2 Assessment exercises week 2

Note: Submitting functions as solutions to exercises requires that requested function names are strictly adhered to in order for the assessment tool to correctly call your functions. Parameter names are irrelevant and up to you but the number and order of parameters to give to the function should be as stipulated in the exercises. Requested names for functions are given as **'function_name()'** in the exercise text, implying that input parameters are still to be determined by the exercise. **Note** that safe-guarding your functions when requested requires you to **return None**, i.e. the 'None' type in Python on failure so that the assessment tool can test your functions for non-physical values where appropriate.

1. Write two functions converting between Celsius scale and Fahrenheit using
$$F = C\,\frac{9}{5} + 32$$
for the first function from Celsius to Fahrenheit and
$$C = \frac{5}{9}\,(F - 32)$$
for the second function converting Fahrenheit to Celsius. Name the first function **'c2f()'** with one input parameter, the Celsius number, and the second function **'f2c()'** with the Fahrenheit number as input. Let each function return the result explicitly instead of printing it to screen.

2. Write a function **'satellite()'** that calculates and returns the altitude in metres above the Earth's surface that a satellite must have for a given, input, orbital period T. Make certain that your function excludes non-sensical orbital periods, for instance those orbits with a zero or negative altitude or orbital period. The formula to program and required constants are:
$$h = \left(\frac{G\,M\,T^2}{4\,\pi^2}\right)^{1/3} - R$$
where $G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, $M = 5.97 \times 10^{24}$ kg and $R = 6371$ km.

3. Write a coordinate conversion function **'p2c()'**: convert polar coordinates to Cartesian coordinates, i.e. expect two polar coordinate values, r and θ (in degrees, not radians), as input and return the corresponding two Cartesian coordinate values
$$x = r\,\cos\theta \quad \text{and} \quad y = r\,\sin\theta\,,$$
in form of a tuple, i.e. one return statement for both values. Safe-guard against non-sensical input values.

4. Consider the quantum mechanical problem of a particle encountering a potential step of height V. The particle with wave number

$$k_1 = \frac{\sqrt{2\,m\,E}}{\hbar}$$

enters from the left and meets the potential step at $x = 0$. If the kinetic energy E of the particle is larger than V, it can either pass the step and continue with a smaller wave number

$$k_2 = \frac{\sqrt{2\,m\,(E - V)}}{\hbar}$$

or be reflected keeping its kinetic energy. The formulae for the probability of transmission (T) or reflection (R) are given as

$$T = \frac{4\,k_1\,k_2}{(k_1 + k_2)^2} \quad ; R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 .$$

Write a function **'trprob()'** that calculates and returns the transmission and reflection probabilities (in that order) for an electron with mass $m = 511 \times 10^3$ eV c^{-2}, meeting a potential step of height $V = 9$ eV depending on the input kinetic energy E in units of eV (take $\hbar = 4.135667$ eV s and $c = 3 \times 10^8$ m s^{-1}). Safe-guard your function against (for this problem) not-permitted kinetic energy values $E < V$.

5. Orbits of one body around another, such as a planet around the Sun, need not be circular. In general, they take the form of an ellipse. If you are given the distance of closest approach, l_1, the perihelion, of the orbiting body and its linear velocity v_1 at that point, then any other property of the orbit can be calculated from these two. Write a function **'orbit()'** that takes these two input values in the order distance, velocity, and calculates and returns the following numbers:

   - Ellipse semi-major axis a,
   - semi-minor axis b,
   - orbital period T and
   - orbital eccentricity e.

   Test your function by confirming that the orbital period of Earth is one year (perihelion distance: 1.471×10^{11} m, $v_1 = 3.0287 \times 10^4$ m/s, $G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} and the Sun's mass is $M = 1.9891 \times 10^{30}$ kg). The speed at the most distant point of the orbit is the smaller root of the quadratic equation

$$v_2^2 - \frac{2\,G\,M}{v_1\,l_1}\,v_2 - \left[v_1^2 - \frac{2\,G\,M}{l_1}\right] = 0 .$$

Get l_2 from $l_2 = l_1 v_1 / v_2$. For the remaining terms the formulae are:

$$a = \frac{1}{2}\,(l_1 + l_2)$$

$$b = \sqrt{l_1\, l_2}$$

$$T = \frac{2\,\pi\,a\,b}{l_1\,v_1}$$

$$e = \frac{l_2 - l_1}{l_2 + l_1}$$