CS280 Computer Vision: HW4

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1 Optical Flow

The code for generating images for a given plane, translation, and rotation is shown in Figure 1.

Figure 1: Generating Images

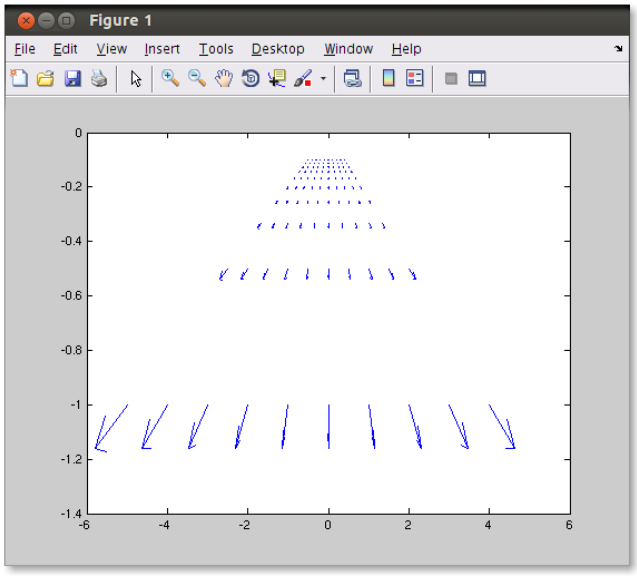
```
function draw_dynamic(X, Y, Z, t, w)
    % t, w: 3 x 1 column vectors
    % X Y Z: m x m matrices s.t. cols correspond to x values, rows to y values
    x = X ./ Z
    y = Y ./ Z
    % x y: m x m matrices
    u = zeros(size(x,2));
    v = zeros(size(y,2));
    for ix = 1:size(x,2)
        for iy = 1:size(y,2)
            cx = x(iy, ix);
            cy = y(iy, ix);
            cz = Z(iy, ix);
            trans =  (1 / cz) * [ -1 0 cx ; 0 -1 cy ] * t;
            ang   = [cx*cy -(1 + (cx*cx)) cy ; (1 + (cy*cy)) -cx*cy -cx] * w;
            u(iy,ix) = trans(1) + ang(1);
            v(iy,ix) = trans(2) + ang(2);
        end
    end
    quiver(x, y, u, v)
end
```

The code for each part is shown in Figure 2.

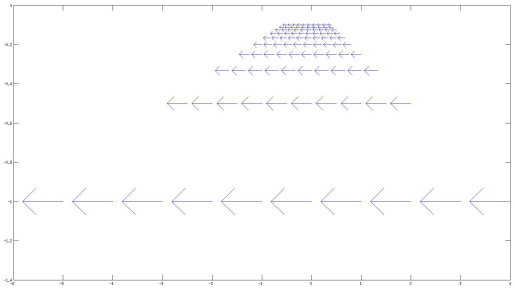
Figure 2: Optical Flow Translations and Rotations

```
% PART A
% speed
ts = 5 % translation
ws = 5 % angular

[X, Y] = meshgrid(-5:4, ones(1,10)*-1);
% e.g. Z(1, 5) = Z value at X = 5, Y = 1
Z = repmat(1:10, 10, 1)';
t = [ 0; 0; ts ];
w = [ 0; 0; 0 ];
figure(1)
draw_dynamic(X, Y, Z, t, w);

% PART B
[X, Y] = meshgrid(-5:4, ones(1,10)*-1);
Z = repmat(1:10, 10, 1)';
t = [ ts ; 0 ; 0 ];
w = [ 0 ; 0 ; 0 ];
figure(2)
draw_dynamic(X, Y, Z, t, w);

% PART C
[X, Y] = meshgrid(-5:4, ones(1,10)*-1);
% e.g. Z(1, 5) = Z value at X = 5, Y = 1
Z = repmat(1:10, 10, 1)';
t = [ 0 ; ts * cos(pi / 4) ; ts * sin(pi / 4) ];
w = [ 0 ; 0 ; 0 ];
figure(3)
draw_dynamic(X, Y, Z, t, w);

% PART D
[X, Y] = meshgrid(-5:4, -5:4);
Z = ones(10)*2;
t = [ 0; 0 ; 0 ];
w = [ 1 / sqrt(2) ; 1 / sqrt(2) ; 0 ];
figure(4)
draw_dynamic(X, Y, Z, t, w);
```

a)



b)



c)

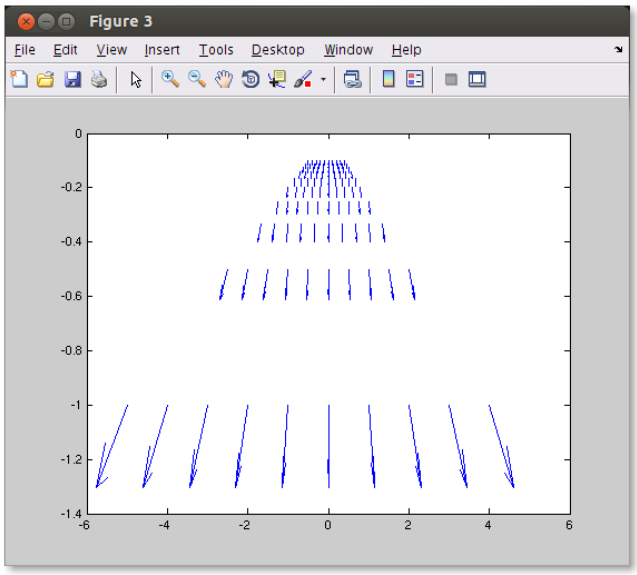


Figure 3
File Edit View Insert Tools Desktop Window Help
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-6
-4
-2
0
2
4
6

d)

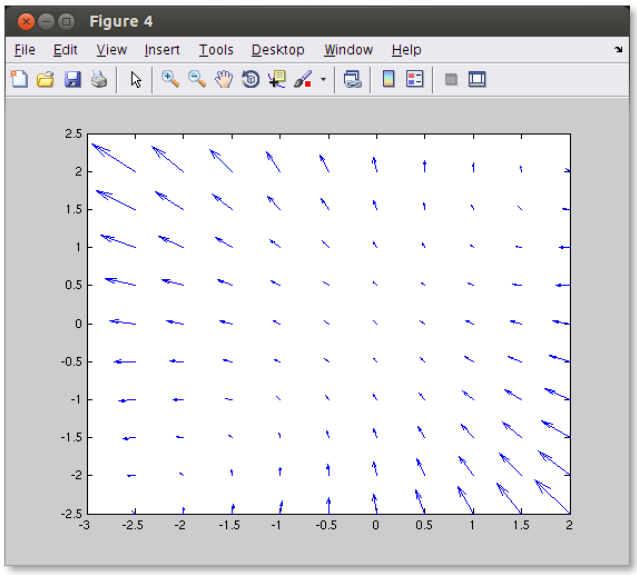


Figure 4
File Edit View Insert Tools Desktop Window Help
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

2 Error Analysis of Binocular Stereopsis

1. First, we calculate d in terms of b, f, Z.

$$Z = \frac{bf}{d}$$
$$d = \frac{bf}{Z}$$

Next, we can calculate δZ in terms of $Z, \delta d$.

$$Z + \delta Z = \frac{bf}{d + \delta d}$$
$$Z + \delta Z = \frac{bf}{\frac{bf}{Z} + \delta d}$$
$$Z + \delta Z = \frac{bfZ}{\frac{bf}{+}(\delta d)Z}$$
$$\delta Z = \frac{bfZ - bfZ + (\delta d)Z^2}{bf + (\delta d)Z}$$
$$\delta Z = \frac{(\delta d)Z^2}{bf + (\delta d)Z}$$

To calculate δd, we can use the equation above to get an equation for δd. Rearranging the terms, we get

$$\delta d = \frac{\delta Z * bf}{Z^2 + Z\delta Z}$$

From here, the code is relatively simple:

```
deltad = (err * b * f) ./ (z .* (z + err));
```

3 Multi-View Reconstruction

3.1 Finding the Fundamental Matrix

First, we normalize all the points by translating them by μ and scaling them such that the mean distance from μ (the new origin) is $\sqrt{2}$. Next, we compute A, a $N\times$ matrix. We want $Af = 0$ as this is just $x_2^t F x_1 = 0$ rewritten, but there is noise. We solve $min_f||Af||_2$, then, by taking the right singular vector of A corresponding to the smallest singular value. This is our F_{est}. We must enforce that the fundamental matrix is of rank 2. So, we solve $min_F||F - F_{est}||_F$, via SVD. If $F_{est} = USV_t$, then $F = U\hat{S}V^t$, where $\hat{S} = diag(s_1, s_2, 0)$. The fundamental matrices we found with this method are shown in Figures 3 and 4.

Thus, to answer the question posed, we are minimizing the residual error. This is because F is defined as $e_2 = Fx_1$, where e_2 is the epipolar line for camera two. $x_2^t F x_1$ should be zero, then, if x_2 lies on e_2. The optimization problem is trying to get x_2 as close as possible to on e_2.

Our definition of the residual is the mean squared distance between the points in the two images and the corresponding lines. Calculation of the residual error is shown in Figure 5. For the library image, we found a residual error of 0.096304, while for the house image, we found a residual error of 0.806142.

Figure 3: Fundamental Matrix for Library

$$\begin{pmatrix} -0.0000 & 0.0004 & -0.0200 \\ -0.0005 & 0.0000 & 0.4109 \\ 0.0697 & -38.96 & -9.2135 \end{pmatrix}$$

Figure 4: Fundamental Matrix for House

$$\begin{pmatrix} 0.0000 & 0.0002 & -0.0523 \\ 0.0003 & 0.0000 & -0.7753 \\ 0.0280 & 0.6763 & 4.6626 \end{pmatrix}$$

Figure 5: Calculation of Residual Error

```
% Compute mean squared distance between points in their two images and their
% corresponding epipolar lines.
dist_sum = 0;
for i = 1:N
    x_1 = [matches(i, 1:2) 1]';
    x_2 = [matches(i, 3:4) 1]';
    el_1 = F' * x_2;
    el_2 = F * x_1;

    a_1 = el_1(1);
    b_1 = el_1(2);
    c_1 = el_1(3);
    dist_1 = abs(a_1*x_1(1) + b_1*x_1(2) + c_1) / sqrt(a_1^2 + b_1^2);

    a_2 = el_2(1);
    b_2 = el_2(2);
    c_2 = el_2(3);
    dist_2 = abs(a_2*x_2(1) + b_2*x_2(2) + c_2) / sqrt(a_2^2 + b_2^2);

    % calc distance of x_1 from el_1, and x_2 from el_2
    dist_sum = dist_sum + dist_1^2 + dist_2^2;
end

res_err = dist_sum / (2*N);
```

3.2 Finding the Camera Rotation and Translation

All possible possibilities for t and R for the two images are shown in Figures 6 and 7.

Figure 6: ts and Rs for Library

$$t = \begin{pmatrix} -0.7882 \\ 0.1562 \\ 0.5953 \end{pmatrix}, \begin{pmatrix} 0.7882 \\ -0.1562 \\ -0.5953 \end{pmatrix}$$

$$R = \begin{pmatrix} -0.9941 & -0.0568 & 0.0924 \\ 0.0525 & -0.9975 & -0.0482 \\ -0.0949 & 0.0430 & -0.9946 \end{pmatrix}, \begin{pmatrix} -0.1649 & 0.1915 & 0.9675 \\ 0.1772 & 0.9708 & -0.1619 \\ 0.9703 & -0.1447 & 0.1940 \end{pmatrix}$$

3.3 Reconstruction of 3D Point Cloud

We are directly trying to minimize the reconstruction error. We rewrite

$$x = PX$$
$$y = PY$$

Figure 7: ts and Rs for House

$$t = \begin{pmatrix} 0.9975 \\ 0.0326 \\ 0.0621 \end{pmatrix}, \begin{pmatrix} -0.9975 \\ -0.0326 \\ -0.0621 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.9822 & 0.0627 & -0.1771 \\ -0.0641 & 0.9979 & -0.0017 \\ 0.1766 & 0.0130 & 0.9842 \end{pmatrix}, \begin{pmatrix} 0.9903 & 0.1286 & -0.0534 \\ 0.1285 & -0.9917 & -0.0058 \\ -0.0537 & -0.0011 & -0.9986 \end{pmatrix}$$

into $Ax = 0$ and try to find $x = \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$ that minimizes Ax. Calculation of the reconstruction error for one pair of points is shown in Figure 8. Reconstruction errors for the library and house are 4.204884 and 2.323244.

Figure 8: Calculation of Reconstruction Error

```
1        proj_1 = P1 * point;
2        proj_1 = proj_1(1:2, :) ./ proj_1(3,1);
3
4        proj_2 = P2 * point;
5        proj_2 = proj_2(1:2, :) ./ proj_2(3,1);
6
7        % Compute distances.
8        errs(1+i*2) = sqrt(sum((x_1.' - proj_1).^2));
9        errs(2+i*2) = sqrt(sum((x_2.' - proj_2).^2));
```

3.4 Plotting the 3D Point Cloud

The scatter plots for each image are shown in Figures 9 and 10. In Figure 9, we note that the points lie mostly in one plane, which corresponds to the library wall. We also notice the spots around the bushes at the base of the library wall. In Figure 10, we present a bird's eye view. To the left, we note the points on the rug. Especially noticable are corner and walls of the house.

Figure 9: 3D Point Cloud for Library

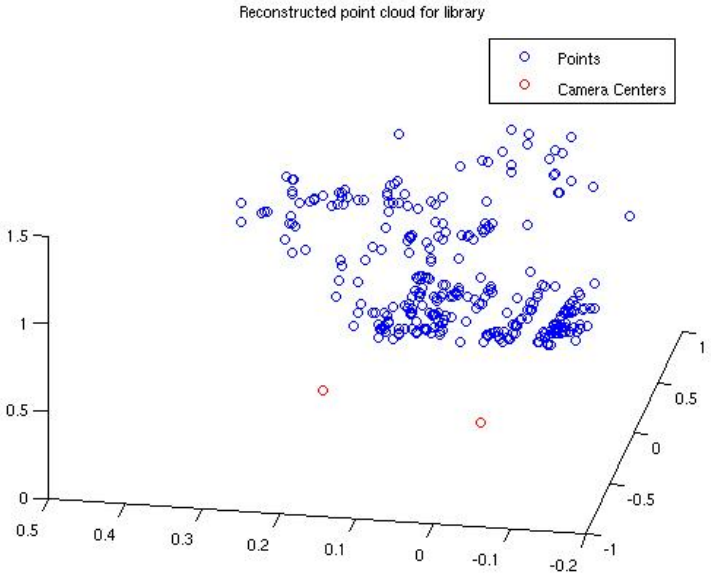


Figure 10: 3D Point Cloud for House