical processes to generate randomness, PRNGs use mathematical formulas or algorithms to produce sequences of numbers that exhibit statistical properties of randomness. These sequences are repeatable given the same initial conditions or seed value.

A PRNG starts with an initial seed value. This seed value determines the starting point of the sequence. The PRNG has a mathematical algorithm to generate a sequence of numbers based on the initial seed. This algorithm typically involves iterative calculations or bitwise operations. In this project, we have generated the PRNGs with the help of `np.random.rand` from the python library `numpy` [8]. The output of a PRNG is a sequence of numbers that appears to be random. The time versus value plot is shown in the figure 2.2. However, since the algorithm is deterministic, the sequence will eventually repeat after a certain period [2]. This means, with the seed and the algorithm, we can recreate the entire sequence of numbers. The noise generated by PRNGs is often referred to as ”pseudo-random noise”. This type of noise is deterministic and lacks the true unpredictability found in truly random processes. PRNGs produce sequences that pass certain statistical tests for randomness, making them suitable for many applications where statistical randomness is sufficient [9].

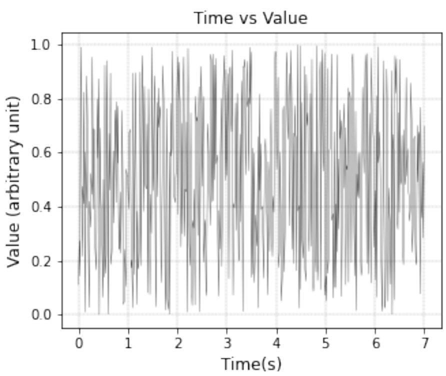

Figure 2.2: Signal generated by PRNGs

2.2 Bipolar Junction transistor (BJT) based Noise Generator

A Bipolar Junction Transistor (BJT)-based noise generator is a device that exploits the random processes occurring within the transistor to produce noise signals. This type of noise generator is mostly used in electronic testing, communications, and research applications where a known and controlled noise source is required [10].

For this project, we have did the data acquisition from BJT based noise generator with using lowpass filter as well as without lowpass filter. It has been sampled at a high rate of 1 MSPS, which stands for one million samples per second. This substantial sampling rate is essential for capturing the intricate details and high-frequency components of the noise generated by the BJT. The four datasets of both with and without lowpass filter, encompassing a total duration of ten seconds, resulting in a comprehensive collection of ten million samples in each dataset. The signal plot of data acquired from BJT with lowpass filter and no lowpass filter is shown in figure 2.3 and figure 2.4 respectively. Such detailed and high-resolution data is invaluable for analyzing the noise characteristics of the BJT, which can be crucial for applications in electronic circuit design and testing where understanding noise behavior impacts performance and reliability. The extensive sample size and high sampling rate make this dataset particularly suited for detailed spectral analysis.

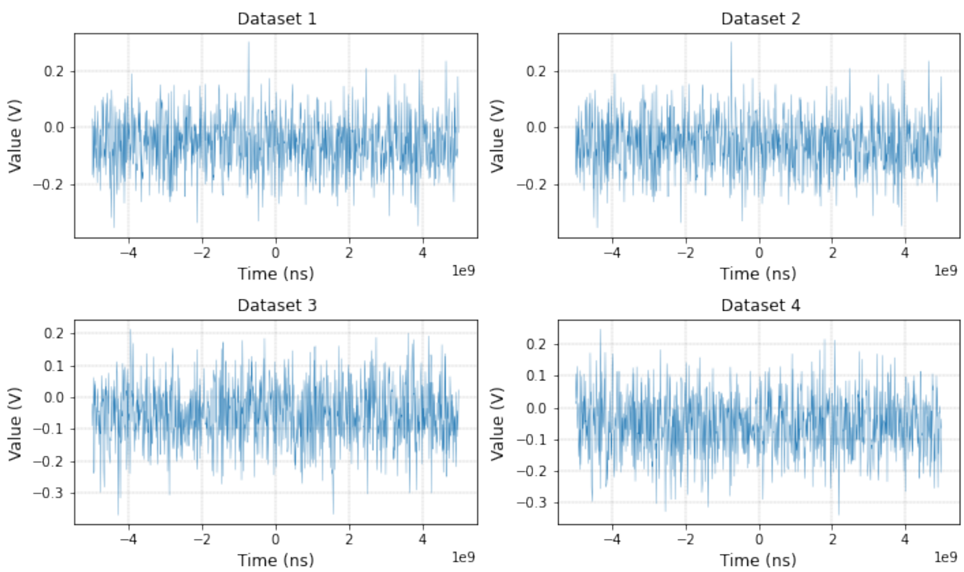

Figure 2.3: Signal generated by BJT based noise generator with lowpass filter

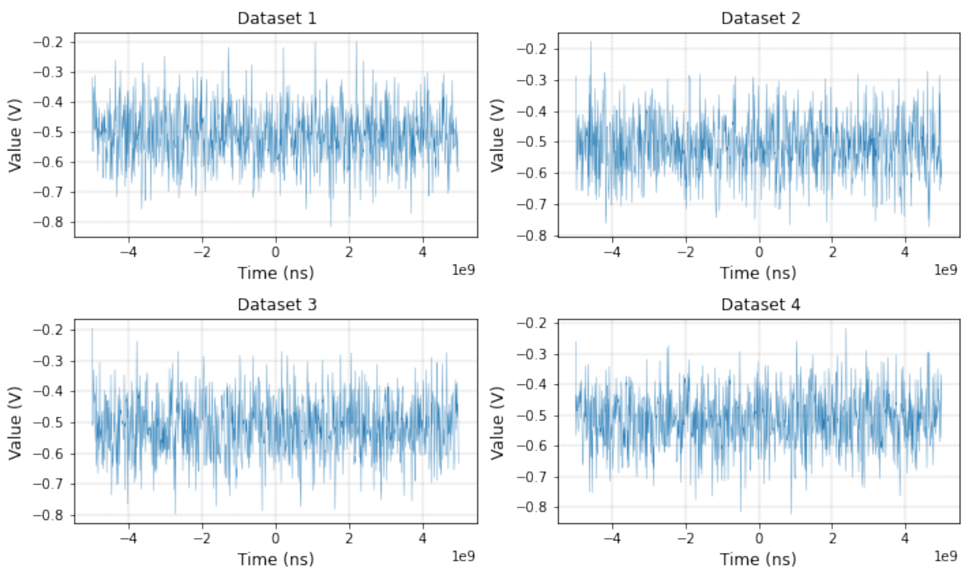

Figure 2.4: Signal generated by BJT noise generator without lowpass filter

Although the signals from these figures 2.3 and 2.4 appears very similar, there is a minute difference between them, which has been covered with the help of basic statistical methods below in the table 2.1 and 2.2.

Table 2.1: Statistical Summary of BJT based noise generator without lowpass filter

Dataset	Mean	Median	Minimum Value	Maximum Value
1	-0.052396	0.0	-10.24	8.32
2	-0.058731	0.0	-10.24	8.24
3	-0.053649	0.0	-10.24	8.64
4	-0.053226	0.0	-10.00	8.48

Table 2.2: Statistical Summary of BJT based noise generator with lowpass filter

Dataset	Mean	Median	Minimum Value	Maximum Value
1	-0.090172	-0.08	-10.24	8.40
2	-0.093982	-0.08	-9.44	8.32
3	-0.088524	-0.08	-10.24	9.52
4	-0.089632	-0.08	-10.24	9.20

2.3 Noise generator 3722-A by Hewlett Packard (HP)

The Hewlett-Packard Model 3722-A is a low frequency broadband noise generator designed primarily for use in control systems evaluation and applications requiring the simulation of random disturbances. The model provides two types of random noise output – binary signal and continuous analog waveform. This continuous analog waveform is an approximate Gaussian amplitude distribution [11].

The data that we have acquired from HP 3722-A has been sampled at a high rate of 1 MSPS, which stands for one million samples per second. This substantial sampling rate is essential for capturing the intricate details and high-frequency components of the noise generated by this noise generator. The four datasets encompasses a total duration of ten seconds, resulting in a comprehensive collection of ten million samples in each dataset. The signal plot of data acquired from HP 3722-A is shown in figure 2.5.

The HP 3722-A also generates pseudo-random versions of the binary and Gaussian signals, in which the signals are repeated noise patterns or sequences of known content or duration. In the random mode, the output have continuous spectra extending down to dc. The signal plots of data acquired from the HP noise generators in pseudo random version is shown in figure 2.6.

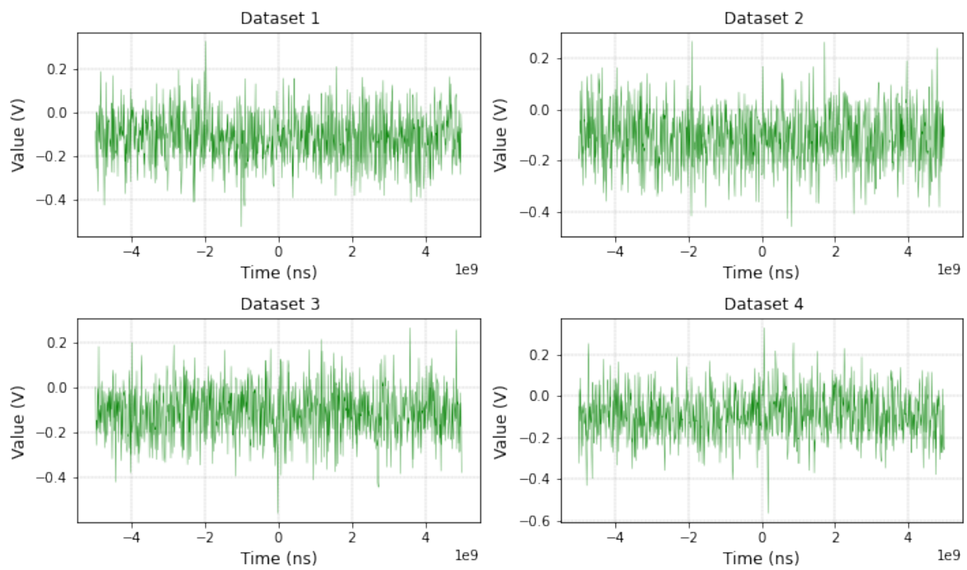

Figure 2.5: Signal generated by HP 3722-A noise generator

The signals 1, 2, 3, 4 from HP 3722-A noise generators looked very similar and hard to bifurcate, we have shown a basic statistical difference between them in the table 2.3 below.

Table 2.3: Statistical Summary HP 3722-A

Dataset	Mean	Median	Minimum Value	Maximum Value
1	-0.110464	-0.2	-13.4	13.4
2	-0.107827	-0.2	-13.4	13.4
3	-0.108640	-0.2	-13.4	13.8
4	-0.088524	0.0	-13.2	13.6

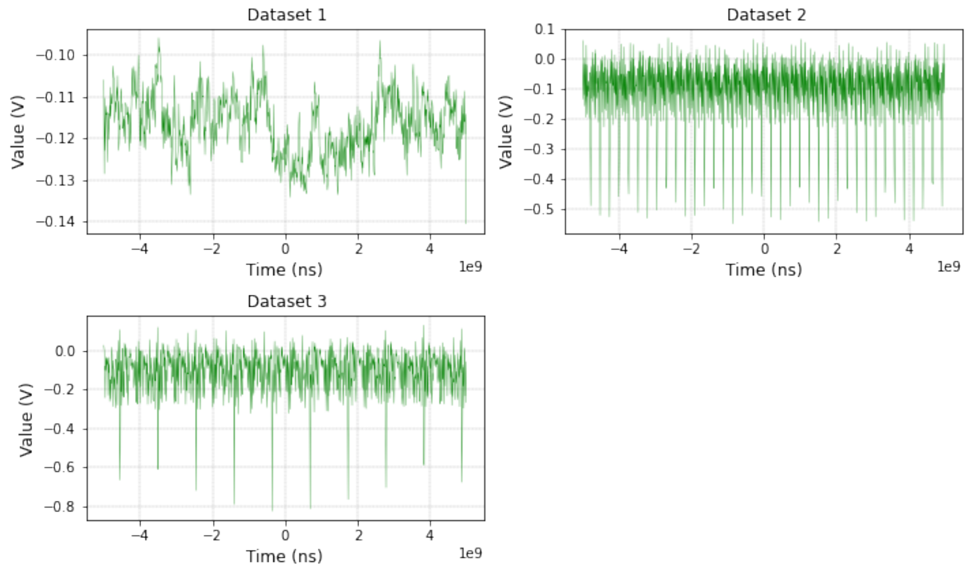

Figure 2.6: Signal generated by HP 3722-A noise generator as pseudo random

As we can observe some patterns in the pseudo random number generator datasets produced by HP 3722-A, we have also done the basic statistic analysis of this signal in the table 2.4 below.

Table 2.4: Statistical Summary HP 3722-A PRNG

Dataset	Mean	Median	Minimum Value	Maximum Value
1	-0.109975	-0.2	-13.4	13.4
2	-0.116682	-0.2	-11.8	11.6
3	-0.105092	-0.2	-12.0	13.8

2.4 Wandel and Goltermann RG-1 noise generator

Wandel and Goltermann is a notable manufacturer of network testing and analysis equipment, produced noise generators used extensively in telecommunications and network testing.

One of such device is RG-1, that we are using for this project. It could generate noise across a wide frequency spectrum. In the context of acquiring data from such a noise generator, measurements were taken at a resolution of 10 bits, providing a decent dynamic range for capturing the noise profile, shown in the figure 2.7. The data was sampled across the frequency ranges, from 16 Hz to 22 kHz, shown in the figure 2.8, extended to 100 kHz, shown in figure 2.9. This broad spectrum is essential for ensuring the type of noise this noise generator creates and its performance at all previously mentioned frequencies.

Even though there is a similarity seen within the plots, it is still different in statistical means. This differences for signal from 10bit shown in figure 2.7, from 16Hz to 22kHz shown in figure 2.8, and 100kHz shown in figure 2.9 and from 1 is explained in tables 2.5, 2.6 and 2.7 respectively.

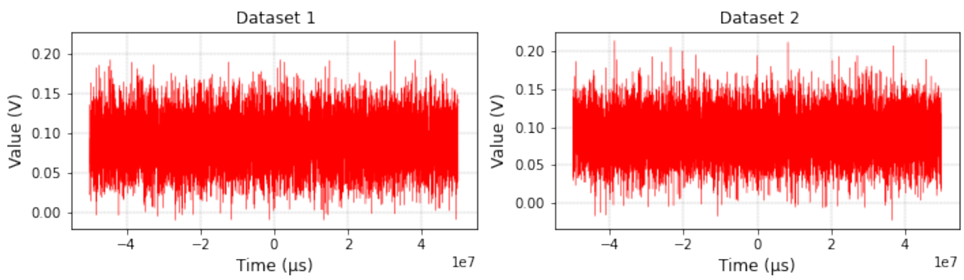

Figure 2.7: Signal generated by Wandel and Goltermann RG-1 noise generator at 10 bit

Table 2.5: Statistical Summary of Wandel and Goltermann RG-1 noise signal at 10 bit

Dataset	Mean	Median	Minimum Value	Maximum Value
1	0.089879	0.1	-8.533333	8.466667
2	0.089769	0.1	-8.200000	8.466667

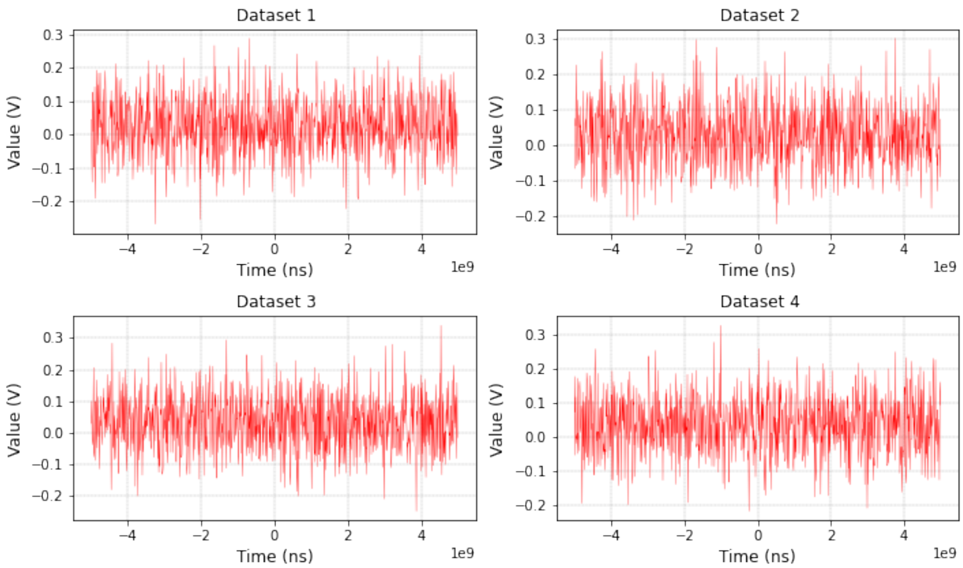

Figure 2.8: Signal generated by Wandel and Goltermann RG-1 noise generator at 16 Hz to 22 kHz

Table 2.6: Statistical Summary of Wandel and Goltermann RG-1 noise signal from 16Hz to 22kHz

Dataset	Mean	Median	Minimum Value	Maximum Value
1	0.025686	0.8	-9.52	9.28
2	0.032030	0.8	-9.52	9.92
3	0.029385	0.8	-9.68	9.68
4	0.036845	0.8	-9.60	10

Table 2.7: Statistical Summary of Wandel and Goltermann RG-1 noise signal 100kHz

Dataset	Mean	Median	Minimum Value	Maximum Value
1	0.090883	0.8	-8.96	9.20
2	0.092262	0.8	-8.56	9.12
3	0.096446	0.8	-8.96	9.36
4	0.089113	0.8	-8.80	9.36

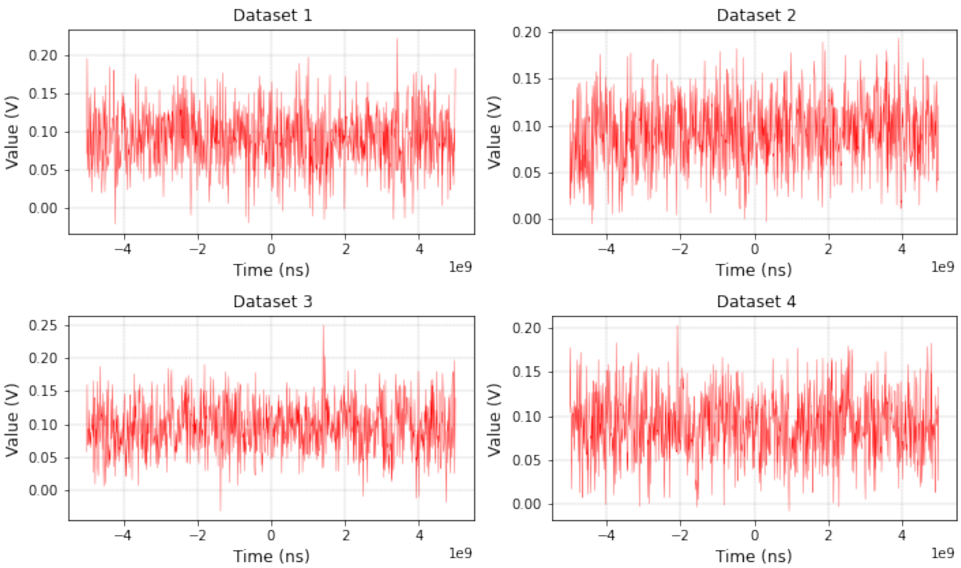

Figure 2.9: Signal generated by Wandel and Goltermann RG-1 noise generator at 100 kHz

2.5 Zener diodes based noise generator

A Zener diode-based noise generator is a circuit that utilizes the avalanche breakdown phenomenon in Zener diodes to produce random electrical noise. The noise generated by such a circuit is often referred to as Zener noise or avalanche noise. This type of noise is useful in various applications, including testing and calibration of electronic equipment, as well as for providing a random signal source in certain electronic systems. We are using this noise generator because of its capability of generating time-continuous noise signals.

Zener diodes exhibit a voltage-controlled avalanche breakdown characteristic. When the reverse voltage across a Zener diode exceeds its Zener voltage (also called the breakdown voltage), a rapid and random generation of electron-hole pairs occurs due to impact ionization. The impact ionization process leads to the creation of charge carriers, and the resulting current fluctuations produce random electrical noise. The noise signal generated by the Zener diode is usually very low in amplitude. Therefore, it is often amplified using an amplifier circuit to achieve a usable output level. Filters or other circuit elements may be employed to control the bandwidth of the noise signal if a specific frequency range is desired.

The primary type of noise generated by a Zener diode-based noise generator is avalanche noise, also known as Zener noise. This type of noise is broadband and exhibits a relatively flat power spectral density across a wide range of frequencies, making it suitable for applications that require a broad-spectrum random signal [12].

Observing the figure 2.10, datasets 1, 2, 3, 4 can show a significant difference within the pattern. We have quantified it with the help of basic statistical methods to differentiate it more in the table 2.8.

Table 2.8: Statistical Summary of Zener diode based noise generator 1 signals

Dataset	Mean	Median	Minimum Value	Maximum Value
1	0.016737	0.08	-5.60	5.84
2	0.008776	0.00	-5.52	5.52
3	0.012036	0.00	-5.36	5.52
4	0.010302	0.00	-5.52	5.60

Observing the figure 2.13, datasets 1, 2, 3, 4 is not able to show a significant difference within the pattern. We have quantified it with the help of basic statistical methods to differentiate it more in the table 2.9.

Observing the figure 2.12, datasets 1, 2, 3, 4 can show a significant difference within the pattern. We have quantified it with the help of basic statistical

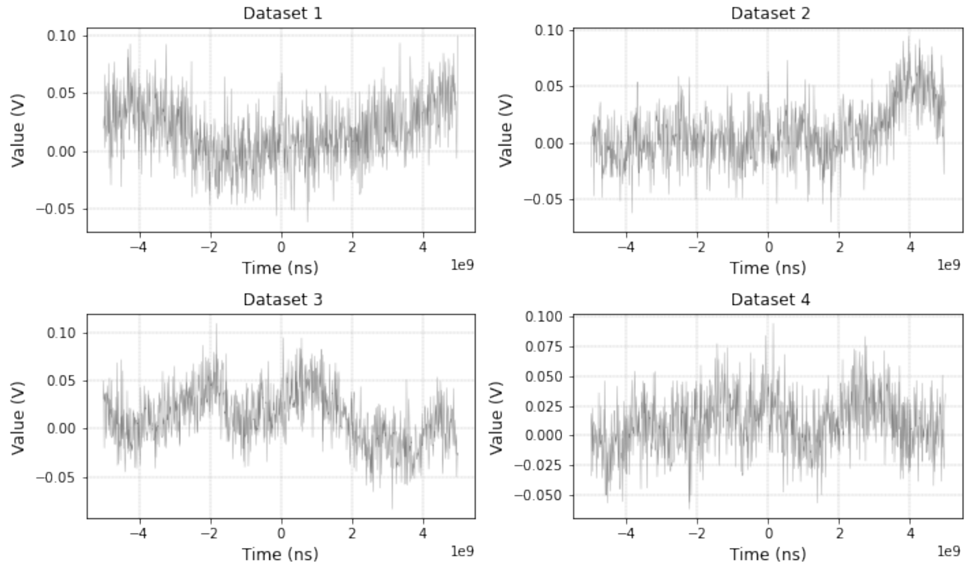

Figure 2.10: Signal generated by Zener based noise generator 1 at 100 kHz

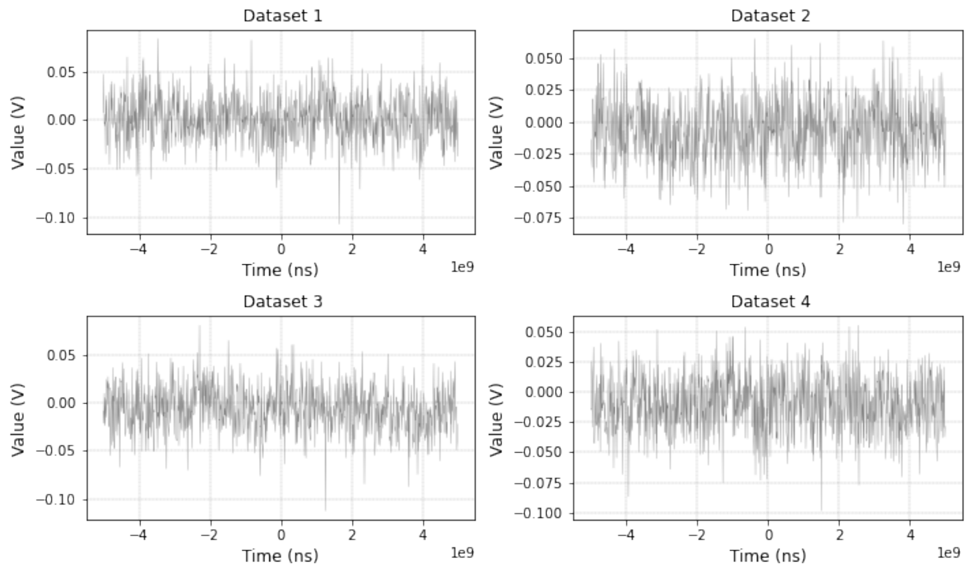

Figure 2.11: Signal generated by Zener based noise generator 1 at 100 kHz without lowpass filter

methods to differentiate it more in the table 2.10.

Observing the figure 2.12, we have also quantified it with the help of basic statistical methods to differentiate it more in the table 2.11.

Table 2.9: Statistical Summary of Zener diode based noise generator 1 signals without lowpass filter

Dataset	Mean	Median	Minimum Value	Maximum Value
4	-0.009714	0.0	-5.36	6.08
1	0.001728	0.0	-5.52	6.08
3	-0.005611	0.0	-5.44	5.84
2	-0.006124	0.0	-5.68	6.00

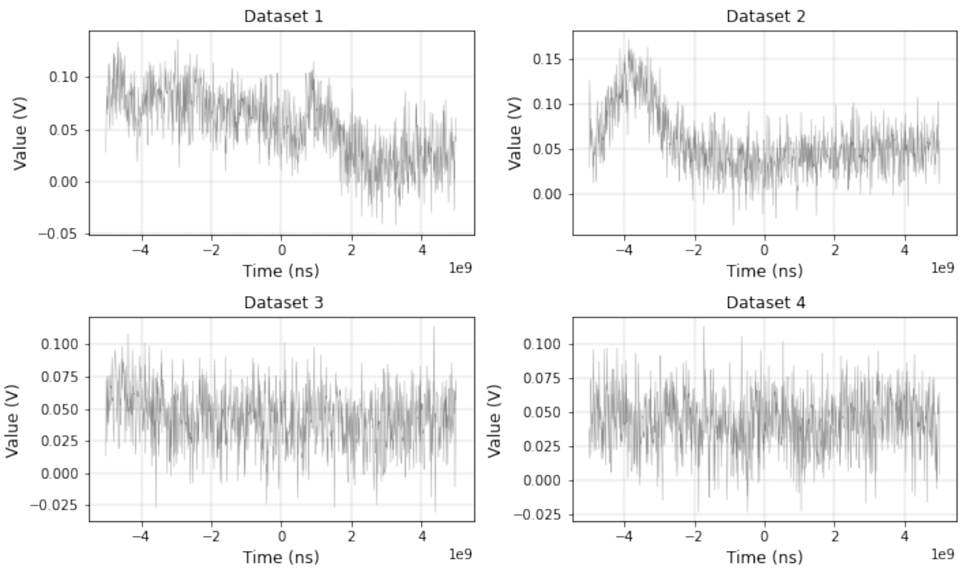

Figure 2.12: Signal generated by Zener based noise generator 2 at 100 kHz

Table 2.10: Statistical Summary of Zener diode based noise generator 2 signals

Dataset	Mean	Median	Minimum Value	Maximum Value
4	0.042974	0.16	-3.68	4.40
1	0.053850	0.16	-3.68	4.32
3	0.043207	0.16	-3.68	4.32
2	0.056288	0.16	-3.68	4.48

Table 2.11: Statistical Summary of Zener diode based noise generator 2 signals without lowpass filter

Dataset	Mean	Median	Minimum Value	Maximum Value
4	0.025764	0.16	-3.76	4.48
1	0.026881	0.16	-3.84	4.48
3	0.028325	0.16	-3.84	4.64
2	0.025286	0.16	-3.76	4.64

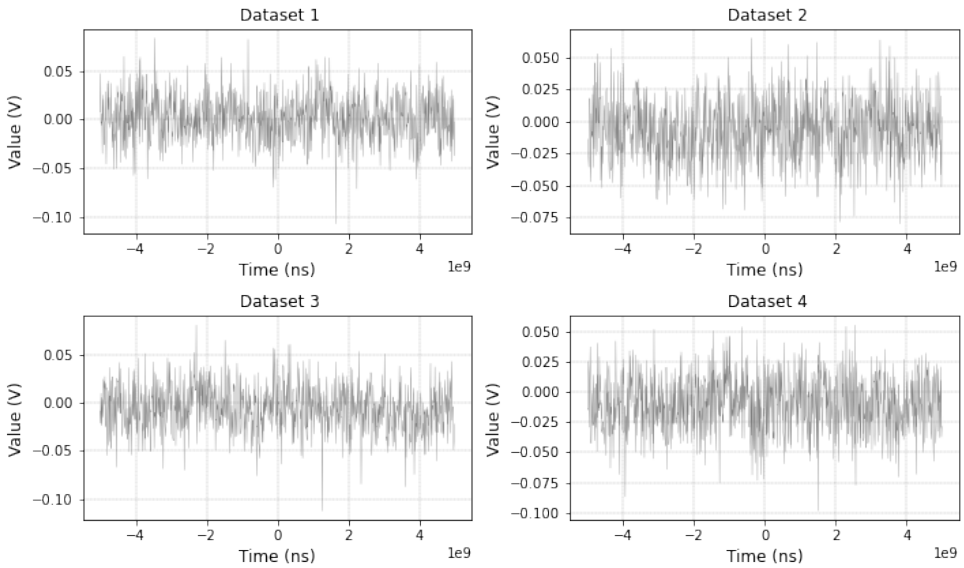

Figure 2.13: Signal generated by Zener based noise generator 2 at 100 kHz without lowpass filter

Chapter 3

Methods for Analysing Signals

As discussed in the chapter 2, that when the signal gets recorded at time stamps followed by processing, the continuous time signal gets converted into discrete time signals. This means, the signal has certain value at a particular time stamp. Here are the methods explained in detail that we will be using for the analysis of signals.

1. **Fourier Transform:** This technique will likely result in a frequency domain representation of the signal, elucidating its periodic features and dominant frequency components.
2. **Autocorrelation:** The expected outcome is a depiction of the signal's correlation with itself over varying time lags, revealing any periodicity and the degree of predictability or memory within the signal.
3. **Wavelets:** Wavelet analysis is anticipated to provide a multi-resolution decomposition of the signal, pinpointing both the frequency and location of singularities and transient phenomena.

These methods will furnish a multifaceted view of the noise signal's characteristics, with each method explaining different aspects of the signal's behavior. This comprehensive framework of analysis will be instrumental in evaluating and optimizing noise generators for the applications we will be covering in the later part of project.

3.1 Fourier Transform

The Fourier Transform is a mathematical technique that decomposes a function or dataset in the time domain into its constituent frequencies, called frequency spectrum. It is a fundamental concept in the field of signal processing, allowing us to analyze the frequency components of signals that vary over time [13]. Basically, the Fourier Transform transitions our perspective from observing how a signal evolves over time to understanding which frequencies make up the signal. This transformation is crucial for analyzing and processing signals in various fields such as engineering, physics, and now, in our project, the characterization of analog noise sources.

The Discrete Fourier Transform (DFT) of a sequence $x[n]$ is represented by $X[k]$, where n denotes the time index and k the frequency index. The transformation is given by the sum:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{kn}{N}}$$

Here, $e^{-j2\pi \frac{kn}{N}}$ is the complex exponential function, which oscillates at frequency index k, and j is the square root of -1. N is the total number of points in the discrete sequence. The inverse Discrete Fourier Transform (IDFT) allows us to reconstruct $x[n]$ from $X[k]$ using the inverse formula:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{kn}{N}}$$

These equations show that any time-domain signal can be represented as a sum of sinusoids of different frequencies, amplitudes, and phases.

3.1.1 Application in the analysis of noise signals

In the context of characterizing analog noise sources for applications such as cryptography [14], or to study uncertainty in finance, biology, the Fourier Transform is instrumental. It allows us to dissect the noise signal into its frequency components, providing a clear picture of its spectrum. This spectral analysis is critical for understanding the behavior of noise generators, especially in terms of randomness and unpredictability which are crucial for all of the applications. By analyzing the frequency components, we can identify the presence of any periodic components or dominant frequencies that might compromise the randomness of the noise source.

The Fourier Transform's ability to separate and identify these components makes it possible to evaluate the suitability of a noise generator for a given application. Specifically, we aim to understand if the noise generator offers the required random walk characteristics and to identify the conditions under which the generator performs optimally for our range of applications.

3.1.2 Expected results

By applying the Fourier Transform to analyze analog noise sources, we can expect several outcomes which will be the desired aim of our project:

- Identification of Frequency Components: We will be able to see the distribution of power across different frequencies, identifying any unexpected peaks or patterns.
- Evaluation of Randomness: The analysis will help in assessing the randomness quality of the noise by checking for uniformity in the frequency spectrum.

A more detailed analysis of the noise spectrum from our noise sources involves several key steps to better understand the noise characteristics and their implications for the generator's operation. Here's what such an analysis could include:

1. Wider Frequency Analysis: Expand the frequency analysis to capture the entire bandwidth of interest. Noise characteristics can span a wide range of frequencies, and capturing the full spectrum is vital for a complete characterization.
2. Logarithmic Frequency Scale: Employing a logarithmic scale can more clearly reveal the characteristics of the noise across a broad range of frequencies, particularly when dealing with a spectrum that spans several orders of magnitude.
3. Noise Floor Assessment: Determining the noise floor level across the frequency spectrum. The noise floor gives a baseline for the lowest level of noise within the system.
4. Statistical Properties: Analyzing the statistical nature of the noise, such as by looking at its probability density function to ascertain if it follows Gaussian, uniform, or another distribution.

3.2 Autocorrelation

Autocorrelation is a statistical tool that measures how a signal correlates with itself over different time intervals. It is pivotal for identifying patterns within time series data, helping to uncover how a signal's present value is related to its past or future values.

The autocorrelation function (ACF) for a continuous signal $x(t)$ is defined by the equation:

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot x(t + \tau) \, dt$$

For discrete signals, the Autocorrelation function modifies to:

$$R(k) = \sum_{n=0}^{N-k-1} x(n) \cdot x(n + k)$$

where τ represents the time lag, T is the total observation period, k is the discrete time lag, and N is the total number of observations. These expressions provide insight into the internal structure and predictability of the signal. Since the signal is converted into discrete, we will be using discrete Autocorrelation function.

3.2.1 Application of Autocorrelation in the analysis of noise signals

Autocorrelation is being utilized to assess the randomness and predictability of noise generated by analog sources, especially for critical applications like cryptography. Through Autocorrelation analysis, we can determine whether a noise generator produces output that is sufficiently random and unpredictable, essential for cryptographic security. A desirable characteristic is a low or near-zero Autocorrelation, indicating an output close to true randomness [3]. This analysis aids in selecting noise generators that fulfill the randomness criteria required for secure cryptographic functions and other applications.

3.2.2 Expected results

When using the Autocorrelation method to characterize analog noise signals from noise generators, we can anticipate several important outcomes:

- Randomness Assessment: The primary goal is to assess the randomness of the noise signal. Ideally, the Autocorrelation function should approach zero for all non-zero time lags, indicating a high level of unpredictability in the signal. This characteristic is particularly crucial for cryptographic applications to ensure security.
- Identifying Patterns and Periodicities: Autocorrelation can uncover hidden patterns, periodicities, or regularities within the noise signal. Ideally, these should not exist in a perfect noise generator. Detecting any such patterns is essential for evaluating the quality and suitability of the noise source.
- Signal Consistency and Stability: Analysis over time can reveal the stability and consistency of the noise generator. A stable Autocorrelation function across different time frames indicates a reliable noise source, desirable for long-term application needs.
- Comparative Analysis of Noise Sources: Autocorrelation allows for the comparison between different noise generators. This comparison aids in selecting the best noise source for specific applications, based on the randomness quality and the absence of predictable patterns.
- Correlation Time Determination: The Autocorrelation function aids in determining the noise signal's correlation time, which is indicative of

how long the signal remains correlated with itself. A shorter correlation time, showing a quick decay of predictability, is often preferred for high-quality random signals.

- Identification of Non-Ideal Behaviors: Non-ideal behaviors, including bias, drift, or other signal anomalies, can be detected through Autocorrelation analysis. Identifying these issues is crucial for ensuring the noise generator's effectiveness in applications requiring high levels of randomness.

3.3 Wavelets

Wavelets are mathematical functions that decompose data into different frequency components, allowing analysis of the data at various resolutions. They are particularly useful for analyzing signals that exhibit non-stationary behavior or contain features that vary in time. Unlike traditional Fourier transforms, which represent a signal as a sum of sinusoidal waves, wavelet transforms provide a way to analyze both the frequency and location in time of features within a signal. This dual capability makes wavelets an excellent choice for signal processing tasks, including denoising, compression, and feature detection. CWT analyzes the signal at various scales, which allows us to capture both high-frequency and low-frequency components with different resolutions.

The Continuous Wavelet Transform (CWT) of a signal $x(t)$ is defined as:

$$CWT(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t)\psi^* \left(\frac{t-\tau}{s} \right) dt \tag{3.1}$$

where $\psi(t)$ is the wavelet function, τ is the translation parameter (indicating the time shift), s is the scale parameter (related to the frequency), and * denotes complex conjugation. The choice of wavelet function $\psi(t)$ depends on the specific requirements of the analysis, such as the need to detect specific types of features in the signal.

The Discrete Wavelet Transform (DWT), a more computationally efficient version, involves choosing scales and positions based on powers of two (dyadic scales and translations). This leads to a Fast Wavelet Transform (FWT), which simplifies analysis and reconstruction of the signal.

3.3.1 Application in the analysis of noise signals

In the context of characterizing analog noise sources for applications like cryptography, wavelets are used because of their ability to isolate the characteristics of noise at multiple scales. This multi-resolution analysis allows us to identify patterns, trends, and anomalies in the noise signals that are crucial for assessing their randomness and suitability for security applications [15]. By examining the time-frequency representations of the noise, we can understand the behavior of different noise generators, evaluate their performance under various conditions, and determine their effectiveness in producing the required random walk characteristics.

3.3.2 Expected results

By employing wavelet analysis, we expect to:

- **Time-Frequency Characteristics**: Wavelet analysis will reveal the time-frequency characteristics of the noise signals, providing insights into how the energy of the noise signal is distributed across different frequency bands over time. This is crucial for understanding the behavior of the noise source.
- **Transient Features**: Identification of transient features or sudden changes in the noise signal that might not be apparent with other analysis methods. Wavelets are particularly adept at detecting and characterizing short-duration phenomena in signals.
- **Statistical Properties**: Analysis of the statistical properties of the noise signal at various scales. Wavelets can help in examining the distribution, variance, and other statistical measures of the noise signals, which are important for assessing their randomness and unpredictability.
- **Comparison of Noise Generators**: The method can compare different noise generators based on their time-frequency profiles and statistical properties. This comparison might include the effectiveness of the noise generators in producing truly random signals suitable for applications like cryptography.
- **Anomalies and Patterns**: Detection of any anomalies or regular patterns within the noise signals. While true noise should be random and unpredictable, any regularities or anomalies detected could indicate issues with the noise generator or external influences on the signal.

The outcome of this analysis will provide a comprehensive understanding of the suitability of various noise generators for applications requiring high degrees of randomness and unpredictability.

Chapter 4

Noise Spectrum

In this chapter, we will be discussing the noise spectrum produced by analog noise generators. As the scope of this thesis project we will be focusing on three primary types of noise: white, pink, and brown. Each type of noise is characterized by its unique spectral density, which has profound implications in various fields.

We will also explore the generation mechanisms behind these noise types, discuss their statistical properties, and examine their applications in various scientific and technological domains. Understanding these noise types enhances our ability to manipulate and apply them in both theoretical and practical scenarios, paving the way for innovations in sound design, signal processing, and beyond.

These noise spectrum are essential for understanding the nature and characteristics of noise within a system. We are characterising noise signals that are obtained from various noise generators (discussed in chapter 2) into different types of noise, that are available in the spectrum, and are discussed below.

4.1 White Noise

White noise is one of the random signal that is generated by analog generators which has equal intensity at different frequencies, giving it a constant Power Spectral Density (PSD), denoted by $\varphi_{xx}(e^{j\omega}) = \sigma_x^2$. This implies that white noise has equal energy at all frequencies and is particularly useful in quantization error analysis because it represents a signal with zero mean and its power is evenly distributed across the frequency spectrum. The average power of a white noise signal is given by σ_x^2, calculated by integrating its power spectrum over all frequencies [4].

In signal processing and physics, white noise is used for analyzing and improving the robustness of systems against noise, among other applications.

It gives an ideal spectrum of values generated which can be further used for the applications being talked in the previous chapters.

We have generated a white noise with the help of PRNGs by using the function `np.random.normal` from the library `numpy` [8]. This function generates a sequence of random samples following a Gaussian distribution with a mean of 0 and standard deviation of one, reflecting the random and uncorrelated nature of white noise. To estimate the PSD, Welch's method is employed through the `welch()` function from the `scipy.signal` module, in `scipy` library [16], consistent with the theoretical approach to average power calculation.

4.1.1 Fundamental Characteristics of White Noise

White noise is characterized by several fundamental properties, making it a unique and widely applicable signal in various fields. These characteristics include:

1. **Flat Power Spectral Density (PSD):** We have used one of the method to visualize the white noise and a semi-logarithmic plot of the PSD using the function `plt.semilogy` by using the `matplotlib` library [17] in figure 4.1. The PSD of white noise is flat, meaning that its energy is distributed equally across all frequencies. This flatness is why it's termed "white," by analogy to white light that contains all visible wavelengths of light at equal intensity.

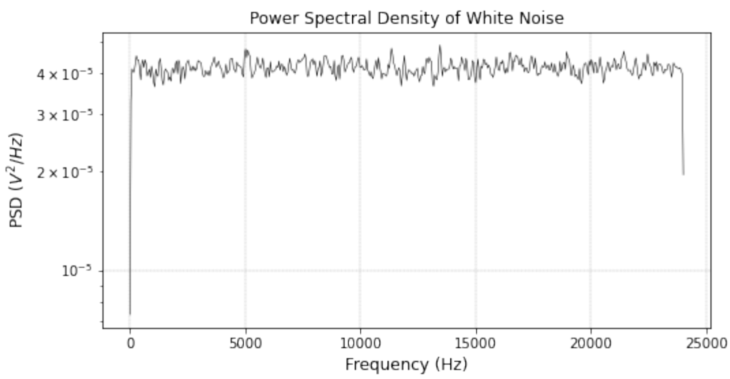

Figure 4.1: Power Spectral Density Plot of White Noise by PRNG

   This graphical representation from figure 4.1 confirms the flatness of the white noise spectrum and aligns with the theoretical description, where the power of white noise is constant and unrelated to frequency.

2. **Randomness:** White noise is a random signal, with its amplitude varying unpredictably over time. The randomness is often modeled

using a Gaussian distribution, where amplitude values follow the normal distribution curve.

3. **No Correlation:** Samples in white noise are statistically independent and uncorrelated. The auto-correlation function of white noise is a delta function, indicating perfect non-correlation between any two samples.
4. **Uniform Energy Across Frequencies:** Owing to its flat PSD, white noise contains all frequencies within its range, allocating equal energy to each. This property is similar to white light, which includes all visible wavelengths at equal intensity.
5. **Lack of a Dominant Frequency:** White noise does not exhibit a dominant frequency or tone, a direct consequence of its equal representation of all frequencies.

4.2 Pink Noise

A Gaussian white noise signal is one where each signal element is drawn from an independent and identically distributed Gaussian distribution, with fixed parameters. If we Fourier transform a two-dimensional white noise image, then multiply the constituent amplitudes with the inverse of their frequencies, i.e., filter the amplitude spectrum with an $\frac{1}{\sqrt{f_x^2+f_y^2}}$ filter (for two-dimensional signals, $f = \sqrt{f_x^2 + f_y^2}$), then transform it back, the resulting signal is called $\frac{1}{f}$ noise, or pink noise. Longer wavelengths are more dominant in this noise, i.e., it has some correlations over space, unlike white noise. Fig. 5.2 shows an example of two-dimensional pink noise texture for PSD [4].

4.2.1 Generation of Pink Noise

Pink noise can be also generated with PRNGs, which involves the use of algorithms that modify a sequence of pseudo-random numbers to adhere to a $1/f$ power spectrum. Common digital algorithms include the Voss-McCartney algorithm, which overlaps white noise from multiple PRNGs across different octaves, and the filter method, where white noise is processed through a digital filter that conforms to a $1/f$ attenuation pattern. The filter method often employs Fast Fourier Transform (FFT) `np.fft.rfftfreq(samples, 1/fs)` technique to shape the spectrum and generate the pink noise from `numpy` library [8], replicating the work as done in [3].

Analog pink noise can also be generated with the help of electronic circuits. A frequent approach uses a noise source, such as a Zener diode (and other noise generators which we have discussed in the conclusion), followed by an analog filter that reduces the higher frequency components, usually a low-pass filter, which results into $1/f$ characteristic.

4.2.2 Fundamental Characteristics of Pink Noise

1. **Power Spectral Density (PSD):** Pink noise has a power spectral density that decreases with the increase in frequency. On a log-log plot, the PSD of pink noise decreases linearly, exhibiting a slope that corresponds to a $1/f$ relationship, and can be seen in 4.2. The figure shows there is a less power at higher frequency components.

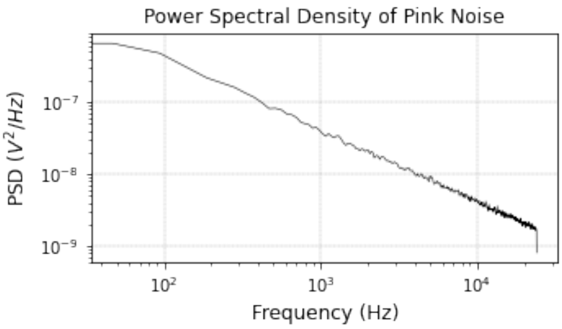

Figure 4.2: Power Spectral Density Plot of Pink Noise by PRNG

2. **Randomness:** The amplitude of pink noise is random and follows Gaussian distribution, similar to white noise. Which will be seen in the later part of this report. However, the distribution of energy across frequencies follow a specific pattern, which correlates to the output that is shown in 4.2, unlike white noise which is equal across all frequencies.
3. **No Correlation:** Although there is a dependency between the frequencies in pink noise, resulting in the $1/f$ PSD, each individual sample point in time is typically uncorrelated with its neighbouring points. This means, the noise is unpredictable in its waveform.
4. **Uniform Energy Across Octaves:** Unlike white noise, which has uniform energy across linear frequency bands, that can be seen in 4.1, pink noise has uniform energy across logarithmic bands (octaves).

4.3 Brown Noise

Brown noise, also known as Brownian noise, gets its name not from its color but from the random motion described by Robert Brown, also known as Brownian motion [4].

The power spectral density (PSD) of Brownian noise decreases in proportion to the square of the frequency, following a $1/f^2$ trend. This creates a sound profile with greater intensity at lower frequencies, much more so than pink noise ($1/f$), and even more so compared to white noise, which has a constant PSD across frequencies.

Mathematically, Brownian noise can be described as the integral of a white noise signal, where white noise has a flat PSD. In the frequency domain, this integration process transforms the flat spectrum of white noise into a power spectrum that falls off as $1/f^2$, yielding the characteristic PSD of Brownian noise. When a white noise signal, denoted as $dW(t)$, is integrated over time, the resulting signal $W(t)$ represents Brownian noise.

An important aspect of the Fourier transform, denoted by F, is that it transforms the derivative of a signal in a manner that is proportional to the frequency. Therefore, when the derivative of a white noise signal is taken and then transformed, it is scaled by $i\omega$, where ω is the angular frequency. Consequently, when the power spectrum of this derived and transformed signal is considered, it is found to be proportional to S_0/ω^2, with S_0 being a constant amplitude.

4.3.1 Generation of Brown Noise

For the generation of brown noise, similar to pink and white noise, both digital methods using PRNGs and analog methods using electronic noise generators can be employed. Digital generation often involves filtering or integrating white noise generated by PRNGs, while analog generation might use electronic circuits with low-pass filters designed to achieve the $1/f^2$ spectral fall-off [18].

We used the function `cumtrapz` from the library `scipy` [16] to transform white noise into brown noise. Then, Welch's method is used to estimate the PSD of the resulting brown noise, which is plotted on a log-log scale. The `plt.loglog` [17] function is particularly useful for visualizing PSDs that follow power-law relationships.

4.3.2 Fundamental Characteristics of Brown Noise

Brown noise, also referred to as Brownian noise or red noise, possesses unique characteristics that distinguish it from other types of noise such as white or pink noise. Here are its fundamental characteristics based on the provided parameters:

1. **Power Spectral Density (PSD):** Brown noise has a PSD that decreases with the square of the frequency, which can be expressed as proportional to $1/f^2$. This results in most of the signal's power being distributed at lower frequencies. The PSD of brown noise is not flat, and when plotted on a logarithmic scale, it shows a steeper slope than pink noise, indicating a faster decrease in power with increasing frequency.

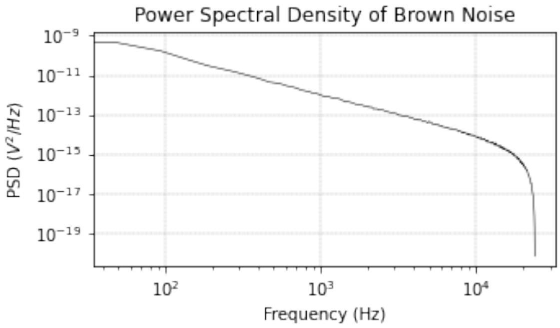

Figure 4.3: Power Spectral Density Plot of Brown Noise by PRNG

2. **Randomness:** Although brown noise is random in nature and has a Gaussian probability distribution in its amplitude over time, its spectrum is not uniformly random across frequencies due to its $1/f^2$ relationship. The randomness in brown noise is the product of accumulating random steps, which can be modeled as a random walk, hence its alternative name, random walk noise.

3. **No Correlation:** Brown noise exhibits a strong correlation between successive samples because it is a cumulative process, akin to the integral of white noise. This correlation means that the current value is dependent on the previous values, making it significantly different from white noise, which has no correlation between successive samples [19].

4. **Non-Uniform Energy Across Frequencies:** Unlike white noise, brown noise does not have uniform energy across linear frequency bands. Instead, it has more energy in lower frequencies, decreasing as the frequency increases. It does not maintain equal energy across octaves as pink noise does.

5. **Lack of a Dominant Frequency:** There is no single frequency at which the power of brown noise is maximized. The power is most concentrated at lower frequencies and diminishes as the frequency increases, and can be seen in the figure 4.3.

Chapter 5

Analysis of Signals

In this chapter, we are going to analyse the noise generators mentioned in chapter 2 with the help of methods that are explained in chapter 3. The analysis of all the noise generators and the data acquired from these noise generators is done on the basis of methods that has been used to characterize the noise generator. Hence, the sections ahead are divided into methods that we discussed in chapter 3. These section will explain how did the noise generators performed when the fore-mentioned methods were applied to it. For reference, we are considering all the datasets that were acquired from the noise generators, whose plots are also shown in chapter 2.

5.1 Fourier Transform

With the reference to chapter 3, we used Fourier Transform as one of the methods to characterise the analog signals. We followed the following procedure to do the analysis.

1. Loading data into pandas DataFrame
2. Computing the time differences to find the sampling rate
3. Applying Fourier transform to the recorded voltages
4. Calculating corresponding frequencies using the sampling rate
5. Identifying the dominant frequency or analyze the spectrum

We used the function `np.fft.fft()` function, from the library `numpy` [8] for this analysis. It computes the Fast Fourier Transform (FFT) of the signal to calculate Discrete Fourier Transform (DFT). To analyse the frequencies in the signal, our python script calculated the frequency components associated with the FFT result. The Nyquist frequency, which is taken as the half of sampling rate, was determined by the reciprocal of twice the delta, which is a the time interval between two consecutive samples in the time variable. Further, the frequencies for FFT are calculated using the function `np.fft.fftfreq`, which generates an array of frequencies corresponding to the FFT result. After observing multiple results, the FFT was appearing symmetric around the zero frequency, and the negative frequencies do not contain additional information, hence we filtered the positive frequency.

5.1.1 FFT analysis of PRNG

For comparative purposes, we are incorporating the Fast Fourier Transform (FFT) of a sequence generated by a pseudo-random number generator. This is achieved using the function `np.random.rand()` with the function `np.linspace()` from the library `numpy` [8], assuming that it generates pure white noise. The FFT of this sequence is displayed in figure 5.1

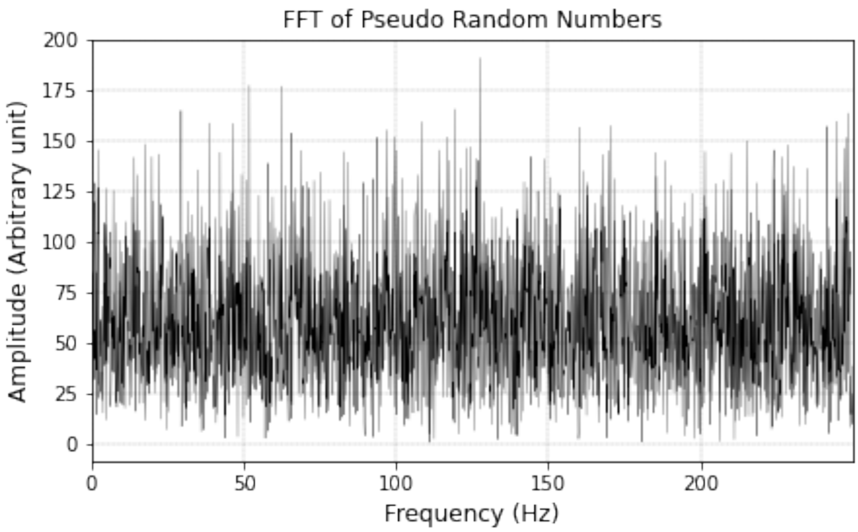

Figure 5.1: FFT of PRNG

5.1.2 FFT Analysis of BJT based noise generators

As mentioned in the chapter 2, we have acquired 4 datasets from BJT based noise generators 1 and 2, both with lowpass filter and without lowpass filter. The following plots were acquired after doing the Fast Fourier transform on those datasets.

Since these are FFT plots, on x-axis we present frequency component of the signal acquired, and on y-axis we present amplitude at that particular frequency.

Analysis of BJT based noise generator 1 with lowpass filter

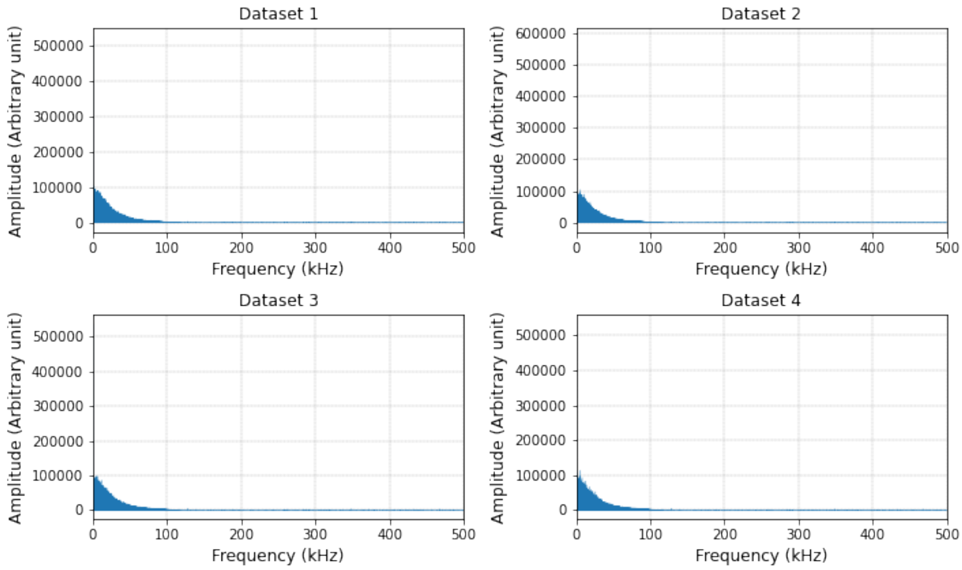

Figure 5.2: FFT of BJT based noise generator 1 with lowpass filter

From the plots created in the figure 5.2, we can observe that these plots show the distribution of signal amplitudes across a range of frequencies, from 0 to 500 kHz.

From figure 5.2, Dataset 1 presented an amplitude peak at the lower end of the spectrum, with a swift roll-off beyond approximately 50 kHz. This indicates a dominant low-frequency component in the noise signal, with the lowpass filter effectively attenuating higher frequencies. Dataset 2 exhibited a relatively flat spectrum up to around 50 kHz, suggesting a more even distribution of frequency components within the passband of the filter. Datasets 3 and 4 similarly displayed peak amplitudes at the lower frequencies, with a steep descent past the cut-off frequency, indicative of the lowpass filter's frequency response. The amplitude measurements, in arbitrary units, reached upwards of 500,000 units, reflecting the significant energy present at lower frequencies.

Analysis of BJT based noise generator 1 without lowpass filter

The figure 5.3 indicates a set of Fast Fourier Transform (FFT) plots of BJT based noise generator 1 without lowpass filter.

In these plots in figure 5.3, the amplitudes are marked in arbitrary units, with peaks observed in the range up to approximately 500,000 units, denoting the high energy at lower frequencies. Unlike the previously analyzed data

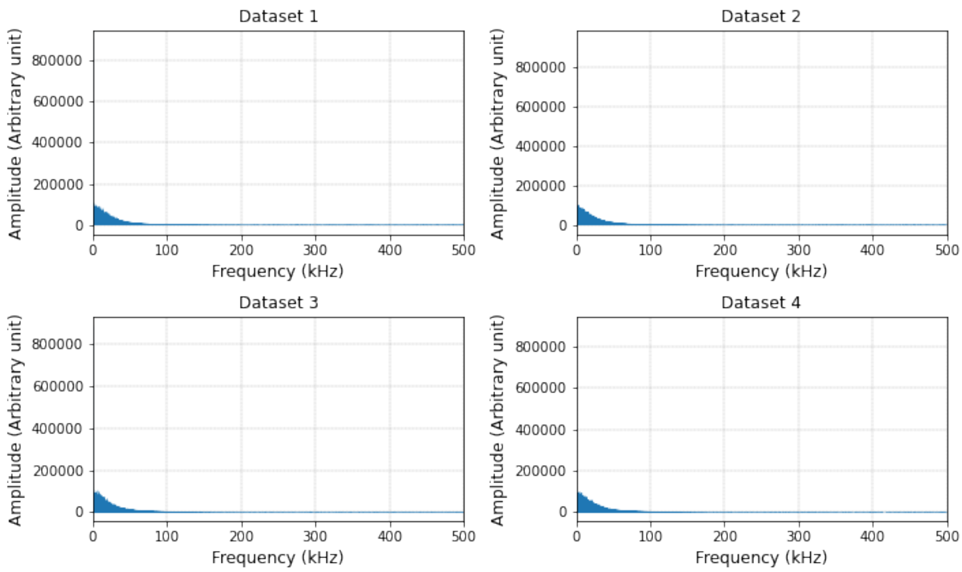

Figure 5.3: FFT of BJT based noise generator 2 without lowpass filter

with lowpass filtering shown in figure 5.2, these plots display a more gradual roll-off without a sharp cutoff, illustrating the wide bandwidth of the noise signal.

Analysis of BJT based noise generator 2 with lowpass filter

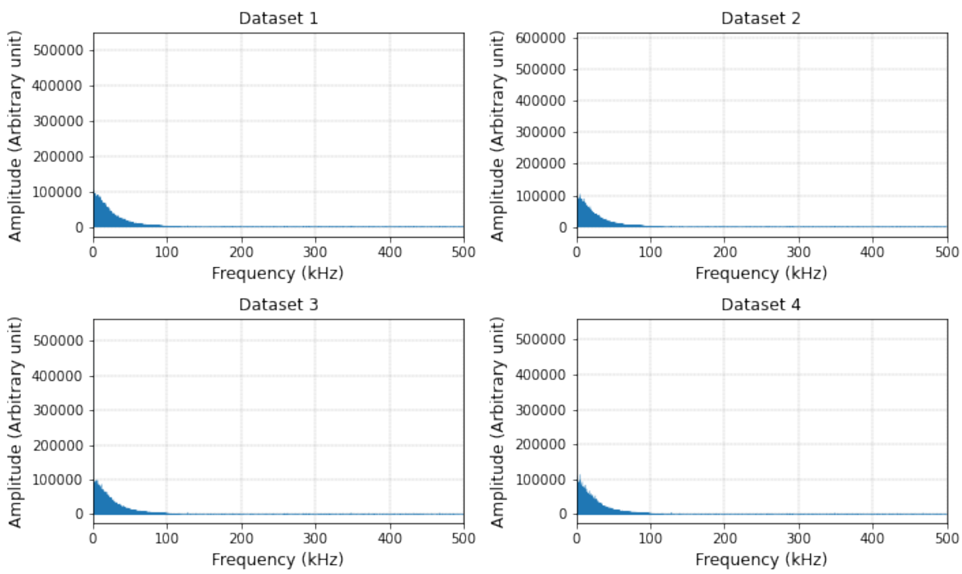

Figure 5.4: FFT of BJT based noise generator 2 with lowpass filter

Each dataset's FFT plot from figure 5.4 also shows a non-linear relationship between frequency and amplitude. Numerically, the amplitude starts at near-zero for the lowest frequencies, escalating to peaks possibly exceeding 500,000 arbitrary units before a frequency of 50 kHz. This also suggests a strong presence of low-frequency components. Post the peak, the amplitude exhibits a marked exponential decay, which indicates the filter's attenuation effect.

The exact numerical amplitudes at specific frequencies, such as at 100 kHz, 200 kHz, and beyond, illustrates the filter's attenuation rate. For example, if the amplitude at 100 kHz is 50 percent of the peak amplitude, the filter's cut-off frequency might be near this value. Subsequent halving of the amplitude could be observed with each octave, which is characteristic of a standard lowpass filter's behavior.

Analysis of BJT based noise generator 2 without lowpass filter

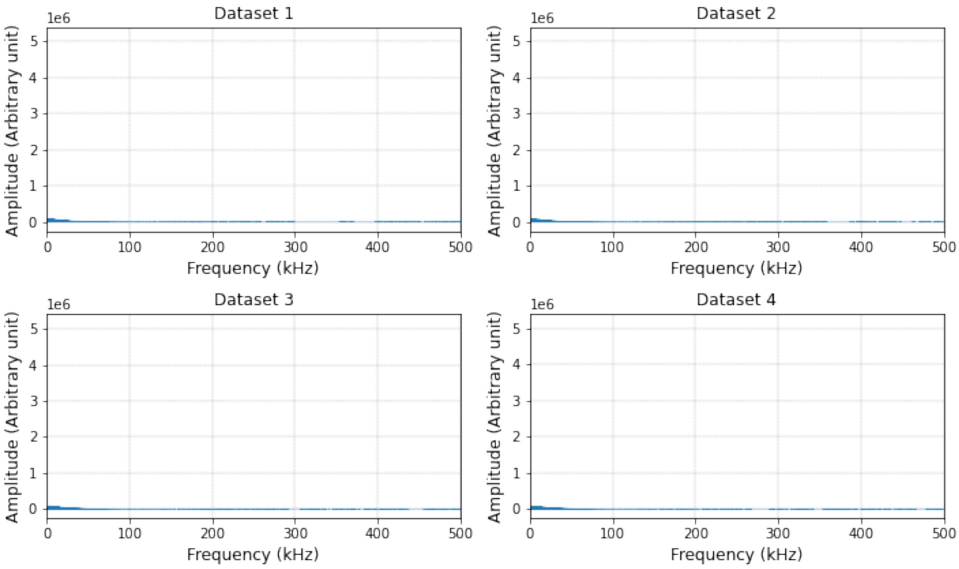

Figure 5.5: FFT of BJT based noise generator 2 without lowpass filter

In this analysis shown in figure 5.5, the absence of lowpass filter indicates a more uniform distribution of energy across the frequency spectrum due to the absence of filtering.

Dataset 1 exhibits a maximum amplitude value that potentially reaches 5e5 arbitrary units, likely occurring at the lower end of the frequency spectrum. The amplitude gradually declines, but remains significantly above the baseline across all frequencies observed, suggesting a uniform distribution of signal power without a steep roll-off. While Dataset 2 appears to follow a similar trend, with its peak amplitude perhaps slightly lower than that of Dataset 1, indicating a possible variance in the noise level or generator output between the two datasets. The decline in amplitude is again gradual, with no discernible cut-off frequency.

Across all datasets within 5.5, the amplitude does not drop to half its maximum value within the displayed frequency range, which would typically indicate the -3dB point commonly used to define the cut-off frequency in filter design. The absence of this feature is consistent with the expected output of an unfiltered noise source.

Analysis of BJT based noise generators (1 and 2)

As a conclusion for BJT based noise generator 1 (with and without lowpass), refer the following table 5.1.

For a conclusion for BJT based noise generator 2 (with and without lowpass), refer the following table 5.2.

5.1.3 FFT Analysis of HP 3722-A noise generator

All the signals generated from HP 3722-A are at 50kHz, and sampled at 1 MSPS (10e6 samples per second), 10 seconds of data.

Analysis of HP 3722 A

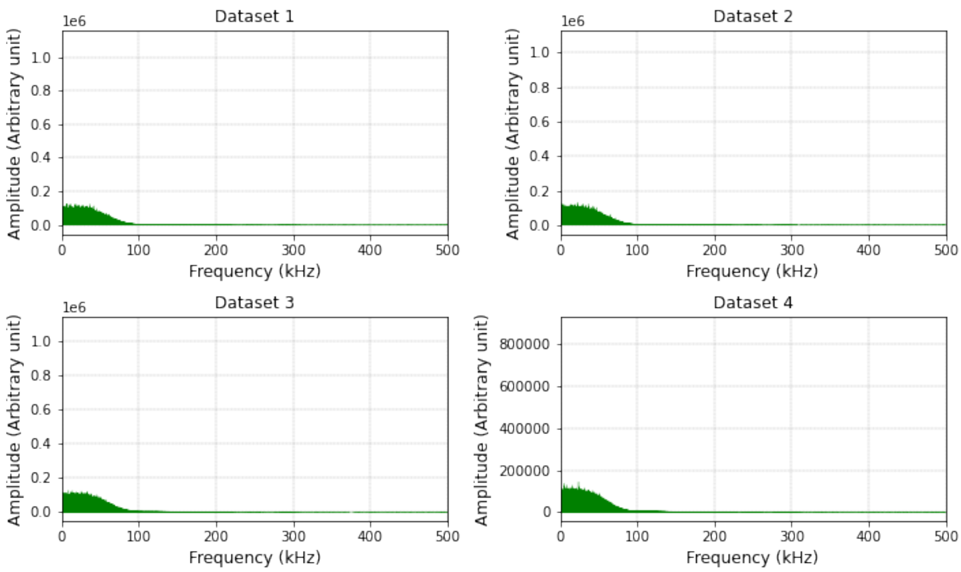

Figure 5.6: FFT of HP3722-A noise generator

Sr. No.	Noise Generator	Analysis	Related Figures	Type of noise
1	BJT based noise generator 1	1. All four datasets show that the amplitude is highest at lowest frequencies, and suggest common pattern across the dataset, where low frequency components are dominant. 2. There is a rapid amplitude decay after the Nyquist frequency. 3. Similar shapes although the scales of y-axes differ slightly. 4. This behaviour of FFT plots suggests that the noise produced by mentioned noise generator produces Pink Noise.	Figure 5.2 Datasets 1, 2, 3 and 4	Pink noise
2	BJT based noise generator 1 without lowpass filter	1. High amplitude at lower frequency indicates that low frequency components are predominant. 2. The steepness of the roll-off after a certain frequency point indicates the cutoff frequency of the lowpass filter. 3. It suggests that frequencies above the cutoff are significantly attenuated, which is expected behaviour for a lowpass filter. 4. This behaviour of FFT plots suggests that the noise produced by mentioned noise generator produces Pink Noise.	Figure 5.3 Datasets 1, 2, 3 and 4	Pink noise

Table 5.1: Analysis of BJT based noise generator 1 (with and without lowpass filter)

The set of FFT plots shown represent the frequency spectrum of signals from figure 5.6.

From the figure 5.6, Dataset 1 and 2 demonstrates an amplitude spectrum starting at the noise floor and increasing to a peak amplitude that may be close to 1e5 arbitrary units. The amplitude in Dataset 3 and 4 starts at the noise floor and peaks at a value that is lower than those observed in Datasets 1 and 2, potentially around 5e4 arbitrary units. The lower peak amplitude in Dataset 3 could indicate a variation in the output level of the noise generator for this dataset.

In all the plots, the lack of significant amplitude below 50 kHz suggests that the signals are undergoing highpass filtering, with a cutoff around 50 kHz. This is supported by the observed absence of significant energy below

Sr. No.	Noise Generator	Analysis	Related Figures	Type of noise
1	BJT based noise generator 2	1. All plots show a high amplitude at lower frequencies. 2. This suggests that the noise has a strong low-frequency component. 3. There is a sharp decline in amplitude as the frequency increases, indicating a clear cutoff point, very close to 0kHz.	Figure 5.4 Datasets 1, 2, 3 and 4	Pink noise
2	BJT based noise generator 2 with lowpass filter	1. High amplitude at lower frequency and are dominant in the noise signal. 2. The use of lowpass filter is reflecting in the amplitude of the signal which is generated from the same noise generator, as the amplitude is dropped to near 0 value. 3. The use of lowpass filter does not change the characteristic of the signal, and the amplitude decay can be seen, which is a characteristic of pink noise.	Figure 5.5 Datasets 1, 2, 3 and 4	Pink noise

Table 5.2: Analysis of BJT based noise generator 2 (with and without lowpass filter)

this frequency threshold across all datasets. Above the 50 kHz mark, the amplitude rises sharply, indicative of the noise generator's broad frequency components being transmitted beyond the highpass filter's cutoff point.

All the FFT plots in 5.6 indicates that the HP 3722A noise generator produces a wide frequency range of noise.

Analysis of HP 3722-A (PRNG)

Figure 5.7 shows signal generated with the help of PRNG source from the same noise generator.

Dataset 1 shows a steep initial drop in amplitude, starting from near the maximum of the plot's amplitude scale, which appears to be in the order of 1.25e6 arbitrary units. The amplitude sharply decreases to a level close to the noise floor within the first 50 kHz, indicating the filter's cutoff frequency is within this range. While, Dataset 2 presents a lower peak amplitude compared to Dataset 1, suggesting a different signal strength or measurement conditions. The amplitude declines rapidly and becomes negligible beyond approximately 50 kHz, supporting the presence of a highpass filter with a

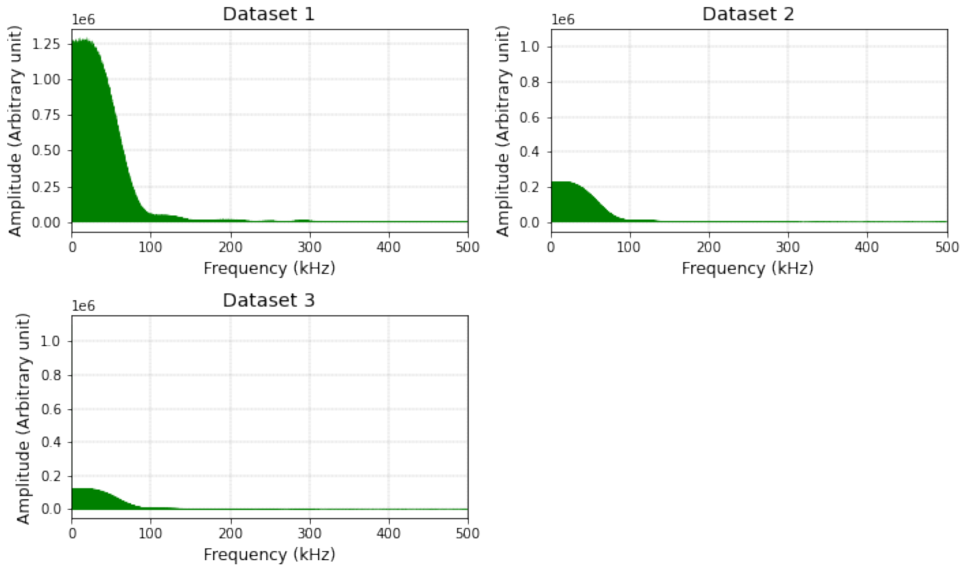

Figure 5.7: FFT of HP3722-A pseudo random noise generator

similar cutoff frequency as observed in Dataset 1. Dataset 3 performs in a similar way as Dataset 2.

For a conclusion for HP 3722-A noise generator, refer the following table 5.3.

5.1.4 FFT Analysis of Wandel and Goltermann RG-1 noise generator

We acquired three kind of datasets from Wandel and Goltermann RG-1 noise generator. These are shown in the table 5.4 shown below.

FFT Analysis of RG-1 at 10bit

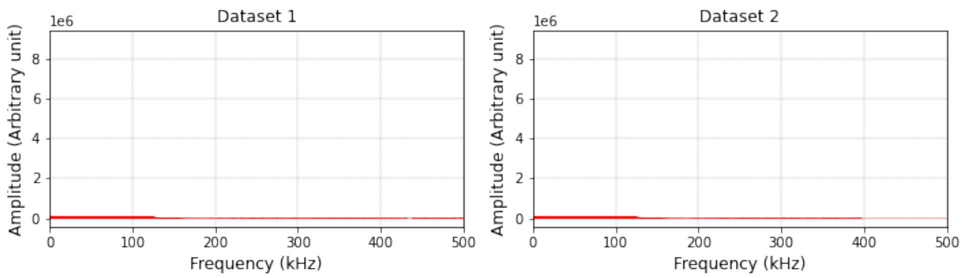

Figure 5.8: FFT of Wandel and Goltermann WG-RG1 noise generator at 10bit

The two FFT plots provided from RG1 at 10 bit in figure 2.7 show flat lines at the noise floor across the displayed frequency spectrum from 0 to 500

Sr. No.	Noise Generator	Analysis	Related Figures	Type of noise
1	HP 3722-A	1. The frequency appears stable until 50kHz and then reduces gradually until 100kHz and attains noise floors at close to 0 amplitude. 2. This variation in the frequency suggests that the signal attains white noise until 50kHz and then pink noise after that until 100kHz.	Figure 5.6 Datasets 1, 2, 3 and 4	White noise until 50kHz, Pink Noise until 100kHz.
2	HP 3722-A (PRNG)	1. The frequencies for these plots are varying according to the sequence length, and so is affecting the FFT plots. 2. These plots tends to have a smoother curve, indicating constant frequencies. 3. This further implies that the noise generator can show different frequencies at different sequence lengths.	Figure 5.7 Datasets 1, 2 and 3	Cannot conclude

Table 5.3: Analysis of HP 3722-A

Sr. No.	Noise Generator	Variation	Number of Datasets
1	Wandel and Goltermann RG1	10 bit	2
2	Wandel and Goltermann	16Hz to 22kHz	4
3	Wandel and Goltermann	100kHz	4

Table 5.4: Summary of noise generator datasets

kHz, with no significant peaks or variations in amplitude that would indicate signal presence. The FFT plots for both datasets numerically indicate a flat amplitude response, consistent with a measurement where no discernible signal exceeds the noise floor, which shows the characteristics of pure white noise, as shown in 4.1.

FFT Analysis of RG-1 at 16Hz to 22kHz

All datasets within the figure 5.9, generated from RG-1 from 16Hz to 22kHz, show a prominent peak in amplitude reaching 200,000 to 300,000 arbitrary units. The frequency appears constant until around 25kHz and decreases immediately after that to close to zero.

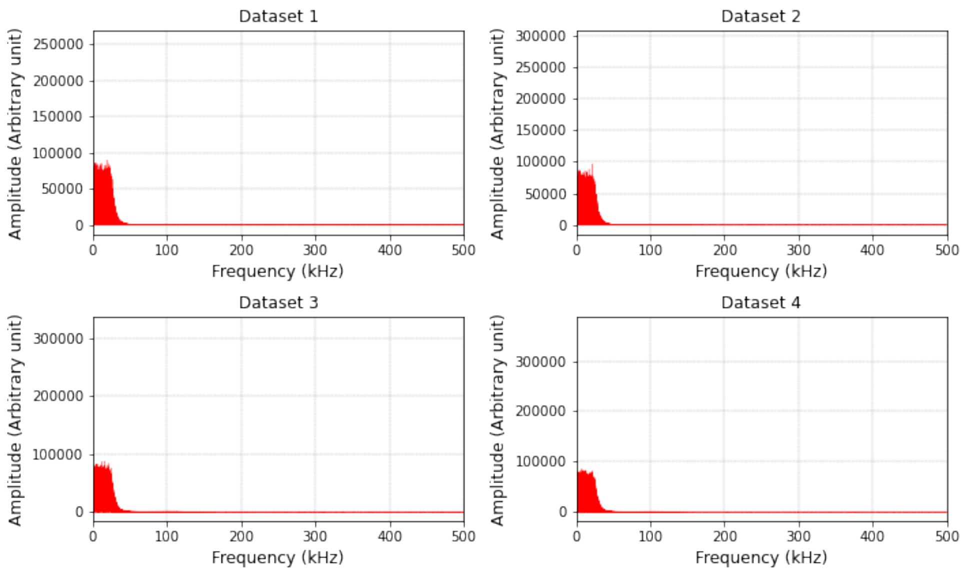

Figure 5.9: FFT of Wandel and Goltermann WG-RG1 noise generator at 16Hz to 22kHz

The presence of these peaks near the start of the frequency axis suggests that the signals have a significant component at a frequency likely below 10 kHz. The variance in peak amplitude between the datasets may reflect differences in signal generation or measurement processes.

FFT Analysis of RG-1 at 100kHz

All the dataset from figure 2.9 generated from RG1 at 100kHz shows an amplitude that barely exceeds the baseline, peaking slightly above zero arbitrary units throughout the spectrum. This uniformity suggests either an extremely low signal strength or a measurement primarily capturing electronic noise. The amplitude does not show significant peaks or specific patterns that would indicate the presence of distinct frequency components or noise characteristics. Instead, the plots suggest that the entire measured frequency spectrum is at a similar energy level, which is minimal. This satisfies the characteristics of possessing white noise, with the reference as mentioned in 4.

Based on the performance of Wandel and Goltermann RG-1 noise generator at 10bit, 16Hz to 22kHz and at 100kHz on FFT plots, we can characterise the behaviour of its signal in the table 5.5 below.

5.1.5 FFT Analysis of Zener diode based noise generators

We acquired the data from two Zener diode based noise generators, using both with and without lowpass filter. Brief plots of time versus signal from

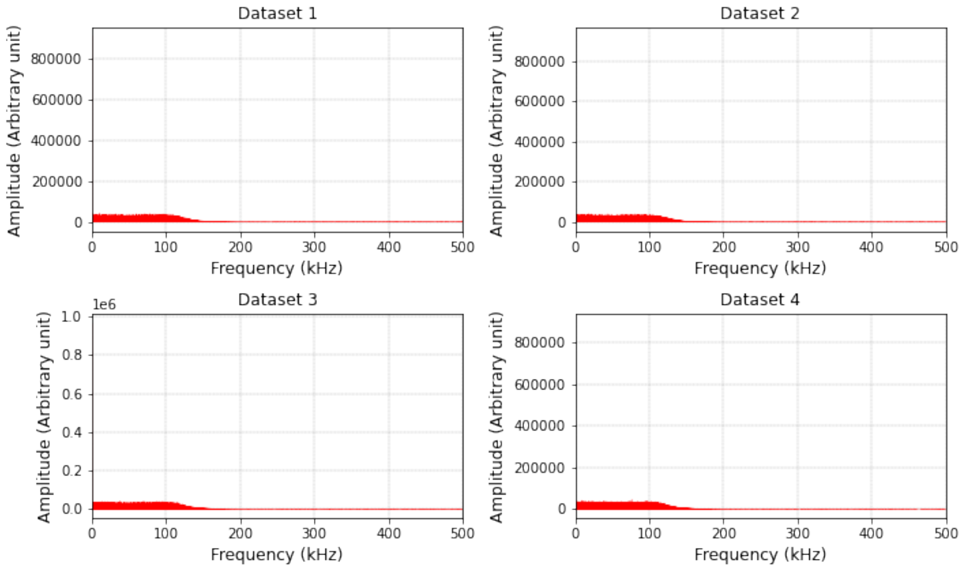

Figure 5.10: FFT of Wandel and Goltermann WG-RG1 noise generator at 100kHz

Sr. No.	Noise Generator	Analysis	Related Figures	Type of noise
1	Wandel and Goltermann RG-1 at 10bit	1. Both of the datasets generates frequency at very low amplitude. 2. There is no spike in the frequency except at 0 frequency. 3. This totally satisfies the characteristics of pure white noise.	Figure 5.8 Datasets 1 and 2	White noise
2	Wandel and Goltermann RG-1 at 16Hz to 22kHz	1. All the dataset show similar plots, in which the amplitude is high and constant at lower frequency. 2. The amplitude decreases gradually after around 25kHz. 3. Since the frequency appears constant until 25kHz, it suggests that the generator produces white noise below 25kHz.	Figure 5.9 Datasets 1, 2, 3 and 4	White noise until 25kHz
3	Wandel and Goltermann RG-1 at 100kHz	1. Both of the datasets generates frequency at very low amplitude. 2. There is no spike in the frequency except at 0 frequency. 3. This totally satisfies the characteristics of pure white noise.	Figure 5.8 Datasets 1 and 2	White noise

Table 5.5: Analysis of Wandel and Goltermann RG1 noise generator

all 16 datasets are shown in 2. Here, we will see the performance of Fourier transform on the signal these noise generator has produced. The data we acquired was sampled at 1 MSPS (10e6 samples per second), and we have total 10 seconds of data.

FFT Analysis of Zener diode based noise generators 1 with lowpass filter

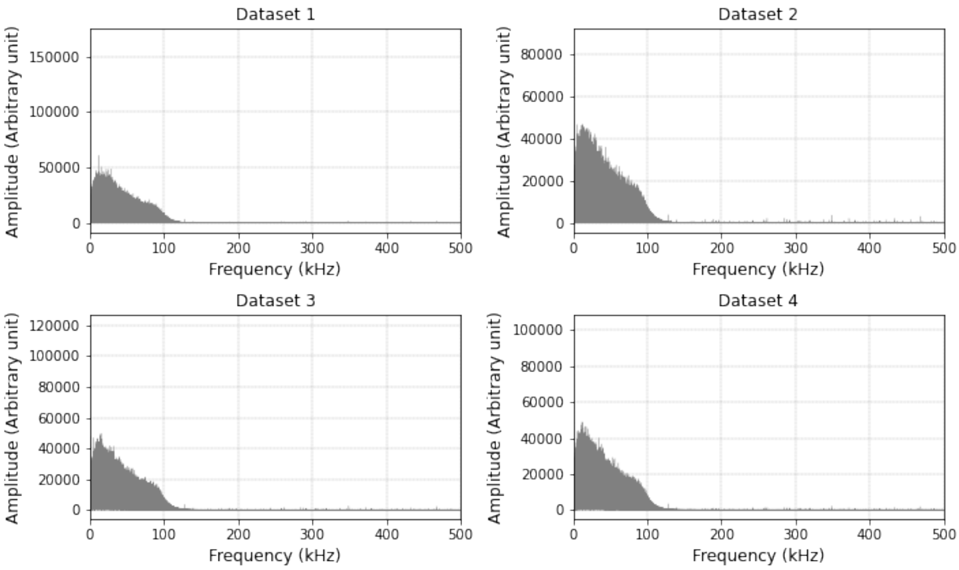

Figure 5.11: FFT of Zener diode based noise generator 1 with lowpass filter

The figure 5.11 shows the first 4 datasets acquired from Zener diode based noise generator 1. Dataset 1 demonstrates an initial high amplitude, peaking at approximately 100,000 arbitrary units, and exhibits a consistent decay in amplitude as the frequency increases. This decay becomes particularly noticeable past 100 kHz, indicating the filter's effective cutoff frequency may be around this point. Dataset 2 follows a similar pattern to Dataset 1, with its peak amplitude slightly lower, around 80,000 arbitrary units. Dataset 3 and Dataset 4 also display a gradual decrease in amplitude with increasing frequency. The peak amplitude in Dataset 3 is around 100,000 arbitrary units, similar to Dataset 1, while Dataset 4 peaks slightly lower at about 70,000 arbitrary units. The amplitude drop-off in both datasets is consistent with that observed in Datasets 1 and 2, suggesting similar filtering parameters are used across all datasets.

The general trend across all datasets is a clear demonstration of lowpass filtering, with the highest amplitudes observed at the lowest frequencies, progressively declining as the frequency increases, and nearly reaching the noise floor by 500 kHz.

FFT Analysis of Zener diode based noise generators 1 without lowpass filter

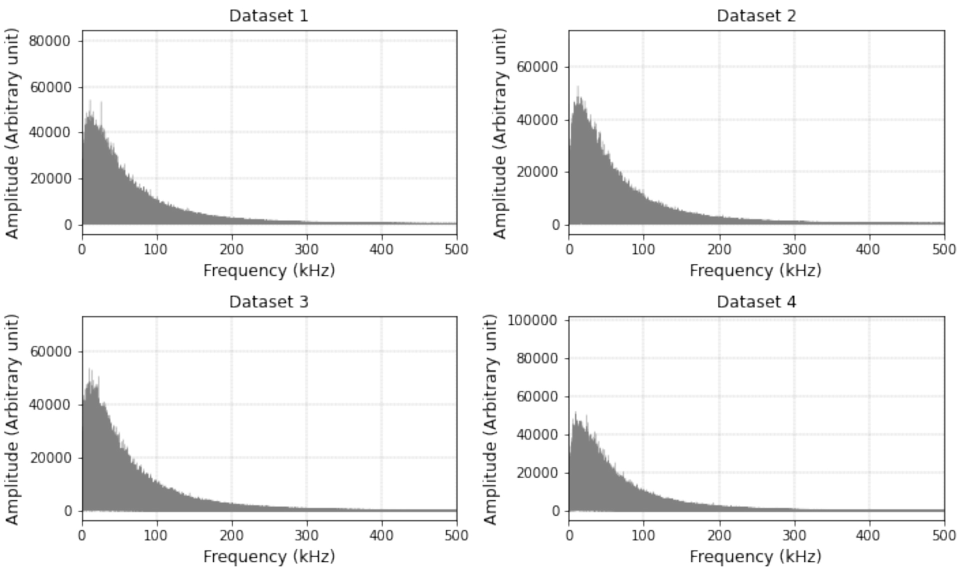

Figure 5.12: FFT of Zener diode based noise generator 1 with lowpass filter

The figure 5.12 shows 4 datasets acquired from Zener diode based noise generator 1 without lowpass filter. Dataset 1 shows an amplitude that starts at just under 8,000 arbitrary units and gradually declines to near-zero levels as frequency approaches 500 kHz, suggesting a natural attenuation or a broad filter response without a sharp cut-off. Dataset 2 displays a peak amplitude slightly lower than Dataset 1, beginning around 6,000 arbitrary units, with a similar downward trend as frequency increases. Dataset 3 and Dataset 4 both demonstrate the same overall declining trend in amplitude with frequency, with Dataset 3 starting at approximately 8,000 arbitrary units, akin to Dataset 1, and Dataset 4 starting slightly lower, similar to Dataset 2.

FFT Analysis of Zener diode based noise generators 2 with lowpass filter

The figure 5.13 shows 4 datasets acquired from Zener diode based noise generator 2 with lowpass filter. Dataset 1 presents a peak amplitude that exceeds 100,000 arbitrary units. This peak is situated in the very low-frequency range, potentially below 50 kHz. Beyond this peak, the amplitude decreases steadily, nearly reaching the noise floor at approximately 500 kHz. While, Dataset 2

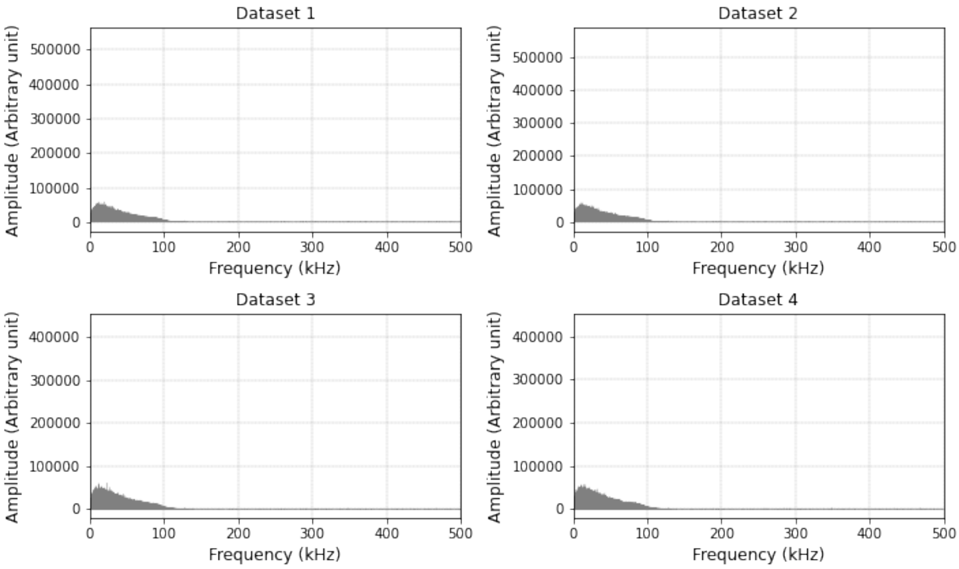

Figure 5.13: FFT of Zener diode based noise generator 1 with lowpass filter

also shows a peak in the low-frequency region, albeit with a lower maximum amplitude around 10,000 arbitrary units, followed by a similar pattern of attenuation across the frequency spectrum. Dataset 3 has a peak amplitude around 30,000 arbitrary units, showing that the signal's strength varies across the datasets but maintains a comparable attenuation pattern, with amplitude levels approaching the noise floor towards the higher frequencies. Dataset 4 exhibits a pattern like that of Dataset 2, with a peak also near the 10,000 arbitrary unit mark and a similar rate of decline in amplitude as the frequency increases.

FFT Analysis of Zener diode based noise generators 2 without lowpass filter

The figure 5.14 shows 4 datasets acquired from Zener diode based noise generator 2 without lowpass filter. Dataset 1 begins with an amplitude just below 20,000 arbitrary units and shows a consistent downward trend, approaching close to zero as it nears 500 kHz. The plot suggests a smooth decline without a distinct cut-off point, which is typical of a natural roll-off or a signal passed through a source without a filter. Dataset 2 has a similar initial amplitude to Dataset 1 and follows the same pattern of decline across the frequency range, ending near the noise floor at the higher frequencies. Dataset 3 starts with a slightly higher initial amplitude, potentially around 25,000 arbitrary units, and decreases in a similar fashion to the first two datasets, again suggesting a smooth frequency-dependent roll-off. And Dataset 4 shows the lowest initial amplitude among the datasets, starting

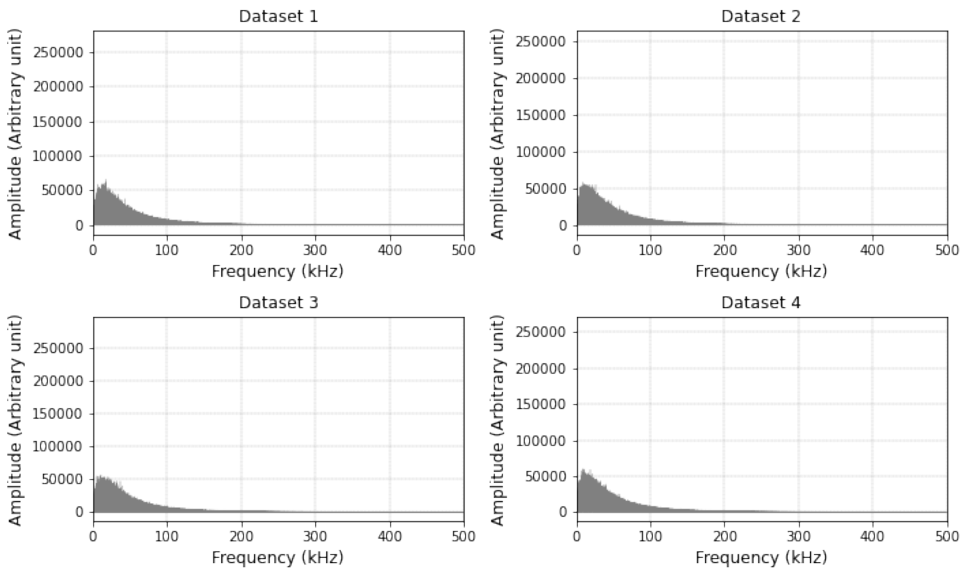

Figure 5.14: FFT of Zener diode based noise generator 2 without lowpass filter

near 10,000 arbitrary units, but still follows the same general trend of decrease towards the higher frequencies.

Based on the performance of Zener diode based noise generators 1 and 2, with and without lowpass filter on FFT plots, we can characterise the behaviour of its signal in the table 5.6 below.

Sr. No.	Noise Generator	Analysis	Related Figures	Type of noise
1	Zener diode based noise generator 1	1. All of the datasets generates similar FFT plots, but with different initial amplitude. 2. The amplitude then decreases linearly with the increasing frequency. 3. Such linear decrease in amplitude with increasing frequency can be seen in Pink Noise.	Figure 5.11 Datasets 1, 2, 3 and 4	Pink noise
2	Zener diode based noise generator 1 without lowpass filter	1. The FFT plot generated by noise generator without lowpass filter shows a curve with the increasing frequency. 2. This type of curve characterizes Brown noise with the reference to chapter 4.	Figure 5.12 Datasets 1, 2, 3 and 4	Brown noise
3	Zener diode based noise generator 2	1. All of the datasets generates similar FFT plots, but with different initial amplitude, similar to generator 1 with lowpass filter. Just the difference is in the amplitude. 2. The amplitude then decreases linearly with the increasing frequency. 3. Such linear decrease in amplitude with increasing frequency can be seen in Pink Noise.	Figure 5.13 Datasets 1, 2, 3 and 4	Pink noise
4	Zener diode based noise generator 2 without lowpass filter	1. The FFT plot generated by noise generator without lowpass filter shows a curve with the increasing frequency. 2. This type of curve characterizes Brown noise with the reference to chapter 4.	Figure 5.14 Datasets 1, 2, 3 and 4	Brown noise

Table 5.6: Analysis of Zener diode based noise generator 1 and 2 (with and without lowpass filter)

5.2 Autocorrelation

With the reference to chapter 3, we used Autocorrelation as one of the methods to characterise the analog signals. We followed the following procedure to do the analysis.

1. Loading data into pandas DataFrame.

2. Downsampling the data by a factor of '500' to enhance the computational speed.
3. Computed the Autocorrelation of the sampled data using `np.correlate()` function from `numpy` library [8] with 'full' mode, then normalizes it by dividing by the maximum value of the Autocorrelation.
4. Calculated the lag values which correspond to the indices of the Autocorrelation array.
5. We plotted the obtained Autocorrelation with and without lag 0, to observe the Autocorrelation values that are close to zero.

5.2.1 Autocorrelation analysis of PRNG

For comparative purposes, we are incorporating the Autocorrelation of a sequence generated by a pseudo-random number generator. This is achieved using the function `np.random.rand()` with the function `np.linspace()` from the library `numpy` [8], assuming that it generates pure white noise. The Autocorrelation plot of this sequence is displayed in figure 5.15.

As we have discussed the examination of noise on the basis of Autocorrelation plot in chapter 3, we will be looking at the following parameters to examine the noise.

1. Randomness assessment
2. Periodicity
3. Signal stability
4. Correlation time
5. Non-ideal behaviors

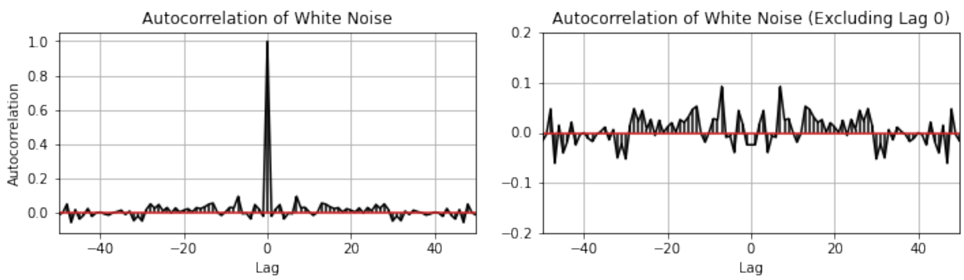

Figure 5.15: Autocorrelation plot of PRNG

Since these are Autocorrelation plots, on x-axis we present Lags in nanoseconds, and on y-axis we present the Autocorrelation factor. Since the correlation at lag 0 is the highest, the plot shows a spike at 0 lag and appears to be close to zero in positive and negative lag. Which suggests that there is no correlation between the signals, when correlated by its own value after and before certain time lag.

Here, the figure 5.15 checks all the points, and hence we will consider figure 5.15 for further reference.

5.2.2 Autocorrelation analysis of BJT based noise generators

As mentioned in the chapter 2, we have acquired 4 datasets from BJT based noise generators 1 and 2, both with lowpass filter and without lowpass filter. The following plots were acquired after performing Autocorrelation on those datasets.

Analysis of BJT based noise generator 1 with lowpass filter

The Autocorrelation plots of BJT based noise generator 1 with lowpass filter is interpreted in the table 5.7

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.7: Autocorrelation analysis of BJT based noise generator 1 with lowpass filter based on figure 5.16

Through this analysis from table 5.7, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by BJT based noise generator 1 with lowpass filter generates white noise.

Analysis of BJT based noise generator 1 without lowpass filter

The Autocorrelation plots of BJT based noise generator 1 without lowpass filter is interpreted in the table 5.8

Through this analysis from table 5.8, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help

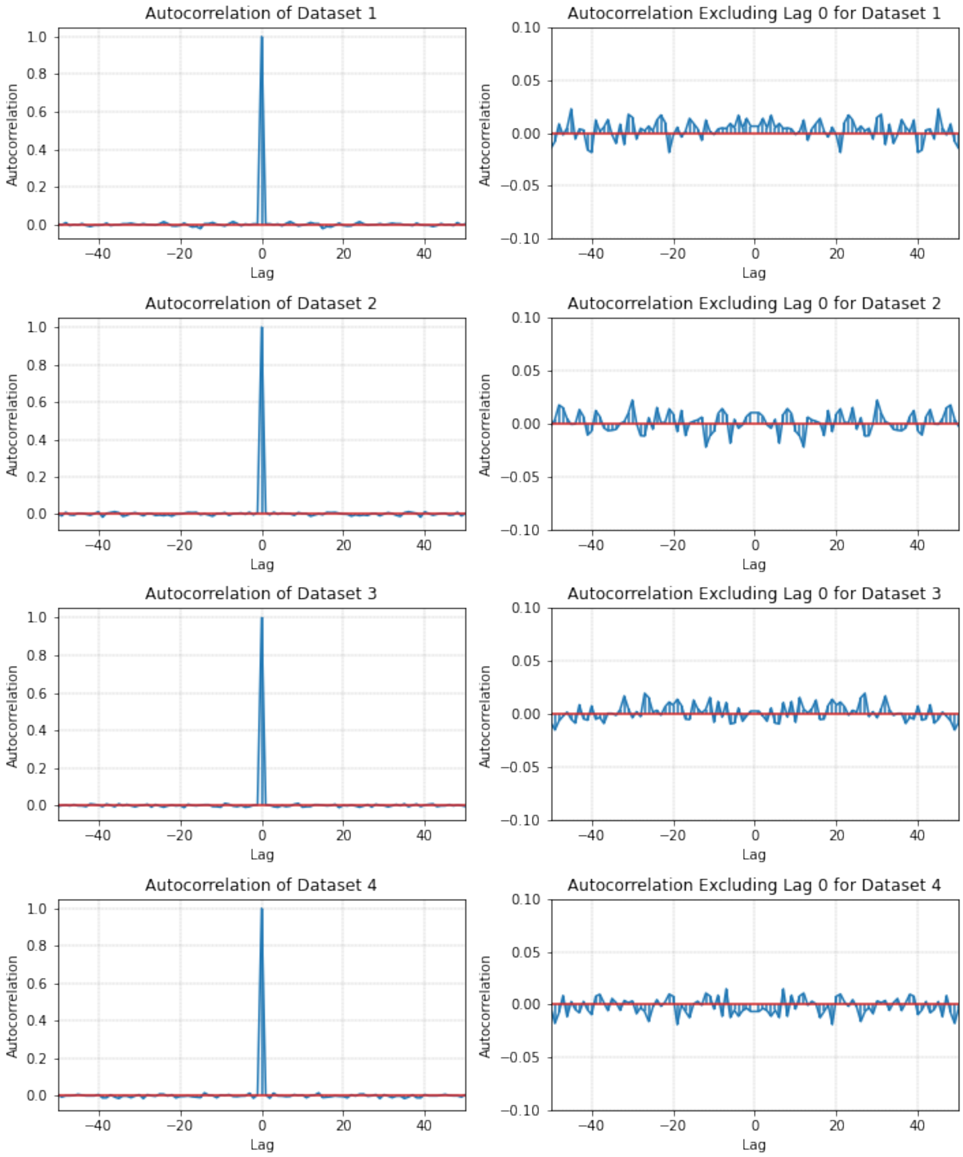

Figure 5.16: Autocorrelation plot of BJT based noise generator 1 with lowpass filter

of this analysis, the noise signal generated by BJT based noise generator 1 without lowpass filter also generates white noise.

Analysis of BJT based noise generator 2 with lowpass filter

The Autocorrelation plots of BJT based noise generator 2 with lowpass filter is interpreted in the table 5.9

Through this analysis from table 5.9, the Autocorrelation plot meets all the

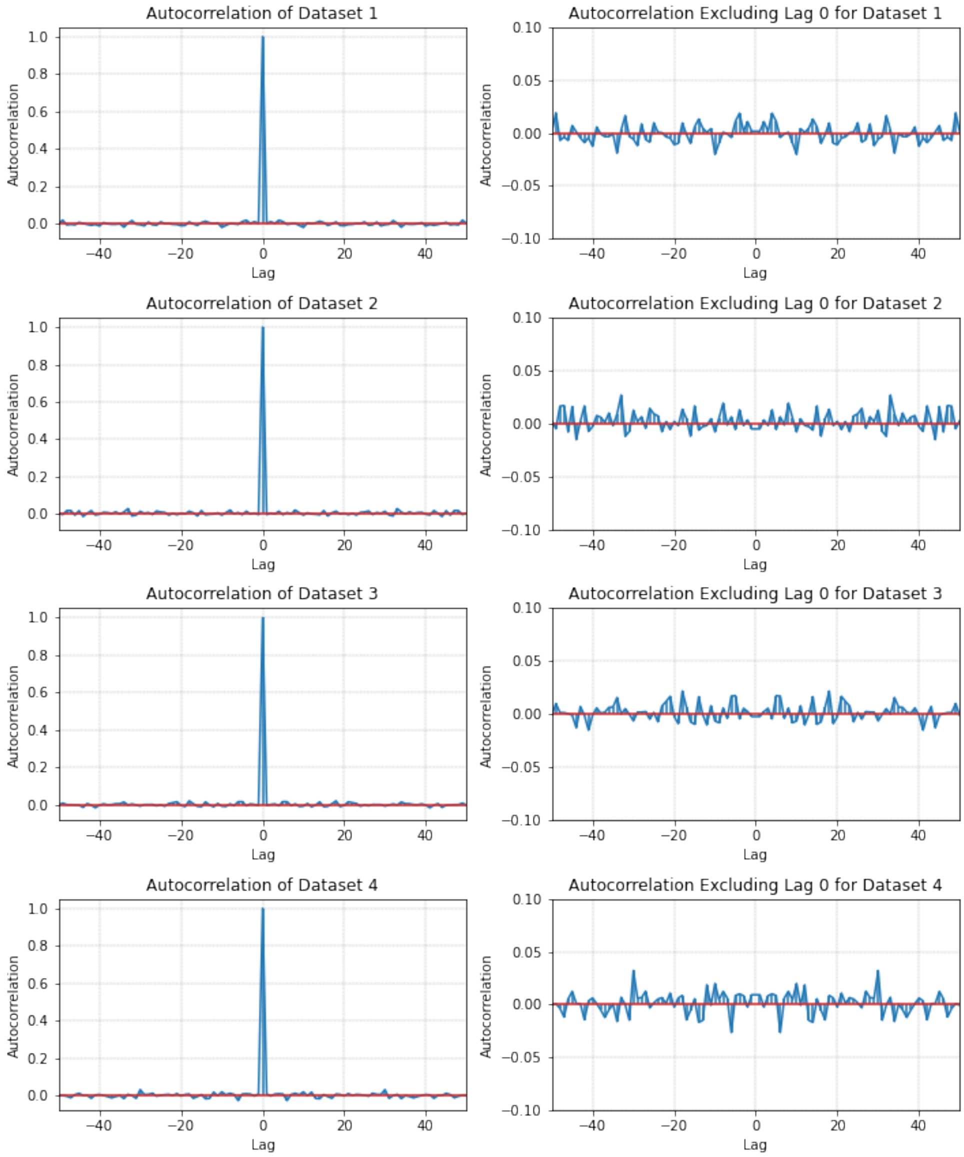

Figure 5.17: Autocorrelation plot of BJT based noise generator 1 without lowpass filter

requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by BJT based noise generator 2 with lowpass filter generates white noise.

Analysis of BJT based noise generator 2 without lowpass filter

The Autocorrelation plots of BJT based noise generator 2 without lowpass filter is interpreted in the table 5.10

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.8: Autocorrelation analysis of BJT based noise generator 1 without lowpass filter based on figure 5.16

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.9: Autocorrelation analysis of BJT based noise generator 2 with lowpass filter based on figure 5.18

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	Oscillatory behavior at non zero lag
2	Periodicity	Periodic components identified
3	Signal stability	Consistent pattern of oscillations
4	Correlation time	Longer correlation time
5	Non-ideal behaviors	Influenced by systematic errors, non-ideal

Table 5.10: Autocorrelation analysis of BJT based noise generator 2 without lowpass filter based on figure 5.19

Through this analysis from 5.10, the Autocorrelation plot does not meet the requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by BJT based noise generator 2 without lowpass filter does not generate white noise.

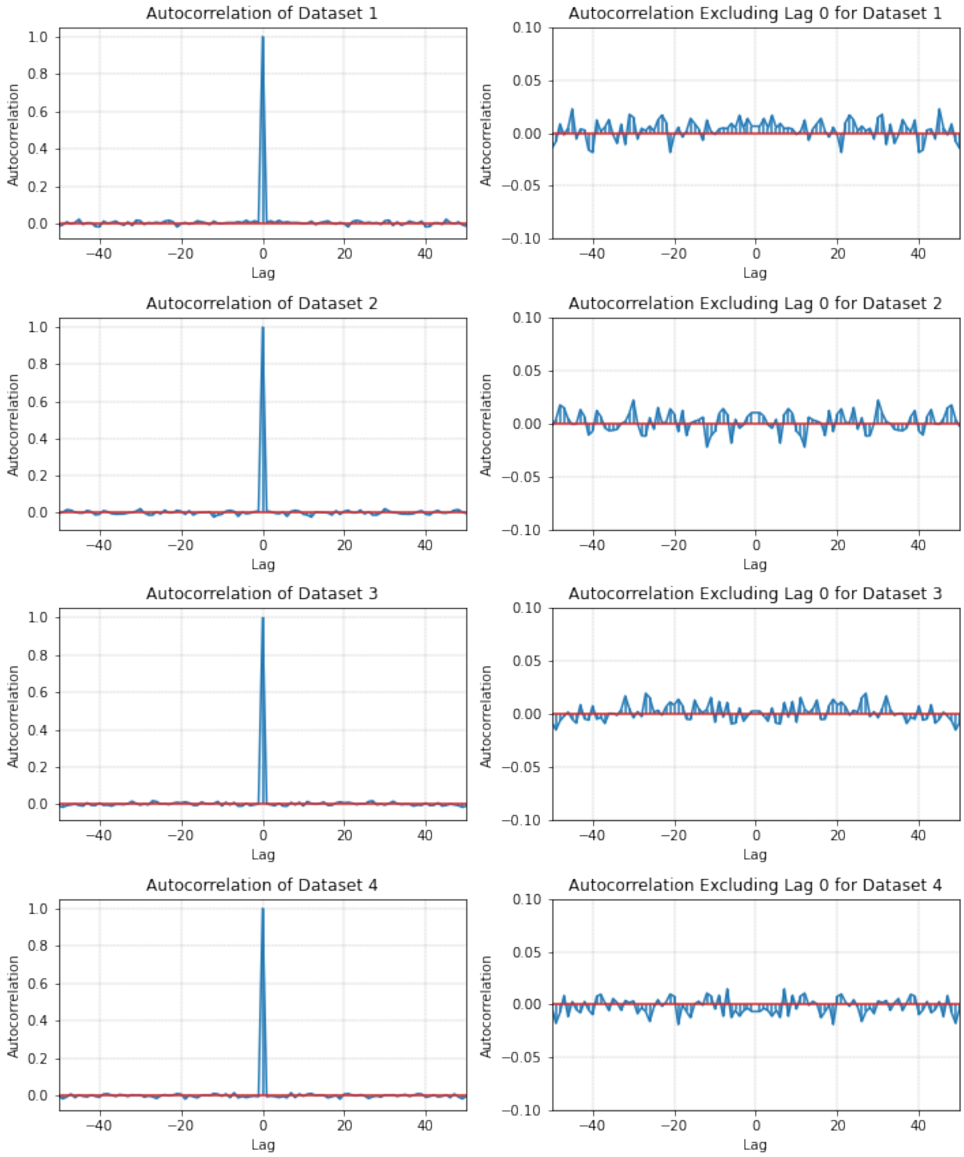

Figure 5.18: Autocorrelation plot of BJT based noise generator 2 with lowpass filter

5.2.3 Autocorrelation analysis of HP 3722-A

As mentioned in the chapter 2, we have acquired 4 datasets from HP 3722-A noise generator and 2 PRNG datasets with the same model. The following plots were acquired after performing Autocorrelation on those datasets. These signals were sampled at 1 MSPS (10e6 samples per second), 10 seconds of data.

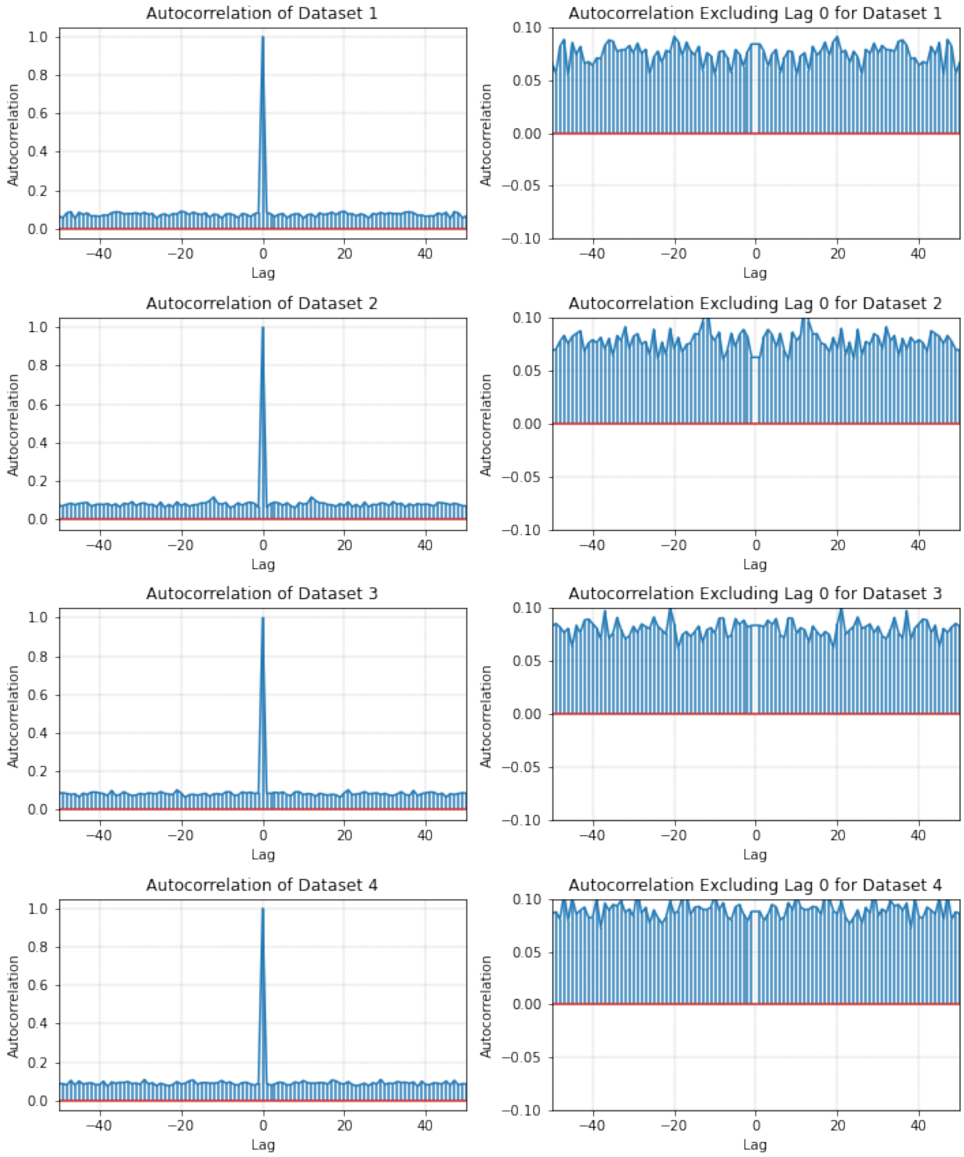

Figure 5.19: Autocorrelation plot of BJT based noise generator 2 without lowpass filter

Analysis of HP 3722-A

The Autocorrelation plots of HP 3722-A is interpreted in the table 5.11

Through this analysis from table 5.11, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by HP 3722-A noise generator generates white noise.

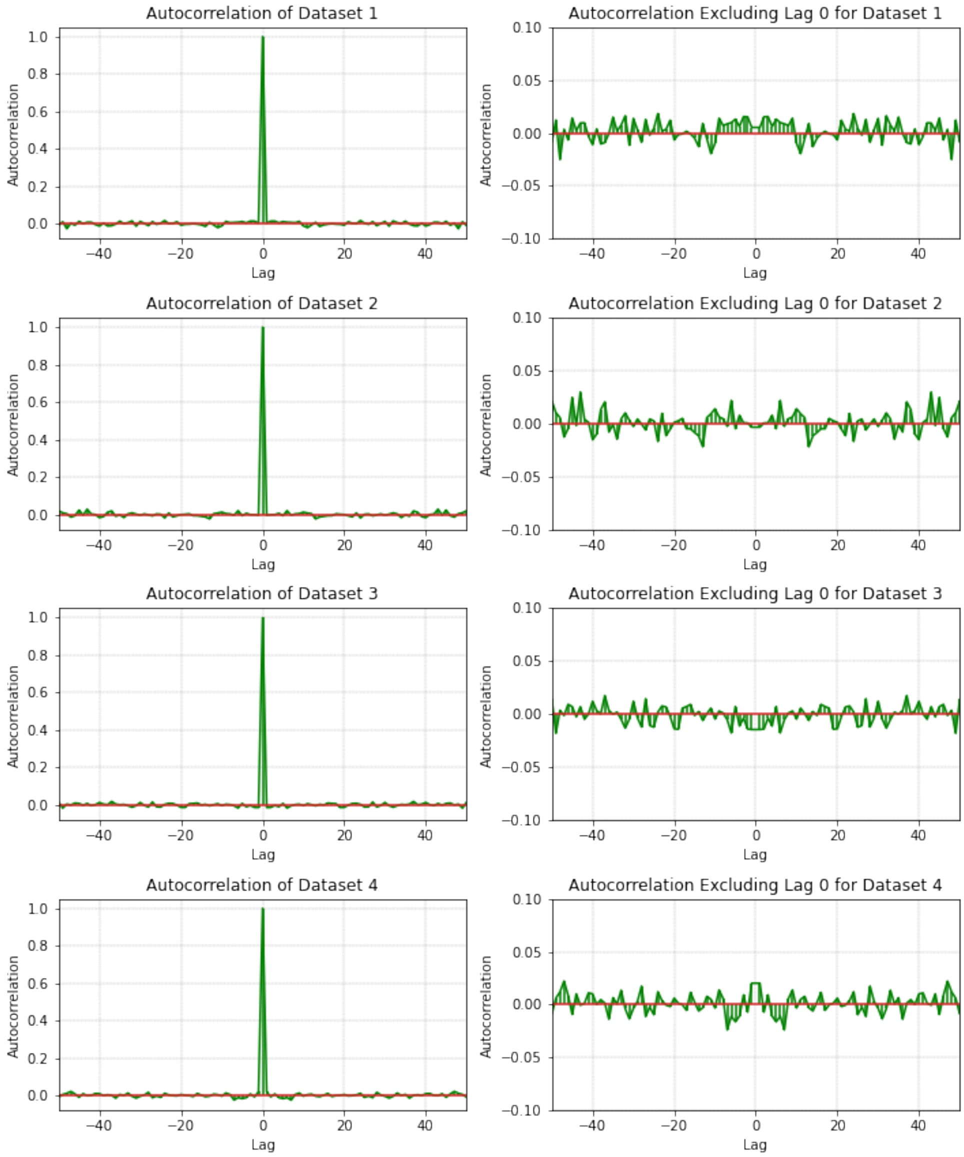

Figure 5.20: Autocorrelation plot of HP 3722-A noise generator

Analysis of HP 3722-A (PRNG)

As mentioned in chapter 2, we also had 3 datasets from the same noise generator producing PRNG. We will see the correlation analysis of those signal below in figure 5.21.

The autocorrelation plots of HP 3722-A (PRNG) is interpreted in the table 5.12

Through this analysis from table 5.11, the Autocorrelation plot meets all the

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.11: Autocorrelation analysis of HP 3722-A noise generator based on figure 5.20

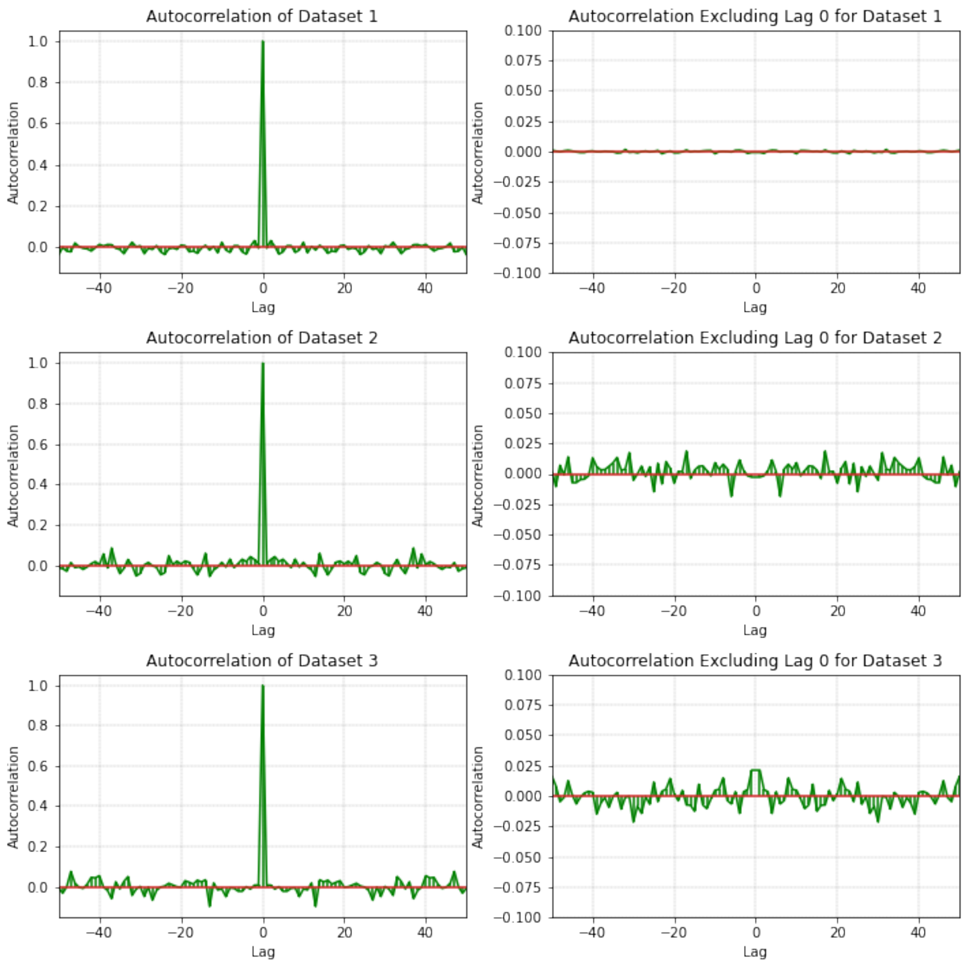

Figure 5.21: Autocorrelation plot of HP 3722-A noise generator with PRNG

requirements of white noise. Hence, we can conclude that with the help

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.12: Autocorrelation analysis of HP 3722-A noise generator based on figure 5.21

of Autocorrelation analysis, the noise signal generated by HP 3722-A noise generator (PRNG) generates white noise.

5.2.4 Autocorrelation analysis of Wandel and Goltermann RG-1

As mentioned in the chapter 2, we have acquired 2 datasets from RG-1 at 10 bits, 4 datasets at 16Hz to 22kHz and 4 datasets at 100kHz. The following plots were acquired after performing Autocorrelation on those datasets. These signals were sampled at 1 MSPS (10e6 samples per second), 10 seconds of data.

Analysis of Wandel and Goltermann RG-1 at 10 bit

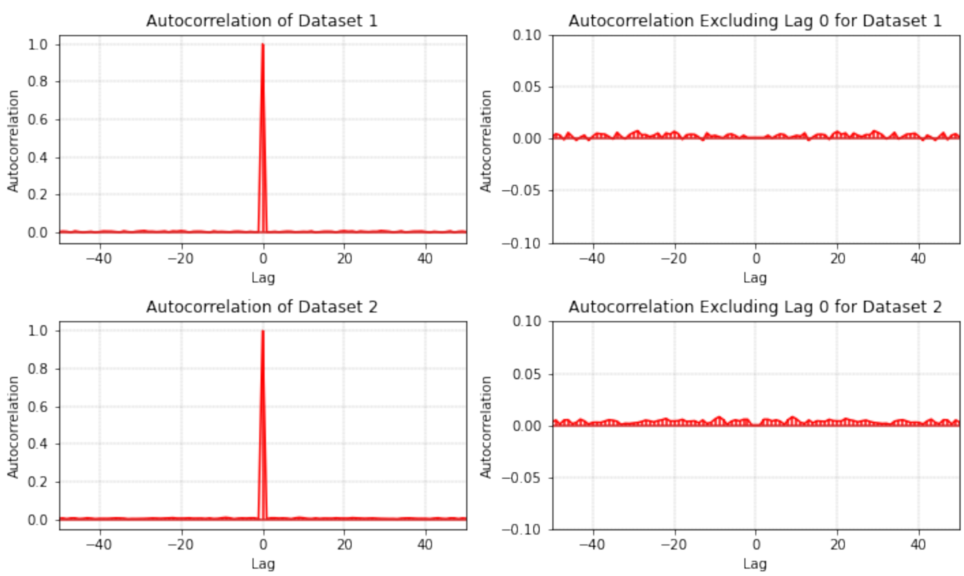

Figure 5.22: Autocorrelation plot of Wandel and Goltermann RG-1 noise generator at 10 bit

The Autocorrelation plots of Wandel and Goltermann RG-1 noise generator at 10 bit is interpreted in the table 5.13

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	Very High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Very Stable signal
4	Correlation time	Very Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.13: Autocorrelation analysis of Wandel and Goltermann RG-1 noise generator at 10 bit based on figure 5.22

Through this analysis from table 5.13, the Autocorrelation plot meets all the requirements of pure white noise. Hence, we can conclude that with the help of Autocorrelation analysis, the noise signal generated by Wandel and Goltermann RG-1 noise generator at 10 bit generates white noise.

Analysis of Wandel and Goltermann RG-1 from 16Hz to 22kHz

The Autocorrelation plots of Wandel and Goltermann RG-1 noise generator from 16Hz to 22kHz is interpreted in the table 5.14

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	Very High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Very Stable signal
4	Correlation time	Very Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.14: Autocorrelation analysis of Wandel and Goltermann RG-1 noise generator from 16Hz to 22kHz based on figure 5.23

Through this analysis from table 5.14, the Autocorrelation plot meets all the requirements of pure white noise. Hence, we can conclude that with the help of Autocorrelation analysis, the noise signal generated by Wandel and Goltermann RG-1 noise generator from 16Hz to 22kHz generates white noise.

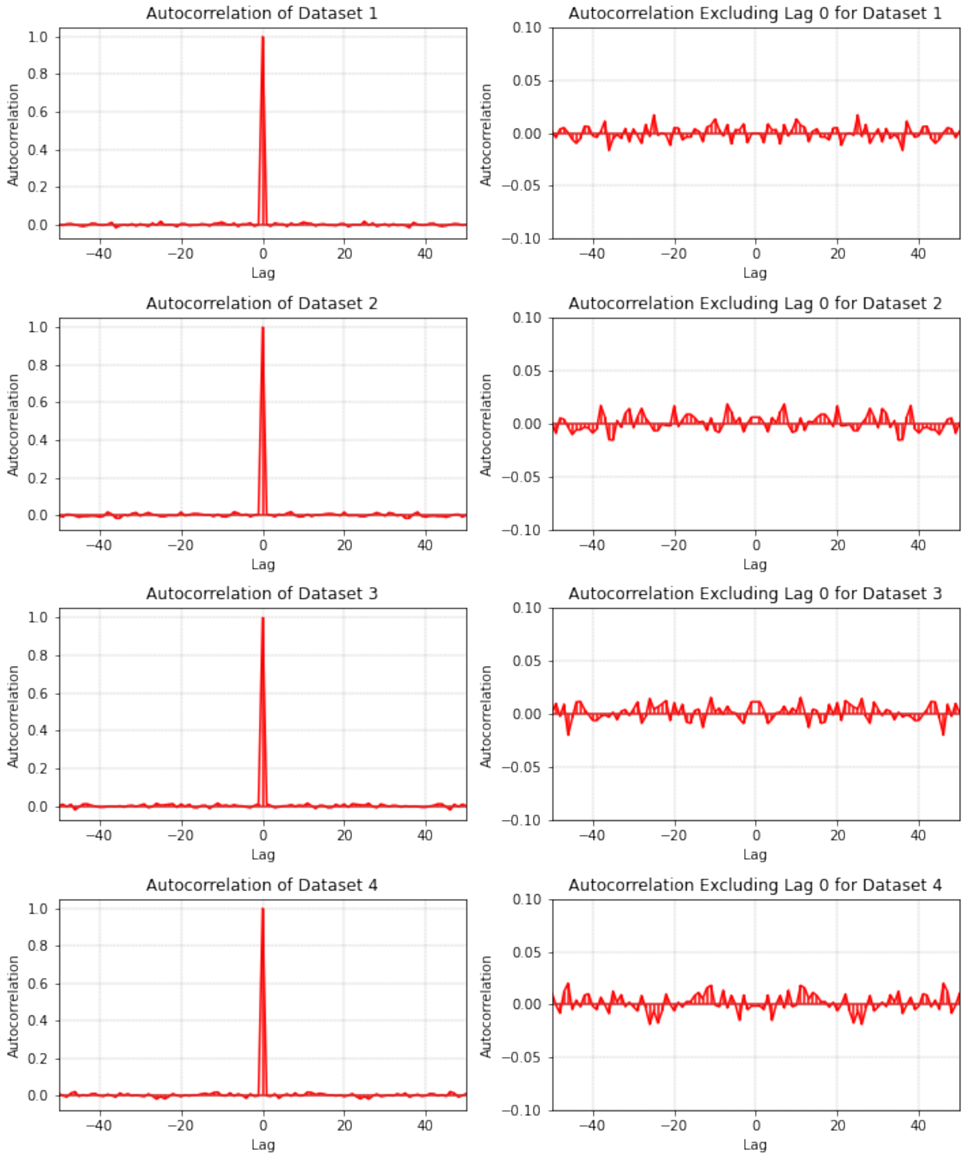

Figure 5.23: Autocorrelation plot of Wandel and Goltermann RG-1 noise generator from 16Hz to 22kHz

Analysis of Wandel and Goltermann RG-1 at 100kHz

The Autocorrelation plots of Wandel and Goltermann RG-1 noise generator at 100kHz is interpreted in the table 5.15

Through this analysis from table 5.15, the Autocorrelation plot meets all the requirements of pure white noise. Hence, we can conclude that with the help of Autocorrelation analysis, the noise signal generated by Wandel and

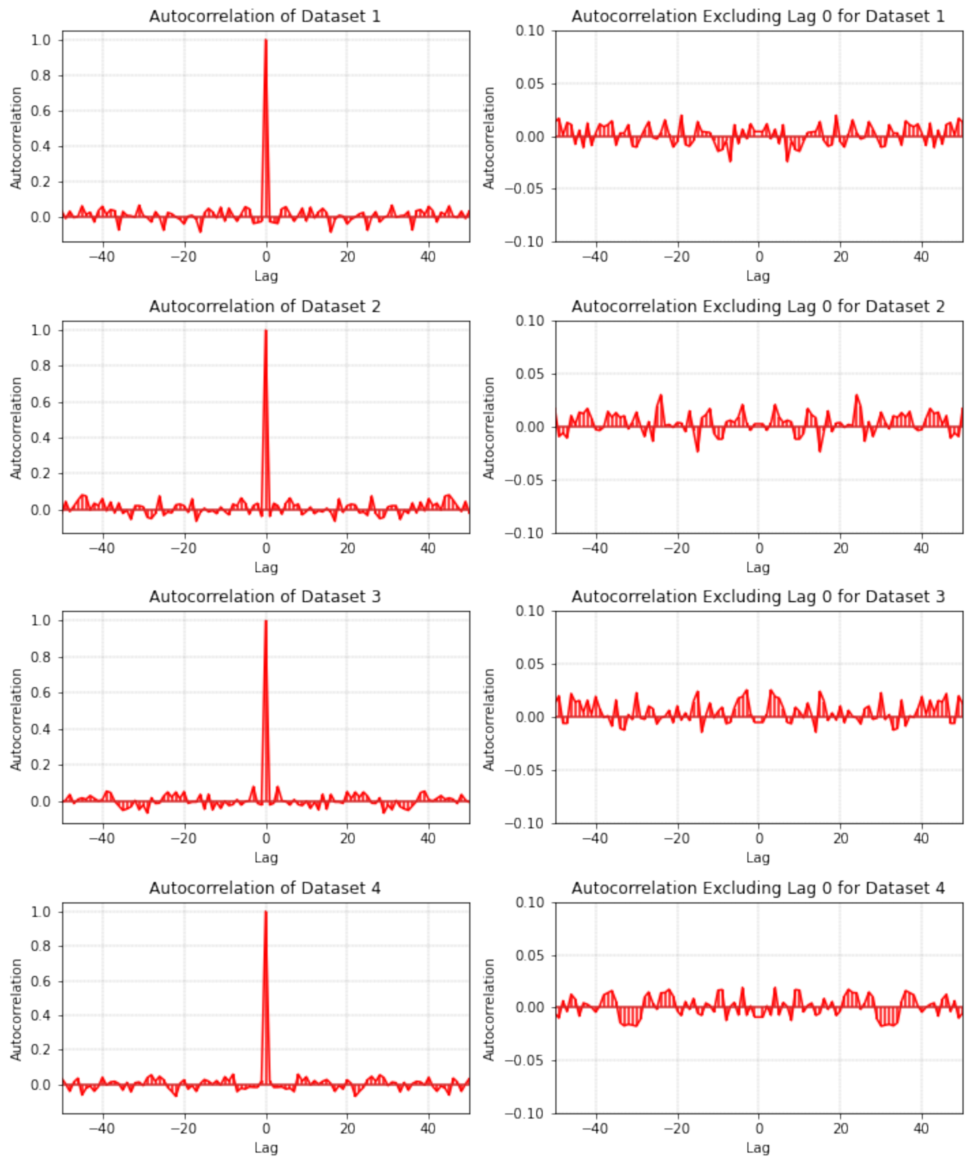

Figure 5.24: Autocorrelation plot of Wandel and Goltermann RG-1 noise generator at 100kHz

Goltermann RG-1 noise generator at 100kHz generates white noise.

5.2.5 Autocorrelation analysis of Zener diode based noise generators

As mentioned in the chapter 2, we have acquired 4 datasets from Zener diode based noise generators with and without lowpass filter. The following plots were acquired after performing Autocorrelation on those datasets. These signals were sampled at 1 MSPS (10e6 samples per second), 10 seconds of

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	Very High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Very Stable signal
4	Correlation time	Very Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.15: Autocorrelation analysis of Wandel and Goltermann RG-1 noise generator at 100kHz based on figure 5.24

data.

Analysis of Zener diode based noise generators 1 with lowpass filter

The Autocorrelation plots of Zener diode based noise generators 1 with lowpass filter is interpreted in the table 5.16

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.16: Autocorrelation analysis of Zener diode based noise generators 1 with lowpass filter based on figure 5.25

Through this analysis from table 5.16, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of Autocorrelation analysis, the noise signal generated by Zener diode based noise generators 1 with lowpass filter generates white noise.

Analysis of Zener diode based noise generators 1 without lowpass filter

The Autocorrelation plots of Zener diode based noise generators 1 without lowpass filter is interpreted in the table 5.17

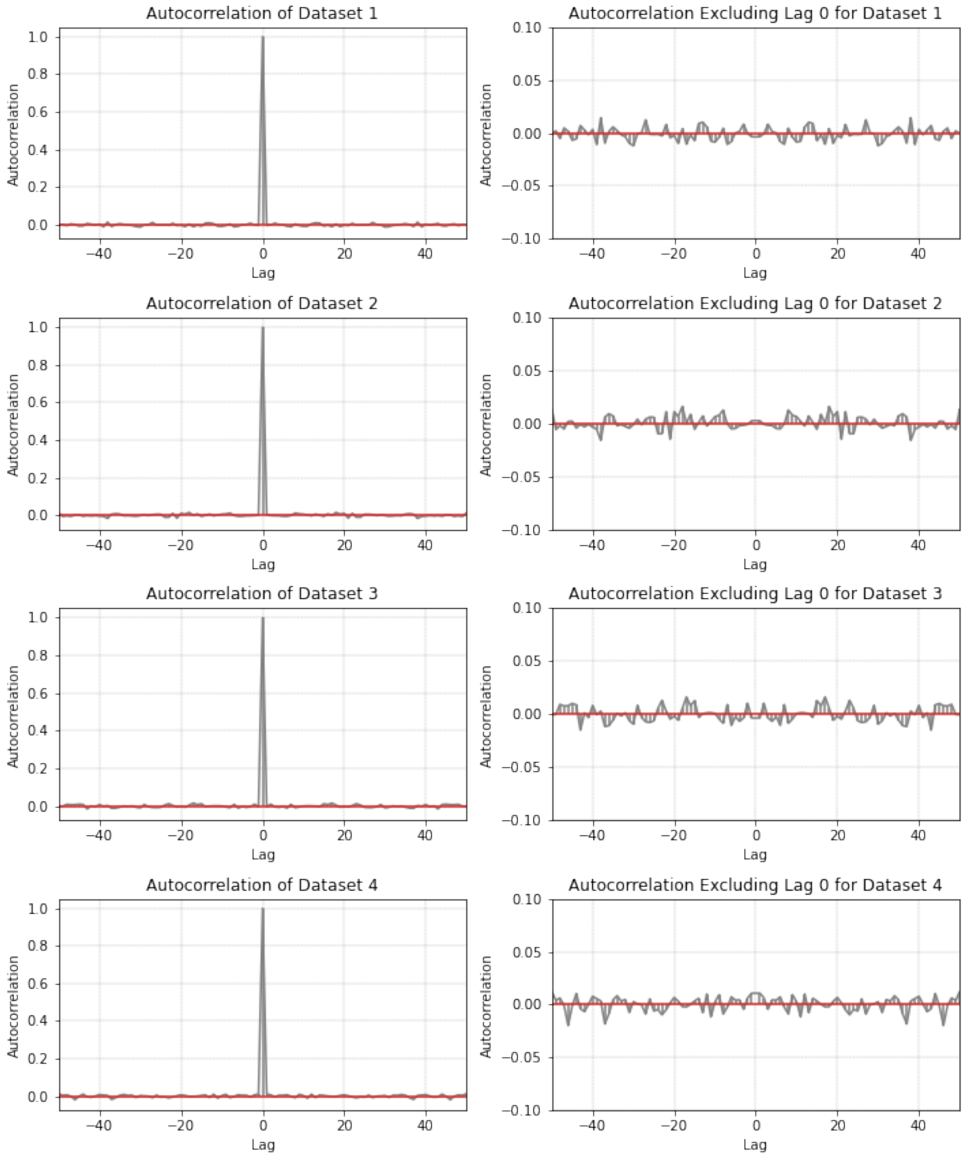

Figure 5.25: Autocorrelation plot of Zener diode based noise generators 1 with lowpass filter

Through this analysis from table 5.17, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of Autocorrelation analysis, the noise signal generated by Zener diode based noise generators 1 without lowpass filter generates white noise.

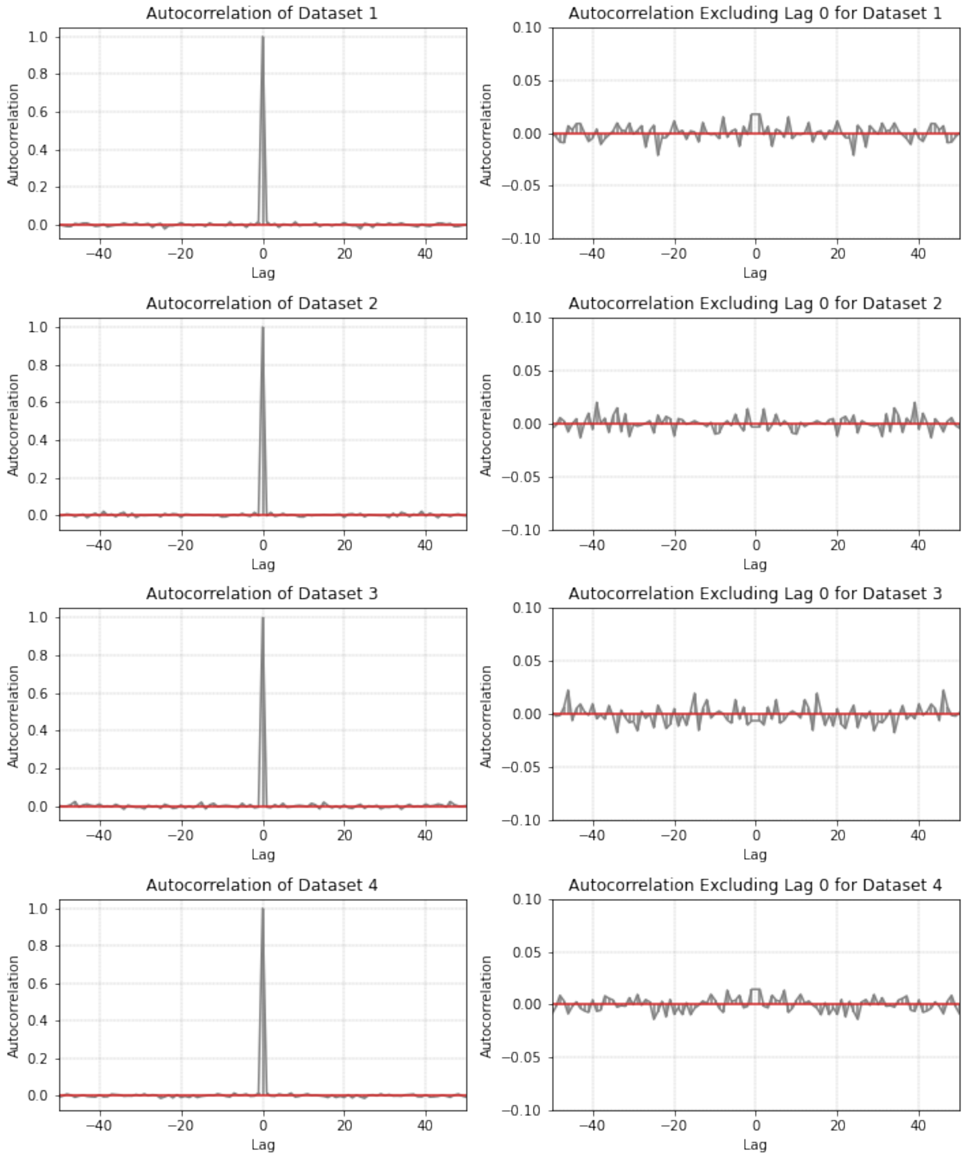

Figure 5.26: Autocorrelation plot of Zener diode based noise generators 1 without lowpass filter

Analysis of Zener diode based noise generators 2 with lowpass filter

The Autocorrelation plots of Zener diode based noise generators 2 with lowpass filter is interpreted in the table 5.18

Through this analysis from table 5.18, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by Zener diode based noise generators 2 with lowpass filter generates white noise.

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.17: Autocorrelation analysis of Zener diode based noise generators 1 with lowpass filter based on figure 5.26

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.18: Autocorrelation analysis of Zener diode based noise generators 2 with lowpass filter based on figure 5.27

Analysis of Zener diode based noise generators 2 without lowpass filter

The Autocorrelation plots of Zener diode based noise generators 2 without lowpass filter is interpreted in the table 5.19

Through this analysis from table 5.19, the Autocorrelation plot meets all the requirements of white noise. Hence, we can conclude that with the help of this analysis, the noise signal generated by Zener diode based noise generators 2 without lowpass filter generates white noise.

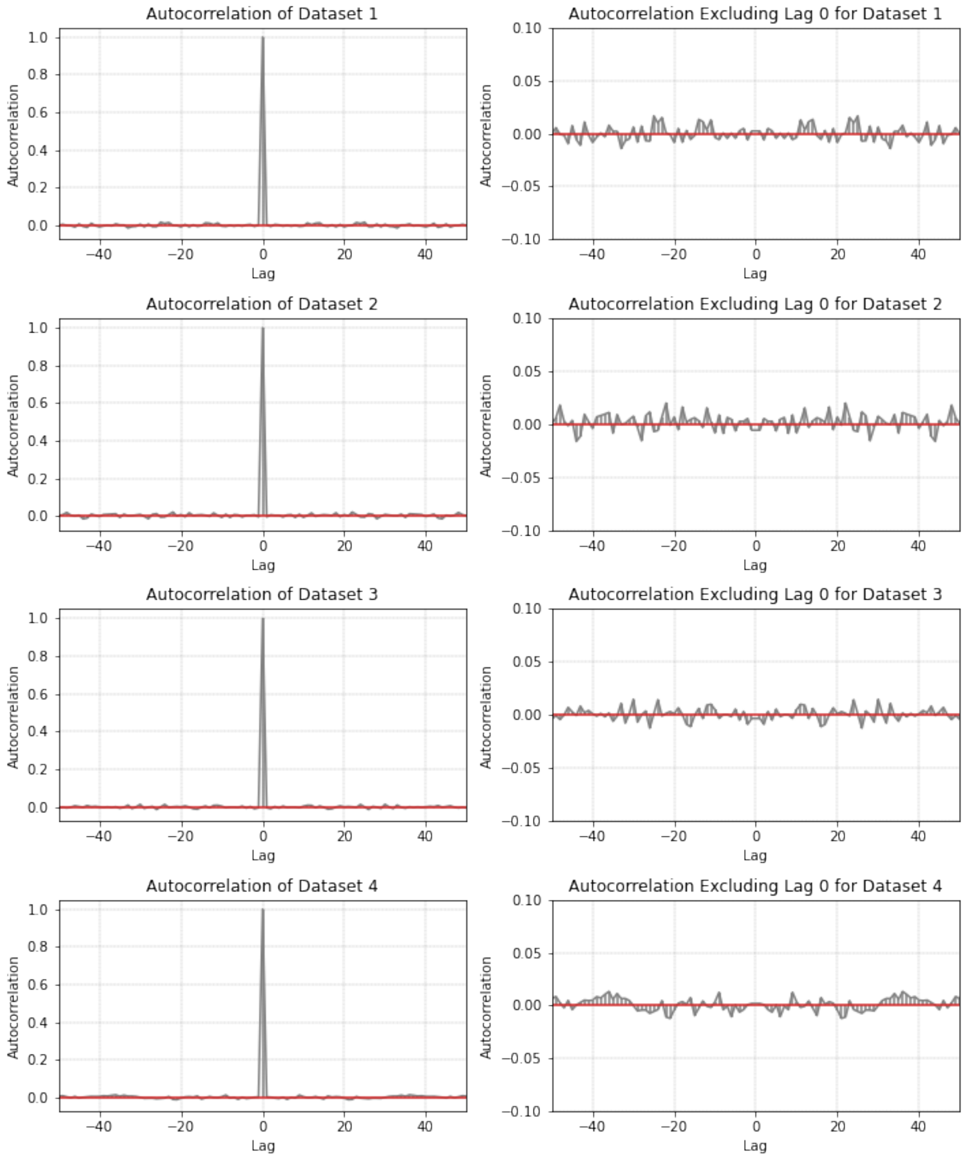

Figure 5.27: Autocorrelation plot of Zener diode based noise generators 1 with lowpass filter

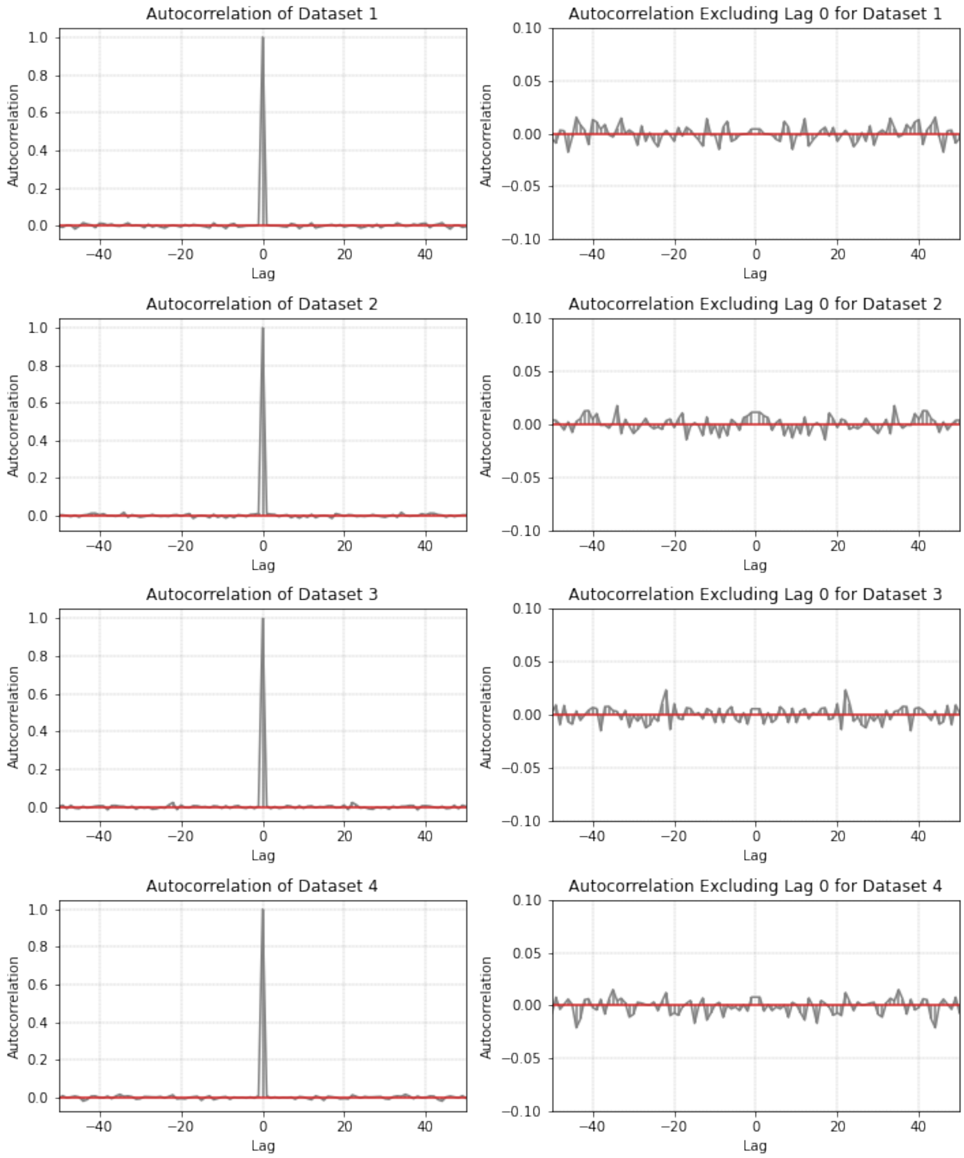

Figure 5.28: Autocorrelation plot of Zener diode based noise generators 2 without lowpass filter

Sr. No.	Parameter	Dataset 1, 2, 3 and 4
1	Randomness assessment	High randomness
2	Periodicity	Lack of periodicity
3	Signal stability	Stable signal
4	Correlation time	Short
5	Non-ideal behaviors	No Non-Ideal Behaviors Observed

Table 5.19: Autocorrelation analysis of Zener diode based noise generators 2 with lowpass filter based on figure 5.28

5.3 Wavelets

When dealing with very high-frequency data, selecting the right type of wavelet transform and an appropriate wavelet function is crucial to effectively capture the characteristics of the signal, especially the high-frequency components. In such cases, the Continuous Wavelet Transform (CWT) is often more suitable because it provides a time-frequency representation of the signal, allowing for a detailed analysis of the signal's frequency content at different times as explained in 3.3. We can adjust the scales used in the CWT to focus on higher frequencies more closely, which is beneficial for high-frequency signal analysis. For this purpose, we have set the scale from 32 to 256 after numerous trails and could see some patterns within the wavelets plot.

With the reference to chapter 3, we used Wavelet analysis as one of the methods to characterise the analog signals. We followed the following procedure to do the analysis.

The choice of the wavelet function in CWT is significant, especially for high-frequency data [20].

1. Loading data into pandas DataFrame.
2. Used the function `compute_cwt()` to perform the Continuous Wavelet Transform on the value series using the specified scales, wavelet name, and sampling period derived.
3. After numerous trials, it was discovered that the hyperparameter 'scale' is suitable from 32 to 255 to see the patterns within the wavelets.
4. With reference to [20], we used Morlet wavelet by mentioning `'morl'`.
5. To make the code capable of easy processing, we downsampled the data to 100x.
6. Plotted wavelet plots.

Morlet Wavelet is a commonly used wavelet for CWT when analyzing high-frequency components because it closely resembles a sinusoidal wave modulated by a Gaussian envelope, making it sensitive to sinusoidal signals at various scales. We used the library `pywv` [21], which had the wavelets already created. The dimensions of the wavelets were limited, hence we could not morph the wavelet. The one that we have used for this analysis is shown in the figure 5.30.

For the comparison purpose, we generated a CWT plot of signal generated using psuedo-random numbers, and tried to visualize how does white noise looks like, on the CWT plot, in the figure 5.30.

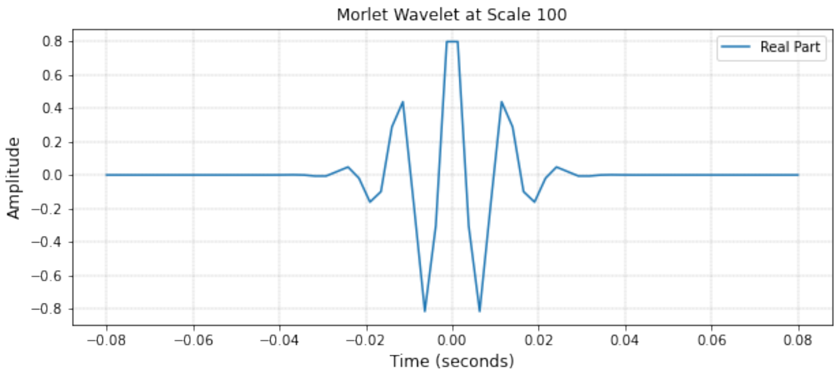

Figure 5.29: Morlet Wavelet used for the analysis

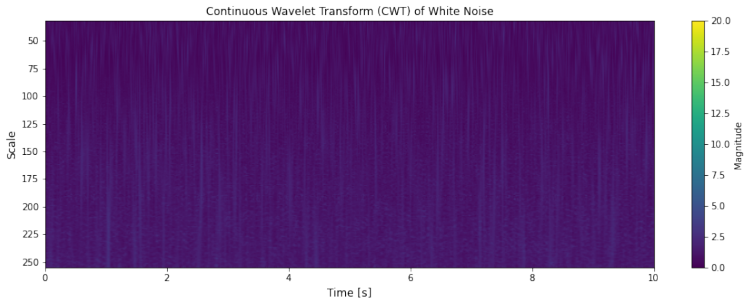

Figure 5.30: Wavelet Analysis plot ideal white noise by pseudo random numbers

For the interpretation of this plot, the horizontal axis represents the time of the signal. For the plot generated from PRNG signal in figure 5.30, the time goes from 0 to 1000 seconds, representing the sample points. The vertical axis represents the scales used in the CWT. The scale is inversely related to frequency - lower scale corresponds to higher frequency and higher scales corresponds to lower frequency. The color coding represents the magnitude of the wavelet coefficients. The darker bands indicate where the wavelet coefficients are large, suggesting that the wavelet closely matches the signal at these scales and times. In the plot, the darker bands are consistent throughout the scale and time. It is not possible to find any brighter bands at any scale in the plot since the signal is pure white noise. But, the brighter areas indicate where the wavelet coefficients are small or near zero, suggesting that the wavelets does not match the signal well at these scales and times.

In the plot 5.30, we see dark bands at consistent intervals along the entire time axis, which means that the high-frequency components are present throughout the signal's duration. The CWT plot can also be used to understand the distribution of energy across different frequencies over time [20].

Interpreting Continuous Wavelet Transformss

1. Interpretation based on color intensity: Colors in a CWT plot typically represent the amplitude or power of the wavelet coefficients at each scale and time point. Commonly, warmer colors (reds, yellows) indicate higher values, while cooler colors (blues, purples) indicate lower values. High intensities can indicate significant or dominant frequency components at specific times.
2. Interpretation based localized events: Look for areas with distinct color changes, such as bursts of intense colors. These can indicate the presence of transient events or features within the signal, like spikes, bursts, or sudden changes.
3. Horizontal bands: Consistent color bands along a particular scale (frequency) indicate the presence of that frequency component throughout the observed time interval.
4. Vertical Stripes: Sudden changes in color across many scales at a specific time point might indicate an event affecting multiple frequencies.

In this analysis, we will first create CWT plots from all the datasets from different noise generators, and will do the comparison based on the above shared visual parameters.

In the visual interpretation of the figures 5.31, 5.32, 5.33, 5.34, 5.35, 5.38, 5.40, 5.41, 5.42, 5.43, 5.44 we can observe vertical bright strips getting more visible till the scale reaches 256. This means, lower frequency components are present and will be visible beyond the scale of 256. However, when tried for the higher scales i.e. from 256 to 512, there occurred a computational limit, due to which the analysis was not complete.

But based on the result with the scale 32 to 264, we can observe that there is a some presence of lower frequency components, which could have been discovered if we would have used the scale from 256 to 512. This also supports our conclusion from FFT analysis on the same datasets in the same chapter. Hence, we can conclude through this analysis that the signal generated by BJT based noise generators, HP 3722-A noise generator at 50kHz, Wandel and Goltermann RG-1 noise generator at 16Hz-22kHz and 100kHz, and Zener diode based noise generators are not the correct noise generators to produce pure white noise.

However, in the other hand, if we observe CWT of Dataset 1 of figure 5.36 which is generated by HP 3722-A (PRNG), we can see a smooth dark gradient that is constant from scale 32 to scale 256, and does not have any vertical bright band. We can assume that this suggest that some smaller bands may or may not appear beyond scale 512. Similarly for Wandel and Goltermann

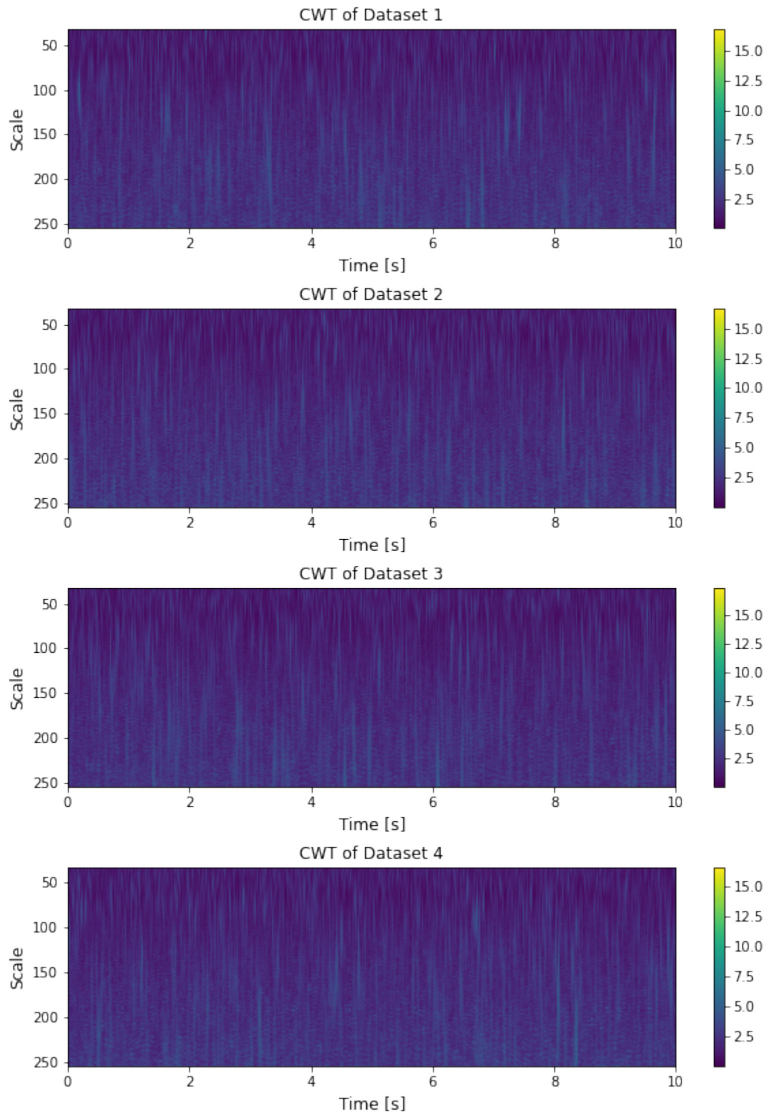

Figure 5.31: Wavelet Analysis plot of BJT based noise generator 1 with lowpass filter

RG-1 at 10bit in figure 5.37. We iterated reducing and increasing the scale, but the smooth gradient was constant. Hence, based on this analysis, we can conclude that the Dataset 1 from HP 3722-A (PRNG) shown in 5.36 and both

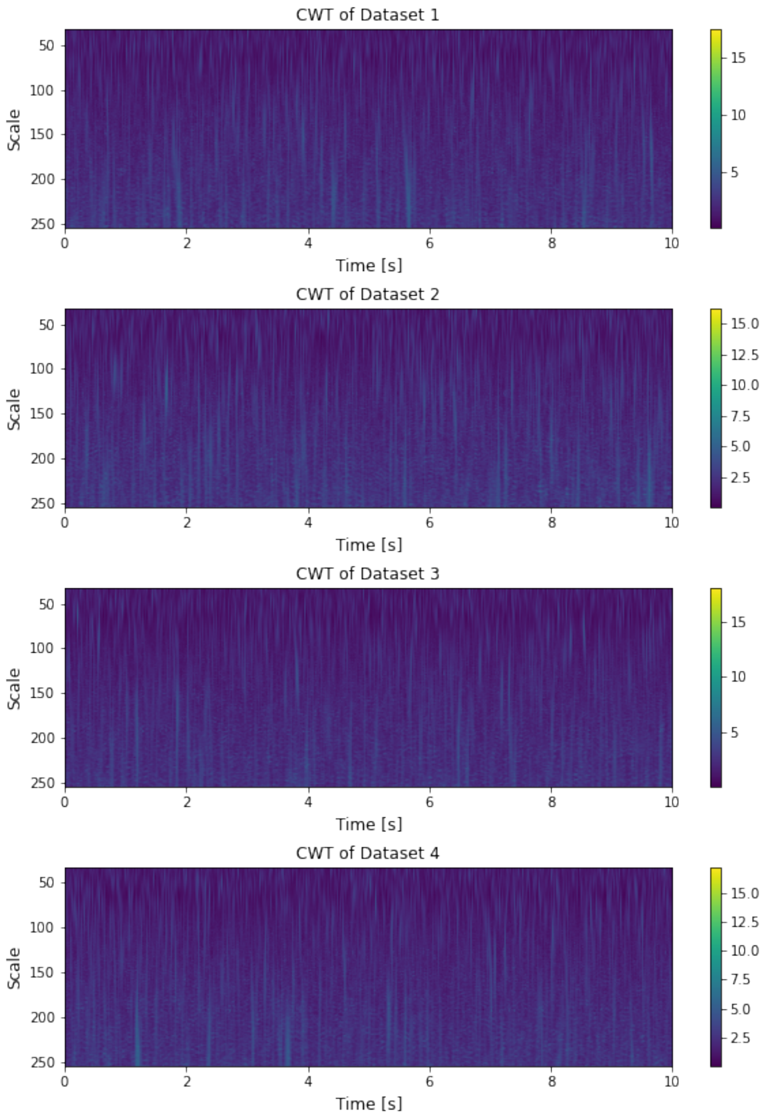

Figure 5.32: Wavelet Analysis plot of BJT based noise generator 1 without lowpass filter

the datasets from Wandel and Goltermann RG-1 at 10bit shown in figure 5.37 produce pure white noise.

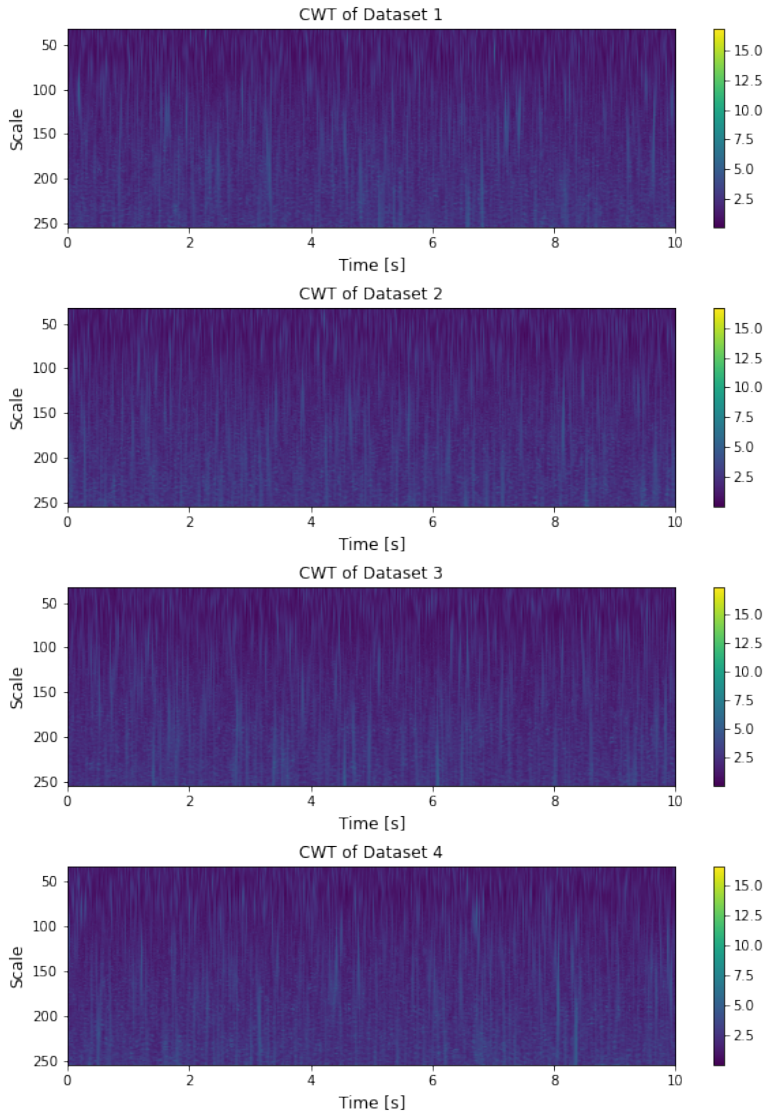

Figure 5.33: Wavelet Analysis plot of BJT based noise generator 2 with lowpass filter

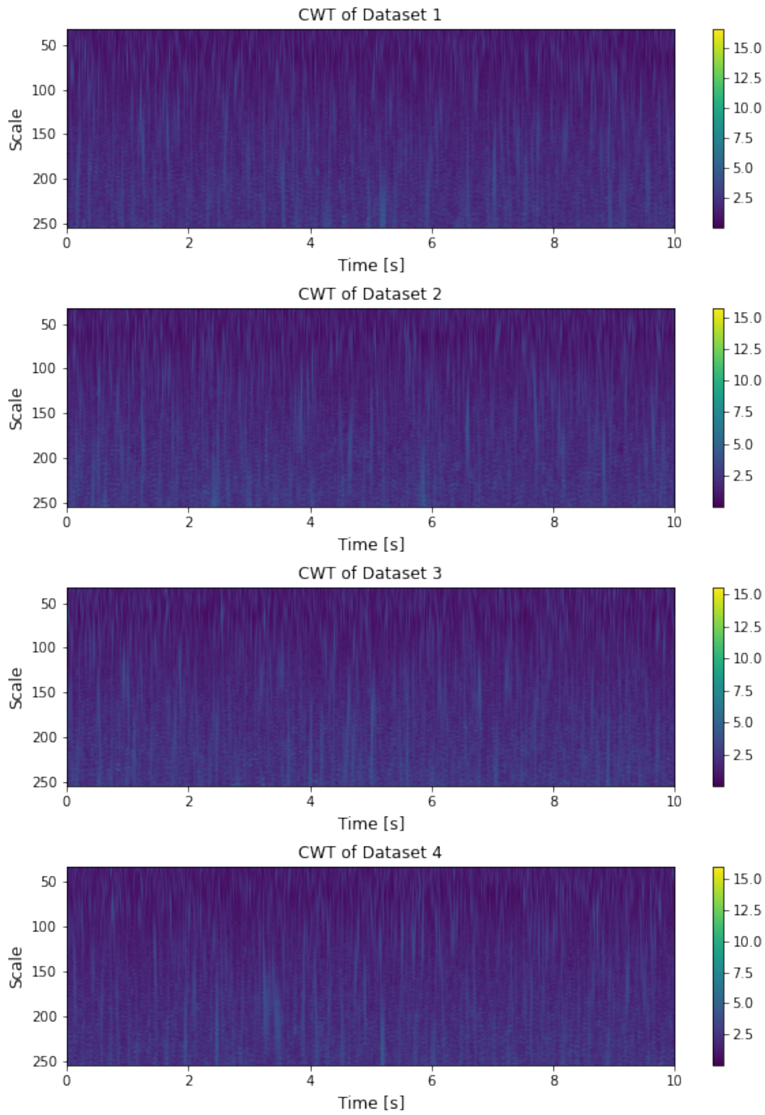

Figure 5.34: Wavelet Analysis plot of BJT based noise generator 2 without lowpass filter

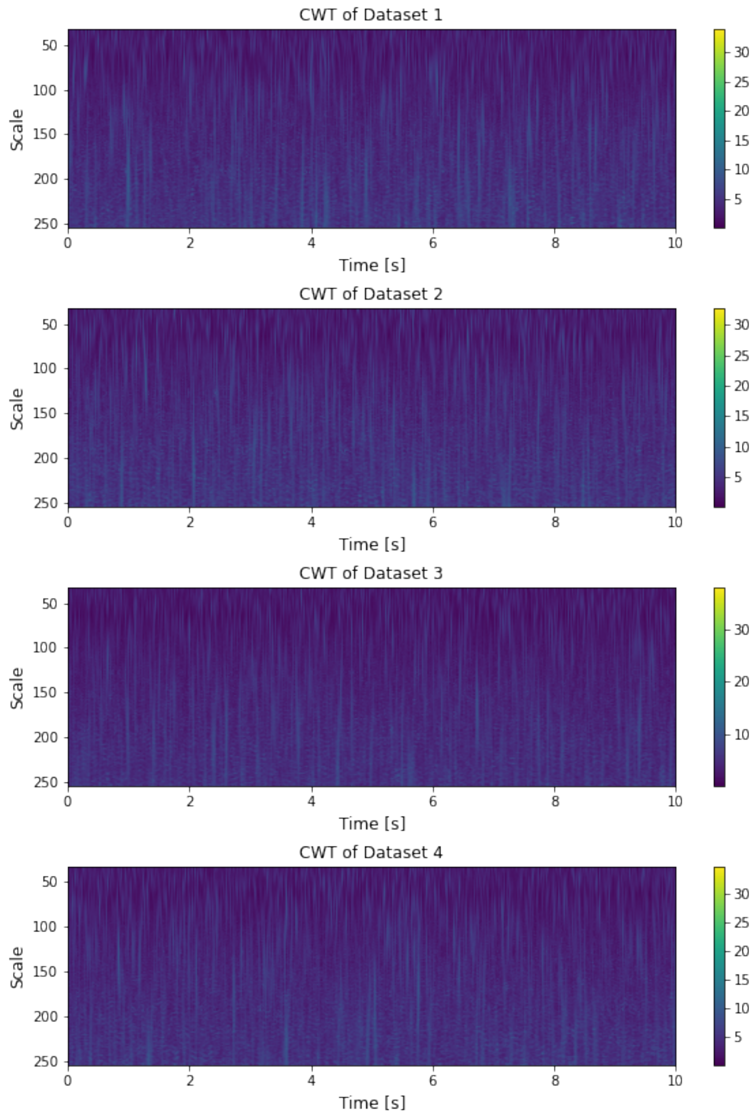

Figure 5.35: Wavelet Analysis plot of HP 3722-A

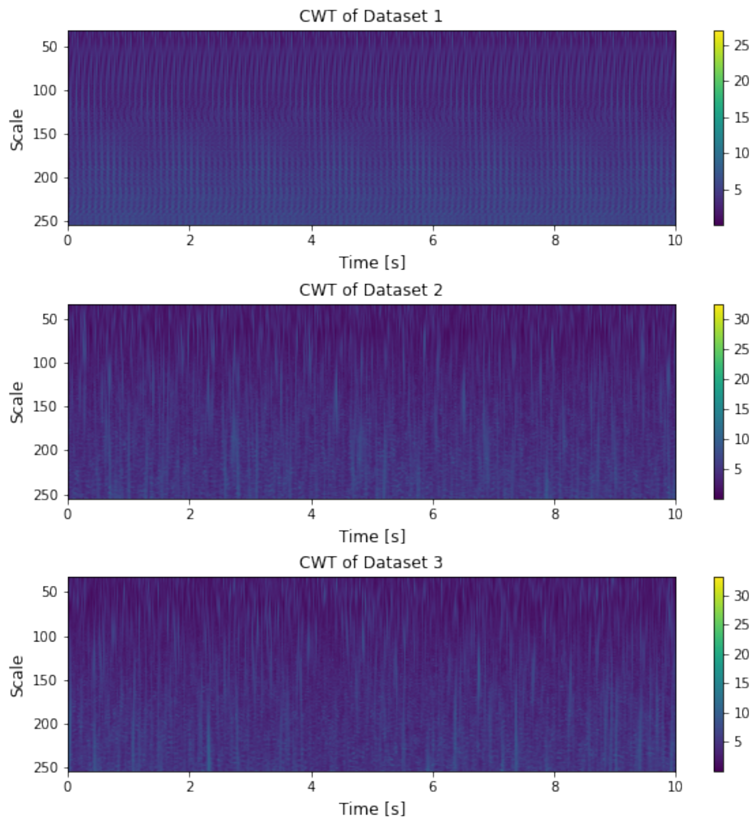

Figure 5.36: Wavelet Analysis plot of HP 3722-A (PRNG)

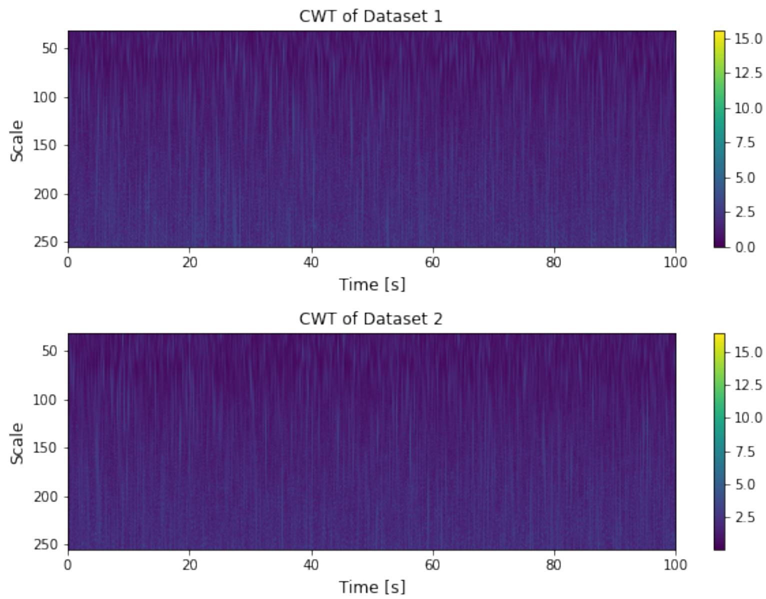

Figure 5.37: Wavelet Analysis plot of Wandel and Goltermann RG-1 at 10bits

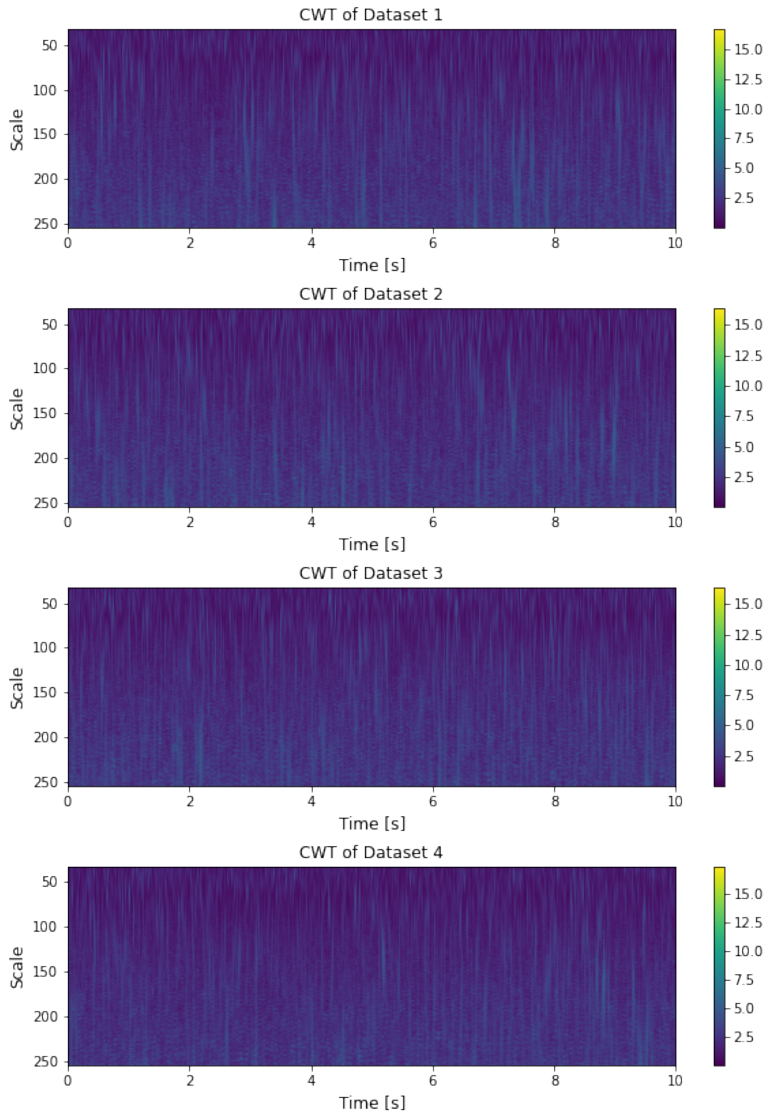

Figure 5.38: Wavelet Analysis plot of Wandel and Goltermann RG-1 from 16Hz to 22kHz

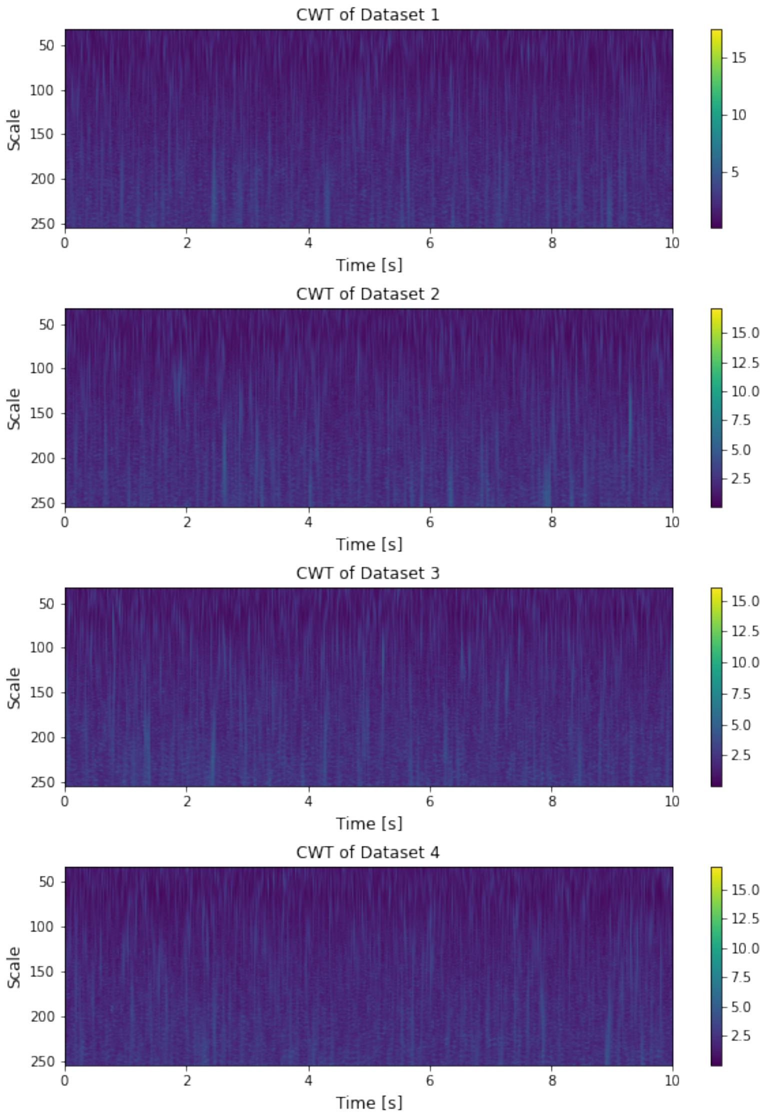

Figure 5.39: Wavelet Analysis plot of Wandel and Goltermann RG-1 at 100kHz

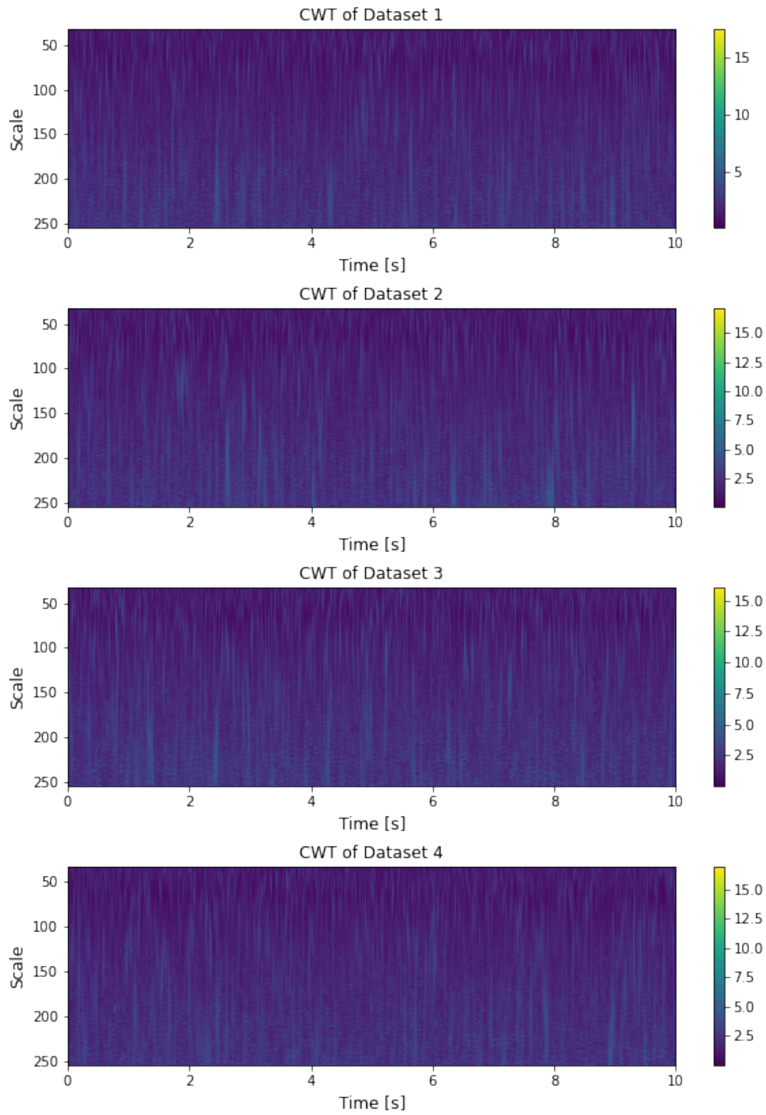

Figure 5.40: Wavelet Analysis plot of Wandel and Goltermann RG-1 at 100kHz

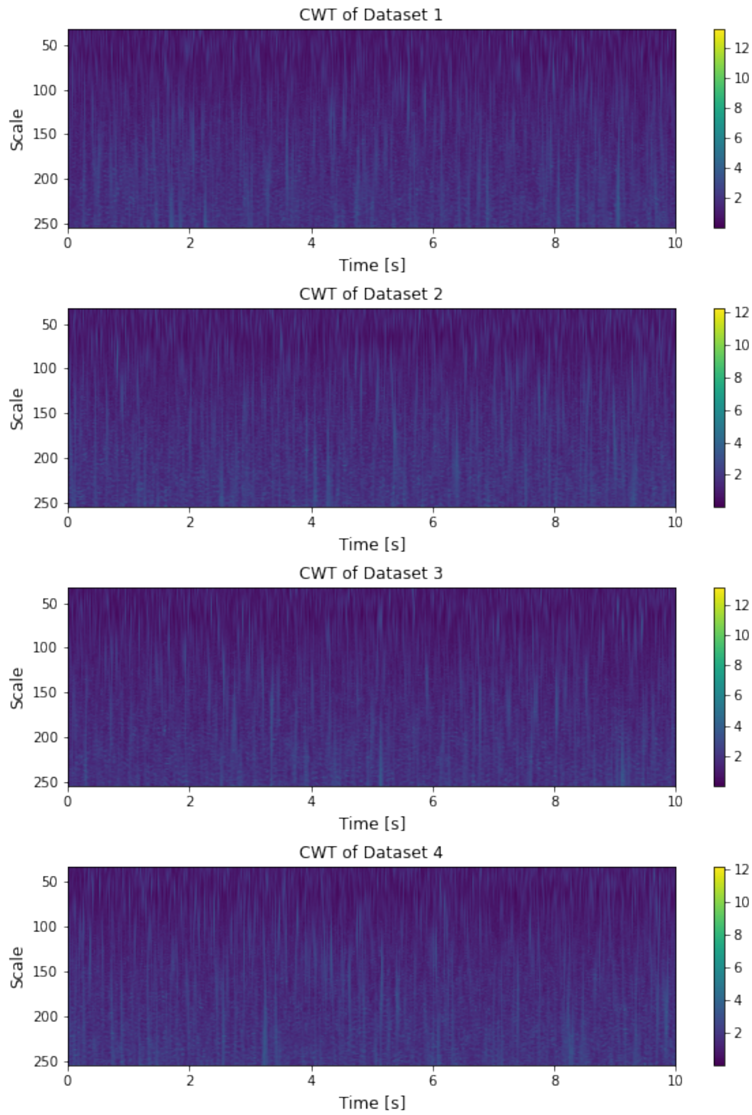

Figure 5.41: Wavelet Analysis plot of Zener diode based noise generator 1 with lowpass filter

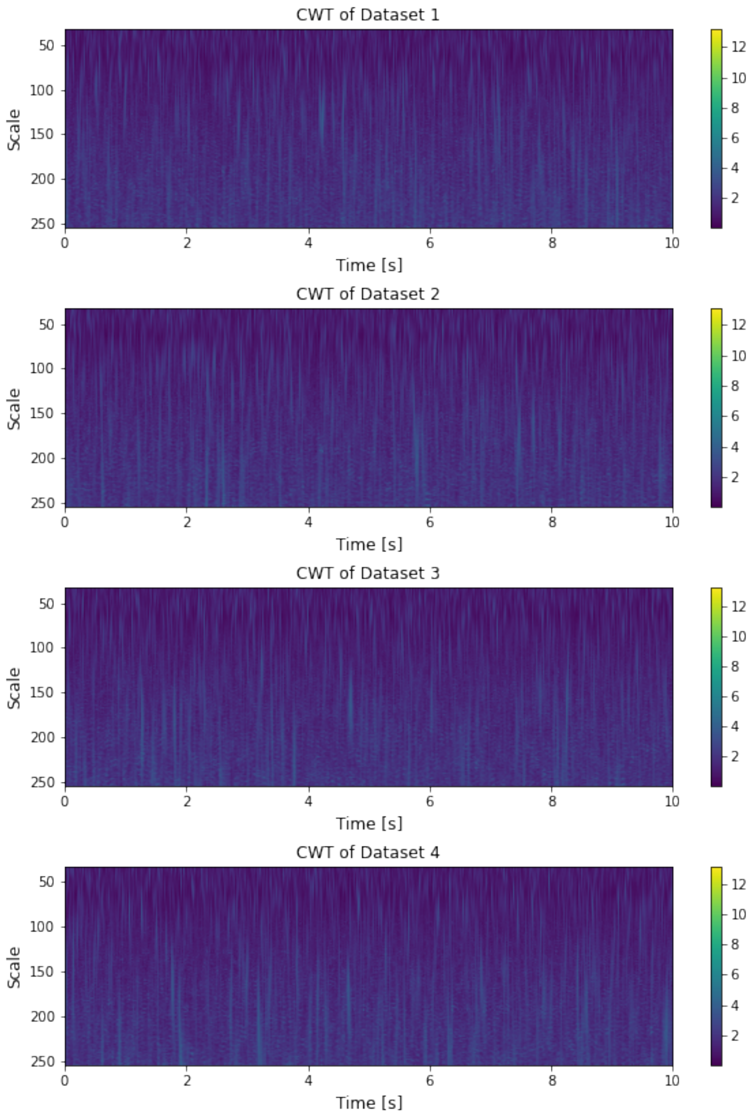

Figure 5.42: Wavelet Analysis plot of Zener diode based noise generator 1 without lowpass filter

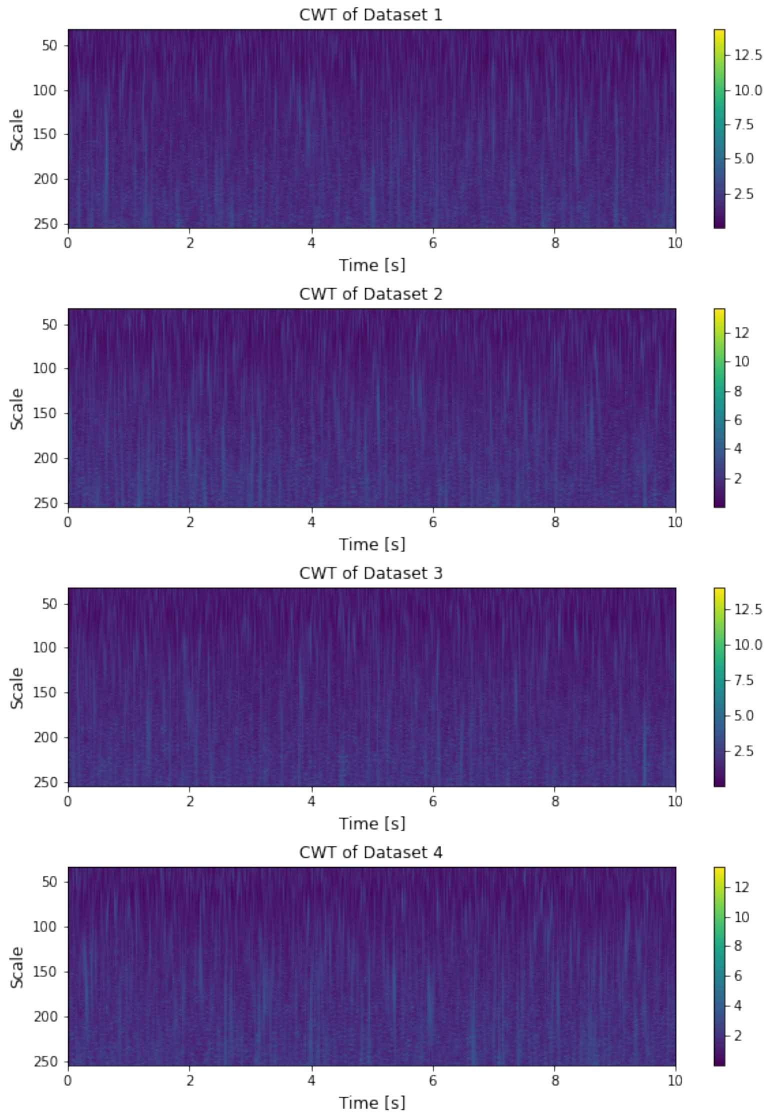

Figure 5.43: Wavelet Analysis plot of Zener diode based noise generator 1 with lowpass filter

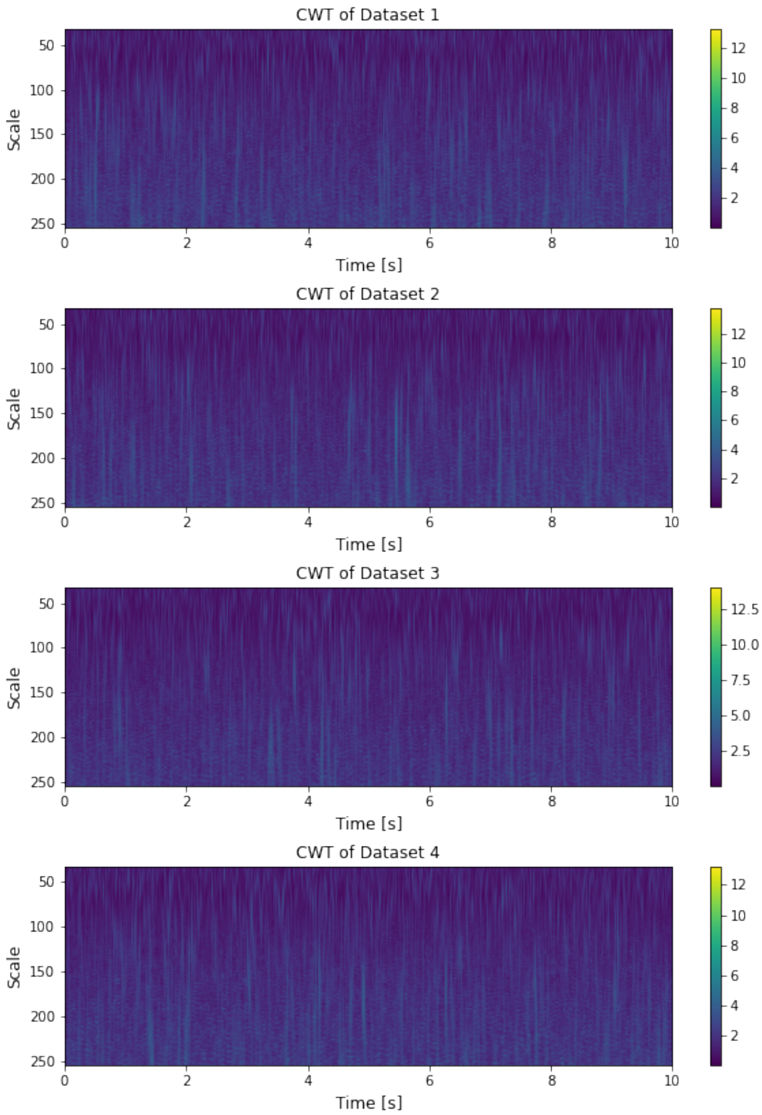

Figure 5.44: Wavelet Analysis plot of Zener diode based noise generator 2 without lowpass filter

Chapter 6

Summary

In this project, we worked on a comprehensive exploration of analog noise sources, aiming to deepen the understanding of their characteristics, their signals and were able to characterize the signal using statistical methods. Our work employed a robust methodology, started after the acquisition of data from various noise generators, utilizing theoretical analysis, to dissect the nuances of various noise types including white, pink, and brown. Through a series of detailed statistical methodologies, we quantified the behaviors of these noise generators based on their limited time signal and studied their performance in the context of type of noise they produce.

This chapter synthesizes the insights gained from our research, revisiting the objectives set out at the commencement of this study. We will revisit the critical findings, discuss the theoretical and practical implications of our work, and suggest avenues for future research. By advancing our knowledge of analog noise sources, this research not only contributes to the field of electronics but also paves the way for the development of more resilient and efficient systems. The following sections will provide a structured overview of our conclusions and the potential impact of our findings on the design and optimization of electronic circuits.

6.1 Summary of Noise Generators and the Noise Type

In this section, we will map the analysis that we have done in the chapter 5 and various methods that were used in that chapter. This summary in 6.1 includes the noise generators discussed in 2 and the noise types (discussed in 4) they produced based on the statistical methods we have used in chapter 5.

Sr. No.	Noise Generator	Variation	Fourier Transform	Autocorrelation	Wavelets
1	BJT based noise generator 1	Lowpass filter	Pink noise	White noise	Cannot conclude
2	BJT based noise generator 1	No lowpass filter	Pink noise	White noise	Cannot conclude
3	BJT based noise generator 2	Lowpass filter	Pink noise	White noise	Cannot conclude
4	BJT based noise generator 2	No lowpass filter	Pink noise	No white noise	Cannot conclude
5	HP 3722-A	50kHz	White noise until 50kHz, Pink noise until 100kHz	White noise	Cannot conclude
6	HP 3722-A	PRNG	Cannot conclude	White noise	White noise
7	Wandel and Goltermann RG-1	10 bit	White noise	White noise	White noise
8	Wandel and Goltermann RG-1	16Hz-22kHz	White noise until 25kHz	White noise	Cannot conclude
9	Wandel and Goltermann RG-1	100kHz	White noise	White noise	Cannot conclude
10	Zener diode based noise generator 1	Lowpass filter	Pink noise	White noise	Cannot conclude
11	Zener diode based noise generator 1	No lowpass filter	Brown noise	White noise	Cannot conclude
12	Zener diode based noise generator 2	Lowpass filter	Pink noise	White noise	Cannot conclude
13	Zener diode based noise generator 2	No lowpass filter	Brown noise	White noise	Cannot conclude

Table 6.1: Summary of Noise Generators and the Noise Type

6.2 Limitations and challenges during the research

No matter how easy or tough the research project is, it cannot be completed without challenges.

1. **Data Size**: Since the data was acquired from physical noise generators, even after sampling the data was over 250 Mega Bites. This gave raise to another challenge that is mentioned below.

2. **Slow Computation Speed**: Due to large data size, during processing of the plots that are used in all the chapters, the computational speed was getting impacted. To overcome this challenge, we tried to sample the data in optimal way, to make the figures visually clear.

3. **Literature**: The overall body of this research has been invented and discovered in and around 1900. This includes statistical techniques like Fourier Transform and Autocorrelation, and all the analog noise generators. Because of this, it was critical to find reliable and up-to-date references for the literature.

4. **Visual interpretation**: Due to time constraint, instead of suggesting some mathematical proofs, we had to conclude our results based on visual interpretation. Since the generated figures were critical to understand, we could not conclude some of the requirements and also had to rely on visual interpretations.

But nevertheless, such challenges gives raise to future opportunities. In the next section, we will share the future scope one access while working on this thesis project.

6.3 Future Scope

In this section, we will briefly discuss the future scope in this project.

1. **Introducing more statistical methods**: You can introduce more statistical methods like Spectral density analysis, Amplitude distribution, Shift register (Cube test) to perform such statistical analysis on noise signals.

2. **A test suite for discrete-time signal**: With these fore-mentioned statistical tests, you can create a test suite for discrete-time signal, just like Die Hard test suite.

3. **Analysis of continuous-time signal**: In some cases, these statistical methods also hold potential to analysis continuous-time signal in real time. This can have tremendous application in the field of signal processing.

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Appendix A

Appendix

Please find the python programmes I have used to generate all the plots in my GitHub Repository:

https://github.com/rhushikezh/Characterising-Analog-Noise-Sources.git

Appendix B

Statement of Certification

I hereby confirm that this thesis constitutes my own work, produced without aid and support from persons and/or materials other than the ones listed. Quotation marks indicate direct language from another author. Appropriate credit is given where I have used ideas, expressions or text from another public or non-public source.

The paper in this or similar form has never been submitted as an assessed piece of work in or outside of Germany. It also has not yet been published.

Frankfurt, 30.04.2024

City, Date

Author's Signature