CS280 Homework 1

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September 17, 2013

1 Perspective Projection

1. Take two arbitrary lines lying in the same plane, $\vec{a_1} + \lambda\vec{d}$ and $\vec{a_2} + \lambda\vec{e}$. We know the vanishing points of these two lines are $f\frac{\vec{d}}{d_z}$ and $f\frac{\vec{e}}{e_z}$. Furthermore, because they lie in the same plane, we know $\vec{d} \times \vec{e} = \vec{n}$, where $\vec{n}$ is the normal vector of the plane. Finally, the vanishing line of a plane is $xn_x + yn_y + fn_z = 0$. Using these facts, we will show the vanishing point of the lines are in fact on the vanishing line of the plane.

$$\vec{n} = \begin{pmatrix} d_y e_z - d_z e_y & d_z e_x - d_x e_z & d_x e_y - e_x d_y \end{pmatrix}$$

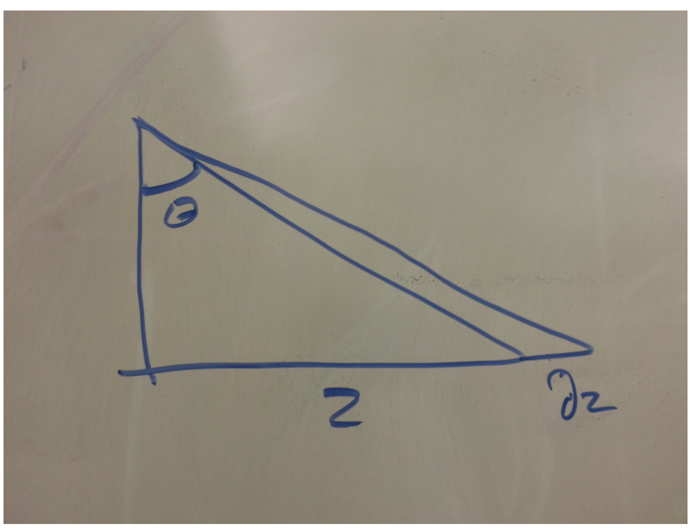
Plugging the vanishing points of the lines into the vanishing line...

$$f\frac{d_x}{d_z}(d_y e_z - d_z e_y) + f\frac{d_y}{d_z}(d_z e_x - d_x e_z) + f(d_x e_y - e_x d_y) = 0$$
$$\frac{d_x d_y e_z}{d_z} - \frac{d_x d_z e_y}{d_z} + \frac{d_y d_z e_x}{d_z} - \frac{d_x d_y e_z}{d_z} + d_x e_y - e_x d_y = 0$$
$$0 = 0$$

$$f\frac{e_x}{e_z}(d_y e_z - d_z e_y) + f\frac{e_y}{e_z}(d_z e_x - d_x e_z) + f(d_x e_y - e_x d_y) = 0$$
$$\frac{e_x d_y e_z}{e_z} - \frac{e_x d_z e_y}{e_z} + \frac{e_y d_z e_x}{e_z} - \frac{e_y d_x e_z}{e_z} + d_x e_y - e_x d_y = 0$$
$$0 = 0$$

2.

3. Define θ as the angle between the observer's body and the point.



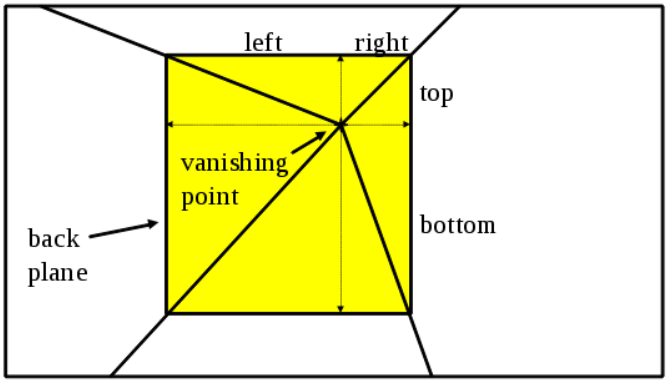
$$\theta = arctan(\frac{z}{h})$$

$$tan(\theta + 1') = \frac{x + \delta z}{h}$$

$$\delta z = h * tan(tan^{-1}(\frac{z}{h}) + 1') - z$$

2 Tour into the Picture

1. To estimate the 3D geometry of the room, we take advantage of the fact that the back plane is parallel to the image plane, and the other walls are perpendicular to it. If we know the back plane along with the vanishing point, we know the relative camera position in the room. Furthermore, if we assume a focal length, we can calculate the depth of the room, using similar triangles.



2. To compute each of the homographies, we solve for H given s and d, source and destination points. A homography must map s to d.

$$\vec{d} = H\vec{s}$$

Writing this out explicitly:

$$\begin{pmatrix} wx' \\ wy' \\ w \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$wx' = ax + by + c$$
$$wy' = dx + ey + f$$
$$w = gx + hy + 1$$

If we plug in the equation for w into the first two, we get:

$$ax + by + c - gxx' - hyx' = x'$$
$$dx + ey + f - gxy' - hyy' = y'$$

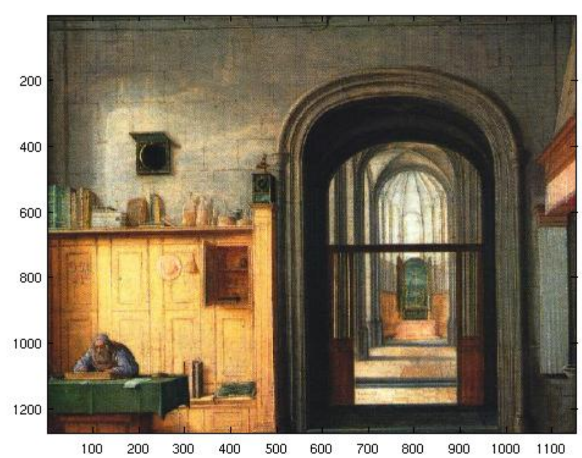
Given a pair of source/destination points, we get two equations. Thus, we need four pairs of points to generate the eight equations we need to solve for the homography. Writing those equations in matrix form, with (x_i', y_i') as the ith destination point:

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & y_1x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & y_1y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & y_2x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & y_2y_2' \\ \dots \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{pmatrix} = \begin{pmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ \dots \end{pmatrix}$$

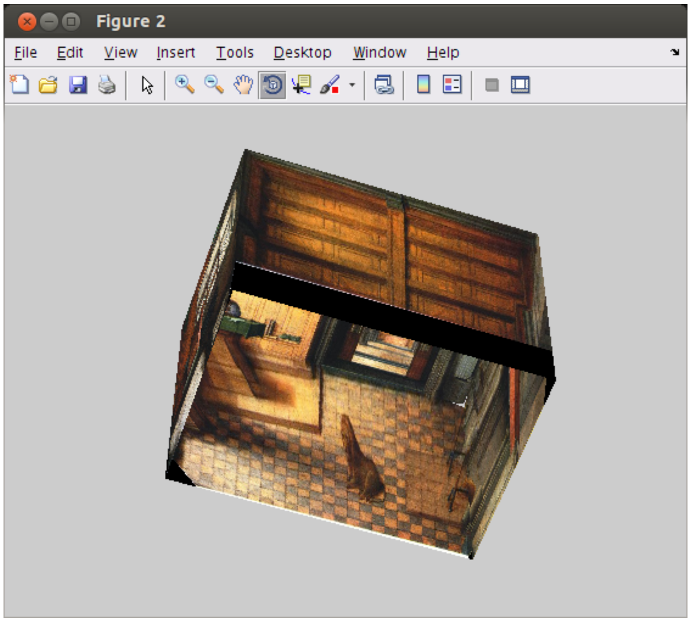
$$Ah = b$$
$$h = A^{-1}b$$

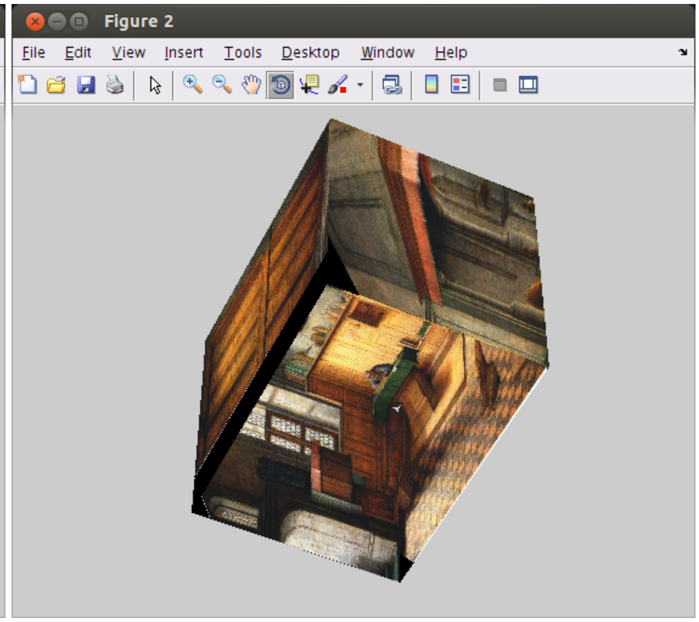
We perform this operation for each of the planes. The vertices obtained from TIP_get5rects are the source, and we create destination vertices such that the resulting rectangle is proportioned correctly according to the 3d geometry calculated.

3. Here we show the fronto-parallel views of the ceiling, floor, left wall, right wall, and back wall.

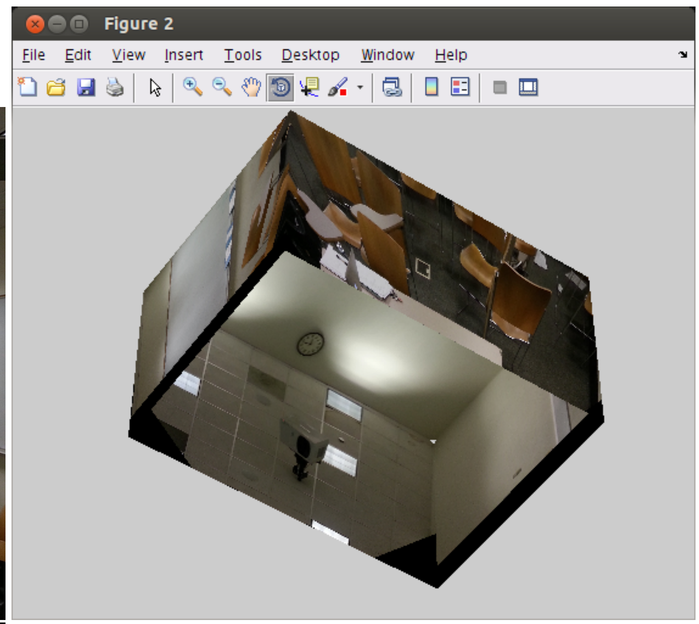


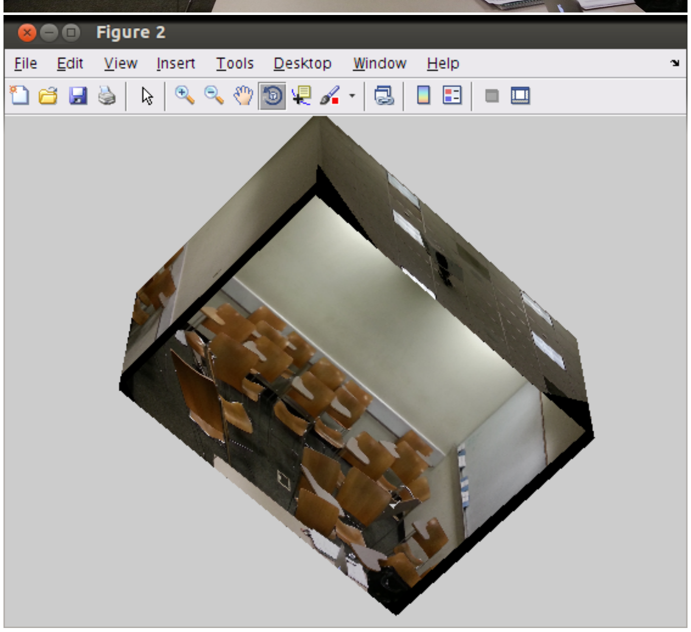
4. Views from sjerome.jpg:



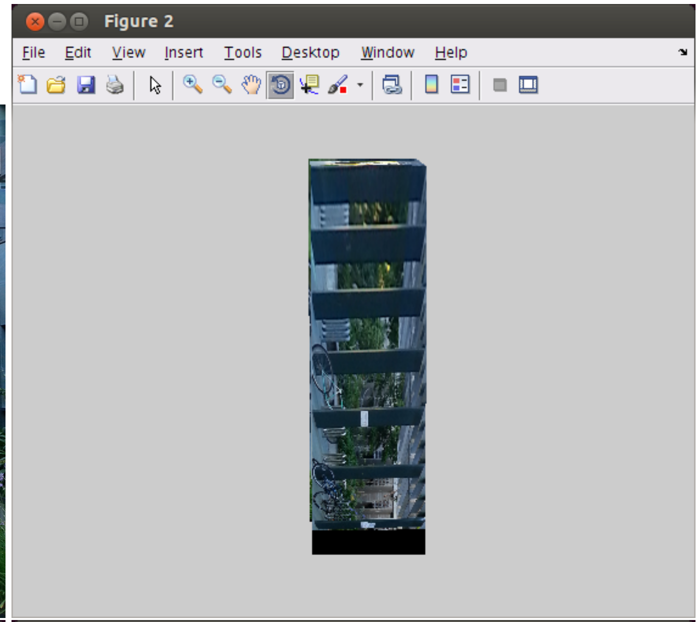


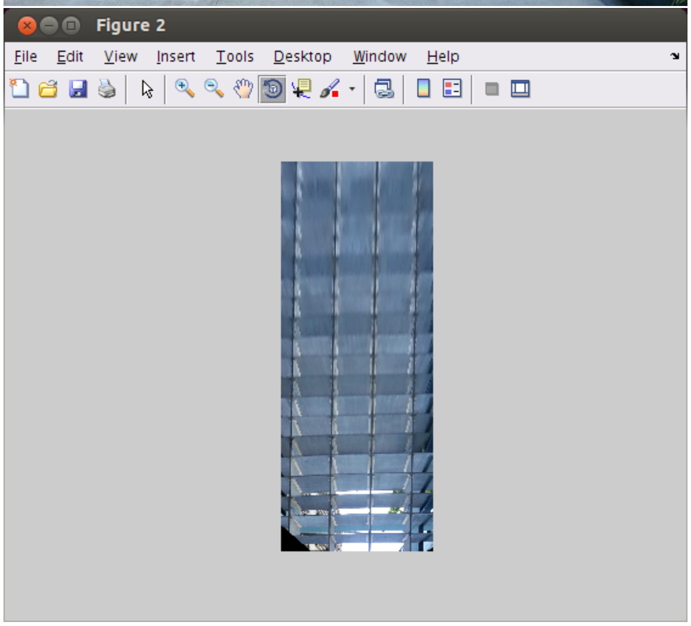
Views from 320 Soda:

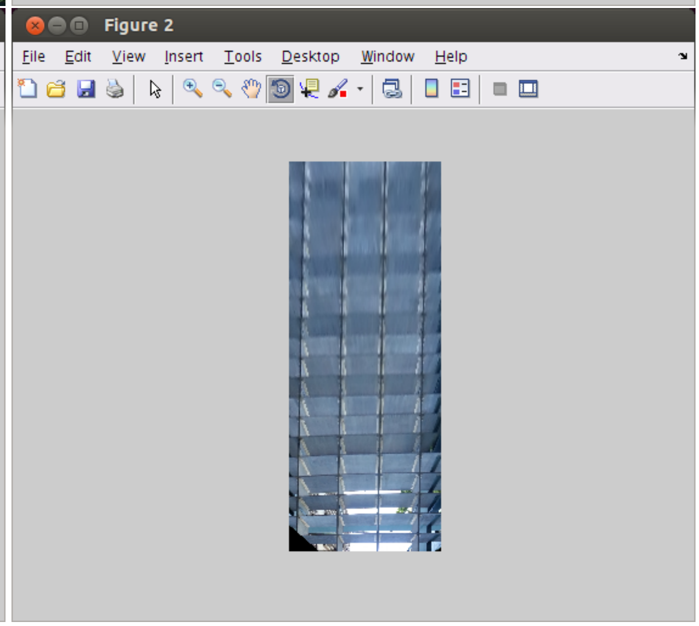




Views from the Soda Etcheverry Breezeway:







3 Geometric Transformations

1. We multiply the two reflection matrices and then use trigonometric sum/difference formulas.

$$\begin{pmatrix} cos(2\beta) & sin(2\beta) \\ sin(2\beta) & -cos(2\beta) \end{pmatrix} \begin{pmatrix} cos(2\alpha) & sin(2\alpha) \\ sin(2\alpha) & -cos(2\alpha) \end{pmatrix} =$$

$$\begin{pmatrix} cos(2\beta)cos(2\alpha) + sin(2\beta)sin(2\alpha) & sin(2\alpha)cos(2\beta) - cos(2\alpha)sin(2\beta) \\ sin(2\beta)cos(2\alpha) - cos(2\beta)sin(2\alpha) & cos(2\alpha)cos(2\beta) + sin(2\alpha)sin(2\beta) \end{pmatrix} =$$

$$\begin{pmatrix} cos(2(\beta - \alpha)) & -sin(2(\beta - \alpha)) \\ sin(2(\beta - \alpha)) & cos(2(\beta - \alpha)) \end{pmatrix}$$

2. Consider a skew-symmetric matrix and its powers.

$$\hat{s} = \begin{pmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{pmatrix}$$

$$\hat{s}^2 = \hat{s}^2$$
$$\hat{s}^3 = (a^2 + b^2 + c^2) * -\hat{s}$$
$$\hat{s}^4 = (a^2 + b^2 + c^2) * -\hat{s}^2$$
$$\hat{s}^5 = (a^2 + b^2 + c^2)^2 * \hat{s}$$
$$...$$

We know $||s|| = 1$, so we recognize a pattern.

$$\hat{s} = \hat{s}$$
$$\hat{s}^2 = \hat{s}^2$$
$$\hat{s}^3 = -\hat{s}$$
$$\hat{s}^4 = -\hat{s}^2$$
$$...$$

Now, we consider Roderigues' formula.

$$R = e^{\phi \hat{s}}$$
$$R = I + \phi\hat{s} + \frac{\phi^2\hat{s}^2}{2!} + \frac{\phi^3\hat{s}^3}{3!} + \frac{\phi^4\hat{s}^4}{4!}...$$

Subbing in the powers of $\hat{s}$,

$$R = I + \phi\hat{s} + \frac{\phi^2\hat{s}^2}{2!} - \frac{\phi^3\hat{s}}{3!} - \frac{\phi^4\hat{s}^2}{4!}...$$
$$R = I + (\phi - \frac{\phi^3}{3!} + ...)\hat{s} + (\frac{\phi^2}{2!} - \frac{\phi^4}{4!} + ...)\hat{s}^2$$
$$R = I + sin(\phi)\hat{s} + (1 - cos(\phi))\hat{s^2}$$

3. The general form of a Euclidean planar transformation is E:

$$\begin{pmatrix} cos\theta & -sin\theta & t_x \\ sin\theta & cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

And a least squares problem is formulated as

$$X\beta = y$$

where β consists of the unknown parameters. Thus, we massage the problem into that form, saying $cos\theta$, $sin\theta$, T_x, and T_y are our parameters, so the problem is linear.

$$\begin{pmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ \cdots & & & \\ x_n & -y_n & 1 & 0 \\ y_n & x_n & 0 & 1 \end{pmatrix} \begin{pmatrix} cos\theta \\ sin\theta \\ T_x \\ T_y \end{pmatrix} = \begin{pmatrix} x'_1 \\ y'_1 \\ \cdots \\ x'_n \\ y'_n \end{pmatrix}$$

Now we can plug in the given points to find an estimate for E.

$$\beta = \begin{pmatrix} cos\theta \\ sin\theta \\ T_x \\ T_y \end{pmatrix} = (X^T X)^{-1} X^T y$$

For the given set of u and v, we get:

$$\beta = \begin{pmatrix} 0 \\ -0.7857 \\ 0 \\ 0.75 \end{pmatrix}$$

Thus, our Euclidean planar transformation is:

$$E = \begin{pmatrix} 0 & 0.7857 & 0 \\ -0.7857 & 0 & 0.75 \\ 0 & 0 & 1 \end{pmatrix}$$

4.

5.