Assignment Set for Laboratory 1

ATSC 409: Hand-in answers to questions 1 and 2.
EOSC 511/ATSC 506: Hand-in answers to questions 1 and 3

All questions should be done by hand (not by computer) and show your steps. Upload your solutions to CANVAS

1. Given the equation
$$\frac{\partial y}{\partial t} = y(y+t) \tag{1}$$
write down
   (a) forward Euler difference formula
   (b) backward Euler difference formula
   (c) centered difference formula

2. The equation
$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0,\ y = \cos(x) \text{ at } t = 0,\ \frac{\partial y}{\partial t} = c\sin(x) \text{ at } t = 0 \tag{2}$$
has a solution $y = \cos(x - ct)$.
   (a) Expand both derivatives as centred differences. *Be very clear about indexing in x and t separately. Notation is up to you as long as it is clear, but I suggest, for example* $y(x = dx, t = 0)$
   (b) Show that the algebraic solution is an exact solution of the difference formula if we choose $\Delta x = c\Delta t$. *Remember for proofs (or shows) like this question, it is important not to assume what you are trying to prove. Work the left-hand-side and right-hand-side separately and show they are equal*

3. Given
$$\frac{\partial y}{\partial t} = -\alpha y,\ y = 1 \text{ at } t = 0 \tag{3}$$
   (a) Show that the forward Euler method gets a smaller answer than the backward Euler method for all $t > 0$, provided that $0 < \alpha^2\Delta t^2 < 1$.
   (b) Solve the equation analytically.
   (c) Show that the forward Euler always under-estimates the answer provided that $\alpha\Delta t < 1$ and $\alpha\Delta t \neq 0$.