MM204 Programming Assignment Report

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Group no. G22

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1 Problem Statement 5

Given problem is based on 1-D unsteady state or transient conduction since the temperature is varying with time and space across the complete length of the element.
Concepts used to solve the problem are:

1.1 Transient 1-D Conduction

Three principles used here are:
I)Heat or Thermal Energy with uniform properties

$$HeatEnergy = Cm\Delta T \tag{1}$$

C: Specific Heat Capacity
m: Mass
T: Temperature

II)Fourier's law of heat transfer: rate of heat transfer proportional to a negative temperature gradient

$$\frac{HeatTransferRate}{Area} = -k\frac{\partial T}{\partial x} \tag{2}$$

k: Thermal Conductivity
Heat is transferred from High Temperature to Low Temperature

III)Conservation of energy.
Rate of Accumulation = Rate in - Rate Out + Generation
Using Fourier's Law we obtain,

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \tag{3}$$

1.2 Explicit Finite-Difference Equation Method:

The technique used here is the finite volume technique by dividing space discretely (symmetrically) into a finite number of control volumes. A staggered grid (scalar properties at centre and vector properties at boundaries) is created and heat balance is carried out on each control volume. First-order central difference is used to convert differential equations to numerical equations which are further solved to obtain the required temperatures at each node at a specific time t.

2 Given

Properties of Fuel Element of the nuclear reactor

- Shape: Plane wall
- Thickness of wall: 10 mm or 0.01 m
- Thermal conductivity k = 30 W/m.K
- Thermal diffusivity
$$\alpha = 5 * 10^{-6} m^2/s \tag{4}$$
- Heat transfer coefficient h = 1100 W/m2.K
-
$$T = 250^0 C = 523K \tag{5}$$
- Volumetric heat generation rate q = 108 W/m3

3 To find

The temperature profile of the fuel element at time 1.5s

4 Assumptions

1. Constant properties
2. Transient 1-D conduction
3. Uniform rate of heat generation

Since the wall is symmetric about the Mid-plane, we have obtained the solution for Mid-plane to the right edge.

5 Conditions and variables

Initial Conditions:

$$T(t = 0, x) = 250^0 C \tag{6}$$

Boundary Conditions:

Since there is a convective heat transfer, we apply the boundary condition at the surface i.e. at the last node.

Variables:

- Let n = number of nodes (1 and n being the mid-plane and surface node respectively).
  The number of nodes is kept variable.
- Let x be the space increment.
  Hence, x = L/(n-1)
- Let A be the cross-section area of the fuel element.

A numerical solution is obtained using a space increment of x. Since there is symmetry about the midplane, the nodal network yields n unknown nodal temperatures. Using the energy balance method, an explicit finite-difference equation is derived for any interior node m. The temperature at all nodes will be obtained at time intervals of t from t=0 to t=1.5s marching in steps along the time axis.

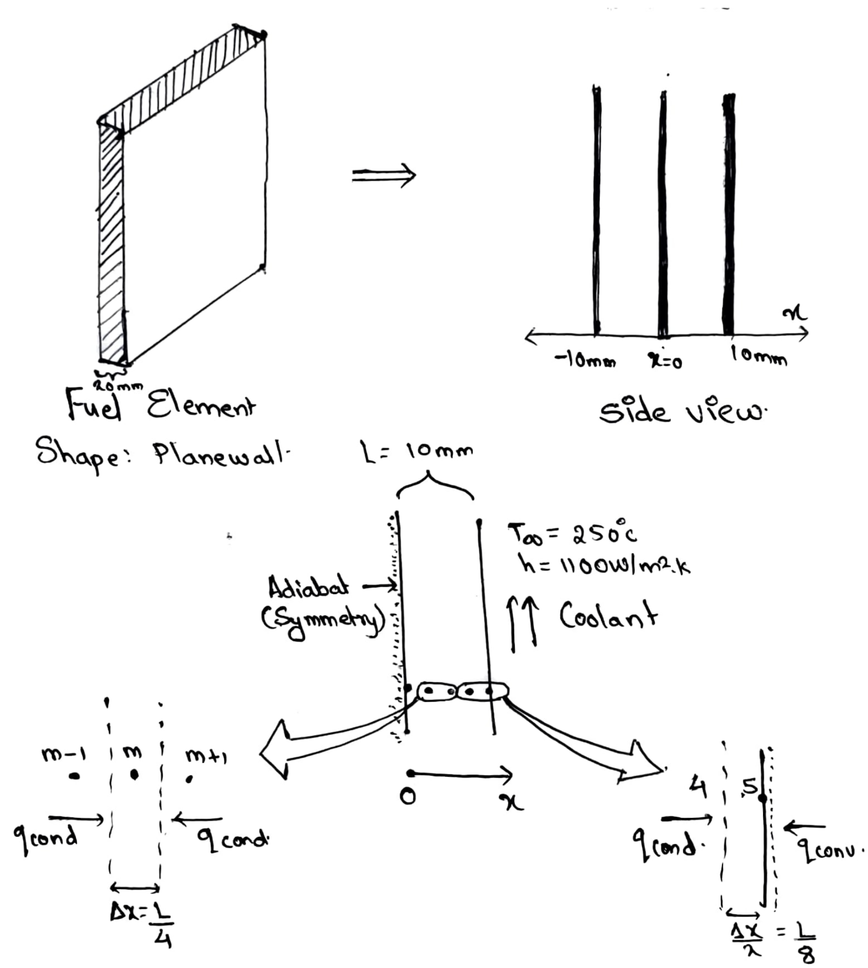

Figure 1: Schematic illustration with 5 nodes

6 Derivation:

Let number of nodes be 5

Control Volumes:

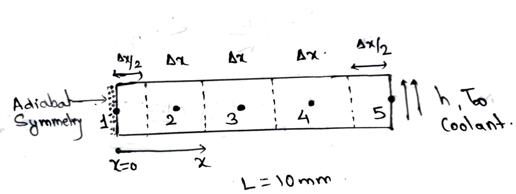

Figure 2: Control volumes

Total: 5
Full control volume: Corresponding to interior nodes (2, 3 and 4)
Half control volume: Corresponding to Mid-plane and surface nodes (1 and 5)

Heat Balance in third control Volume:
Rate of Accumulation = Rate in - Rate Out + Generation rate

$$\rho CA\Delta x\frac{T_3}{\partial t} = Ak(-\frac{\partial T}{\partial x})_{2.5} - Ak(-\frac{\partial T}{\partial x})_{3.5} + q'''A\Delta x \tag{7}$$

$$(\frac{\partial T}{\partial x})_{2.5} = \frac{T_3 - T_2}{\Delta x} \tag{8}$$

$$\rho CA\Delta x\frac{T_3}{\partial t} = -k(\frac{T_3 - T_2}{x}) - (-k)(\frac{T_4 - T_3}{x}) + q'''\Delta x \tag{9}$$

$$\alpha = \frac{k}{\rho C}(Constant) \tag{10}$$

$$\frac{\partial T_3}{\partial t} = \frac{\alpha}{\Delta x^2}(T_2 - 2T_3 + T_4) + q''' \tag{11}$$

$$\frac{\partial T_3}{\partial t} = \frac{(T_3^(t + \Delta t) - T_3^(t)}{\Delta t} - \tag{12}$$

Equation obtained (Explicit):

$$T_3^(t + \Delta t) = \frac{\alpha\Delta t}{\Delta x^2}T_2^t + [1 - 2\frac{\alpha\Delta t}{\Delta x^2}]T_3^t + \frac{\alpha\Delta t}{\Delta x^2}T_4^t + q'''\Delta t \tag{13}$$

Equation can be generalized for n number of nodes where T(m,p+1), the temperature of node m at time t + delta t can be found using the known nodal temperatures at time t.

$$T_m^{t+\Delta t} = \frac{\alpha\Delta t}{\Delta x^2}(T_{m-1}^t + T_{m+1}^t + q'''\frac{\Delta x^2}{k}) + (1 - 2\frac{\alpha\Delta t}{\Delta x^2})T_m^t \tag{14}$$

Eqn.(12) can be used for node 1 (with T(m-1,t)=T(m+1,t) to n-1.
Now, applying energy conservation to last node we obtain following equation:

$$hA(T_\infty - T_n^t) + kA(\frac{T_{n-1}^t - T_n^t}{\Delta x}) + q'''A\frac{\Delta x}{2} = \rho A\frac{\Delta x}{2}C\frac{(T_5^{t+\Delta t} - T_5^t)}{\Delta t} \tag{15}$$

on further simplifying we obtain;

$$T_n^{t+\Delta t} = 2\frac{\alpha\Delta t}{\Delta x^2}[T_{n-1}^t + \frac{h\Delta x}{k}T_\infty + q'''\frac{\Delta x^2}{2k}] + [1 - 2\frac{\alpha\Delta t}{\Delta x^2} - 2\frac{h\Delta x}{k}\frac{\alpha\Delta t}{\Delta x^2}]T_n^t \tag{16}$$

$$\frac{\alpha\Delta t}{\Delta x^2}(1 + \frac{h\Delta x}{k}) \leq \frac{1}{2} \tag{17}$$

we chose a suitable delta-t

Initialize
at t equal to 0,

$$T_m^0 = 250^0C \tag{18}$$

for all nodes.

7 Results

7.1 Table Nodal Temperatures

$$\Delta t = 0.5 seconds \tag{19}$$

T_1	T_2	T_3	T_4	T_5
523.00	523.00	523.00	523.00	523.00
531.34	531.34	531.34	531.44	531.34
539.67	539.67	539.67	539.67	539.06
548.00	548.00	548.00	547.76	546.78

8 Plot

Plot 1

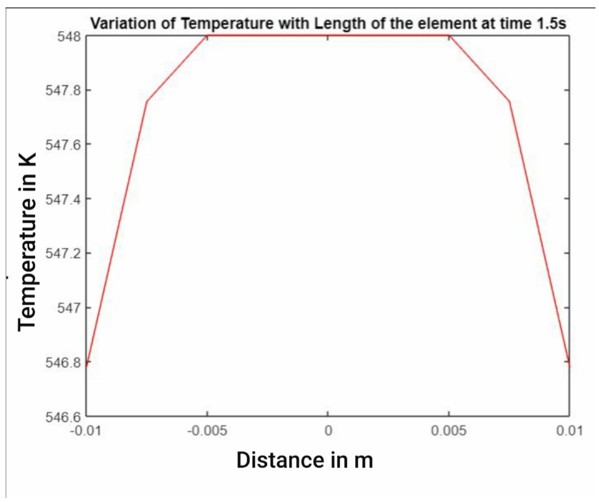

Figure 3: Temp vs x for n = 5

Plot 2

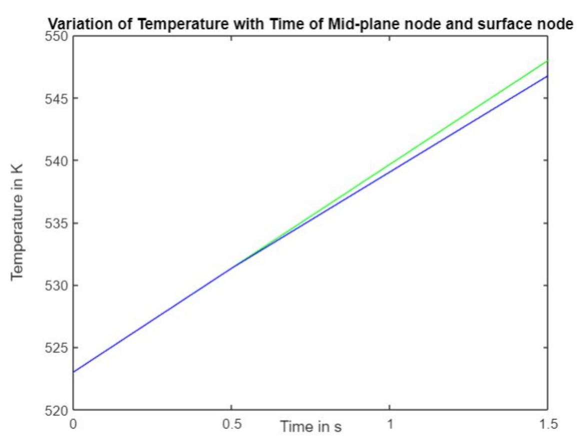

Figure 4: Temp vs t for n=5,Green:Mid Plane nodeBlue:Surface node

Plot 3

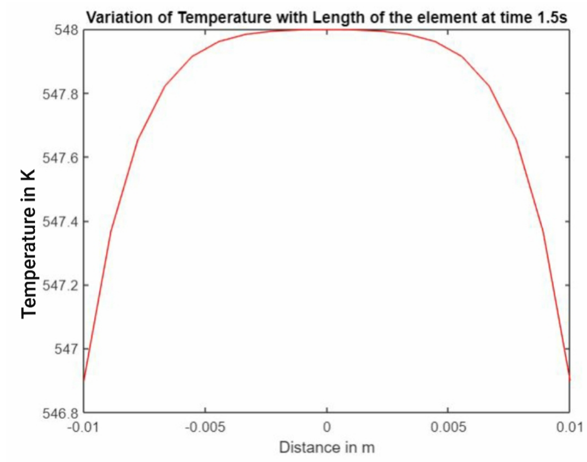

Figure 5: Temp vs x for n = 10

Plot 4

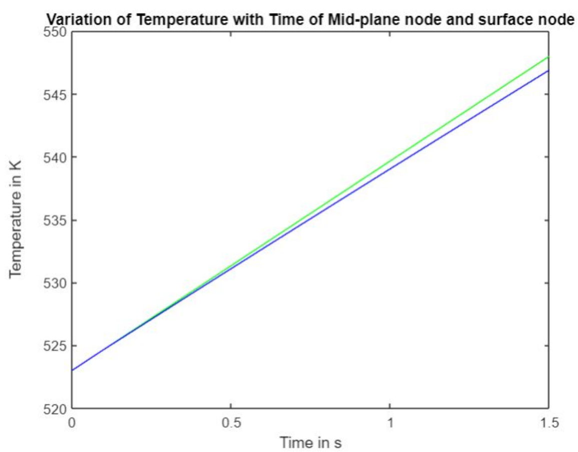

Figure 6: Temp vs t for n = 10Green:Mid Plane nodeBlue:Surface node

9 Remarks

The accuracy of the finite-difference solution may be improved by decreasing the values of x and t.
As the number of nodes increases, x decreases, hence accuracy improves.
Time interval should be reduced until the computed results become constant or independent of further reduction in t.
The choice of x is typically based on a compromise between accuracy and computational requirements. Once this selection has been made, however, the value of t may not be chosen independently. It is, instead, determined by stability requirements.
The time required to reach the new steady state can be determined, with temperature history computed for the Mid-plane node (1st node) and Surface node (nth node).
When t tends to infinity steady state is achieved.

Figure 7 illustrates the steady state temperature profile along the thickness of wall.

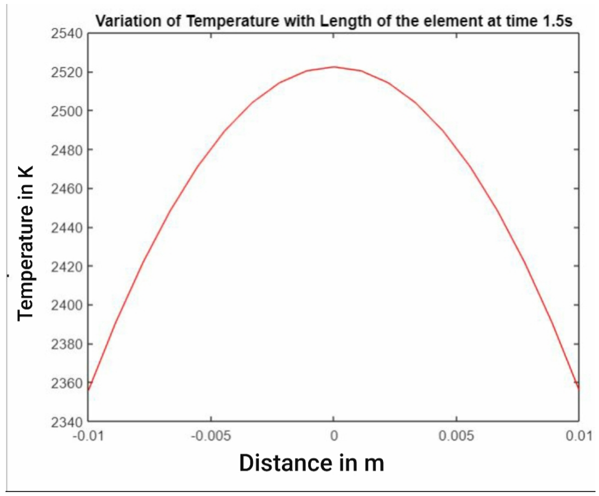

Figure 7: Temp vs x for n = 10

Figure 8 illustrates the temperature change of mid plane node and surface node with time resp.

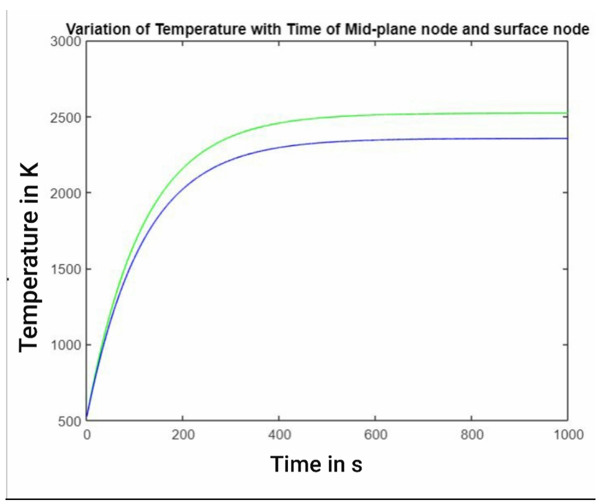

Figure 8: Temp vs t for n = 10Green:Mid Plane nodeBlue:Surface node

As observed from figure 8 ,
The steady state is achieved at t=450 seconds.
The steady state temperatures are 2350 K for surface plane and 2520 K for mid plane.

10 Contributions

Research: Primary Research by Nishigandha
Code: Written by Rhythm in MATLAB with contributions from other team members for its further development. The code converted to README file and uploaded on Github by Rhythm.
Report: Written in LaTeX by Neha.Results summarized and plots obtained by Neha with help of Nishigandha

11 GitHub link

Find code and README file here.

12 Reference

Fundamentals of Heat and Mass Transfer - Seventh Edition
By *Theodore L. Bergman, Adrienne S. Lavine, Frank P. Incropera and David P. Dewitt*