In [1]:
```
import warnings
warnings.filterwarnings("ignore")
```

COURSE PROJECT :

Predicting Final Grades from Student Data

Introduction

The objective of this project is to predict the final grade of students using multiple linear regression on a dataset of student achievement in two Portuguese schools. We use the Statsmodels and Patsy modules for this task with Python version >= 3.8. The dataset was sourced from the UCI Machine Learning Repository at http://archive.ics.uci.edu/ml/datasets/Student+Performance (http://archive.ics.uci.edu/ml/datasets/Student+Performance) (FUBUTEC 2008). This report is organized as follows:

- Overview section describes the dataset used and the features in this dataset.
- Data Preparation section covers data cleaning and data preparation steps.
- Data Exploration section explores dataset features and their inter-relationships.
- Statistical Modeling & Performance Evaluation section first fits a full multiple linear regression model and performs diagnostic checks. Next, we perform backwards variable selection using p-values to obtain a reduced model, after which we perform another set of diagnostic checks on the reduced model.
- Summary & Conclusions section provides a summary of our work and presents our findings.

Overview

Data Source

Our dataset contains data on the achievement of students in secondary education of two Portuguese schools. The dataset was collected from http://archive.ics.uci.edu/ml/datasets/Student+Performance (http://archive.ics.uci.edu/ml/datasets/Student+Performance), using `student-por.csv`. Our dataset has 649 instances, with 33 attributes, which can be verified with `df.dtypes`.

In [2]:
```
import pandas as pd

df = pd.read_csv('Data.csv', sep=';')
pd.set_option('display.max_columns', None)
```

Our dataset was only one column of data including the column title with only the semi-colon (;) symbol seperating values, and so it needed to be seperated when being read in order to create the seperate columns with their respective row values using

```
sep = ';'
```

This is how our dataset currently looks:

In [3]:
```
df.sample(10)
```

Out[3]:

	school	sex	age	address	famsize	Pstatus	Medu	Fedu	Mjob	Fjob	reaso
596	MS	F	17	U	GT3	T	4	2	other	other	cours
603	MS	F	18	R	LE3	A	4	2	teacher	other	reputatic
571	MS	M	19	U	GT3	T	2	1	at_home	other	cours
517	MS	F	16	R	LE3	T	1	2	other	other	reputatic
238	GP	F	16	U	GT3	T	4	4	teacher	services	hom
38	GP	F	15	R	GT3	T	3	4	services	health	cours
484	MS	F	16	U	LE3	A	2	2	other	other	hom
392	GP	F	17	U	GT3	T	3	2	other	other	hom
455	MS	F	15	U	GT3	T	2	1	at_home	other	hom
291	GP	M	18	U	LE3	T	2	1	services	other	cours

Project Objective

Our goal is to predict our target feature, `G3` , within an acceptable margin of error using linear regression.

Target Feature

Our target feature is `G3` , which is a discrete numerical feature, and represents the students final grade for a specific course project (Math or Portuguese).

Descriptive Features

The variable descriptions below are from the `student-por` file:

- `school` - student's school (binary: 'GP' - Gabriel Pereira or 'MS' - Mousinho da Silveira)
- `sex` - student's sex (binary: 'F' - female or 'M' - male)
- `age` - student's age (numeric: from 15 to 22)
- `address` - student's home address type (binary: 'U' - urban or 'R' - rural)
- `famsize` - family size (binary: 'LE3' - less or equal to 3 or 'GT3' - greater than 3)
- `Pstatus` - parent's cohabitation status (binary: 'T' - living together or 'A' - apart)
- `Medu` - mother's education (numeric: 0 - none, 1 - primary education (4th grade), 2 - 5th to 9th grade, 3 - secondary education or 4 - higher education)
- `Fedu` - father's education (numeric: 0 - none, 1 - primary education (4th grade), 2 - 5th to 9th grade, 3 - secondary education or 4 - higher education)
- `Mjob` - mother's job (nominal: 'teacher', 'health' care related, civil 'services' (e.g. administrative or police), 'at_home' or 'other')
- `Fjob` - father's job (nominal: 'teacher', 'health' care related, civil 'services' (e.g. administrative or police), 'at_home' or 'other')
- `reason` - reason to choose this school (nominal: close to 'home', school 'reputation', 'course' preference or 'other')
- `guardian` - student's guardian (nominal: 'mother', 'father' or 'other')
- `traveltime` - home to school travel time (numeric: 1 - <15 min., 2 - 15 to 30 min., 3 - 30 min. to 1 hour, or 4 - >1 hour)
- `studytime` - weekly study time (numeric: 1 - <2 hours, 2 - 2 to 5 hours, 3 - 5 to 10 hours, or 4 - >10 hours)
- `failures` - number of past class failures (numeric: n if 1<=n<3, else 4)
- `schoolsup` - extra educational support (binary: yes or no)
- `famsup` - family educational support (binary: yes or no)
- `paid` - extra paid classes within the course subject (Math or Portuguese) (binary: yes or no)
- `activities` - extra-curricular activities (binary: yes or no)
- `nursery` - attended nursery school (binary: yes or no)
- `higher` - wants to take higher education (binary: yes or no)
- `internet` - Internet access at home (binary: yes or no)
- `romantic` - with a romantic relationship (binary: yes or no)
- `famrel` - quality of family relationships (numeric: from 1 - very bad to 5 - excellent)
- `freetime` - free time after school (numeric: from 1 - very low to 5 - very high)
- `goout` - going out with friends (numeric: from 1 - very low to 5 - very high)
- `Dalc` - workday alcohol consumption (numeric: from 1 - very low to 5 - very high)
- `Walc` - weekend alcohol consumption (numeric: from 1 - very low to 5 - very high)
- `health` - current health status (numeric: from 1 - very bad to 5 - very good)
- `absences` - number of school absences (numeric: from 0 to 93)

these grades are related with the course subject, Math or Portuguese:

- `G1` - first period grade (numeric: from 0 to 20)
- `G2` - second period grade (numeric: from 0 to 20)
- `G3` - final grade (numeric: from 0 to 20, output target)

Feature Set

Inspection of the feature descriptions from the `student-por` file allows the removal of features that represent similar data or deemed to have low predictive power.

For instance, features `freetime` and `goout` are deemed to represent similar data, thus feature `goout` is removed preliminarily.

A description of each feature that we will be using in our dataset is presented below in a table format:

name	datatype	units	description
school	binary	'GP' - Gabriel Pereira or 'MS' - Mousinho da Silveira	Student's school
sex	binary	binary: 'F' - female or 'M' - male	Student's gender
address	binary	'U' - urban or 'R' - rural	Student's address (urban/rural)
Medu	numeric	0 - none, 1 - primary education (4th grade), 2 - 5th to 9th grade, 3 - secondary education or 4 - higher education	Highest education achieved by student's mother
Fedu	numeric	0 - none, 1 - primary education (4th grade), 2 - 5th to 9th grade, 3 - secondary education or 4 - higher education	Highest education achieved by student's father
traveltime	numeric	1 - <15 min., 2 - 15 to 30 min., 3 - 30 min. to 1 hour, or 4 - >1 hour	Time spent travelling to and from school
studytime	numeric	1 - <2 hours, 2 - 2 to 5 hours, 3 - 5 to 10 hours, or 4 - >10 hours	Time spent studying over a week
failures	numeric	n if 1<=n<3, else 4	Student's total number of past class failures
schoolsup	binary	yes/no	Extra educational support from the school
famsup	binary	yes/no	Extra educational support from the family
paid	binary	yes/no	Extra tutoring classes for Math/Portuguese
higher	binary	yes/no	Student's intention of higher education post -high school
internet	binary	yes/no	Student's access to internet from home
famrel	numeric	1-5	Quality of Student's relationships with his/her family
freetime	numeric	1-5	Amount of free time student has after school
health	numeric	1-5	Current health status
absences	numeric	0-93	Total number of school absences
G3	numeric	0-20	Final grade

Data Preparation

Preliminaries

In [4]:
```
# Importing modules
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
import statsmodels.formula.api as smf
import scipy.stats as stats
import patsy
%matplotlib inline
%config InlineBackend.figure_format = 'retina'
plt.style.use("ggplot")
```

Data Cleaning and Transformation

In [5]:
```
print(f"Shape of the dataset is {df.shape} \n")
print(f"Data types are below where 'object' indicates a string typ
e: ")
print(df.dtypes)
```

```
Shape of the dataset is (649, 33)

Data types are below where 'object' indicates a string type:
school        object
sex           object
age            int64
address       object
famsize       object
Pstatus       object
Medu           int64
Fedu           int64
Mjob          object
Fjob          object
reason        object
guardian      object
traveltime     int64
studytime      int64
failures       int64
schoolsup     object
famsup        object
paid          object
activities    object
nursery       object
higher        object
internet      object
romantic      object
famrel         int64
freetime       int64
goout          int64
Dalc           int64
Walc           int64
health         int64
absences       int64
G1             int64
G2             int64
G3             int64
dtype: object
```

Since our response variable is the final grade (G3), we do not need data on the first two periods so we will remove them:

In [6]:
```
df = df.drop(['G1', 'G2'], axis = 1)
```

- The student's age is irrelevant information as the range is too small for there to be any significant impact to the student's final age
- The parent's cohabitation status, parent's job, and the reason for selecting the school the student attends, provides little to no value to our dataset, hence we will remove them.
- The guardian of the student would only be relevant if the parents were apart, and considering that not all student's parents are apart, we will also remove this.
- The columns: freetime, studytime, traveltime, goout, and activities all refer to how time was spent outside of school hours. Since we dont need all the specific details of time spent outside of school hours, only keeping freetime, traveltime and studytime would give us all the information we need.
- Weather a student attended nursery school or not would be relevant to their current grades, especially if they're receiving school support, family support and/or paid extra classes.

In [7]:
```
df = df.drop(['age', 'Pstatus', 'Mjob', 'Fjob', 'reason', 'guardian', 'goout', 'activities', 'nursery'], axis = 1)
```

Workday alcohol consumption (Dalc) and weekend alcohol consumption (Walc) can be added together to create weekly alcohol consumption (Wkalc), a rating out of 10 (from 0-very low to 8-very high):

In [8]:
```
df['Wkalc'] = df['Walc'] + df['Dalc'] - 2
df = df.drop(columns=['Walc', 'Dalc'])
df['Wkalc'].describe()
```

Out[8]:
```
count    649.000000
mean       1.782743
std        1.992411
min        0.000000
25%        0.000000
50%        1.000000
75%        3.000000
max        8.000000
Name: Wkalc, dtype: float64
```

Fixing numerical variables that don't begin with 0:

Our model would make a lot more sense if our numerical variables (eg: famrel is a rating between 1 and 5) began with 0 instead of 1. If variables began with 0, our equation for our data model would be a lot more simplified and our intersept can then be explained as the expected age without the influence of the other variables.

Such variables are:

- traveltime
- studytime
- famrel
- freetime
- health

In [9]:
```
df['traveltime'] = df['traveltime'] - 1
df['studytime'] = df['studytime'] - 1
df['famrel'] = df['famrel'] - 1
df['freetime'] = df['freetime'] - 1
df['health'] = df['health'] - 1
```

Discretising very large ranged numerical variables:

We will check the variable `absences` using the `value_counts` method in Pandas.

In [10]:
```
df['absences'].value_counts().sort_index()
```

Out[10]:
```
0     244
1      12
2     110
3       7
4      93
5      12
6      49
7       3
8      42
9       7
10     21
11      5
12     12
13      1
14      8
15      2
16     10
18      3
21      2
22      2
24      1
26      1
30      1
32      1
Name: absences, dtype: int64
```

Let's save a copy of `df['absences']` to `absences` just in case we need to visualise the unmodified data at another time.

In [11]:
```
absences = df['absences'].copy()
```

Since the range is such a large value in the variable `absences` , we would need to discretise the data into bins for it to have more of a significant impact on the model.

In [12]:
```
df['absences'] = pd.cut(df['absences'], bins = 5, labels=['very lo
w', 'low', 'medium', 'high', 'very high'])
```

Let's see how our values look now using the `value_counts` method in Pandas.

In [13]:
```
df['absences'].value_counts()
```

Out[13]:
```
very low     527
low           90
medium        24
high           5
very high      3
Name: absences, dtype: int64
```

It seems perfect. Let's now perform integer encoding such that *very low* is 0, *low* is 1, *medium* is 2, *high* is 3 and *very high* is 4.

In [14]:
```
level_mapping = {'very low': 0, 'low': 1, 'medium': 2, 'high': 3,
'very high': 4}
df['absences'] = df['absences'].replace(level_mapping)

df.sample(5)
```

Out[14]:

	school	sex	address	famsize	Medu	Fedu	traveltime	studytime	failures	schoolsu
584	MS	F	R	GT3	0	0	1	0	0	n
128	GP	M	R	GT3	4	4	0	0	0	n
620	MS	F	U	LE3	4	4	0	1	0	n
12	GP	M	U	LE3	4	4	0	0	0	n
303	GP	F	U	GT3	3	3	0	2	0	n

Checking for Missing Values

In [15]:
```
print(f"\nNumber of missing values for each feature:")
print(df.isnull().sum())
```

```
Number of missing values for each feature:
school        0
sex           0
address       0
famsize       0
Medu          0
Fedu          0
traveltime    0
studytime     0
failures      0
schoolsup     0
famsup        0
paid          0
higher        0
internet      0
romantic      0
famrel        0
freetime      0
health        0
absences      0
G3            0
Wkalc         0
dtype: int64
```

No missing attributes for any of the features so no need to remove any rows.

In [16]:
```
print(f'Now the number of columns are {df.shape[1]}. The dataset c
urrently looks like:')
df.head()
```

```
Now the number of columns are 21. The dataset currently looks lik
e:
```

Out[16]:

	school	sex	address	famsize	Medu	Fedu	traveltime	studytime	failures	schoolsup
0	GP	F	U	GT3	4	4	1	1	0	yes
1	GP	F	U	GT3	1	1	0	1	0	no
2	GP	F	U	LE3	1	1	0	1	0	yes
3	GP	F	U	GT3	4	2	0	2	0	no
4	GP	F	U	GT3	3	3	0	1	0	no

Summary Statistics

In [17]:
```
from IPython.display import display, HTML
display(HTML('<b>Table 1: Summary of continuous features</b>'))
df.describe(include='int64')
```

Table 1: Summary of continuous features

Out[17]:

	Medu	Fedu	traveltime	studytime	failures	famrel	freetime
count	649.000000	649.000000	649.000000	649.000000	649.000000	649.000000	649.000000
mean	2.514638	2.306626	0.568567	0.930663	0.221880	2.930663	2.180277
std	1.134552	1.099931	0.748660	0.829510	0.593235	0.955717	1.051093
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	2.000000	1.000000	0.000000	0.000000	0.000000	3.000000	2.000000
50%	2.000000	2.000000	0.000000	1.000000	0.000000	3.000000	2.000000
75%	4.000000	3.000000	1.000000	1.000000	0.000000	4.000000	3.000000
max	4.000000	4.000000	3.000000	3.000000	3.000000	4.000000	4.000000

In [18]:
```
display(HTML('<b>Table 2: Summary of categorical features</b>'))
df.describe(include='object')
```

Table 2: Summary of categorical features

Out[18]:

	school	sex	address	famsize	schoolsup	famsup	paid	higher	internet	romanti
count	649	649	649	649	649	649	649	649	649	64
unique	2	2	2	2	2	2	2	2	2	
top	GP	F	U	GT3	no	yes	no	yes	yes	n
freq	423	383	452	457	581	398	610	580	498	41

Data Exploration

Numerical features

Searching for Outliers

We will check if any of the numerical features have any outliers based on `Table 1: Summary of continuous features` .

- Medu is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- Fedu is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- traveltime is expected to contain values between `0 to 3`. Based on Table 1, the minimum is `0` and the maximum is `3` and, hence, contains `no outliers`.
- studytime is expected to contain values between `0 to 3`. Based on Table 1, the minimum is `0` and the maximum is `3` and, hence, contains `no outliers`.
- failures is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `3` and, hence, contains `no outliers`.
- famrel is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- freetime is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- health is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- absences is expected to contain values between `0 to 4`. Based on Table 1, the minimum is `0` and the maximum is `4` and, hence, contains `no outliers`.
- G3 is expected to contain values between `0 to 20`. Based on Table 1, the minimum is `0` and the maximum is `19` and, hence, contains `no outliers`.
- Wkalc is expected to contain values between `0 to 8`. Based on Table 1, the minimum is `0` and the maximum is `8` and, hence, contains `no outliers`.

Catagorical Features

In [19]:
```
categoricalColumns = df.columns[df.dtypes==object].tolist()

for col in categoricalColumns:
    print('Unique values for ' + col)
    print(df[col].unique())
    print('')
```

```
Unique values for school
['GP' 'MS']

Unique values for sex
['F' 'M']

Unique values for address
['U' 'R']

Unique values for famsize
['GT3' 'LE3']

Unique values for schoolsup
['yes' 'no']

Unique values for famsup
['no' 'yes']

Unique values for paid
['no' 'yes']

Unique values for higher
['yes' 'no']

Unique values for internet
['no' 'yes']

Unique values for romantic
['no' 'yes']
```

Each catagorical feature contains only two unique values each, such as 'yes' or 'no'.

It seems like no accidental symbol, such as a full stop (.) is in any of the rows for any of the catagorical columns, so we dont need to use:

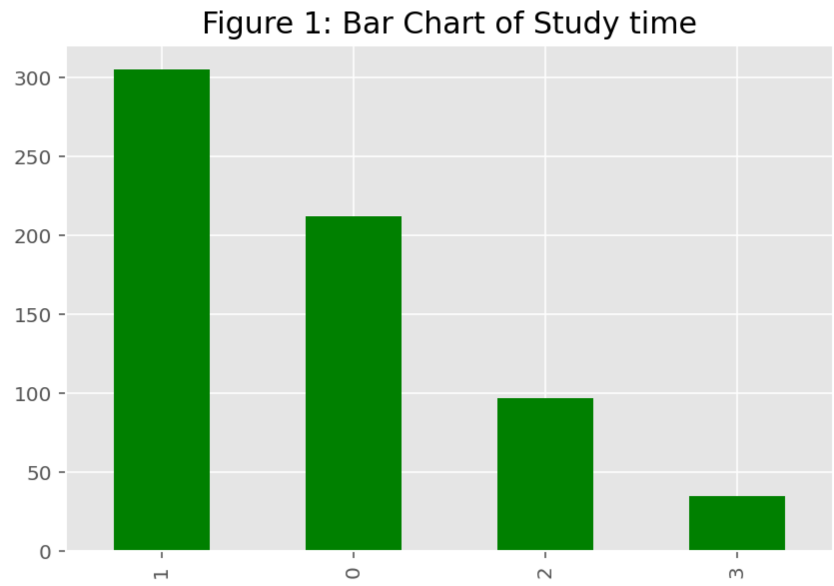
```
df['column_name'].str.rstrip(".")
```

We can now consider our dataset 'clean' & ready for visualisation & data modelling.

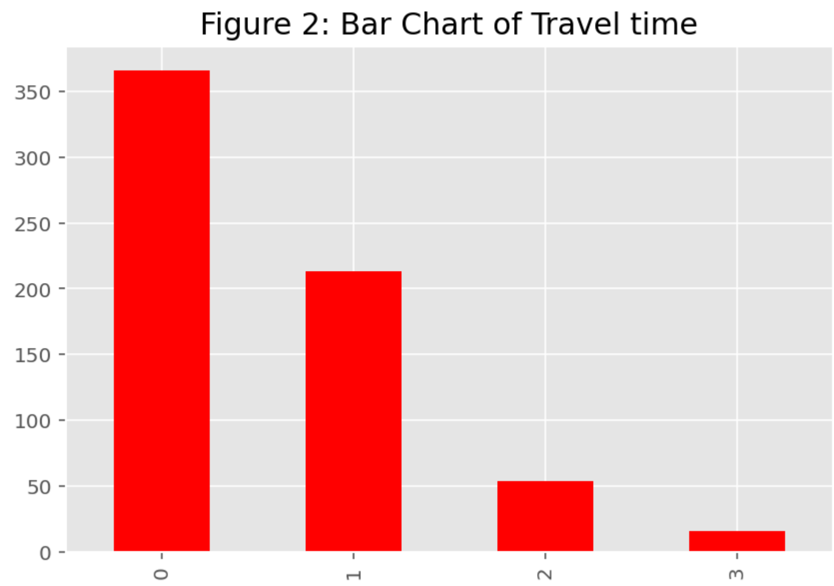
Univariate Visualisation

Lets get a histogram of Study time & Travel times.

In [20]:
```
ax = df['studytime'].value_counts().plot(kind = 'bar', color = 'green')
ax.set_xticklabels(ax.get_xticklabels(), rotation = 90)
plt.tight_layout()
plt.title('Figure 1: Bar Chart of Study time', fontsize = 15)
plt.show();
```

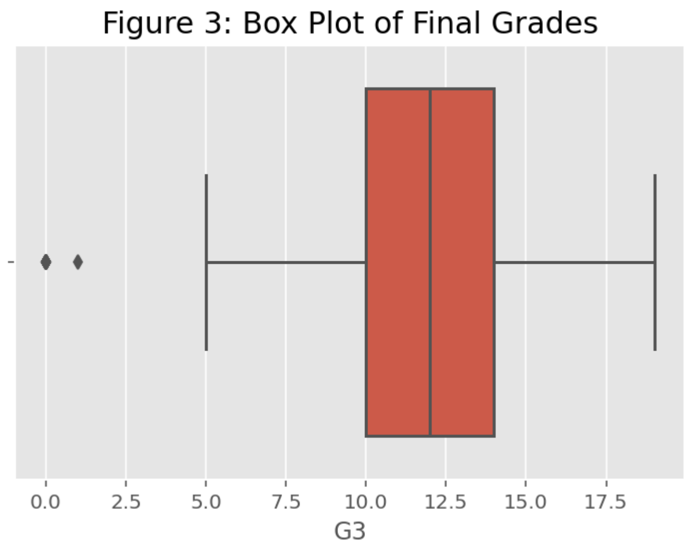


In [21]:
```
ax = df['traveltime'].value_counts().plot(kind = 'bar', color = 'red')
ax.set_xticklabels(ax.get_xticklabels(), rotation = 90)
plt.tight_layout()
plt.title('Figure 2: Bar Chart of Travel time', fontsize = 15)
plt.show();
```

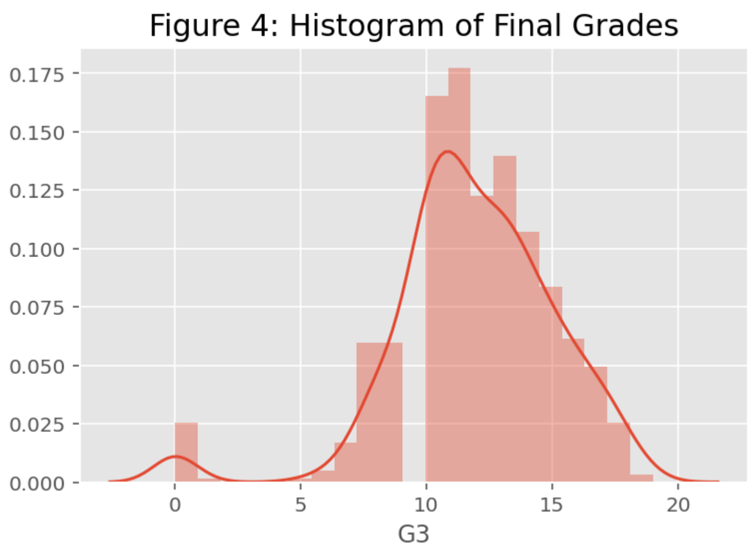


Let's display a boxplot and histogram for Final Grades. Figure 3 shows that this variable is left-skewed.

In [22]:
```
# get a box plot of final grades
sns.boxplot(df['G3']).set_title('Figure 3: Box Plot of Final Grades', fontsize = 15)
plt.show();
```



In [23]:
```
# get a histogram of age with kernel density estimate
sns.distplot(df['G3'], kde = True).set_title('Figure 4: Histogram of Final Grades', fontsize = 15)
plt.show();
```

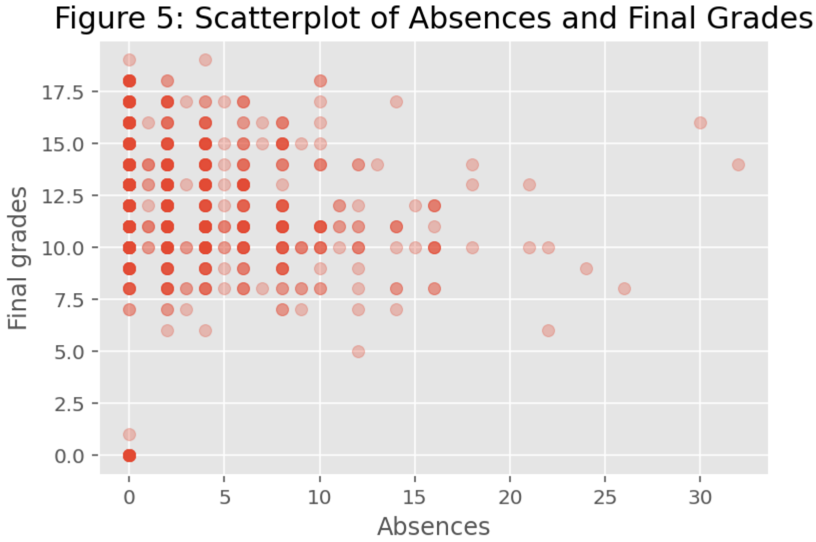


Multivariate Visualisation

Scatterplot of Numerical Features & Final Grades

We will make a scatterplot between absences and final grades using the `absences` from the copy of the unmodified dataframe.

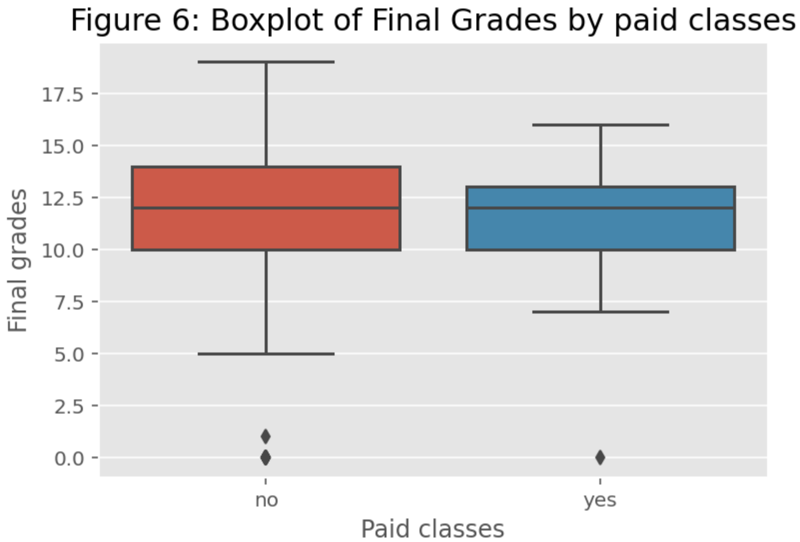
In [24]:
```
plt.scatter(absences, df['G3'], alpha = 0.3)
plt.title('Figure 5: Scatterplot of Absences and Final Grades', fo
ntsize = 15)
plt.xlabel('Absences')
plt.ylabel('Final grades')
plt.show();
```



The scatterplot in Figure 5 shows slightest negative correlation between the absences and final grades numeric variables.

Catagorical attributes by Final Grades

In [25]:
```
# Creating a boxplot
sns.boxplot(df['paid'], df['G3']);
plt.title('Figure 6: Boxplot of Final Grades by paid classes', fon
tsize = 15)
plt.xlabel('Paid classes')
plt.ylabel('Final grades')
plt.show();
```



The distribution of students taking paid classes and those that aren't does not differ significantly, but have a similiar median as seen in `Figure 6`. The whiskers suggest that students with a final grade of over 16 are more likely to not be involved in any sort of paid classes.

Statistical Modeling & Performance Evaluation

Full Model

We begin by fitting a multiple linear regression that predicts final grades using all of the avaliable features. We call this the full model. First let's take a quick peak at the clean data.

In [26]:
```
df.sample(10)
```

Out[26]:

	school	sex	address	famsize	Medu	Fedu	traveltime	studytime	failures	schoolsu
557	MS	F	R	GT3	3	1	1	1	3	n
417	GP	F	U	GT3	3	2	0	2	0	n
318	GP	F	R	GT3	3	2	1	1	0	n
552	MS	M	U	GT3	1	1	0	1	2	n
276	GP	M	U	GT3	2	1	2	0	0	n
310	GP	F	R	GT3	2	1	1	1	0	n
570	MS	M	R	GT3	2	2	1	0	0	n
371	GP	F	U	GT3	2	2	0	0	0	n
553	MS	M	U	LE3	1	0	0	0	0	n
378	GP	M	U	GT3	3	3	0	0	0	n

When constructing the regression formula, we can manually add all the independent features.

In [27]:
```
dependant_var = 'G3'
independant_var = ' + '.join(df.drop(columns=['G3']).columns)
formula_string = dependant_var + ' ~ ' + independant_var
print('formula_string: ', formula_string)
```

```
formula_string:  G3 ~ school + sex + address + famsize + Medu + Fe
du + traveltime + studytime + failures + schoolsup + famsup + paid
+ higher + internet + romantic + famrel + freetime + health + abse
nces + Wkalc
```

The formula string above works just fine with the Statsmodels module. The problem, however, is that we cannot do automatic variable selection with this formula. What we need for this purpose is "one-hot-encoding" of categorical features.

In the code chunk below, we first use the get_dummies() function in Pandas for one-hot-encoding of categorical features and then we construct a new formula string with the encoded features.

In [28]:
```
data_encoded = pd.get_dummies(df, drop_first=True)
data_encoded.head()
```

Out[28]:

	Medu	Fedu	traveltime	studytime	failures	famrel	freetime	health	absences	G3	Wk
0	4	4	1	1	0	3	2	2	0	11	
1	1	1	0	1	0	4	2	2	0	11	
2	1	1	0	1	0	3	2	2	0	12	
3	4	2	0	2	0	2	1	4	0	14	
4	3	3	0	1	0	3	2	4	0	13	

In [29]:
```
formula_string_indep_vars_encoded = ' + '.join(data_encoded.drop(c
olumns='G3').columns)
formula_string_encoded = 'G3 ~ ' + formula_string_indep_vars_encod
ed
print('formula_string_encoded: ', formula_string_encoded)
```

```
formula_string_encoded:  G3 ~ Medu + Fedu + traveltime + studytime
+ failures + famrel + freetime + health + absences + Wkalc + schoo
l_MS + sex_M + address_U + famsize_LE3 + schoolsup_yes + famsup_ye
s + paid_yes + higher_yes + internet_yes + romantic_yes
```

Now that we have defined our statistical model formula as a Python string, we fit an OLS (ordinary least squares) model to our encoded data.

In [30]:
```
model = sm.formula.ols(formula = formula_string_encoded, data = da
ta_encoded)
model_fitted = model.fit()
print(model_fitted.summary())
```

```
                            OLS Regression Results
==================================================================
============
Dep. Variable:                     G3   R-squared:
0.339
Model:                            OLS   Adj. R-squared:
0.318
Method:                 Least Squares   F-statistic:
16.10
Date:                Sun, 01 Nov 2020   Prob (F-statistic):
2.13e-44
Time:                        22:39:47   Log-Likelihood:
-1547.2
No. Observations:                 649   AIC:
3136.
Df Residuals:                     628   BIC:
3230.
Df Model:                          20
Covariance Type:            nonrobust
==================================================================
===============
                    coef    std err          t      P>|t|      [0.
025      0.975]
------------------------------------------------------------------
---------------
Intercept        10.6367      0.697     15.265      0.000       9.
268      12.005
Medu              0.1405      0.128      1.094      0.274      -0.
112       0.393
Fedu              0.2053      0.127      1.614      0.107      -0.
044       0.455
traveltime        0.0649      0.156      0.416      0.677      -0.
241       0.371
studytime         0.4255      0.136      3.135      0.002       0.
159       0.692
failures         -1.2913      0.193     -6.694      0.000      -1.
670      -0.912
famrel            0.1176      0.114      1.035      0.301      -0.
106       0.341
freetime         -0.1304      0.104     -1.255      0.210      -0.
334       0.074
health           -0.1704      0.075     -2.283      0.023      -0.
317      -0.024
absences         -0.3173      0.185     -1.717      0.087      -0.
680       0.046
Wkalc            -0.1549      0.059     -2.630      0.009      -0.
271      -0.039
school_MS        -1.3571      0.255     -5.323      0.000      -1.
858      -0.857
sex_M            -0.5728      0.243     -2.355      0.019      -1.
051      -0.095
address_U         0.2845      0.256      1.110      0.267      -0.
219       0.788
famsize_LE3       0.2858      0.233      1.225      0.221      -0.
172       0.744
schoolsup_yes    -1.4168      0.353     -4.010      0.000      -2.
111      -0.723
famsup_yes       -0.0118      0.224     -0.053      0.958      -0.
451       0.427
paid_yes         -0.5291      0.453     -1.167      0.244      -1.
```

```
420       0.361
higher_yes       1.6709      0.373      4.474      0.000       0.
937       2.404
internet_yes     0.3667      0.268      1.369      0.171      -0.
159       0.893
romantic_yes    -0.3742      0.223     -1.676      0.094      -0.
813       0.064
==================================================================
============
Omnibus:                     108.875   Durbin-Watson:
1.882
Prob(Omnibus):                 0.000   Jarque-Bera (JB):
328.833
Skew:                         -0.806   Prob(JB):
3.93e-72
Kurtosis:                      6.092   Cond. No.
45.4
==================================================================
============

Warnings:
[1] Standard Errors assume that the covariance matrix of the error
s is correctly specified.
```

The equation of the regression model that includes all of the variables rounded to two decimal places is:
10.64 + (0.14 *Medu) + (0.21* Fedu) + (0.06 *traveltime) + (0.43* studytime) + (-1.29 *failures) + (0.12* famrel) + (-0.13 *freetime) + (-0.17* health) + (-0.32 *absences) + (-0.15* Walc) + (-1.36 *School_MS) + (-0.57* sex_M) + (0.28 *address_U) + (0.29* famsize_LE3) + (-1.42 *schoolsup_yes) + (-0.01* famsup_yes) + (-0.53 *paid_yes) + (1.67* higher_yes) + (0.37 *internet_yes) + (-0.37* romantic_yes)

The intercept in this case, in simple terms, refers to the expected final grade if everything else was 0, such as all of the nominal catagorical features are all 'no' (eg: famsup_yes = 0).
Overall, a student is expected to receive 10.6367 as their final grade if their mother and father has no education, home to school travel time is less than 15 minutes, study time is less than 2 hours, they have 0 previous failures, their family relationship is very bad, freetime afterschool is very low, their current health status is very bad, their absences from school is very low, weekly alcohol consumption is very low, they attend Gabriel Pereira school, are females, live in rural areas, have a family size greater than 3, receive no school support, no family support, no extra paid classes, does not want to go into higher education, does not have internet and is not in a romantic relationship.

In [31]:
```
residuals_full = pd.DataFrame({'actual': df['G3'],
                               'predicted': model_fitted.fittedvalue
s,
                               'residual': model_fitted.resid})
residuals_full.head(10)
```

Out[31]:

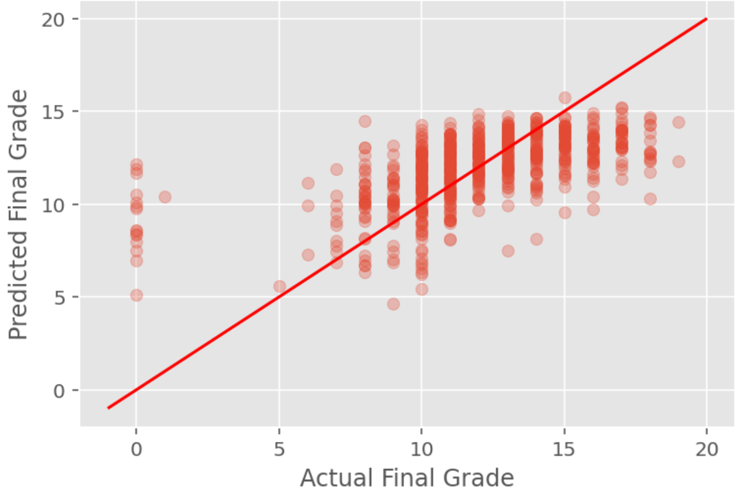
	actual	predicted	residual
0	11	12.800059	-1.800059
1	11	13.587117	-2.587117
2	12	11.885661	0.114339
3	14	13.819668	0.180332
4	13	13.298714	-0.298714
5	13	13.506113	-0.506113
6	13	13.409647	-0.409647
7	13	13.389911	-0.389911
8	17	14.140013	2.859987
9	13	13.309535	-0.309535

Let's plot actual final grade values vs. predicted values.

In [32]:
```
def plot_line(axis, slope, intercept, **kargs):
    xmin, xmax = axis.get_xlim()
    plt.plot([xmin, xmax], [xmin*slope+intercept, xmax*slope+inter
cept], **kargs)

plt.scatter(residuals_full['actual'], residuals_full['predicted'],
alpha=0.3);
plot_line(axis=plt.gca(), slope=1, intercept=0, c="red");
plt.xlabel('Actual Final Grade');
plt.ylabel('Predicted Final Grade');
plt.title('Figure 7: Scatter plot of actual vs. predicted Final Gr
ade for the Full Model', fontsize=15);
plt.show();
```

Figure 7: Scatter plot of actual vs. predicted Final Grade for the Full Model



From `Figure 7` , we observe that the model never produces a prediction above 16 even though the highest final grade in the dataset is 19.

We will now check the diagnostics for the full model.

Full Model Diagnostic Checks

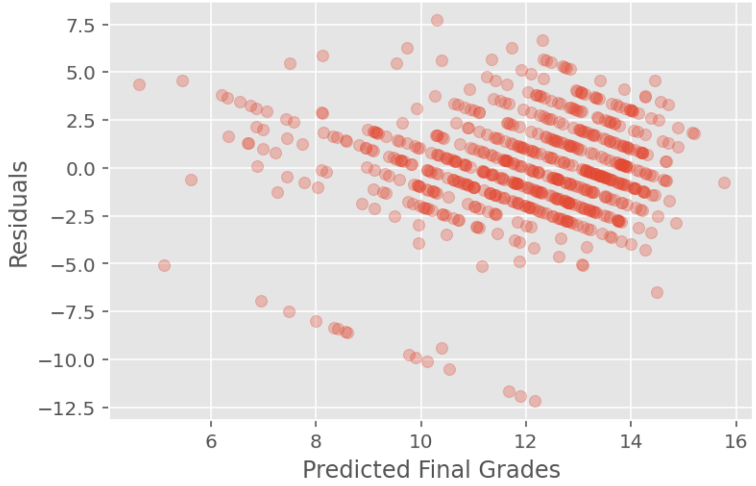
We would like to check whether there are indications of violations of the regression assumptions, which are

1. linearity of the relationship between target variable and the independent variables
2. constant variance of the errors
3. normality of the residual distribution
4. statistical independence of the residuals

Let's first get a scatter plot of residuals (as a function of predicted final grades).

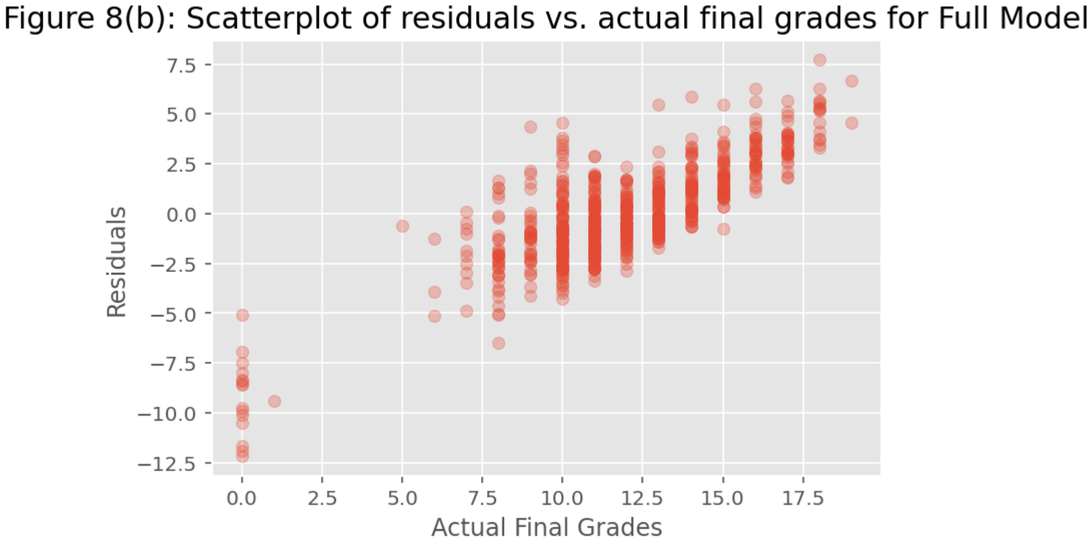
In [33]:
```
plt.scatter(residuals_full['predicted'], residuals_full['residual'], alpha=0.3);
plt.xlabel('Predicted Final Grades');
plt.ylabel('Residuals')
plt.title('Figure 8(a): Scatterplot of residuals vs. predicted final grades for Full Model', fontsize=15)
plt.show();
```

Figure 8(a): Scatterplot of residuals vs. predicted final grades for Full Model



Let's now plot actual age vs. residuals.

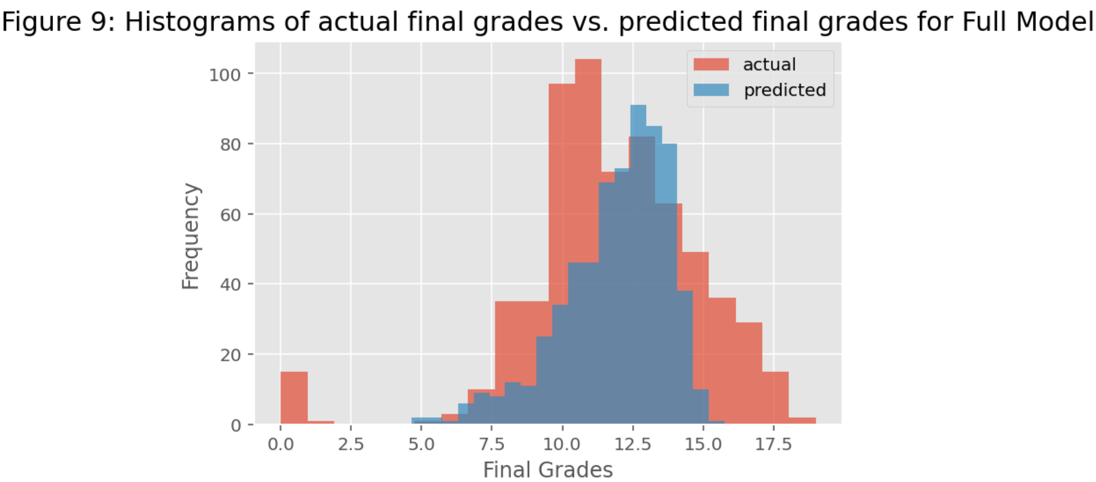
In [34]:
```
plt.scatter(residuals_full['actual'], residuals_full['residual'],
alpha=0.3);
plt.xlabel('Actual Final Grades');
plt.ylabel('Residuals')
plt.title('Figure 8(b): Scatterplot of residuals vs. actual final
grades for Full Model', fontsize=15)
plt.show();
```



We notice that the model overestimates lower final grades. In particular, for those with a final grade less than 5, the model predicts much higher final grades.

Let's overlay the histograms of actual vs. predicted final grades on the same plot.

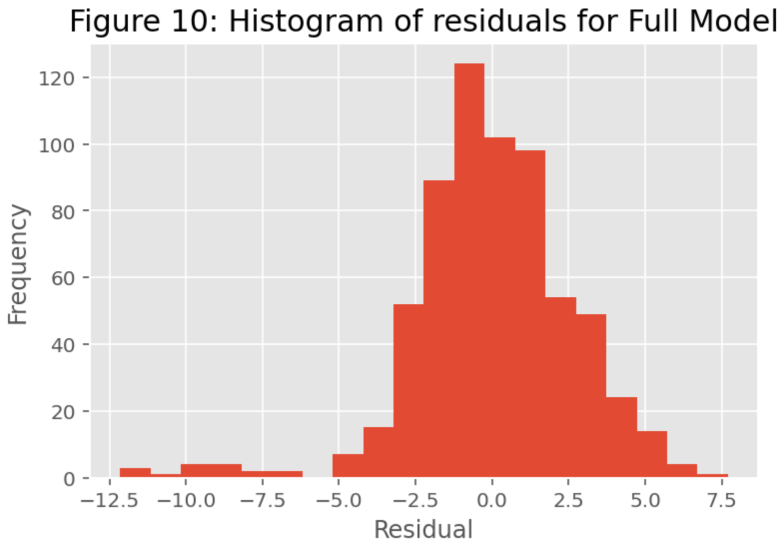
In [35]:
```
plt.hist(residuals_full['actual'], label='actual', bins=20, alpha=
0.7);
plt.hist(residuals_full['predicted'], label='predicted', bins=20,
alpha=0.7);
plt.xlabel('Final Grades');
plt.ylabel('Frequency');
plt.title('Figure 9: Histograms of actual final grades vs. predict
ed final grades for Full Model', fontsize=15);
plt.legend()
plt.show();
```



We notice that their distributions are quite different. In particular, the model's predictions are highly clustered around mid-13's.

Let's now have look at the histogram of the residuals.

In [36]:
```
plt.hist(residuals_full['residual'], bins = 20);
plt.xlabel('Residual');
plt.ylabel('Frequency');
plt.title('Figure 10: Histogram of residuals for Full Model', font
size=15);
plt.show();
```



From Figure 10, the histogram of residuals looks somewhat symmetric, though slightly left-skewed. Nonetheless, it seems the normality assumption of linear regression is not significantly violated in this particular case.

Backwards Feature Selection

We now perform backwards feature selection using p-values. It appears Statsmodels does not have any canned code for automatic feature selection, so we wrote one ourselves.

In [37]:
```
patsy_description = patsy.ModelDesc.from_formula(formula_string_en
coded)
linreg_fit = model_fitted
p_val_cutoff = 0.05

print('\nPerforming backwards feature selection using p-values:')

while True:
    pval_series = linreg_fit.pvalues.drop(labels='Intercept')
    pval_series = pval_series.sort_values(ascending=False)
    term = pval_series.index[0]
    pval = pval_series[0]
    if (pval < p_val_cutoff):
        break
    term_components = term.split(':')
    print(f'\nRemoving term "{term}" with p-value {pval:.4}')
    if (len(term_components) == 1):
        patsy_description.rhs_termlist.remove(patsy.Term([patsy.Ev
alFactor(term_components[0])]))
    else:
        patsy_description.rhs_termlist.remove(patsy.Term([patsy.Ev
alFactor(term_components[0]),
                                                            patsy.Eval
Factor(term_components[1])]))

    linreg_fit = smf.ols(formula=patsy_description, data=data_enco
ded).fit()


model_reduced_fitted = smf.ols(formula = patsy_description, data =
data_encoded).fit()

print("\n***")
print(model_reduced_fitted.summary())
print("***")
print(f"Regression number of terms: {len(model_reduced_fitted.mode
l.exog_names)}")
print(f"Regression F-distribution p-value: {model_reduced_fitted.f
_pvalue:.4f}")
print(f"Regression R-squared: {model_reduced_fitted.rsquared:.4
f}")
print(f"Regression Adjusted R-squared: {model_reduced_fitted.rsqua
red_adj:.4f}")
```

```
Performing backwards feature selection using p-values:

Removing term "famsup_yes" with p-value 0.958

Removing term "traveltime" with p-value 0.6775

Removing term "famrel" with p-value 0.2995

Removing term "address_U" with p-value 0.3248

Removing term "Medu" with p-value 0.2688

Removing term "paid_yes" with p-value 0.2576

Removing term "freetime" with p-value 0.2651

Removing term "famsize_LE3" with p-value 0.1636

Removing term "internet_yes" with p-value 0.1073

Removing term "absences" with p-value 0.1222

Removing term "romantic_yes" with p-value 0.07934

Removing term "sex_M" with p-value 0.05772

***
                            OLS Regression Results
==================================================================
============
Dep. Variable:                     G3   R-squared:
0.318
Model:                            OLS   Adj. R-squared:
0.310
Method:                 Least Squares   F-statistic:
37.34
Date:                Sun, 01 Nov 2020   Prob (F-statistic):
1.06e-48
Time:                        22:39:48   Log-Likelihood:
-1557.2
No. Observations:                 649   AIC:
3132.
Df Residuals:                     640   BIC:
3173.
Df Model:                           8
Covariance Type:            nonrobust
==================================================================
===============
                    coef    std err          t      P>|t|      [0.
025      0.975]
------------------------------------------------------------------
---------------
Intercept        10.9878      0.481     22.825      0.000      10.
043      11.933
Fedu              0.2755      0.100      2.749      0.006       0.
079       0.472
studytime         0.4806      0.133      3.605      0.000       0.
219       0.742
failures         -1.4381      0.190     -7.574      0.000      -1.
811      -1.065
```

```
health          -0.1912      0.074     -2.597      0.010      -0.
336      -0.047
Wkalc           -0.2162      0.055     -3.953      0.000      -0.
324      -0.109
school_MS       -1.4241      0.231     -6.170      0.000      -1.
877      -0.971
schoolsup_yes   -1.3615      0.349     -3.899      0.000      -2.
047      -0.676
higher_yes       1.8616      0.370      5.028      0.000       1.
135       2.589
===================================================================
============
Omnibus:                      103.221   Durbin-Watson:
1.918
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
301.889
Skew:                          -0.775   Prob(JB):
2.79e-66
Kurtosis:                       5.960   Cond. No.
23.7
===================================================================
============

Warnings:
[1] Standard Errors assume that the covariance matrix of the error
s is correctly specified.
***
Regression number of terms: 9
Regression F-distribution p-value: 0.0000
Regression R-squared: 0.3182
Regression Adjusted R-squared: 0.3097
```

Similar to what we did for the full model, let's define a new data frame for actual final grade vs. predicted final grade and the residuals for the reduced model.

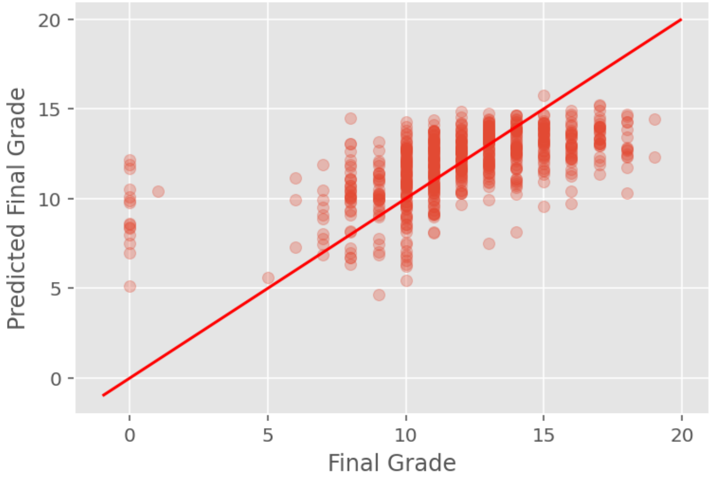
In [38]:
```
residuals_reduced = pd.DataFrame({'actual': data_encoded['G3'],
                            'predicted': model_fitted.fittedvalue
s,
                            'residual': model_fitted.resid})
residuals_reduced.head(10)
```

Out[38]:

	actual	predicted	residual
0	11	12.800059	-1.800059
1	11	13.587117	-2.587117
2	12	11.885661	0.114339
3	14	13.819668	0.180332
4	13	13.298714	-0.298714
5	13	13.506113	-0.506113
6	13	13.409647	-0.409647
7	13	13.389911	-0.389911
8	17	14.140013	2.859987
9	13	13.309535	-0.309535

In [39]:
```
plt.scatter(residuals_reduced['actual'], residuals_reduced['predic
ted'], alpha=0.3);
plot_line(axis=plt.gca(), slope=1, intercept=0, c="red");
plt.xlabel('Final Grade');
plt.ylabel('Predicted Final Grade');
plt.title('Figure 11: Scatter plot of actual vs. predicted Final G
rade for Reduced Model', fontsize=15);
plt.show();
```

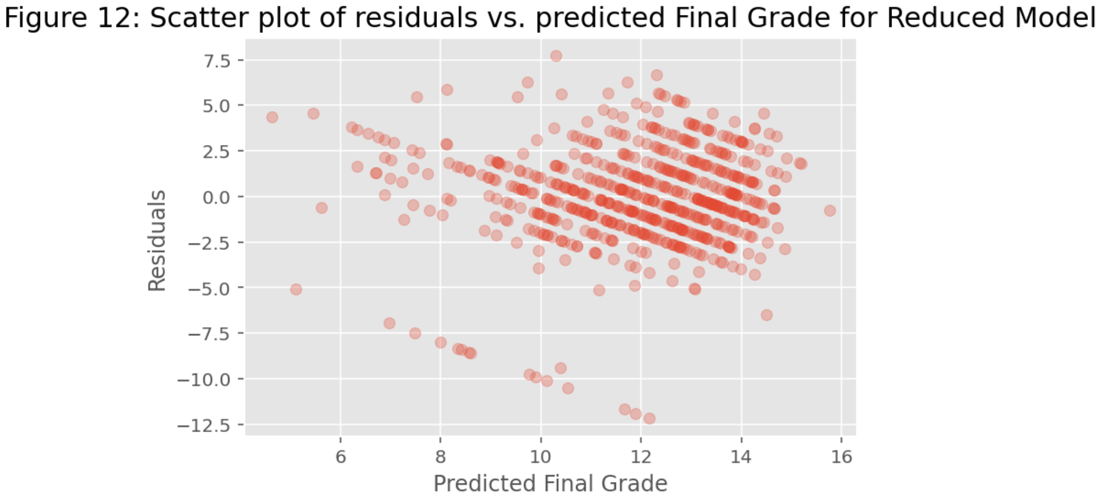
Figure 11: Scatter plot of actual vs. predicted Final Grade for Reduced Model



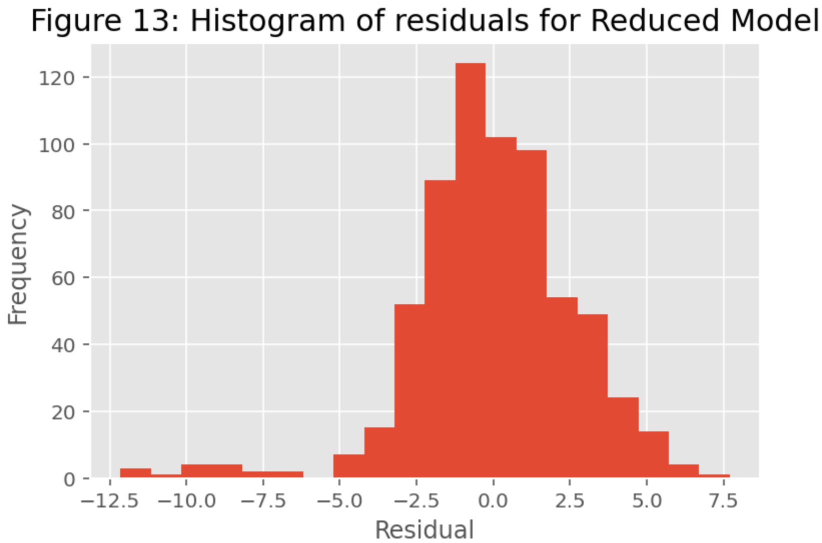
This model returns an Adjusted R-squared of 0.3097, meaning the reduced model still explains about 31% of the variance, but with 6 less variables. Looking at the p-values, they are all significant at the 5% level, as expected. From Figure 11, we still have the same issues with our model. That is, the model overestimates higher grades and underestimates lower grades. We will now perform the diagnostic checks on this reduced model.

Reduced Model

In [40]:
```
plt.scatter(residuals_reduced['predicted'], residuals_reduced['res
idual'], alpha=0.3);
plt.xlabel('Predicted Final Grade');
plt.ylabel('Residuals')
plt.title('Figure 12: Scatter plot of residuals vs. predicted Fina
l Grade for Reduced Model', fontsize=15)
plt.show();
```



In [41]:
```
plt.hist(residuals_reduced['residual'], bins = 20);
plt.xlabel('Residual');
plt.ylabel('Frequency');
plt.title('Figure 13: Histogram of residuals for Reduced Model', f
ontsize = 15)
plt.show();
```



Summary & Conclusions

Using our independent variables, we were able to get a full model with an Adjusted R-squared value of about 31%. After backwards variable selection with a p-value cutoff value of 0.05, we were able to maintain the same performance but with 12 less variables. Our final model has 9 variables all together with a model p-value of 0.

The final multiple linear regression model has an Adjusted R-squared value of about 31%, which is significantly low. So, it appears that the variables we used are not enough for accurately estimating the final grade of students from a dataset of student achievement from two Portuguese schools. Next time we should add some more interaction terms and maybe some other higher order terms to see if this would result in some improvement for the Adjusted R-squared value. Nonetheless, it might be the case that nonlinear models such as a neural network might be more appropriate for the task at hand rather than a linear regression model. Our regression model appears to predict final grade correctly within 10-13 marks in general, though this is clearly a huge margin of error for the model to be useful for any practical purposes.

Furthermore, our model has some quite significant problems. More specifically, our model consistently underestimates lower final grades and overestimates higher final grades. In particular, for those who received a final grade below 8, the model predicts much lower grades. Also, for those who received a final grade above 15, the model predicts significantly higher final grades.

References

- P. Cortez and A. Silva. Using Data Mining to Predict Secondary School Student Performance. In A. Brito and J. Teixeira Eds., Proceedings of 5th FUture BUsiness TEChnology Conference (FUBUTEC 2008) pp. 5-12, Porto, Portugal, April, 2008, EUROSIS, ISBN 978-9077381-39-7. Available at http://archive.ics.uci.edu/ml/datasets/Student+Performance (http://archive.ics.uci.edu/ml/datasets/Student+Performance) [Accessed 2020-20-07]
- Regression Case Study : Predicting Age in Census Data. Available at https://www.featureranking.com/tutorials/statistics-tutorials/regression-case-study/ (https://www.featureranking.com/tutorials/statistics-tutorials/regression-case-study/) [Accessed 2020-20-07]
- Data Preparation for Statistical Modeling and Machine Learning. Available at https://www.featureranking.com/tutorials/machine-learning-tutorials/data-preparation-for-machine-learning/ (https://www.featureranking.com/tutorials/machine-learning-tutorials/data-preparation-for-machine-learning/) [Accessed 2020-20-07]

In [ ]: