Question 3(c)

Prove that $\dfrac{\partial l(w_0, w)}{\partial w} = 2Z^T(y-t)$

$$\frac{\partial l(w_0, w)}{\partial w} = \frac{\partial \sum_{n=1}^{N} \left[y(x^{(n)}) - t^{(n)} \right]^2}{\partial w}$$

$$= \sum_{n=1}^{N} \frac{\partial \left[y(x^{(n)}) - t^{(n)} \right]^2}{\partial w}$$

$$= \sum_{n=1}^{N} 2\left(y(x^{(n)}) - t^{(n)} \right) \cdot \frac{\partial \left(y(x^{(n)}) - t^{(n)} \right)}{\partial w}$$

$$= 2 \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) \cdot \frac{\partial \left(\sum_{m=1}^{M} w_m z_m - t^{(n)} \right)}{\partial w}$$

$$= 2 \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) \cdot \phi_m(x^{(n)})$$

$$= 2 \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) \cdot Z_{nm}$$

$$= 2 \begin{bmatrix} \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) Z_{n0} \\ \vdots \\ \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) Z_{nM} \end{bmatrix}$$

$$= 2 \begin{bmatrix} \sum_{n=1}^{N} \left(y(x^{(n)}) - t^{(n)} \right) \phi_0(x^{(n)}) \\ \vdots \\ \sum \left(y(x^{(n)}) - t^{(n)} \right) \phi_M(x^{(n)}) \end{bmatrix}$$

$\boxed{\text{Note: } \phi_0(x) = 1}$

$$= 2 \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ \phi_M(x^{(1)}) & \cdots & \phi_M(x^{(n)}) \end{bmatrix} \cdot \begin{bmatrix} y(x^{(1)}) - t^{(1)} \\ \vdots \\ y(x^{(n)}) - t^{(n)} \end{bmatrix}$$

$$= 2Z^T(y-t) \quad \blacksquare$$