## Question 3(a)

Prove that $\ell(w_0, w) = ||y - t||^2$,

where $t = [t^{(1)}, t^{(2)}, \dots, t^{(N)}]$
$y = [y^{(1)}, y^{(2)}, \dots, y^{(N)}]$

Note: Definition of magnitude of a vector

$$||v||^2 = \sum_n v_n^2$$

$$||y - t||^2 = \sum_n \left[y^{(n)} - t^{(n)}\right]^2$$

we can reorder $\left[y^{(n)} - t^{(n)}\right]^2$ b/c $x^2 = (-x)^2 \Rightarrow (a-b)^2 = [-1(a-b)]^2 = (b-a)^2$

$$= \sum_n \left[t^{(n)} - y^{(n)}\right]^2$$

$$= \ell(w_0, w)$$ (def. of loss function (4)) ∎

## Question 3(b)

Prove that $y = w_0 \vec{1} + Zw$,

where $y$ and $w$ are treated as column vectors

$$w_0 \vec{1} + Zw = \begin{bmatrix} \phi_1(x^{(1)}) & \phi_2(x^{(1)}) & \cdots & \phi_M(x^{(1)}) \\ \phi_1(x^{(2)}) & & & \\ \vdots & & \ddots & \\ \phi_1(x^{(N)}) & & & \phi_M(x^{(N)}) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} + w_0 \vec{1}.$$

$$= \begin{bmatrix} w_1 \phi_1(x^{(1)}) + w_2 \phi_2(x^{(1)}) + \ldots + w_M \phi_M(x^{(1)}) \\ w_1 \phi_1(x^{(2)}) + w_2 \phi_2(x^{(2)}) + \ldots + w_M \phi_M(x^{(2)}) \\ \vdots \\ w_1 \phi_1(x^{(N)}) + w_2 \phi_2(x^{(N)}) + \ldots + w_M \phi_M(x^{(N)}) \end{bmatrix} + w_0 \vec{1}$$

let $a_i$ represent row $i$ of $Zw$ where $i \in [1, N]$

$$= \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} + \begin{bmatrix} w_0 \\ w_0 \\ \vdots \\ w_0 \end{bmatrix} = \begin{bmatrix} w_0 + \sum_{m=1}^{M} w_m z_m \\ w_0 + \sum_{m=1}^{M} w_m z_m \\ \vdots \\ w_0 + \sum_{m=1}^{M} w_m z_m \end{bmatrix} \quad \text{(by eqn. \#5)}$$

where $x = 1$

where $x = 2$

$\vdots$

where $x = N$

$$= \begin{bmatrix} y(x^{(1)}) \\ y(x^{(2)}) \\ \vdots \\ y(x^{(N)}) \end{bmatrix} = y \quad \blacksquare$$

Note: $y(x) = w_0 + \sum_{m=1}^{M} w_m z_m \quad (5)$