Assignment I – CSC376 – Fall 2019

Fundamentals of Robot Design
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Abstract

The primary purpose of the assignment is to reinforce the concepts of rotation and transformation of rigid bodies. This assignment consists of two parts. In the first part, rotations in Cartesian space are considered in order to make you aware of various rotation properties and rotation representations. In the second part, transformations between different objects/frames have to be determined.

Remember to write your full name and student number prominently on your submission. To avoid suspicions of plagiarism: at the beginning of your submission, clearly state any resources (people, print, electronic) outside of your group, the course notes, and the course staff, that you consulted.

Please upload your submission as a single `.pdf` file on Quercus. By uploading, you confirm that this is your original work without using any references other than the listed ones.

Answer each question completely, always justifying your claims and reasoning and explaining your calculation/derivation. Your solution will be graded not only on correctness, but also on clarity. Answers that are technically correct that are hard to understand will not receive full marks. Mark values for each question are contained in the [square brackets].

______________________________ ______________________________

Name Date

Due Date: Oct 4, 2019 (11:59pm)

Part I: Rotation

Rotations and different ways of representation are subject of this part of the assignment. Note that a question may be composed of several sub-questions.

1. Are the matrices R_1 and R_2 rotation matrices? Give mathematical proof!

$$R_1 = \begin{bmatrix} 0.6314 & 0.6301 & 0.4520 \\ 0.3267 & -0.7448 & 0.5818 \\ -0.7033 & 0.2197 & 0.6761 \end{bmatrix} \quad \text{and} \quad R_2 = \begin{bmatrix} -7 & 5 & 3 \\ 6 & -9 & 2 \\ 1 & 3 & -4 \end{bmatrix}.$$

[6 points]

2. Compute the rotation matrix R_3 to rotate a point by π rad about an unit axis with $\hat{\omega} = [1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]^T$. Show that rotating a point twice by R_3 returns it to its original position. Afterwards, compute the corresponding quaternion $\xi\,(R_3)$ and exponential coordinates.

[4 points]

3. From the explicit formula (B.12) in Lynch & Park [1], it can be seen that both quaternions ξ and $-\xi$ produce the same rotation matrix and, therefore, lead to same rotated point. Show that both quaternions ξ and $-\xi$ produce the same rotated point by using the sandwich product $\xi\rho\xi^{-1}$, where $\rho = 0 + x\imath + y\jmath + zk$ is a pure quaternion embedding the position vector $p = [x, y, z]^T \in \mathbb{R}^3$.

[2 points]

4. Use $\xi\rho\xi^{-1}$ to rotate the vector $p = [0, 1, 0]^T$ about the x-axis by 90°. Check your results by converting the angle and axis to an elemental rotation matrix and perform a matrix multiplication.

[8 points]

5. Convert the matrices R_4 and R_5 into the corresponding quaternion by using the method proposed by Shepperd [2]. Then convert both matrices directly to angle-axis representation by considering one of the three elemental rotation matrices. Afterwards, construct the corresponding quaternion by using the angle-axis-representation and check your previous results.

$$R_4 = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

[10 points]

6. Compute $R_z\,(\alpha)\,R_y\,(\beta)\,R_x\,(\gamma)$ for the angles $\alpha = \beta = \gamma = \pi/2$. Can you describe $R_z\,(\pi/2)\,R_y\,(\pi/2)\,R_x\,(\pi/2)$ with an elemental rotation? Determine the angle and rotation axis of this elemental rotation. Give an alternative set of Euler angles for the ZYX sequence based on your finding. Now, compute the Euler angles for the ZYX sequence based only on the computed matrix. According to [1], the angles can be determined with

$$\alpha = \operatorname{atan2}\,(r_{21}, r_{11})\,, \tag{1}$$

$$\beta = \operatorname{atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right), \text{and} \tag{2}$$

$$\gamma = \operatorname{atan2}\,(r_{32}, r_{33})\,, \tag{3}$$

respectively. What is the name of this phenomenon which arises through an unfortunate rotation sequence and angles?

[10 points]

Part II: Transformations between Frames

The scene illustrated in Fig. 1 depicts a common test bed for a peg-in-hole task. Six frames are shown in the robot workspace: the base frame $\{b\}$, the tool-centre-point frame $\{g\}$ of the gripper, an intermediate hover frame $\{i\}$, the peg frame $\{p\}$ of the peg, the operational frame $\{h\}$ for the peg-and-hole task, and an auxiliary frame $\{a\}$. It is assumed that the bar with three holes has the dimension K, $4B$, and $2L$, where $0 < K < 2L < 4B$ holds. Further, it is assumed that the frame of the peg $\{p\}$ is h_p units above the base frame $\{b\}$ and it is rotated around the z-axis of the base frame $\{b\}$ by $135°$.

1. Write down the homogeneous transformation matrices T_{bg}, T_{bp}, T_{ba}, T_{bi} and T_{bh} in terms of the dimension given in Fig. 1.

   [5 points]

2. Compute the transformation matrix T_{gp} using the results of II-1 and determine the Euclidean distance between the tool-center-point (origin of frame $\{g\}$) and the handle of the peg (origin of frame $\{p\}$).

   [8 points]

3. Compute the transformation matrices T_{ai} and T_{ih} in terms of the dimensions given in Fig. 1.

   [12 points]

4. Find the Euclidean distance between the origin of frame $\{h\}$ and the origin of frame $\{a\}$. Now suppose that someone moves the bar with the three holes within the workspace of the robot such that T_{ba} computed in II-1 does not hold any more. Is the previous determined Euclidean distance changing? Show mathematically that for any homogeneous transformation matrix $T_{ba}^{\text{(new)}}$ the Euclidean distance between the origin of frame $\{h\}$ and the origin of frame $\{a\}$ is not affected by $T_{ba}^{\text{(new)}}$.

   [15 points]

5. Remember that the bar has been moved, therefore, the transformations T_{ba}, T_{bi}, and T_{bh} are no longer valid. However, all other transformations are still correct. How can $T_{ba}^{\text{(new)}}$, $T_{bi}^{\text{(new)}}$, and $T_{bh}^{\text{(new)}}$ be determined without moving the bar again? It is assumed that the robot's joints can be freely moved by the user and that the robot's software interface can provide T_{bg}. Further, suppose that the peg can be rigidly attached to the gripper and that the orientation between the peg and hole can be determined, if the peg is in the hole. Outline your approach and provide the transformation sequences leading to $T_{ba}^{\text{(new)}}$, $T_{bi}^{\text{(new)}}$, and $T_{bh}^{\text{(new)}}$.

   [20 points]

References

[1] Lynch, K.M. & Park, F.C. , *Modern Robotics: Mechanics, Planning, and Control*, Cambridge University Press, 2017.

[2] Shepperd , S.W. , "Quaternion form Rotation Matrix,"*Journal of Guidance and Control*, Vol. 41, No. 3, pp. 223–224, 1978.

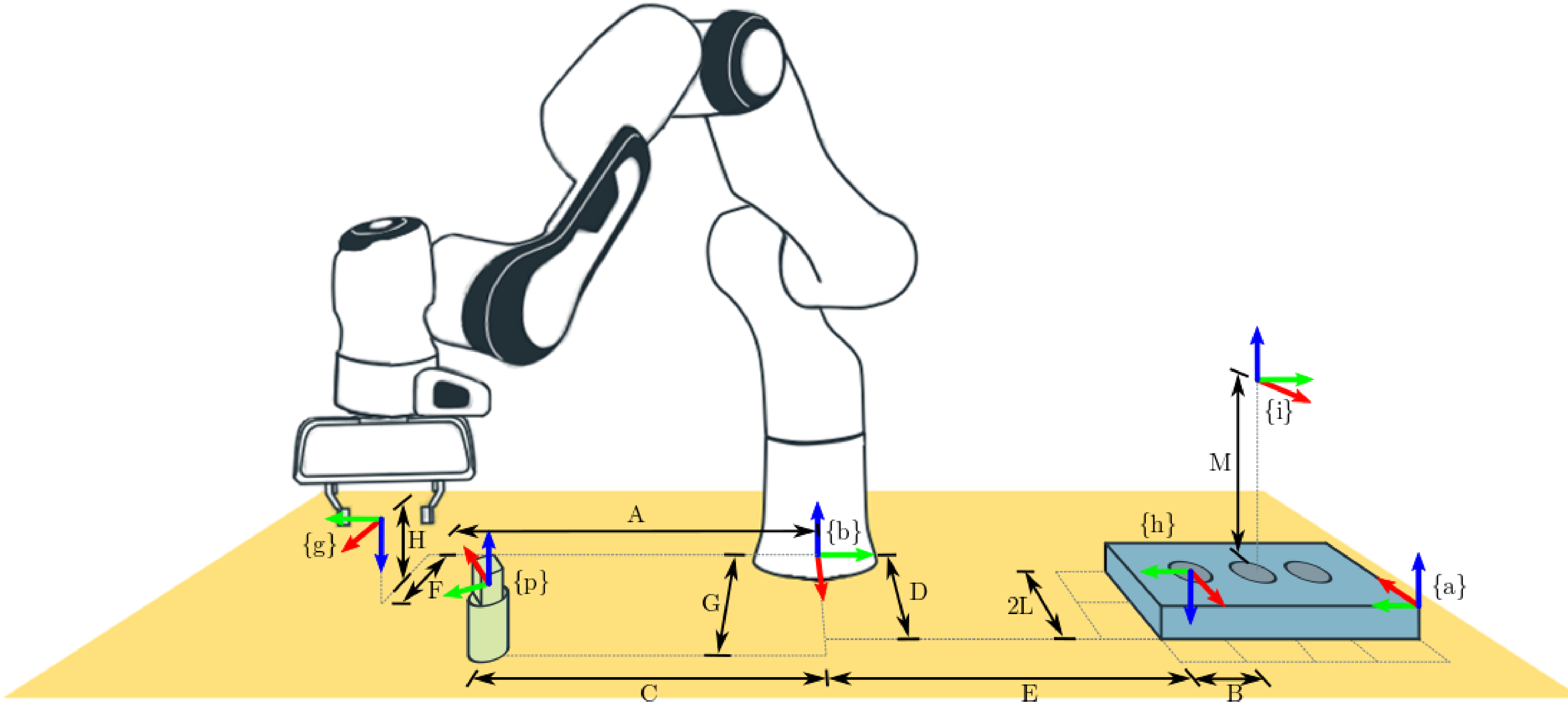

Figure 1: Peg-in-hole testbed. Six frames are shown in the robot workspace: the base frame $\{b\}$, the tool-centre-point frame $\{g\}$ of the gripper, an intermediate hover frame $\{i\}$, the peg frame $\{p\}$ of the peg, the operational frame $\{h\}$ for the peg-and-hole task, and an auxiliary frame $\{a\}$. All shown dashed lines are aligned with one of the axes of the base frame $\{b\}$. The x, y, and z axis of each right-handed frames are coloured with red, green, and blue, respectively. For the sake of clarity, the farthest corner of the bar has the coordinate $[2L, 4B, -K]^T$ w.r.t. the auxiliary frame $\{a\}$. Note that $D \neq 2L$.