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3C09

Q1 Maximum likelihood estimator for Normal distribution

2) mean $= \theta_1 = \mu$
variance $= \theta_2 = \sigma^2$

given :-
Pdf for Normal distribution

$$f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-(x-\theta_1)^2/2\theta_2}$$

$$-\infty \leq x \leq \infty$$
$$-\infty \leq \theta_1 \leq \infty$$
$$\theta_2 > 0$$

$$L(x; \theta_1, \theta_2) = \prod_{i=1}^{n} f(x; \theta_1, \theta_2)$$

$$L(x; \theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-(x-\theta_1)^2/2\theta_2}$$

$$L(x; \theta_1, \theta_2) = \left(\frac{1}{\sqrt{2\pi\theta_2}}\right)^n e^{-\sum_{i=1}^{n} \frac{(x-\theta_1)^2}{2\theta_2}}$$

$$L(x;\theta_1,\theta_2) = \frac{1}{(\sqrt{2\pi\theta_2})^n} e^{-\sum_{i=1}^{n}\frac{(x-\theta_1)^2}{2\theta_2}}$$

$$= \frac{1}{(2\pi)^{n/2}(\theta_2)^{n/2}} e^{-\sum\frac{(x-\theta_1)^2}{2\theta_2}}$$

$$L(x;\theta_1,\theta_2) = (2\pi)^{-n/2}(\theta_2)^{-n/2} e^{-\sum(x-\theta_1)^2/2\theta_2}$$

$$\log L(x;\theta_1,\theta_2) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\theta_2 - \sum\frac{(x-\theta_1)^2}{2\theta_2}$$

$$\frac{d\log L(x;\theta_1,\theta_2)}{d\theta_1} = 0 - 0 - 2\sum_{i=1}^{n}\frac{(x_i-\theta_1)}{2\theta_2}$$

$$\Rightarrow -\sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2} = 0$$

$$-\sum_{i=1}^{n} x_i + n\theta_1 = 0$$

$$\sum_{i=1}^{n} x_i = n\theta_1$$

Divide B.S. by n.

$$\theta_1 = \sum_{i=1}^{n}\frac{x_i}{n} = \bar{X}$$

$$\Rightarrow \hat{\theta}_1 = \bar{X}$$

ii) $\dfrac{d \log L(x; \theta_1, \theta_2)}{d\theta_2} = 0$

$$-n + \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})$$

(g) Suppose $X_1, X_2, \dots X_n \sim B(m, \theta)$

PMF

$$P(x_i | m, \theta) = {}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

$x_i \in (0, 1, 2, \dots m)$

$$L(P) = \prod_{i=1}^{n} P(x_i | m, \theta)$$

$$= \prod_{i=1}^{n} {}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

$$= \cancel{\prod_{1}} \prod_{i=1}^{n} ({}^mC_{x_i}) \prod_{i=1}^{n} \theta^{x_i} \prod_{i=1}^{n} (1-\theta)^{m-x_i}$$

$$= \prod_{i=1}^{n} ({}^mC_{x_i})\, \theta^{\sum_{i=1}^{n} x_i} (1-P)^{\sum_{i=1}^{n} (m-x_i)}$$

$$\Rightarrow L(P) = \prod_{i=1}^{n} {}^mC_{x_i}\, \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}$$

$$\Rightarrow \log L(P) = \log \left[\prod_{i=1}^{n} {}^mC_{x_i}\, \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i} \right]$$

$$= \log \left(\prod_{i=1}^{n} {}^mC_{x_i} \right) + \log \left[\theta^{\sum_{i=1}^{n} x_i} \right] +$$

$$\log \left[(1-\theta)^{mn - \sum_{i=1}^{n} x_i} \right]$$

$$\log L(P) = \log\left[\prod_{i=1}^{n} {}^{m}C_{x_i}\right] + \left(\sum_{i=1}^{n} x_i\right)\log\theta$$

$$+ \left(mn - \sum_{i=1}^{n} x_i\right)\log(1-\theta)$$

Diff w.r.t θ B.S.

$$\frac{d\log L(P)}{d\theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\left(mn - \sum_{i=1}^{n} x_i\right)$$

$$\Downarrow$$

$$\theta$$

$$\Rightarrow \frac{1}{\hat{\theta}}\left(\sum_{i=1}^{n} x_i\right) - \frac{1}{1-\hat{\theta}}\left(mn - \sum_{i=1}^{n} x_i\right) = 0$$

$$\frac{1}{1-\hat{\theta}}\left(mn - \sum_{i=1}^{n} x_i\right) = \frac{1}{\hat{\theta}}\sum_{i=1}^{n} x_i$$

$$\frac{mn - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i} = \frac{1-\hat{\theta}}{\hat{\theta}}$$

$$\Rightarrow \frac{mn}{\sum_{i=1}^{n} x_i} = \frac{1}{\hat{\theta}}$$

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{mn} = \frac{\bar{x}}{m}$$