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Guest Editorial

Developing Interdisciplinary Problem Solving

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Disciplinarity

Academe is fixated on disciplinary depth and specialization. People have a tendency to gravitate to their interests and focus their learning toward one most-comfortable academic discipline. If successful enough, they become experts (specialists) in their chosen discipline. In this disciplinary-based academic world, the more advanced a student's learning, the more narrow (but deeper) are her interests and skills. The common phrase is that she becomes "an inch wide and a mile deep."

Universities and many colleges, through their discipline-based departments, educate these experts at various levels of degrees. Society rewards holders of these degrees with positions that sometimes have something to do with the person's academic discipline. But a tight discipline–skill match does not always happen. Why? Narrow academic disciplines do not always match the broader job-skills and professional needs all that well.

For many universities and colleges, that limitation does not matter — having their graduates obtain a good job is what matters. Traditionally, employers are not as happy with this academic disciplinary focus, but they have little choice but to accept the best available, narrowly educated graduates. Professional education and on-the-job training reduce this discipline-skill gap and eventually create a person who is suited to the job of solving the broader, more complex real-world problems.

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We frequently encounter this discipline-focused academic environment. The first thing one asks a person attending a university or college is what is her major. The hope and expectation is to hear a well-known, highly-regarded discipline in the response. Most reading this editorial would smile if at that inquiry the stated discipline was "mathematics." Perhaps some would frown if the response was "interdisciplinary studies".

Depth-Breadth Considerations

What are the disciplinary requirements of society? Does society really need and use these specialists, or is there something more that meets the needs of society than academic disciplinary focus, degree categorization, professional preparation, and confining specialization? What about multi-disciplinary, interdisciplinary, transdisciplinary, and cross-disciplinary skills, plus the need for more intellectual breadth and knowledge-integration skills, in student learning? Is this depth-breadth issue just a difference between liberal arts and professional education? Do any of the broader academic frameworks have merit or relevance in society?

First, I will consider these questions from problem-solving and conceptual points-of-view. Later I will consider more programmatic answers to these questions.

Transdisciplinarity

Transdisciplinary is when all ideas are immersed or incorporated into the whole of knowledge. It is the most holistic view of knowledge. In a sense, it is the complete absence of disciplinary focus. Transdisciplinary is an extreme view of knowledge, information, and problems that limits the world to a generalist or holistic viewpoint without any role for specialization. While there are instances where the generalist or polymath is valuable to problem solving, it is difficult to imagine that a transdisciplinary approach would be the only way to solve all or even many societal problems. This transdisciplinary concept was derived from the ideal in Renaissance humanism, when several centuries ago it was thought that it was possible to acquire a universal learning without the restrictions of learning specific academic disciplines. A skilled generalist can develop an open mind with the skills to understand and articulate relationships between disciplines. Transdisciplinary is portrayed as producing inch-deep, mile-wide graduates. The concepts and terms "transdisciplinary" and "generalist" directly contrast "disciplinary" and "specialist."

Cross-disciplinarity

Cross-disciplinary is an entirely different concept. It simply defines or explains aspects of one discipline in terms of another. It is merely a way to add a new perspective from a second or different discipline to an existing disciplinary view and so is limited as a problem-solving framework. Cross-disciplinary examples are found in courses such as "mathematics for music" or "history of science." These kinds of course connections may develop some breadth in understanding, but a cross-disciplinary framework, like a strictly disciplinary one, is too limited to be empowering for societal problem solving.

Multidisciplinarity

Let's explore the conceptual frameworks and notions of multidisciplinary and interdisciplinary. We start with a problem-solving approach. Some problems lend themselves to a reductionist, divide-and-conquer, approach. These kinds of problems are perfectly suited to a multidisciplinary methodology or conceptual framework.

Once divided, the component parts can be considered as disciplinary problems, where specialization is an advantage or perhaps a requirement. Once the components are solved, the solution elements are pieced backed together to provide a complete solution to the problem. In this case, the multidisciplinary team approach is a powerful and appropriate method of attack that often leads to success.

This multidisciplinary thinking and problem solving is how society advanced during the Industrial Age and used its science and engineering to enhance commerce, to find and produce energy, and to develop new technologies. Structured processes, such as the scientific method and the engineering design process, often decipher highly reductionist problems that are well-suited for the multiple disciplines that call themselves science or engineering; they use a multidisciplinary process to solve these kinds of societal (often technical) problems.

Certainly, mathematics, as the science of measurement, is used in that way for many problems. For those problems, the multidisciplinary approach of engineering and science methodology is appropriately and effectively used. Similarly, many of the problems in operations research and computer science that involve design optimization of an algorithm or statistical data mining can be effectively attacked using a similar multidisciplinary framework.

Interdisciplinarity

In contrast to the multidisciplinary reductionist approach is holism—the non-reductionist concepts of complexity theory or systems science. In a holistic interdisciplinary framework, the properties of the problem are not explainable or solvable just from the sum or combination of their reduced parts but from a synergy produced from the interrelations of those parts or perspectives. The whole is greater than or significantly different from just the sum or collection of its parts. These properties often include phenomena such as learning, synchronization, self-organization, adaptation, and emergence.

Reductionism and multidisciplinary problem solving are limited for problems with high complexity where the integration of ideas dominates, especially those with the human elements of culture, cognition, and relationships. The reductionist and multidisciplinary approaches are also limited for networked phenomena, where higher levels or subsets of the organization exert causal and indirect influence on other subsets or lower levels. Complexity occurs when many entities interact in different ways so that the whole takes on a life of its own.

The modern 21st-century Information Age world is full of examples of complex or "wicked" problems requiring interdisciplinary approaches and methodologies. Prime examples are problems and questions formed from the social, biological, and informational sciences involving interactions of large numbers of diverse or unique components. These include ecosystems, financial markets, company organizations and functions, urban populations, pandemics, government systems and policies, and warfare. The multidisciplinary framework fails when it inappropriately uses reductionism on these complex systems. These modern-era non-reductive problems need holistic solutions within a pluralistic philosophy that integrate ideas, work with large amounts of data, and use interdisciplinary methodologies by articulating relationships between several disciplines.

One form of this modern approach to problem solving is called eScience [Hey 2009]. This holistic, non-reductionist approach changes problem solving from being analytic (one that "breaks apart") to one that is synthetic, that identifies the patterns, dynamics, influences, and behaviors exhibited by systems of interaction. Interdisciplinarity and holism are not newly discovered phenomenon. "The whole is more than the sum of its parts" was an important holistic component of Aristotle's *Metaphysics* philosophy and of Gestalt psychology. Today, there are numerous writings on the methodologies of complexity theory, systems science and educational programs to develop interdisciplinary problem solving [Strogatz 2003; Barabasi 2003].

A Key Role for Modeling

Modeling is the glue that makes interdisciplinary problem-solving skills powerful and robust.

Of course, we still need to teach reductionist problem solving, and therefore disciplinary learning; and using multidisciplinary methodology to solve appropriate problems has a place in education programs and courses. However, in today's world, we need more interdisciplinary integrators and holistic problem solvers. We need interdisciplinary academic programs and courses that teach and develop complexity theory, systems theory, pluralistic thinking, and holistic problem solving, where students work alone or in concert to meld together their intelligence into a powerful problem solving approach [Gardner 2006]. The complexity of society has increased since the Industrial Age, and therefore we need to teach the more powerful and flexible modeling-based techniques advocated by George Polya [1945]. Polya's methodology addresses more interdisciplinary, society-relevant, complex, quantitative and qualitative problems than the scientific method, engineering design process, reductionism, or multidisciplinary methodology.

Education

Undergraduate learning must reflect the acquisition of knowledge that is both specialized and broad—deep enough for mastery of a discipline and broad enough to build relationships among key subject areas, along with the interdisciplinary integration of knowledge, ideas, theories, methods, practices, and applications. Education should not be limited to just the acquisition of more advanced knowledge in key knowledge areas, but must also include the creative integration of knowledge in science, humanities, culture and society [Adelman 2011].

How do we develop educational programs that produce students with these integrative thinking skills? It is unreasonable to set the goal of undergraduate education to produce graduates of programs that are a mile wide AND a mile deep! However, as advocated in the Degree Qualifications Profile [Adelman 2011], we need a better balance of breadth and depth for many more students. We need to lessen our focus on disciplinary *skills*. We need to give students more experiences in multidisciplinary and interdisciplinary problem solving, and develop graduates with systemic reasoning in multiple disciplines. We need courses that teach students how to collaborate in multidisciplinary teams, and how to think, integrate, and use holistic and interdisciplinary methodology. Interdisciplinarity overcomes the limitations of the reductive, siloed problem solving that we traditionally teach.

The reality is that society's future will driven by holistic, non-reductionist education. While we are making great strides in advancing technologies able to capture vast quantities of complex data, and we are racing to develop tools to make sense of these data, nowhere in the educational system do we systematically deal with this interdisciplinary issue. Our disciplinary-based education system is rapidly falling behind our future needs. The challenge is to transform the way we educate so society can keep pace with complex problems. Systems and holistic approaches must be emphasized over reductive and inductive approaches.

Another example of this educational shortcoming is the existence of many more topics courses than unifying concept courses. Topics courses are small-scale multidisciplinary frameworks, and unifying courses are more likely to expose the student to a holistic-like framework.

How?

How do students learn the interdisciplinary framework and develop holistic problem-solving skills? One way is to develop an understanding of the internal structure of a discipline and then show and give opportunities for its interdisciplinary connections. Another is to alternate disciplinary development with interdisciplinary problem-solving opportunities within the student's academic program.

The result is an educational concept called "T-shaped skills" that describe the abilities of students. The vertical bar of the T represents the depth of skills and expertise in a discipline, and the horizontal bar is the ability to collaborate across disciplines and apply knowledge in disciplines other than one's own. These concepts enable students to see knowledge more as a unified whole, rather than just as a collection of independent topics. These broader experiences develop more sophisticated modes of thought so students can tackle more complex problems. To do this, our courses need to expose students to problems in a variety of interdisciplinary contexts that progressively become more complex and therefore require more multidisciplinary and interdisciplinary problem solving skills.

None of this is to imply that all students need to become interdisciplinary. It is apparent that for some problems diverse teams of intellectual specialists forming multidisciplinary teams can be more powerful than teams of generalists. As explained in Degree Qualifications Profile [Adelman 2011], there are several areas of learning at the college level in addition to the two areas (broad integrative knowledge and specialized knowledge) that we have discussed. These other knowledge areas include applied learning, civic learning, and intellectual skills. Each school and program has its own goals and therefore unique learning profile. And within the schools, every student progresses toward their own academic goals within that profile.

Certainly, liberal arts programs already provide more breadth by exposing students to many disciplines. However, I would suggest that many liberal arts and engineering programs are more multidisciplinary than interdisciplinary, with very little in terms of integrating opportunities. It is therefore balance and diversity in knowledge and skills that are needed, where the education programs are designed to develop students with problem-solving, collaboration, and integration skills that are disciplinary, multidisciplinary, interdisciplinary, transdisciplinary, and cross-disciplinary.

Medicine—An Analogy

In the medical field, we must have specialists for some very disciplinary-like (focused) health problems—cardiac surgeons for a specific operation, for example. But we also need people or teams of doctors to work on two or more physical systems at once—neuromuscular disorders, as an example of a multidisciplinary medical issue.

However, very often we need interdisciplinary doctors, general practitioners or holistic physicians, who can think about all the physical systems, environmental factors, and psychological factors when trying to determine a diagnosis and course of treatment. Once the general practitioner determines the diagnosis, she may call in a specialist or a multispecialist team; but the holistic, integrative understanding of the situation needed to be obtained before the specialist could be used effectively. All these skills are needed.

A Role for Network and Systems Sciences

One way to teach interdisciplinary problem solving is to use network and/or systems science to break down traditional silos, to compare and contrast processes across domains. Interdisciplinarity is the natural state of these sciences! The study of interdisciplinary sciences can help build powerful holistic problem-solving skills for non-reductionist problems [Stichweh 2011]. Using network science, complex systems can be explored and synthesized to find patterns and properties in complex processes [Morrison 2006]. These patterns can help us understand evolutionary processes, system behavior, and modes of structure associated with topics as diverse as epidemics, organizational management, information percolation, and the movement of energy within systems.

Network and systems science with their associated quantitative and qualitative modeling provide insight into complex systems resistant to analysis through other more reductionist approaches. These interdisciplinary approaches use flexible data structures for mining of complex data sets; and they create processes to explore the intersections of social and environmental issues in order to study the factors and causes of famine, war, disease, and poverty.

Mathematics

The reason for interdisciplinary learning is to build the holistic skills to tackle society's complex problems. Mathematics (and its associated prose) is the language of quantitative problem solving. Without the language of mathematics, science, engineering, technology, and humanities do not exist. Without the logic of modeling, in both its multidisciplinary and interdisciplinary forms and quantitative and qualitative perspectives, many problems cannot be solved. However, even more compelling is that by using mathematical, network, and systems science to reveal the complexity of the world, we realize that the disciplines are connected and must be integrated using interdisciplinary methodology to solve the most complex problems we face. In this way, mathematics supports, or connects to, every course in every department. Mathematical language, along with innovative modeling, acts as the interdisciplinary glue that will help us solve our future problems and create the valuable knowledge and powerful ideas for improved organizations and a better society.

Conclusion

Society in 2013 needs its graduates to study, model, and solve problems using network and systems science concepts by embracing the modern world's complexity. Our education programs need to teach holistic problem solving using the interdisciplinary glue of modeling to graduate more students with stronger T-skills. Developing these skills are the educational goals incorporated into the Mathematical Contest in Modeling and the Interdisciplinary Contest in Modeling.

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About the Author

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM)®. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

Editor's Note

About This Issue

This year we had almost 6,600 (!) participating teams in the two contests combined; the 16 Outstanding papers ran to over 500 manuscript pages. Editing and publishing all the Outstanding papers, which we once did, is simply not possible any more.

Hence, as in the past few years, we are able to present in the pages of the *Journal* only one Outstanding entry for each of the MCM and ICM problems. The selection of which papers to publish reflected editorial considerations and was done blind to the affiliations of the teams.

All of the 16 Outstanding papers appear in their original form on the 2012 MCM-ICM CD-ROM, which also has the press releases for the two contests, the results, the problems, unabridged versions of the Outstanding papers, and some of the commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772-6627.

MCM Modeling Forum

Results of the 2013 Mathematical Contest in Modeling

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Introduction

A total of 5,636 teams of undergraduates from hundreds of institutions and departments in 14 countries spent a weekend in February working on applied mathematics problems in the 29th Mathematical Contest in Modeling (MCM)®.

The 2013 MCM began at 8:00 P.M. EST on Thursday, January 31, and ended at 8:00 P.M. EST on Monday, February 4. During that time, teams of up to three undergraduates researched, modeled, and submitted a solution to one of two open-ended modeling problems. Students registered, obtained contest materials, downloaded the problems and data, and entered completion data through COMAP's MCM Website. After a weekend of hard work, solution papers were sent to COMAP on Monday. Two of the top papers appear in this issue of *The UMAP Journal*, together with commentaries.

In addition to this special issue of *The UMAP Journal*, COMAP offers a supplementary *2013 MCM-ICM CD-ROM* containing the press releases for the two contests, the results, the problems, unabridged versions of the Outstanding papers, and judges' commentaries. Information about ordering is at `http://www.comap.com/product/cdrom/index.html` or at (800) 772–6627.

Results and winning papers from the first 28 contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2012). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains the 20 problems used in the first 10 years of the contest and a winning paper for each year. That volume and the special

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MCM issues of the *Journal* for the last few years are available from COMAP. The 1994 volume is also available on COMAP's special *Modeling Resource* CD-ROM. Also available is *The MCM at 21* CD-ROM, which contains the 20 problems from the second 10 years of the contest, a winning paper from each year, and advice from advisors of Outstanding teams. These CD-ROMs can be ordered from COMAP at `http://www.comap.com/product/cdrom/index.html`.

This year, the two MCM problems represented significant challenges:

- Problem A, "The Ultimate Brownie Pan," asked teams to design the size and shape of a pan for cooking brownies that would feature even distribution of heat through the pan and also maximize the number of pans that can fit in the oven.
- Problem B, "Water, Water, Everywhere," asked teams to pick a country from a given list and build a model to determine a strategy to meet its water needs in 2025.

COMAP also sponsors:

- The MCM/ICM Media Contest (see p. 114).
- The Interdisciplinary Contest in Modeling (ICM)®, which runs concurrently with the MCM and next year again will offer a modeling problem involving network science. Results of this year's ICM are on the COMAP Website at `http://www.comap.com/undergraduate/contests`. The contest report, an Outstanding paper, and commentaries appear in this issue.
- The High School Mathematical Contest in Modeling (HiMCM)®, which offers high school students a modeling opportunity similar to the MCM. Further details are at `http://www.comap.com/highschool/contests`.

2013 MCM Statistics

- 5,636 teams participated (with 957 more in the ICM)
- 4 high school teams (<0.1%)
- 375 U.S. teams (7%)
- 5,261 foreign teams (93%), from Canada, China, Finland, Germany, Hong Kong, India, Indonesia, Mexico, Malaysia, Singapore, South Korea, Sweden, and the United Kingdom
- 11 Outstanding Winners (0.2%)
- 13 Finalist Winners (0.2%)
- 858 Meritorious Winners (15%)
- 1,650 Honorable Mentions (29%)
- 3,094 Successful Participants (55%)

Problem A: The Ultimate Brownie Pan

When brownies are baked in a rectangular pan, heat is concentrated in the four corners and the product gets overcooked at the corners (and to a lesser extent at the edges). In a round pan, the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape, using round pans is not efficient with respect to using the space in an oven. Develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes—rectangular to circular and other shapes in between.

Assume:

1. A width to length ratio of W/L for the oven, which is rectangular in shape.
2. Each pan must have area A.
3. Initially two racks in the oven, evenly spaced.

Develop a model that can be used to select the best type of pan (shape) under the following conditions:

1. Maximize the number (N) of pans that can fit in the oven.
2. Maximize even distribution of heat (H) for the pan.
3. Optimize a combination of conditions (1) and (2), with weights p and $(1-p)$ assigned to illustrate how the results vary with different values of W/L and p.

In addition to your MCM formatted solution, prepare a one- to two-page advertising sheet for the new *Brownie Gourmet Magazine* highlighting your design and results.

Problem B: Water, Water, Everywhere

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement, desalination, and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a nontechnical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the "best water strategy choice."

Countries: United States, China, Russia, Egypt, or Saudi Arabia.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at either Appalachian State University (Brownie Pan Problem) or at Carroll College (National Water Problem). At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree, a third judge evaluated the paper.

Additional Regional Judging sites were created at the U.S. Military Academy, the Naval Postgraduate School, and Carroll College to support the growing number of contest submissions.

Final judging took place at the Naval Postgraduate School, Monterey, CA. The judges classified the papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participation	Total
Brownie Pan Problem	6	5	394	720	1,477	2,606
National Water Problem	5	8	464	930	1,617	3,030
	11	13	858	1,650	3,094	5,636

We list here the 11 teams that the judges designated as Outstanding; the list of all participating schools, advisors, and results is at the COMAP Website.

Outstanding Teams

Institution and Advisor	Team Members

Brownie Pan Problem

"Optimize the Shape for a Brownie Pan"
Fudan University
Shanghai, China
Ling Yang

Kunrui Wang
Jing Xu
Wei Zheng

"Solving the Brownie Pan Problem"
Shanghai Jaotong University
Shanghai, China
Yuehui Zhang

Libin Wen
Jingyuan Wu
Cong Wang

"Applied Mathematicans Bake the Best Brownies:
The Ultimate Pan Design"
Peking University — Meng Wu
Beijing, China — Chong Jin
Xufeng Liu — Bowen Liu

"Cooking Up the Optimal Baking Algorithm"
Bethel University — Michael Tetzlaff
Arden Hills, MN — Jake Smith
Nathan M. Gossett — Anthony J. Burand, Jr.

"The Best Rounded Rectangle for Ultimate Brownies"
University of Colorado Boulder — Christopher V. Aicher
Boulder, CO — Tracy Babb
Ann M. Dougherty — Fiona Pigott

"The Design of the Ultimate Brownie Pan "
Shandong University — Yankan Song
Jinan, China — Ke Xu
Hengxu Zhang — Fan Yi

National Water Problem

"Make Wise Use of Every Drop"
Nanjing University — Wei Chen
Nanjing, China — Weizhi Liu
Hui Qu — Cenying Yang

"Quenching China's Thirst in 2025:
A Min-Cost-Max-Flow Network Model"
Tsinghua University — Pengfei Gao
Beijing, China — Boshuo He
Hao Wu — Tianxin Zou

"Sustainable Water Management for Saudi Arabia
in 2025 and Beyond"
University of Colorado — Yue-Ya Hsu
Boulder, CO — Gregory S. Mcquie
Anne M. Dougherty — David Thomas

"Five Models for China's Water Scarcities"
Beijing University of Posts and Telecommunicatons
Beijing, China
Zuguo He

Zhongxin Guo
Fan Wu
Beidan Wang

"Water, Water Everywhere: Meeting the Demands of Saudi Arabia's Water Needs Today"
Colorado College
Colorado Springs, CO
Andrea Bruder

Aradhya Sood
Yukiko Iwasaki
Namgyal Angmo

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, recognized the Outstanding teams from Shandong University (Brownie Pan Problem) as INFORMS Outstanding teams and provided the following recognition:

- a letter of congratulations from the current president of INFORMS to each team member and to the faculty advisor;
- a check in the amount of $300 to each team member;
- a bronze plaque for display at the team's institution, commemorating team members' achievement;
- individual certificates for team members and faculty advisor as a personal commemoration of this achievement; and
- a one-year student membership in INFORMS for each team member, which includes their choice of a professional journal plus the *OR/MS Today* periodical and the INFORMS newsletter.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The SIAM Award teams were from University of Colorado Boulder (Brownie Pan Problem) and the Tsinghua University (National Water Problem). Each team member was awarded a $300 cash prize, and the teams received partial expenses to present their results in a special Minisymposium at the SIAM Annual Meeting in San Diego, CA in July. Their schools were given a framed hand-lettered certificate in gold leaf.

The Mathematical Association of America (MAA) designated one North American team from each problem as an MAA Winner. The Winner for the

Brownie Pan Problem was from Bethel University, and the winner for the National Water Problem was from the University of Colorado Boulder. With partial travel support from the MAA, the teams presented their solution at a special session of the MAA Mathfest in Hartford, CT in August. Each team member was presented a certificate by an official of the MAA Committee on Undergraduate Student Activities and Chapters.

Ben Fusaro Award

One Meritorious, Finalist, or Outstanding paper is selected for the Ben Fusaro Award, named for the Founding Director of the MCM and awarded for the 10th time this year. It recognizes an especially creative approach; details concerning the award, its judging, and Ben Fusaro are in Vol. 25 (3) (2004): 195–196. The Ben Fusaro Award Winner, for the Brownie Pan Problem, was the Finalist team from the Tongji University. A commentary about it appears in this issue.

Frank Giordano Award

For the second time, the MCM is designating a paper with the Frank Giordano Award. This award goes to a paper that demonstrates a very good example of the modeling process in a problem featuring discrete mathematics—this year, the National Water Problem. Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. The Frank Giordano Award for 2013 went to the Outstanding team from Colorado College.

Judging

Director
William P. Fox, Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA
Associate Director
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY

Brownie Pan Problem

Head Judge
Marvin Keener, Oklahoma State University, Stillwater, OK
Associate Judges
William C. Bauldry, Chair-Emeritus, Dept. of Mathematical Sciences, Appalachian State University, Boone, NC (Head Triage Judge)

Kelly Black, Mathematics Dept., Clarkson University, Potsdam, NY
Karen Bolinger, Dept of Mathematics, Clarion University, Clarion, PA
Jim Case, Baltimore, MD (SIAM Judge)
Ben Fusaro, Dept. of Mathematics, Florida State University, Tallahassee, FL (SIAM Judge)
Veena Mendiratta, Lucent Technologies, Naperville, IL
John Scharf, Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT (Fusaro Award Judge)
Yongji Tan, Dept. of Mathematics, Fudan University, Shanghai, China
Michael Tortorella, Dept. of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ

Regional Judging Session at the U.S. Military Academy

Head Judge

Patrick J. Driscoll, Dept. of Systems Engineering

Associate Judges

Tim Elkins, James Enos, Kenny McDonald, and Elizabeth Schott, Dept. of Systems Engineering
Steve Horton, Dept. of Mathematical Sciences
—all from the United States Military Academy at West Point, NY

Paul Heiney, Dept of Mathematics, U.S. Military Academy Preparatory School, West Point, NY

Ed Pohl, Dept. of Industrial Engineering
Tish Pohl, Dept. of Civil Engineering
—both from University of Arkansas, Fayetteville, AR

Triage Session at Appalachian State University

Head Triage Judge

William C. Bauldry, Chair, Dept. of Mathematical Sciences

Associate Judges

Bill Cook, Ross Gosky, Jeffry Hirst, Katie Mawhinney, Trina Palmer, Greg Rhoads, René Salinas, Tracie McLemore Salinas, Kevin Shirley, and Nate Weigl
—all from the Dept. of Mathematical Sciences, Appalachian State University, Boone, NC

Amy H. Erickson and Keith Erickson
—Dept. of Mathematics, Georgia Gwinnett College, Lawrenceville, GA
Steven Kaczkowski and Douglas Meade
—Dept. of Mathematics, University of South Carolina, Columbia, SC

National Water Problem

Head Judge
Maynard Thompson, Mathematics Dept., University of Indiana, Bloomington, IN
Associate Judges
Robert Burks, Operations Research Dept., Naval Postgraduate School, Monterey, CA
Jerry Griggs, University of South Carolina, Columbia, SC
Michael Jaye, Dept. of Defense Analysis, Naval Postgraduate School, Monterey, CA
Mario Juncosa, RAND Corporation, Santa Monica, CA (retired)
Jack Picciuto, Office of Institutional Research, U.S. Military Academy, West Point, NY (retired)
Kathleen M. Shannon, Dept. of Mathematics and Computer Science, Salisbury University, Salisbury, MD (MAA Judge)
Dan Solow, Case Western Reserve University, Cleveland, OH (INFORMS Judge)
Marie Vanisko, Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT (Giordano Award Judge)
Richard Douglas West, Francis Marion University, Florence, SC (Giordano Award Judge)
Jinxing Xie, Dept. of Mathematical Sciences, Tsinghua University, Beijing, China

Regional Judging Session at the Naval Postgraduate School
Head Judges
William P. Fox, Dept. of Defense Analysis
Frank R. Giordano, Dept. of Defense Analysis
Associate Judges
Michael Jaye, Dept. of Defense Analysis
Robert Burks, Greg Mislick, and Scott Nestler, Operations Research Dept.
—all from the Naval Postgraduate School, Monterey, CA
Rich West, (retired) PA

Triage Session at Carroll College
Head Judge
Marie Vanisko
Associate Judges
Terry Mullen and Kelly Cline
—all from Dept. of Mathematics, Engineering, and Computer Science, Carroll College, Helena, MT

Sources of the Problems

The Ultimate Brownie Pan Problem was contributed by Veena Mendiratta (Lucent Technologies, Naperville, IL), who has been an MCM judge for many years. The National Water Problem was contributed by David Olwell (Dept. of Systems Engineering, Naval Postgraduate School, Monterey, CA).

Acknowledgments

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We also thank for their involvement and unflagging support the MCM judges and MCM Board members, as well as

- **Two Sigma Investments.** "This group of experienced, analytical, and technical financial professionals based in New York builds and operates sophisticated quantitative trading strategies for domestic and international markets. The firm is successfully managing several billion dollars using highly-automated trading technologies. For more information about Two Sigma, please visit `http://www.twosigma.com`."

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each paper here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential MCM advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are the only international modeling contests in which students work in teams. Centering its educational philosophy on mathematical modeling, COMAP serves the educational community as well as the world of work by preparing students to become better-informed and better-prepared citizens.

Editor's Note

The complete roster of participating teams and results has become too long to reproduce in the *Journal*. It can now be found at the COMAP Website, in separate files for each problem:

```
http://www.comap.com/undergraduate/contests/
  mcm/contests/2013/results/2013_MCM_Problem_A_Results.pdf
http://www.comap.com/undergraduate/contests/
  mcm/contests/2013/results/2013_MCM_Problem_B_Results.pdf
```

Media Contest

This year, COMAP again organized an MCM / ICM Media Contest.

Over the years, contest teams have increasingly taken to various forms of documentation of their activities over the grueling 96 hours—frequently in video, slide, or presentation form. This material has been produced to provide comic relief and let off steam, as well as to provide some memories days, weeks, and years after the contest. We *love* it, and we want to encourage teams (outside help is allowed) to create media pieces and share them with us and the MCM / ICM community.

The media contest is *completely separate* from MCM and ICM. No matter how creative and inventive the media presentation, it has *no* effect on the judging of the team's paper for MCM or ICM. We do not want work on the media project to detract or distract from work on the contest problems in any way. This is a separate competition, one that we hope is fun for all.

Further information about the contest is at

`http://www.comap.com/undergraduate/contests/mcm/media.html.`

There were 31 entries—27 of them from Dalian Maritime University! (Come on, you other teams!)

Outstanding Winners:

- Dalian Maritime University
  (Kexin Chen, Yunhui Liu, Yu Meng)
- Dalian Maritime University
  (Cheng Ding Junxiao Wang Jiahuan Pei)

Finalists:

- Northwestern Polytechnical University
  (Zhu Bin, Jianhua Zhou, Tinghe Zhang)
- Dalian Maritime University
  (Xin Lin, Haobo Sun, Chaoju Liu)
- Dalian Maritime University
  (Baojin Liu, Yongbi Li, Fan Yang)

The remaining entries were judged Meritorious Winners.

Complete results, including links to the Outstanding videos, are at

`http://www.comap.com/undergraduate/contests/mcm/contests/2013/solutions/index.html.`

Applied Mathematicians Bake the Best Brownies: The Ultimate Pan Design

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Advisor: Xufeng Liu

Abstract

When brownies are baked in a traditional square pan, the corners often get overcooked. However, when a circular pan is used, some of the room inside the oven is wasted during the baking process. We explain the overcooking phenomenon in theoretical models and calculations, and attempt to achieve a trade-off between a square pan and a circular pan. We choose rounded rectangles, with parameters to achieve shapes in between.

We investigate the heat distribution of the air inside the oven, in the pan, and in the batter. We construct two-dimensional heat balance equations with boundary constraints. We give stationary solutions analytically for the circular and square situations, while for an arbitrary rounded rectangle, we employ the finite element method (FEM) to obtain a numerical solution. PDE solutions show that the temperature distribution is uniform in the pan, while the shape of the pan greatly affects heat distribution along the periphery of a horizontal cross section of the brownie batter.

We measure the uniformity of heat distribution on the edge by μ, the ratio of the smallest temperature gradient to the biggest temperature gradient throughout the edge. We check typical shapes to validate that this measure is consistent with our intuition. We construct a function to fit μ using a two-layer feed-forward neural network.

The maximum number of pans that can fit in the oven can be achieved when the shape is a rectangle with either w or l of the pan equal to the length of one side of the oven (though a circular pan undoubtedly attains the most even distribution of heat on the edge). We can get shapes in between by adjusting a parameter p.

The UMAP Journal 34 (2–3) (2013) 115–139.

We perform sensitivity analysis on a couple of parameters and discuss the strengths and weaknesses of our model.

We strongly recommend our model because of its reasonable assumptions, convincing estimation of key parameters, and clear but robust results.

Introduction

When you are ready to invite your family members and friends to enjoy your fascinating brownies, you will probably find with dismay that the hard edges of the brownies are overcooked while the gooey interior is undercooked. This phenomenon is caused by uneven distribution of heat through the pan during the baking process.

To get a heat distribution as uniform as possible and secure a good taste for the brownie, Howe [1990] devised and patented a "controlled heating baking pan." The center and the exterior of his plastic baking pan differ in thickness to slow down the baking of the outer edges of a brownie. Another approach is to use a round pan; but poor space utilization is its inherent downside, for ovens are rectangular.

By modeling what happens inside the oven during a heat-transfer process, mathematicians and physicists can guard brownies against overcooking. We model the baking process of the brownie with two-dimensional heat equations with reasonable boundary conditions. We give an analytical solution of the two-dimensional stationary heat equation, and find a numerical solution by applying the finite element method (FEM) to the equation using MATLAB.

For the pan shape, we choose a spectrum of rounded rectangles that includes the circle and the rectangle as special cases. Given the oven dimensions W and L, and a fixed pan area of A, we maximize the number N of pans that can fit in the oven, the even distribution of heat H, and a suitable combination of these two where weights p and $(1-p)$ are assigned.

We identify the strengths and weaknesses of our model. Finally, we gladly advertise our novel design.

Background

Ingredients of a Brownie

We follow the five-star recipe by Murrin [2003]. The ingredients are listed in **Table 1**. Pour the mixture into a pan, put in the oven, set the timer for 30 minutes at 350°F ≅ 450K (which we take as the operating temperature of the oven afterwards), and patiently wait for the sweet smell of yummy brownies. It is indeed a foolproof recipe, but an inexperienced amateur could spoil the dessert by undercooking or overcooking. To secure the best quality of brownies under any circumstances is our instigation to devise an optimized baking pan.

Table 1.

The ingredients of a brownie, per kg of finished product. Adapted from Murrin [2003].

Item	Quantity (g)
Unsalted butter	185
Best dark chocolate	185
Plain flour	80
Cocoa powder	40
White chocolate	50
Milk chocolate	50
Eggs	3×57
Golden caster sugar	275

A Glance at Heat Transfer and Ovens

Before we delve into heat equations, let's first accustom ourselves to some intuitive concepts about how brownies are baked. Heat, or thermal energy, can be transferred in three fundamental ways: conduction, convection, and radiation.

Conduction happens between objects in physical contact. It is the most direct way to transfer heat when you cook, especially when the substances involved are good thermal conductors.

Convection is distinguished by the flow of thermal energy caused by fluids, like the air when you bake, and the water when you boil. Convection is almost negligible in heat transfer between solids.

Radiation transfers energy by either emission or absorption of electromagnetic waves, so the cooking medium does not take part in energy transmission. When we bake a brownie, thermal radiation absorbed by the pan can be neglected because a metal surface perfectly reflects the electromagnetic waves emitted by radiation sources.

Zimmerman [2007] elaborated on comparisons of common cooking methods by identifying modes of heat transfer. His conclusive table reveals that baking is characterized by high levels of conduction and convection.

A traditional oven usually features heating elements in the bottom, which may result in direct heating to the food (a process undesired in baking). Confronted with the problem, convection ovens were invented to take advantage of forced air to circulate the heat inside the oven cavity. This strategy reduces baking time and temperature, and a more even heating environment is achieved.

The particular oven that we investigate is Siemens HB75GB550B. It is a traditional oven with a cavity width $L = 48.2$ cm, depth $W = 40.5$ cm, and height $D = 32.9$ cm [Siemens Home Appliances 2012].

Values of Parameters

We list some values of parameters in **Table 2**. The sources are Efunda [2013] and Urieli [n.d.].

Table 2.
Some values of parameters.

Parameter	Symbol	Value
Environmental temperature	T_e	450K
Density of air	ρ_0	$0.78\ \mathrm{kg/m^3}$
Density of pan	ρ_1	$2700\ \mathrm{kg/m^3}$
Density of brownie	ρ_2	$475\ \mathrm{kg/m^3} \sim 660\ \mathrm{kg/m^3}$
Thermal conductivity of air	k_0	$0.0365\ \mathrm{W/m \cdot K}$
Thermal conductivity of pan	k_1	$238\ \mathrm{W/m \cdot K}$
Thermal conductivity of brownie	k_2	$0.385\ \mathrm{W/m \cdot K}$
Heat capacity of air	c_0	$1020\ \mathrm{J/kg \cdot K}$
Heat capacity of pan	c_1	$975.15\ \mathrm{J/kg \cdot K}$
Heat capacity of brownie	c_2	$700\ \mathrm{J/kg \cdot K}$
Convective heat transfer coefficient between the pan and the air	h	$15\ \mathrm{W/m^2 \cdot K}$
Size of the (square) pan		$9'' \times 9'' \times 2.25''$ (or $0.2286\ \mathrm{m} \times 0.2286\ \mathrm{m} \times 0.05715\ \mathrm{m}$)
Area of the pan	A_1	$0.05225\ \mathrm{m^2}$

Assumptions

- The bottom of the baking pans are rounded rectangles in shape, with length l, width w, and radius of the arcs r, illustrated in **Figure 1**. We consider a two-dimensional pan bottom with no thickness.

  The pan is shallow enough that the pan borders can be neglected when we study the pan alone. Still, when we establish the model of the brownie baking, we suppose that both the bottom and sides of the batter are surrounded by metal. An empty pan is illustrated in **Figure 2**.
- All of the N pans are identical. In other words, they share the same set of parameters w, l, and r.
- The pans are made of metal, more specifically, aluminum.
- The portion of thermal energy transferred from the wire rack can be neglected. After we confirm the point later in the paper that the temperature is evenly distributed across the pan even without any rack, we can safely neglect the influence of the rack in our model.

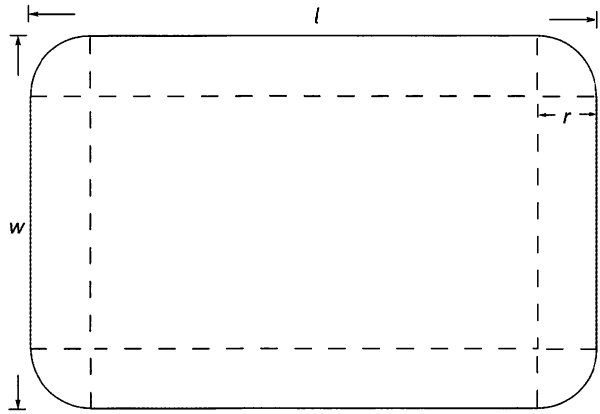


Figure 1. An illustration of the rounded rectangle, with parameters.

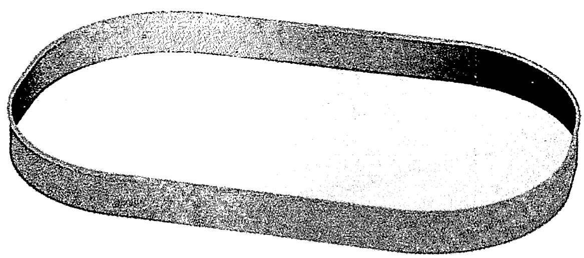


Figure 2. The pan in perspective view.

- All pans are placed in the same direction, parallel to one side of the rectangular rack. Overlap of pans is not permitted.
- The outer boundary of the pan shares the same temperature with the surrounding air, as the heat transfer between steel and air is swift, validated by the record in Wikipedia [2013] of a cooling rate roughly 38°C per minute. Furthermore, we assume that there is always some space between adjacent pans admitting air flow.
- The physical quantities are constant with regard to temperature, an approximation that greatly simplifies our model and calculation while still producing reasonable results.

Objectives

For a rectangular oven with width-to-length ratio W/L and a pan area of A_1 (we leave A_2 for the oven horizontal section area), which we take as 0.05225 m^2, we determine the parameters w, l, and r of the pan, so as to:

- Maximize the number of pans (N) that can fit in the oven, which we denote by $N_{\max}$. Notice that there are two evenly-spaced racks in the oven.
- Construct a measure of how evenly heat is distributed along the outer edges of the brownie batter during the baking process. Denote the relative uniformity of heat distribution of a particular pan shape by μ. We intend to maximize μ by adjusting the shape parameters.
- Optimize a weighted combination of the above two conditions, using weights p and $(1-p)$. Define ν as

$$\nu = \frac{N(\text{the particular shape})}{N_0},$$

where N_0 is the maximal number of pans in our feasible zone and serves to normalize the parameter ν. We seek to maximize

$$\xi = p\nu + (1-p)\mu.$$

Heat Distribution During Baking

Air inside the Oven

According to our recipe, the oven should be preheated to the working temperature. Modern ovens equipped with thermostats perform well in temperature regulation, as demonstrated in Fig. 2 of Purlis and Salvadori [2008].

A question naturally arises: Is there any temperature variation among various parts of the air?

Luckily, the answer is simple. We assert that the temperature of air in the oven is constant everywhere. In the following, we estimate the characteristic time for air to gain thermal equilibrium.

According to the Newton's cooling law [Wikipedia 2013], which originates from the convection of fluids with solids, we have the following equation:

$$\frac{dQ}{dt} = h_c S(T_e - T(t)) = -h_c A \Delta T(t),$$

where

- Q is the thermal energy (J);
- h_c is the heat transfer coefficient (W·m^{-2}·K);
- S is the surface area of the heat being transferred (m^2);
- T is the temperature of the object's surface and interior (since these are the same in this approximation);
- T_e is the temperature of the environment, i.e., the temperature suitably far from the surface;
- $\Delta T(t) = T(t) - T_e$ is the time-dependent thermal variation between environment and object.

In thermal dynamics, the change of thermal energy can be represented as the product of special heat capacity, the mass of the medium, and the change of temperature. That is,

$$\Delta Q = cm\Delta T.$$

Therefore, we can rewrite Newton's cooling law as

$$cm\,\frac{dT}{dt} = h_c S\big(T_e - T(t)\big).$$

Assuming that T_e is independent of time, we can get the analytical solution for $T(t)$:

$$T = T_e + (T_0 - T_e)\exp\left(\frac{-h_c S}{cm}\,t\right).$$

We define the parameter τ as the characteristic time for air to gain thermal equilibrium, so that

$$\tau = \frac{cm}{h_c S}.$$

Then we have

$$\frac{T(t)}{T_e} = 1 + \left(\frac{T_0}{T_e} - 1\right)\exp\left(-\frac{t}{\tau}\right).$$

At the operating temperature (~450K), the density of air is approximately 0.5 kg·m^{-3}. The dimensions of the oven are $0.35 \times 0.25 \times 0.2$ m^3.

From our data, we estimate orders of magnitude as follows:

$$\begin{aligned}
c &\sim 10^3\ \mathrm{J\cdot kg^{-1}\cdot K^{-1}},\\
h_c &\sim 10\ \mathrm{W\cdot m^{-2}\cdot K^{-1}},\\
S &\sim 0.1^2\ \mathrm{m^3},\\
V &\sim 0.1^3\ \mathrm{m^3}.
\end{aligned}$$

Therefore,

$$\tau \sim 1\,\mathrm{s},$$

which confirms our assumption that the air in the oven can gain thermal equilibrium in approximately one second.

Pan: Analytical and Numerical Solutions

Analytical Solution For a Rectangle Pan

Combining our assumption that the air temperature is constant inside the oven with Newton's cooling law, we obtain the heat balance equation

$$\begin{cases} \frac{\partial T}{\partial t} - \frac{k_1}{\rho_1 c_1}\nabla^2 T = \frac{Q}{\rho_1 c_1} + \frac{h}{\rho_1 c_1}(T_e - T), \\ T|_{x=0} = T_e, \quad T|_{x=l} = T_e, \\ T|_{y=0} = T_e, \quad T|_{y=w} = T_e. \end{cases}$$

where

- k_1 is the thermal conductivity of the pan,
- h is the heat transfer coefficient between the pan and the air,
- ρ_1 is the density of the pan,
- c_1 is the specific heat capacity (J/(kg·K)), and
- Q is the power per area unit generated (absorbed) by the internal heat source (drain).

Using the residual temperature $u(x,y) = T(x,y) - T_e$, we get

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{k}{\rho c}\nabla^2 u = \frac{Q}{\rho c} + \frac{h}{\rho c}(-u), \\ u|_{x=0} = u|_{x=l} = 0, \\ u|_{y=0} = u|_{y=w} = 0. \end{cases}$$

A stationary solution of the problem above satisfies

$$k\nabla^2 u = h\left(u - \frac{Q}{h}\right).$$

Then we split $u(x,y)$ into the general solution and a particular solution that depends on only one argument:

$$u(x,y) = v(x,y) + \omega(x),$$

where $v(x, y)$ is defined to be the solution of

$$\begin{cases} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) v = \frac{h}{k}\, v, \\ v\big|_{x=0} = v\big|_{x=l} = 0, \\ v\big|_{y=0} = v\big|_{y=w} = -\omega(x). \end{cases}$$

Employing separation of variables, we compute the coefficients

$$\begin{aligned} C &= -\frac{Q}{h}, \\ D &= \frac{-Q/h + (Q/h)\cosh(\sqrt{h/k}\cdot l)}{\sinh(\sqrt{h/k}\cdot l)}, \\ Q_n M_n &= -\frac{C\cosh(\sqrt{h/k}\cdot x) + D\sinh(\sqrt{h/k}\cdot x) + Q/h}{\sin(\pi n x/l)}, \\ Q_n N_n &= Q_n M_n \cdot \frac{-1 + \cosh(\sqrt{\pi^2 n^2/l^2 + h/k}\cdot w)}{\sinh(\sqrt{\pi^2 n^2/l^2 + h/k}\cdot w)}, \end{aligned}$$

along with the particular solution

$$\omega(x) = C\cosh(\sqrt{h/k}\cdot x) + D\sinh(\sqrt{h/k}\cdot x) + Q/h.$$

Finally, the desired stationary solution is

$$\begin{aligned} u(x, y) = \sum_{n=0}^{\infty} \sin\left(\frac{\pi n x}{l}\right) \Bigg[& Q_n M_n \cosh\left(\sqrt{\frac{\pi^2 n^2}{l^2} + \frac{h}{k}}\cdot y\right) \\ & + Q_n N_n \sinh\left(\sqrt{\frac{\pi^2 n^2}{l^2} + \frac{h}{k}}\cdot y\right) \Bigg] + \omega(x). \end{aligned}$$

Analytical Solution for a Circular Pan

We investigate the heat balance equation of a circular pan with radius R. Again we use the residual temperature $u(r) = T(r) - T_e$. We obtain an ordinary differential equation in polar coordinates:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u}{\mathrm{d}r}\right) = -\frac{Q}{k} + \frac{h}{k}u(r),$$

with general solution

$$u(r) = C_1 J_0\left(i\sqrt{\frac{h}{k}}\,r\right) + C_2 Y_0\left(-i\sqrt{\frac{h}{k}}\,r\right) + \frac{Q}{h},$$

where J_α and Y_α denote the Bessel functions of the first and second kinds. We use the boundary conditions to determine the coefficients C_1, C_2 and obtain the result

$$u(r) = \frac{Q}{h}\left(1 - \frac{J_0\left(i\sqrt{\frac{h}{k}}\,r\right)}{J_0\left(i\sqrt{\frac{h}{k}}\,R\right)}\right).$$

The analytical solution is portrayed in **Figure 3**. The radius of the outermost circle shown is $R = 0.1289$ m, so that the area of the pan, $A_1 = 0.5225$ m^2, remains constant.

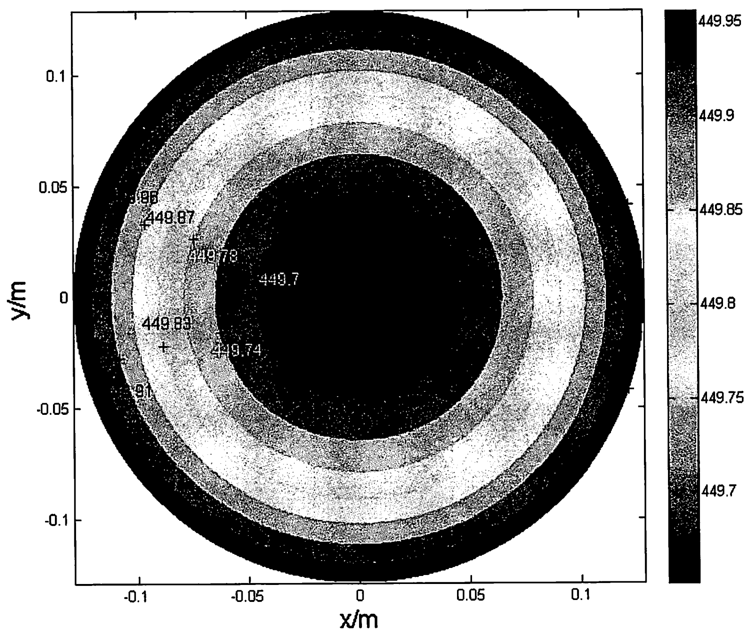


Figure 3. The temperature distribution of a circular pan: analytical solution.

Numerical Solutions for an Arbitrary Pan

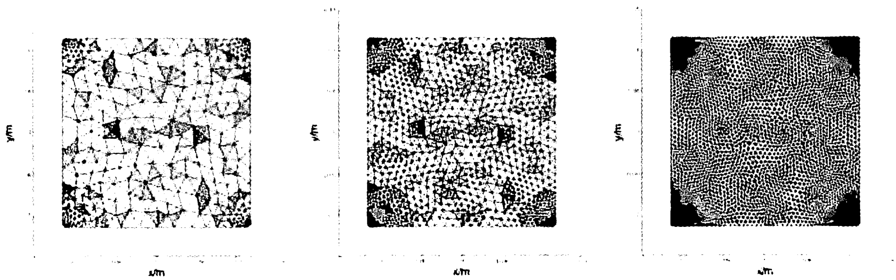
The heat balance equation for an arbitrary rounded rectangle is

$$\begin{cases} \frac{\partial T}{\partial t} - \frac{k}{\rho c}\nabla^2 T = \frac{Q}{\rho c} + \frac{h}{\rho c}(T_e - T), \\ T|_{x \in \partial G} = T_e, \end{cases}$$

where G represents the rounded rectangular region, and Q is the constant $Q = -5000$ W/m^2. The validity for this treatment is illustrated in the sensitivity analysis section.

For a complicated boundary condition like this, a concise analytical solution does not exist. In fact, only in quite a few specific conditions, such as rectangular or circular boundary conditions, can we possibly get an analytic solution.

Therefore, we employ the powerful finite element method (FEM) to calculate a numerical solution of the heat balance equation. With the aid of the MATLAB Partial Differential Equation Toolbox, we initialize the mesh by partitioning the region into small triangles (A in **Figure 4**)). The mesh can be further refined by dividing small triangles into tiny triangles (B), and another repetition generates even smaller triangles (C). In this way, a mesh consisting of more than 10,000 triangles is generated on the rounded rectangle (the figure shows the special case of a square).



(A) (B) (C)

Figure 4. Triangulation of the region: first step of FEM.

This approach enables us to approximate the stationary problem with a system of linear equations and obtain numerical solutions. To pave the way for further calculations, we estimate the values on the rectangular grid by interpolating from the vertices of the triangles. For a square pan with $W = L = 0.2286$ m, the solution is depicted in **Figure 5**.

Discussions

The temperature colorbars in both **Figure 2** and **Figure 3** reveal a temperature range of 0.3K. Because the temperature range of the pan is so small (explained by the excellent thermal conductivity of aluminum), we safely conclude that the temperature throughout the pan is the same as that of the air in the oven.

Brownies: Vertical and Horizontal Cross Section

We model the brownie batter as a three-dimensional mixed substance, whose shape is identical to the pan, with thickness $H = 4.0$ cm. Capable of solving only two-dimensional problems, the MATLAB PDE Toolbox at first

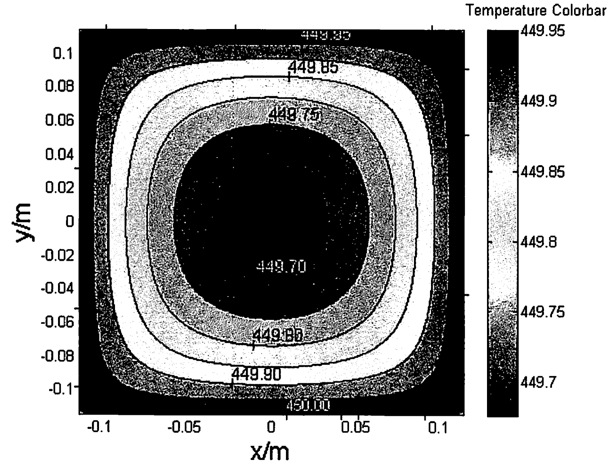


Figure 5. The temperature distribution over a square pan.

glance is not up to the task involved. We need to reduce the dimension of the problem to exploit the potential of the PDE Toolbox. We tackle the problem by analyzing a cross section of the batter, thus obtaining a two-dimensional problem that we are comfortable with.

A Vertical Cross Section

We first study the temperature distribution in a vertical cross section. If we slice parallel to the front face of the oven, we get a rectangular cross section that satisfies the heat balance equation

$$-k_2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) T = Q(x, z),$$

with boundary conditions

$$\begin{cases} T\big|_{x=0} = T\big|_{x=L} = T\big|_{z=0} = T_e, \\ T\big|_{z=H} = T_e - \Delta T, \end{cases}$$

where

- T is the temperature of the cross section with coordinates (x,z);
- ΔT is the temperature difference betwen the edge of the pan and the air right above the brownie;
- $Q(x, z)$ denotes the heat absorption of batter caused by various thermal effects, including water evaporation and chemical reactions. We fit $Q(x, z)$ as

$$Q = \frac{-50000}{\exp\left(-\dfrac{0.1}{\sqrt{\left(\frac{x}{0.2}\right)^2 + \left(\frac{z}{0.02}\right)^2}}\right) + 1}$$

to represent the actual process of baking;

- k_2 is the thermal conductivity of the brownie, approximated by the weighted average of the thermal conductivity of its ingredients, shown in **Table 3**. We compile the table from data in ASAE [1954], Clegg [2001], Mohos [2010], Řezníček [1988], and Williams [n.d.].

Table 3.
Thermal conductivity of ingredients of a brownie (1 kg final product).

Ingredient	Quantity (g)	Thermal conductivity (W/m·K)
Unsalted butter	185	0.21
Chocolate	285	0.26
Plain flour	80	0.37
Cocoa powder	40	0.10
Eggs	171	0.54
Golden caster sugar	275	0.58
Average		0.385

The heat distribution in the vertical cross section is illustrated in **Figure 6**. The graph resembles the temperature distribution of the middle cross section of bread in the literature [Purlis and Salvadori 2008], which strongly supports our model.

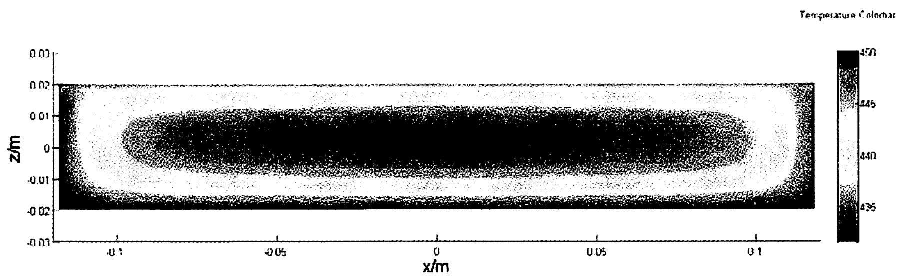


Figure 6. The temperature distribution of a vertical cross section of the brownie.

The Horizontal Cross Section

We study a horizontal cross section of the batter, which bears the same shape as the pan. The layer satisfies the equation

$$-k_2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T=Q,$$

with the boundary condition

$$T|_{\partial G}=T_e\ ,$$

where G represents the rounded rectangular region.

Unlike for the vertical cross section, the temperature distribution across the outer edges in a horizontal cross section fluctuates with changes in shape. We consider three typical shapes in **Table 4**, and plot their temperature distributions and heat flux distributions.

Table 4.
Parameters of three typical shapes (in meters).

	l	w	r
Square	0.2286	0.2286	0.0030
Circle	0.2578	0.2578	0.1286
Rounded rectangle	0.2580	0.2186	0.0695

The heat flux distribution of the three shapes clearly reflects how uniformly heat distributes across the outer edge. What distinguishes the square from the circle is that heat is concentrated at the corners of the square, while for the circle heat distributes uniformly across the outer edge.

Optimizing the Pan Shape

Task 1: Maximize the Number of Pans

Given a constant area A, the pan shape parameters w, l, and radius r of the corner arc are interrelated, which means we can derive the third if any two parameters are given. We choose $l \geqslant w$ because the length and width are symmetric. If we can establish a rounded rectangle, three geometric restrictions are implied:

$$\begin{cases} l \geqslant w, \\ 0 \leqslant 2r \leqslant \min\{w, l\}, \\ wl - r^2(4-\pi) = A. \end{cases}$$

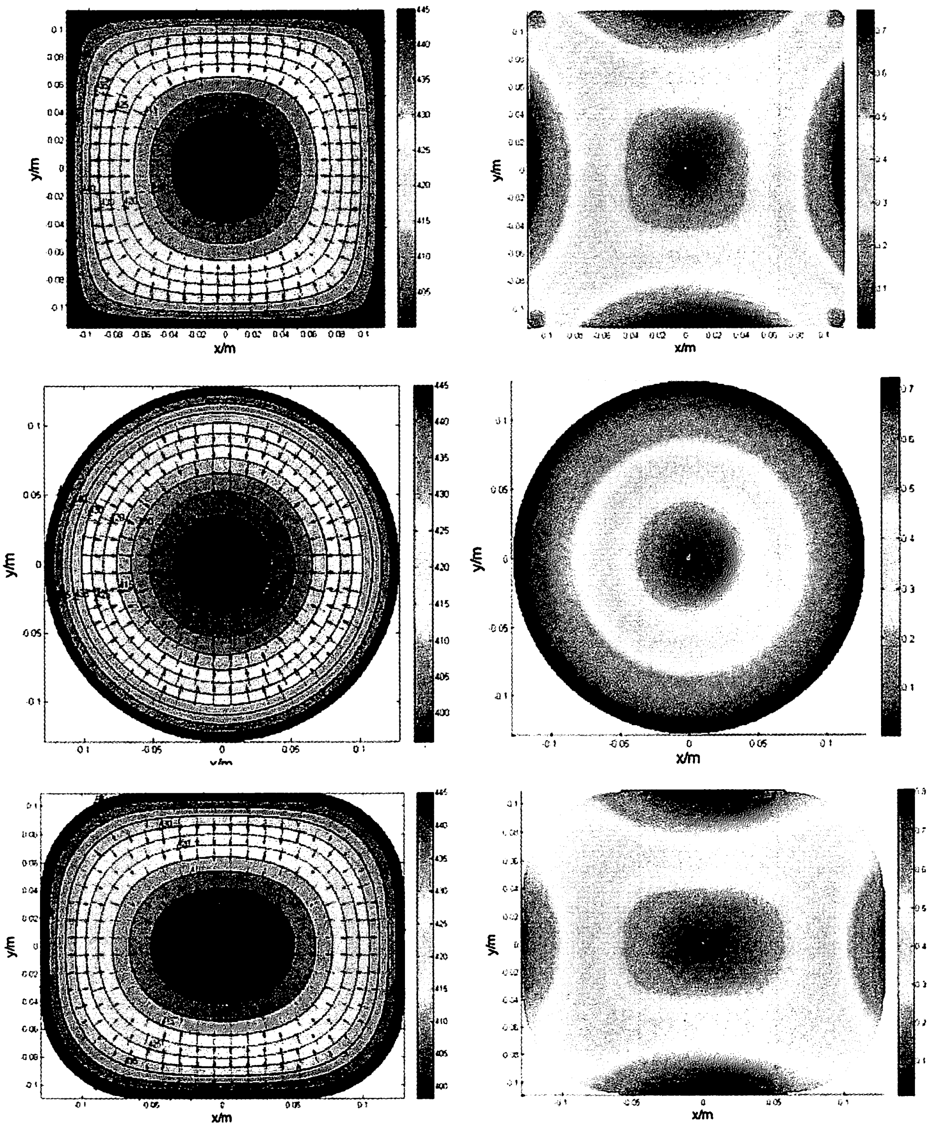


Figure 7. Temperature distribution (left) and heat flux distribution (right) of three typical shapes in **Table 5**: Square (top), circle (middle) and rounded rectangle (bottom). Arrows represent heat flow.

Representing r by w, l, we derive the domain Ω of (w, l):

$$\Omega = \left\{ (w, l) \in \mathbb{R}^2 : wl \geqslant A, \quad l \geqslant w, \quad 2\sqrt{\frac{wl - A}{4 - \pi}} \leqslant \min\{w, l\} \right\}.$$

which is demonstrated in **Figure 8**. The points on the visible branch of the hyperbola represent rectangles, which are special cases of rounded rectangles.

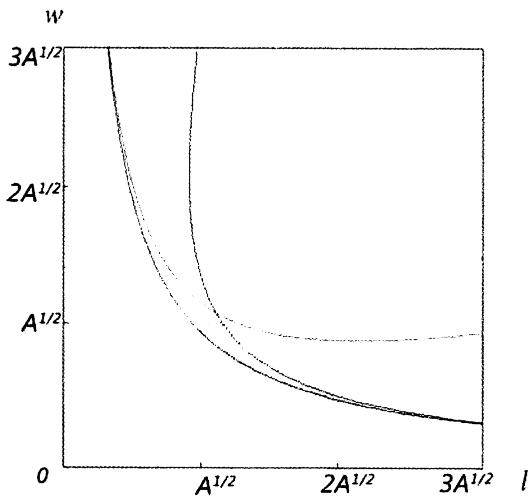


Figure 8. The domain Ω of the parameters w and l, in light cyan (shaded).

We proceed to calculate N, the largest number of pans that can fit in the oven. And don't forget that we have two racks!

$$N = 2 \max_{(w,l)\in\Omega} \left\{ \left\lfloor \frac{W}{w} \right\rfloor \cdot \left\lfloor \frac{L}{l} \right\rfloor, \ \left\lfloor \frac{W}{l} \right\rfloor \cdot \left\lfloor \frac{L}{w} \right\rfloor \right\}.$$

We use MATLAB to find the maximum of N by searching in Ω. Interestingly, without the deduction above, it is easy to recognize that $N = 2\lfloor \frac{WL}{A} \rfloor$, obtained when $r = 0$ and the length of one side of the pan equals W or L.

Task 2: Maximize Uniformity of Heat Distribution

We define a parameter to measure the distribution of heat for the brownie, which is solely determined by the geometry of the aluminum brownie pan. Considering that a brownie tends to be overbaked on the edges and corners, we can reasonably focus our attention on the periphery of the brownie. We define the parameter as

$$\mu = \frac{\min\limits_{(l,w)\in\partial G}\{|\nabla T|\}}{\max\limits_{(l,w)\in\partial G}\{|\nabla T|\}}$$

where ∂G represents the periphery of the brownie.

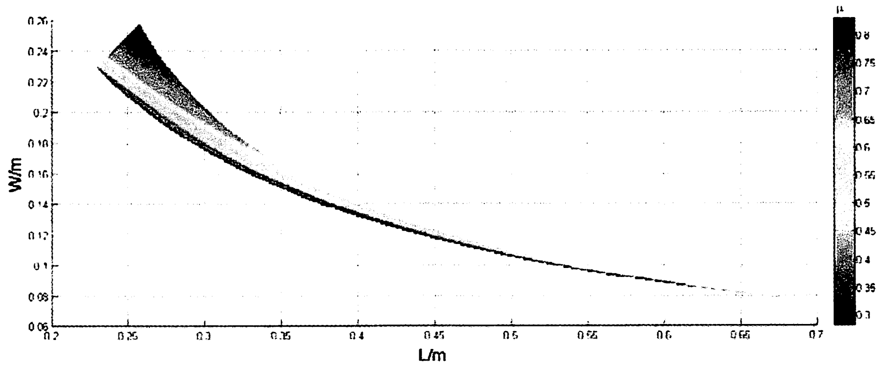


Figure 9. Value of μ in the domain.

In practice, we apply the finite difference method to the temperature field that we get from FEM calculation and get the distribution of the gradient of the temperature. In fact, according to thermodynamics, the negative gradient of temperature is proportional to the heat flux of the temperature field, and it describes the tendency of temperature change. Since the boundary condition requires the brownie edge to share the same temperature as the oven, the smaller the heat flux of some point on the edge is, the hotter the area around that point. Therefore, areas that have a smaller heat flux will be overbaked more quickly. Intuitively, for a circle, $\mu = 1$, while for a rectangle with sharp edges, $\mu \sim 0$; our calculation in **Table 5** confirms these speculations.

Table 5.
Parameter μ for three typical shapes.

	l	w	r	μ
Square	0.2286	0.2286	0.0030	0.09
Circle	0.2578	0.2578	0.1286	0.98
Rounded rectangle	0.2580	0.2186	0.0695	0.64

We still have to fit the function between μ and (w, l), because the size of samples is limited and any later optimization requires high resolution in the (w, l) plane. Moreover, since μ is derived from the solution of a partial differential equation with irregular boundary conditions (compared with rectangles or circles), the relation between μ and (w, l) is highly nonlinear. Based on these two considerations, we introduce the MATLAB Neural Network Toolbox, which can tackle nonlinear fitting problems with a large number of arguments.

In this context, we construct a two-layer feed-forward neural network with sigmoid hidden neurons (10) and linear output neurons (1), which is illustrated in following **Figure 10**.

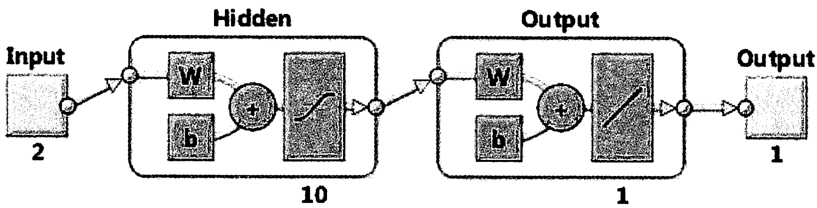


Figure 10. The structure of the neural network.

Moreover, we train the network with the Levenberg-Marquardt back-propagation algorithm and divide the sample into three parts for network training (70%), validation (15%), and testing (15%). The performance of the neural network is measured by the iteration-dependent Mean Squared Error (MSE), Error Histogram (which shows how the error sizes are distributed), and Regression Diagrams (which show the actual network outputs plotted in terms of the associated target values). All these three diagrams prove that the neural network has successfully fitted the function and is ready to be used to generate high-resolution (w, l) samples. Here we provide only the Error Histogram (in **Figure 11**) for conviction.

Task 3: Optimize Combined Conditions

We use the function fitted by the neural network to compute the best pan shape parameters for different levels of p and W/L. Meanwhile, we fix the oven area just as we fix the pan area. The result is illustrated in **Figure 12**.

We observe that when p increases and we are more concerned with the number of pans, the optimal shape is a long rounded rectangle. When more weight is assigned to the even distribution of heat, we obtain circular pans. So p strikes a balance of the two considerations. With regard to W/L, we find that for the upper half of the square, the ratio w/l after optimization correlates strongly with W/L.

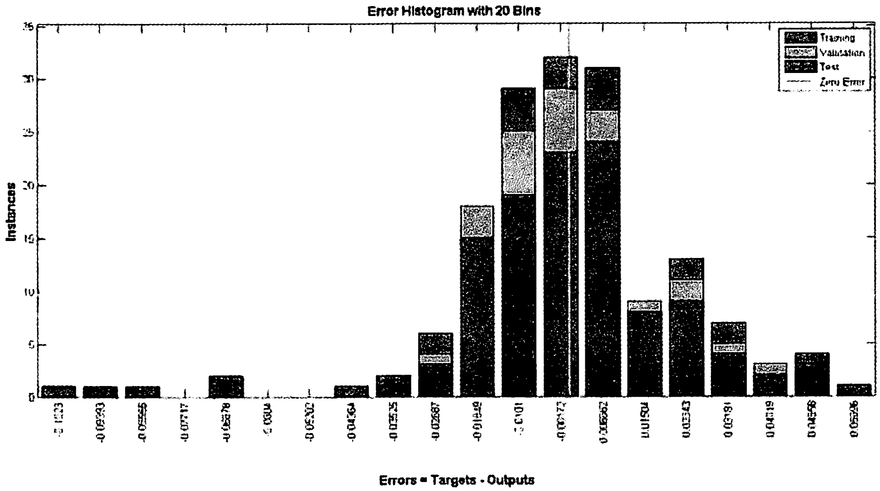


Figure 11. Error Histogram for $A = 0.05225\ \mathrm{m}^2$.

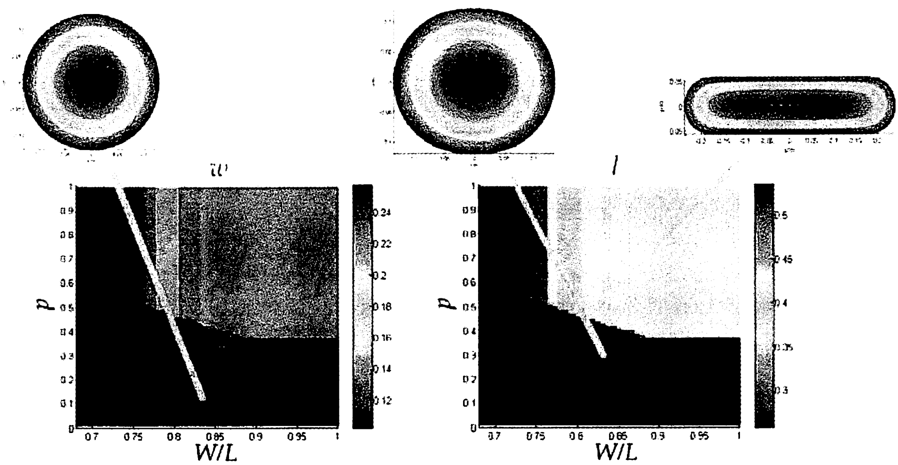


Figure 12. Optimized pan shape parameters w and l, with respect to p and W/L.

Analysis of the Results

Sensitivity Analysis

In our model of baking brownies, we assume that the horizontal section area of the oven (A_2) and the areas of a pan (A_1) are kept constant, while the heat drain Q inside the brownie, the geometry (w, l) of the pans, and the aspect ratio (W/L) of the oven are allowed to change. In this section, we undertake sensitivity analysis by allowing previously fixed qualities to change while fixing the previous variables.

To begin with, we study the influence of the heat drain (Q) inside the brownie on the temperature distribution indicator (μ) during FEM calculation. Remember that Q is a constant during calculation; we set Q from -3000W/m^2 to -8000W/m^2 and run the FEM. All the other parameters, such as A_1, A_2 and the thermal parameters, are set to standard values. We list in **Table 6** the central temperature T_c and μ for each Q.

Table 6.

Sensitivity analysis of Q.

Q (W/m^2)	T_c (K)
−3000	420
−3500	415
−4000	410
−4500	405
−5000	400
−5500	395
−6000	390
−6500	385
−7000	380
−7500	375
−8000	370

Surprisingly, we notice also that μ does not change with Q, while the central temperature T_c depends almost linearly on Q. Hence, we can safely employ Q as an adjustable parameter to achieve a reasonable temperature distribution while at the same time securing our definition of μ as the indicator of uniformity of temperature distribution on the periphery of brownie.

Secondly, we keep the aspect ratio of the oven constant while adjusting its horizontal section area from $0.15\ \text{m}^2$ to $0.25\ \text{m}^2$. (Remember that the standard value is around $0.20\ \text{m}^2$.) Doing this, we can test the influence of oven size on the geometry optimization of pan. We keep $A_1 = 0.05225\ \text{m}^2$, which is the standard value, and utilize the same neural network to finish our optimization. Results are listed in **Table 7**.

The results are reasonable. When we employ a larger oven, the optimized number of pans increases from 4 to 8. Along with the jump of

Table 7.
Sensitivity analysis of A_2.

$A_2(\text{m}^2)$	$l_{\text{opti}}(\text{m})$	$w_{\text{opti}}(\text{m})$	N_{opti}	μ_{opti}
0.15	0.291	0.211	4	0.73
0.16	0.436	0.122	6	0.46
0.17	0.449	0.123	6	0.66
0.18	0.459	0.12	6	0.66
0.19	0.466	0.118	6	0.67
0.2	0.466	0.118	6	0.67
0.21	0.466	0.118	6	0.67
0.22	0.508	0.107	8	0.63
0.23	0.261	0.219	8	0.66
0.24	0.267	0.224	8	0.73
0.25	0.272	0.229	8	0.77

N_{opti}, we also notice a sudden change in $(l_{\text{opti}}, w_{\text{opti}})$ and in μ_{opti}. Besides, as the oven becomes larger than $0.23\ \text{m}^2$, the optimized pan becomes gradually more circular with higher μ_{opti}, while the value of μ_{opti} for A_2 in the range $[0.17,\ 0.21]\ \text{m}^2$ is relatively stabile (at about 0.67), which makes our standard optimization a stable solution.

Finally, we change A_1 while fixing all the other parameters, to test the influence of pan size on the optimized geometry. We set A_1 to values in $[0.04,\ 0.06]\text{m}^2$ and run the FEM. After that, we undertake the neural network fitting procedures to achieve an accurate optimized result. Results are listed in **Table 8**.

Table 8.
Sensitivity analysis of A_1.

$A_1(\text{m}^2)$	p	$l_{\text{opti}}(\text{m})$	$w_{\text{opti}}(\text{m})$	N_{opti}	μ_{opti}
0.03	0.5	0.487	0.063	12	0.80
0.035	0.5	0.215	0.204	8	0.92
0.04	0.5	0.241	0.202	8	0.80
0.05	0.5	0.346	0.160	6	0.64
0.0511	0.5	0.377	0.148	6	0.66
0.052	0.5	0.449	0.123	6	0.66
0.05225	0.5	0.370	0.155	6	0.65
0.053	0.5	0.392	0.147	6	0.67
0.0541	0.5	0.372	0.160	6	0.65
0.055	0.5	0.377	0.160	6	0.60
0.06	0.5	0.473	0.135	6	0.63

Unfortunately, there seems to be no obvious dependence of optimized geometry $(w_{\text{opti}}, v_{\text{opti}}, \mu_{\text{opti}})$ on A_1. Quite interestingly, we can find a local maximum for μ near the standard value, that is , $A_1 = 0.05225\ \text{m}^2$ and such a result could somehow justify our choice of this standard value. Moreover, we can still confirm the validity of the results because smaller

A_1 gives larger N_{opti}, as expected.

Strengths and Weaknesses

Strengths

- We systematically and correctly estimate the order of magnitude of various physical quantities such as Q, τ, and h_1. Through these reasonable estimations, we get a clear physical picture of the problem.
- We find values for the unestimated physical quantities in our model in the literature or at reliable Websites.
- In our model, we reduce the 3D time-dependent thermodynamic and fluid dynamics problem inside the oven into a 2D stationary heat equation with carefully checked boundary condition for brownie. Such a treatment greatly facilitates our numerical calculations and physical analysis.
- We employ various powerful MATLAB toolboxes to facilitate our investigation, and mature and user-friendly methods, such as the finite element method and neural networks provided by MATLAB, give us reliable methods of analysis.
- Throughout our paper, we draw many vivid and informative pictures to illustrate our conclusions.
- We provide a well-organized sensitivity analysis to further verify our model.

Weaknesses

- Our model omits a lot of potentially important parameters, such as the complex thermal convection of air, vapor emission, and mass loss of the brownie, as well as the contribution of radiation.
- Although dimensionality reduction greatly facilitates our calculation, it also brings us difficulties in determining the boundary condition and inner thermal conductivity of the brownie, as revealed in our calculation of the vertical temperature distribution in the brownie.

Conclusion

We base our models on thermodynamics and geometric conditions, and we combine them to determine the optimal pan shape in general situations.

- Before investigations into the brownie batter, we use concrete deduction to prove that the air temperature and the pan temperature are the same.

- Our estimations of parameters in the equations have been carefully checked to ensure that the model reflects the phenomenon in the real world.
- We reduce the dimension of the heat balance problem of the brownie by considering its two dimensional cross section so as to suit our problem to the MATLAB PDE Toolbox.
- We formulate a measure of how evenly the temperature is distributed on the edge by observing the heat flux distributions in various typical shapes.
- We exploit the full potential of analyzing figures to discover underlying principles.
- We explore how our model responds to small changes and assess the advantages and disadvantages of the model.

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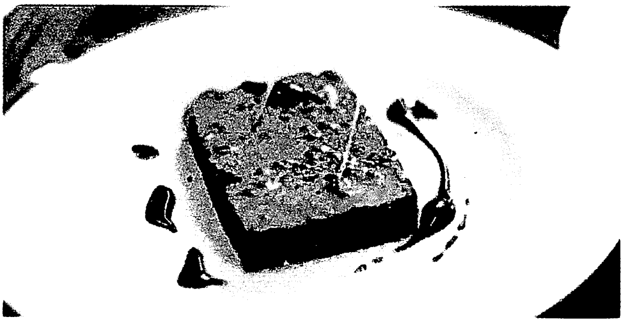
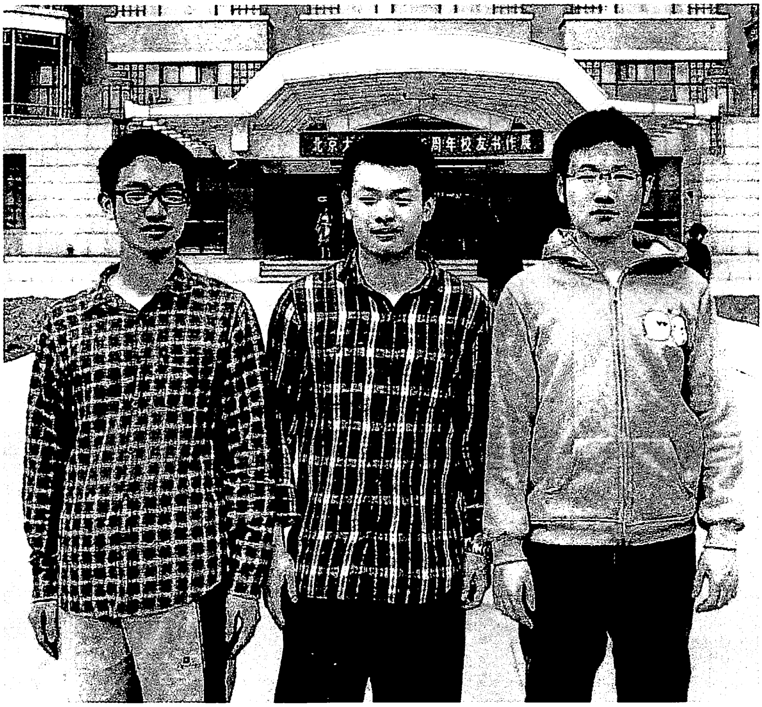
Custom Made Pan

Technology Makes Us The Best



No need to worry about the trade-off between quality and quantity. You can have the BEST OF BOTH WORLDS! Geometric design combined with thermal engineering is guaranteed to satisfy everyone's preferences.



Team members Chong Jin, Meng Wu, and Bowen Liu.

Judges' Commentary: The Ultimate Brownie Pan Papers

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Introduction

The focus of this year's problem was how to balance the compromise between quality versus quantity. The teams were asked to determine the best way to strike the balance between the two competing interests for a given compromise. The context for this decision was to design the pans used to bake brownies.

On the side of quality, the pans themselves should be designed so that the temperature distribution of the brownie mix should be as uniform as possible. The goal was to reduce the large temperature gradients that can occur near the corners of rectangular pans. For this aspect of the problem, a circular pan is the best choice to achieve the most even heating and the best brownies.

On the side of quantity, the pans should be designed to reduce the amount of empty space on one of the racks in the oven. The goal here is to put as many pans as possible into an oven that has a rectangular shape. For this aspect of the problem, the best choice, as a way to achieve the least amount of empty space, is a pan with an aspect ratio that matches the aspect ratio of a horizontal cross section of the oven.

In this commentary, we first describe the judging process, then discuss the three questions that the teams were asked to address. The next topic is the issue of sensitivity and assumptions, followed by a discussion on identifying the strengths and weaknesses of a given approach. Finally, we give a brief discussion on the difference between references and citations.

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The Process

I give an overview of the judging process, which has two primary activities: the triage rounds and the final rounds. The papers that are to be judged to be the best must make it through both sets of rounds, and the criteria used to identify good papers gradually change as the judging progresses. The papers that make it to the final stages of the judging must stand out as the best under a wide variety of criteria.

Triage

The first rounds of the judging are referred to as the *triage rounds*. The primary idea in these initial rounds is to identify the papers that should be given more detailed consideration. Every paper is read at least twice. When reading a paper, a judge's primary question is whether or not the paper contains all of the necessary ingredients that make it a candidate for the most detailed readings.

In these initial rounds, the judges' time is constrained, and we try to give a paper the benefit of doubt. If a paper addresses all of the issues and appears to have a reasonable model, then the judge will likely identify it as a paper that deserves more attention. Some papers in these early rounds receive a mixed review. In those cases, the paper will be read by other judges with the intent to give the paper every chance possible.

In these early rounds, it is most important that a paper be clear and concise. It must be clear what the approach is, and the results should be clearly spelled out. In these early rounds, the importance of the summary is accentuated. In particular, a good summary should have a brief overview of the problem, plus an overview of the paper and approach that the team will discuss, and specific results should be stated or expressed in some way. There should be no surprises as a judge reads a paper.

Small things that can make a difference in the earlier rounds include having a table of contents. It makes it much easier for a judge to look at the paper and decide what to expect while reading the paper.

It is also important that all of the questions asked be answered. If a paper does not address all of the questions, then the judge is more likely to decide that a team's efforts do not compare well with the better papers.

Finally, it is vital that the team express their general approach and results as clearly and concisely as possible. This means providing in the introduction a broad overview of the problem, the paper, and specific results. Additionally, at the start of each section, the team should provide an overview of the section itself.

These small things will make it much easier for a judge to identify what the team did, and will increase the probability that the judge will correctly identify and reward good work. It is important to remember that the competition is about the *process* of modeling and is also about *expressing* the work done. The best models and the best effort are not effective if the results are not adequately communicated.

Final Rounds

For the final rounds of judging, the judges all assemble in the same location. As the rounds progress from the earlier readings, the judging criteria slowly shift away from identifying papers that need further consideration to a process of trying to identify the very best papers. As this transition unfolds, it becomes even more important that the judges are working together and reach an understanding of what is expected.

The first round in the final set of readings begins with a meeting of the judges. In this meeting, the problem is discussed, and the judges share what they think are the key aspects of the question itself. Then each judge reads a large number of papers. These papers have been given a variety of scores ranging from low to high in the previous judging rounds with a relatively uniform distribution.

After examining these papers, the judges get together again and discuss what they think should be included in a "good" paper. The judges are well aware that the teams have limited time to complete their activities, and the purpose of this extra step is to ensure that judges adequately compensate for the limitations and restrictions imposed on the teams. This competition is about rewarding the teams' efforts and recognizing excellent work under tremendous limitations, and we make every effort to consider this situation from the students' point of view.

Once the judges agree on a set of minimal criteria, the final rounds begin in earnest. Each paper is read multiple times. As the rounds progress, the number of papers is decreased, and the entries come under increased scrutiny. Also, the time dedicated to reading each paper steadily increases.

In the very last rounds, the papers that remain are given the highest levels of attention. The amount of time that a judge spends on reading the paper increases, and multiple judges can be reading copies of the same paper at the same time. By this time, the papers that remain generally have excellent summaries and are well-written. The judges are then able to focus almost solely on the modeling process and the mathematical integrity of a paper.

The Questions

This year's problem can be boiled down to three different questions.

- The first question is to determine the heat distribution of the brownies for a given shape of the pan.

- The second question is to determine the best way to load the oven with pans of a given shape.
- The last question is to determine the best pan shape and oven configuration, given the relative trade-off between the quality of the brownies and the quantity that can be made.

We examine each of these three questions separately. We first examine the heat distribution, then address the question of how to best load the oven, and finally examine the best way to determine a compromise.

Heat Distribution

Determining the heat distribution of the pan of brownies after it has been placed in the oven is inherently a question of a relationship in space that changes in time. In particular, the temperature is assumed to change in time and vary with position in the pan.

The most common model for this was the heat equation. The primary question that resulted was whether or not to approximate the brownie mix as a two-dimensional object or use a three-dimensional approximation. Given the time and computational restrictions, either approach was OK; but a team should have provided some kind of discussion as to why they made the decision that they did.

Some teams started with the heat equation and added a body force based on Newton's Law of Cooling. A couple of the Outstanding teams did this. The judges recognized that this causes a significant problem with the model; but given the nature of the mathematics and the short time span, we decided that the balance of the team's other efforts could offset this error.

For most teams, the modeling efforts tended to be on the boundary conditions. Some sort of mixed boundary conditions are most appropriate, since the heat flux through the sides of the pan contributes to the primary exchange of energy between the oven and the brownie mix. Many teams used Dirichlet boundary conditions, and this was acceptable as long as the team provided some kind of justification for their decision.

Another aspect to the problem was how to decide what shape to use for the pan. The majority of entries took one of two approaches. The first approach was to use a regular polygon with n sides and increase n as a way to reduce the temperature gradients that occur at corners. Another common approach was to use rectangular pans with rounded corners. Some groups made use of elongated ellipses or other shapes, but they were in the minority.

The judges did not place any kind of preference on the decision about shape. The primary judging emphasis for this aspect of the problem was to determine if the shape of the pans was adequately described and whether or not a team remained consistent with the other two questions associated with the problem.

Nonrectangular and the noncircular pan shapes required the teams to approximate the heat equation. This was done in a wide variety of ways. Many teams used a finite element approximation and either MATLAB [2013] or COMSOL Multiphysics [2012]. Other teams wrote their own finite-difference approximations; and some teams made up their own technique, of which cell-averaging techniques were the most common approach.

Finally, another aspect of this problem is that the teams were required to define how to measure the temperature variation. A wide variety of techniques were used to determine this aspect of their approximation. A large number of teams simply said that they would use the "standard deviation" of the temperature. This approach was problematic, since it is not clear what this term means in a nonstochastic situation and what it means in terms of interpreting the numerical approximation.

Some teams found ways to use the gradient of the temperature distribution. Some teams just focused on what they decided were critical locations within the brownie mix. The judges did not attribute different weights to the different approaches. The primary criterion was that the technique employed was clearly stated and used in a consistent manner throughout the paper.

Loading the Oven

Another critical aspect of the problem was to decide how to arrange the brownie pans within the oven. The pans had to be distributed on the racks within the oven, and the horizontal cross-section of the oven is assumed to be rectangular with a predetermined aspect ratio. Trying to arrange oddly shaped pans in this context is an extremely difficult problem.

The majority of teams tried to employ a pre-existing technique to this part of the problem. There are many "off the shelf" approaches that are available. Many teams simply tried to use an existing pallet-loading approach. Other teams attempted to make up their own technique or simply made gross approximations.

The judges made every effort to take into consideration the extreme time constraints associated with the event. The judges did not impose any preference over the different approaches employed here. Again, the primary concern was whether or not the method was described well and implemented in a way consistent for the third question.

It was also important that the teams adequately describe the results for the packing algorithm that they employed. For this purpose, a figure was a tremendous aid in conveying to the readers what the team had implemented and what their results were. Additionally, many teams included flowcharts as an additional way to share their algorithm.

The use of figures is a great help for the judges in trying to decide what the team has decided to do, and to help decide if it makes sense and is a reasonable approach. A key aspect of this contest is to communicate complex ideas, so teams should know how to use figures and properly place them in a document.

Simple things like proper labels make a huge difference in figures. Also, every figure should include a brief description to inform the reader what it is and what to look for. Finally, every figure should be discussed and described in the narrative; otherwise the reader has no reason to look at the figure. The reader should know exactly what the team thinks is most important and revealing before searching for the figure.

The Compromise

The last question was how to decide on a shape for the pan, and an arrangement of the pans within the oven, for a given compromise between quality and quantity. This aspect of the event turned out to be one of the parts that helped differentiate the very top papers. The higher-rated papers clearly described what the teams did and clearly stated their conclusions.

The teams had to describe how they determined the balance between the two competing interests. In the original problem statement, a ratio p is briefly stated to help motivate the idea of the compromise. Many teams simply assumed that this ratio was understood and did not do a good job of explaining what it meant to them. Since it was primarily a general way to motivate the idea, the ratio was not clearly defined in the original problem statement, and most teams had a very different idea about what it meant for their approach.

For most of the papers, it was difficult to determine how the team interpreted what they thought the ratio p meant to them. A clear statement of this part of the problem was a great aid to the judges as they read through the results.

Finally, the teams were expected to state their conclusions explicitly for varying compromises between quantity versus quality. This meant for a given balance that the team should provide some way to determine what shape pan to use and a way to place the pans in the oven. Most teams had a difficult time trying to convey this complex piece of information. Those teams that were able to discuss this key piece of information in a consistent and clear manner were more likely to be noticed by the judges and recognized for their efforts.

Sensitivity and Assumptions

Every year, when the judges get together and discuss the what we think is important in a paper, we discuss assumptions and sensitivity. The basic assumptions that a team makes is the starting point for all of their efforts. We do not place many restrictions on the basic assumptions but simply ask that they make sense on some basic level.

Once those assumptions are made and clearly stated, it is expected that everything else in the paper remain consistent with the assumptions. That is a key component of the modeling process. It is important to recognize, however, that those assumptions are also a restriction and should be explicitly examined in a structured and coherent manner.

This year, the teams had to make a large number of assumptions and decisions. This is especially true in light of the computational nature of the first question, about heat distribution.

The issue of examining what happens when a small change in the assumptions takes place is a vital part of the modeling process. This year, the teams had to make decisions about a wide variety of parameters. For example, the teams had to decide the coefficient for heat conduction, how long to bake the brownies, the temperature of the oven, the coefficients associated with the boundary conditions, and a wide variety of other considerations.

The basic question that the teams should have asked is *what happens to their conclusions if they make a small change in any one assumption or make a small change in any one parameter.* If the result is a big change in one of their conclusions, then the team should indicate that as a potential weakness or at least something that deserves more consideration.

For example, the team might try to change the aspect ratio in their oven by a small percentage. The team could then ask if there are a range of values for the aspect ratio that then result in a large change with respect to the recommended shape and loading configuration. The teams could then repeat this exercise for other small changes as well.

We understand that the teams are highly constrained with respect to time and resources. We do not expect the best possible model, but we do expect the teams to explore the models that they develop in a structured way and ask critical questions associated with the models. The model itself is important, but the analysis of the model is also important.

Strengths and Weaknesses

Teams are expected to include a critical analysis of their modeling efforts. The teams should take the time to inform the judges what aspects they think are best about their effort and also what aspects they think should receive further attention. No model is perfect, and the limited time available to the teams to complete their efforts represent a considerable constraint on what they can do.

A vital part of the process of mathematical modeling is to step back and ask basic questions about the model. This ranges from examining the basic assumptions that are made to an examination of the techniques employed in the model itself. When a team explicitly provides a clear and honest examination of its own work, then it is demonstrating that they understand this basic part of the modeling process.

Citations Versus References

Finally, we give a brief word about citations and references. Over the history of this event, the teams have greatly improved on their use of references in their submissions. Most teams clearly indicate a list of resources that they have consulted in their efforts.

The use of citations, though, has not been quite as popular an activity. While we appreciate the greater use of a section of references that lists the resources used, that list is not much help if the teams do not provide citations within the narrative to indicate which resource they used for a particular idea being expressed. A team that provides proper citations that are also coupled to complete references immediately sends a message to the reader that they have paid attention to important details and are taking a responsible and professional approach to their efforts.

This issue is especially important with respect to figures and images. If a paper includes an image that is clearly copied from a source, then the paper should include a citation, together with a reference associated with the citation. The use of an appropriated image without a citation is plagiarism, and in many cases this stands out to the judges reading the papers.

Conclusions

This year, the teams were asked to determine how to balance the competing interests of quantity versus quality. They were asked to determine the best way to design a brownie pan and pack multiple pans in an oven for a given balance between the two considerations. To do this, the teams had to determine the heat distribution within a brownie mix in a pan with a complex shape. They also had to determine how to load those same pans in an oven to maximize the number of pans that could be placed in the oven. Finally, they had to determine the shape and the loading pattern to achieve a specific balance between quantity and quality.

These tasks resulted in a computationally intensive set of activities. The modeling ranged from relatively straightforward (the heat equation with attention to the boundary conditions) to extremely complex (the pallet loading problem). The teams had to bring these two approaches together and provide specific recommendations, and they had to convey complex results within their narrative.

The teams that were able to provide solutions to all three problems and provide a balanced and consistent approach were rated highest by the judges. The exercise required the teams to coordinate the competing demands of the complexity of the models developed with the time and insight required to perform an analysis of their own models. As is usually the case, the most sophisticated modeling did not necessarily result in the highest ratings; but the teams that were able to find the best balance between the model, the analysis, and clarity of their presentation received the highest ratings.

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About the Author

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My Decade with the MCM

Xiaofeng Gao
Guoliang Wu
Meng Wu
alumni of Nankai University
Tianjin, China

Overview

A decade ago, Xiaofeng Gao, Guoliang Wu, and Meng Wu were senior undergraduate students in Nankai University (NKU) in Tianjin, China. All of them majored in mathematics. They first teamed up in May 2003 to contend in the Nankai University Mathematical Contest in Modeling and won the first prize. They then participated in the China Undergraduate Mathematical Contest in Modeling (CUMCM) in September 2003 and won the second prize in Tianjin Province. In February 2004, they competed in COMAP's Mathematical Contest in Modeling (MCM)®, becoming a Meritorious Winner out of 599 teams from all over the world.

Figure 1. The 2004 Meritorious team: Meng Wu, advisor Prof. Qingzhi Yang, Guoliang Wu, and Xiaofeng Gao.

The UMAP Journal 34 (2–3) (2013) 151–161.

In July 2004, all three graduated from Nankai University and drifted apart.

During the decade since then, Xiaofeng Gao received a master's degree from Tsinghua University in 2006 and a Ph.D. degree from the University of Texas at Dallas in 2010. Currently she is an associate professor in the Dept. of Computer Science and Engineering, Shanghai Jiao Tong University.

Guoliang Wu completed his Ph.D. degree from the University of California, Berkeley in 2009 and had a post-doc position at the University of Texas at Austin from 2010 to 2012. Currently he works at J.P. Morgan Chase & Co. in New York.

Meng Wu got his Ph.D. degree from Nankai University in 2009 and joined Tianjin Normal University as an assistant professor.

Both Xiaofeng Gao and Meng Wu have become advisors for COMAP's MCM competitions.

Part I: Xiaofeng Gao's Experience

Figure 2. Xiaofeng Gao.

It has been more than 10 years since I first learned about COMAP's Mathematical Contest in Modeling. At the beginning of 2002, I participated in a table tennis mixed doubles competition in our university; surprisingly, my partner Zhigen Zhao asked me whether I would like to take part in COMAP's MCM in 2003. At that time, I was a sophomore and he was a junior. He had already participated in the MCM in 2002 but did not get a good result, so he wanted to form a new team and try it again. Without hesitation, I joined his team; I already knew the third team member, Zeng Lian, who was a classmate of Zhigen Zhao and also a junior student in the NKU School of Mathematical Science.

We started our journey with the Nankai University Mathematical Contest in Modeling in May 2002, which is used to select teams for the China Undergraduate Mathematical Contest in Modeling (CUMCM). In this competition, we chose an agricultural problem about a locust plague, in which we were asked to design an efficient and economic method to control the plague and reduce the damage. We made a thorough survey about locust plagues from the Internet and summarized several popular methods to control the disaster. After the survey process, we found that different methods have different emphases and it is not easy to choose one intuitively. With the knowledge from our optimization class, we interviewed some experts from this area, got their comments, and applied the Analytic Hierarchy Process (AHP) to achieve a scalable operation expression. According to the situation and localized information, our model can provide a best suggestion to resolve locust plague.

That was the first time that I completed a modeling contest. I thank my team members, who had rich experiences in modeling contests; they taught me almost everything about contests: how to formulate the problem, find appropriate methods, evaluate our design, and write a research paper. Our model was simple but efficient, and thus won a high score in the competition. Our advisor even recommended that we modify it as a research paper, which resulted in our first professional publication [Gao et al. 2002]. Though this paper presents a simple result, I consider it one of my greatest achievements in college—not only because I got my first success in beating a practical challenge, but also because I found where my interest lies: I was indeed captivated by the marvelous process of modeling and the application of mathematics.

We performed so well in this competition that every person around us had high expectations for us in upcoming competitions. However, life is not always a smooth path. In 2002 and 2003, we participated in modeling contests several times, including the China Undergraduate Mathematical Contest in Modeling (CUMCM) in September 2002 and COMAP's Mathematical Contest in Modeling (MCM) in February 2003.

But each time, we faced difficulties. For instance, when we were working on a probabilistic lottery problem in CUMCM 2002, we could hardly propose an effective method to evaluate a sales strategy, due to our lack of knowledge of stochastic decision analysis and marketing evaluation. When we dealt with a surgical problem about gamma-knife treatment planning in the 2003 MCM, we found that English writing became our biggest challenge. I still clearly remember that it was my first time writing a paper in English, which was almost an impossible mission. I could hardly express my ideas clearly, let alone finish a structured and complete report. Finally, we didn't get good awards in either of the games. Frankly speaking, such results were not unexpected but nevertheless really discouraging.

I must say that, because of the two failures, I began to know myself well, and I learned from mistakes. First, I gradually found my strengths and weaknesses. I was actually more "sensitive" to discrete problems than to continuous problems, especially with deterministic variables. This understanding of myself influenced my life greatly: I changed my major from computational mathematics to computer science when I pursued my doctoral degree, and my research interests are optimization problems in data engineering and data communication, which are exactly discrete optimization problems with fixed constraints.

Second, all of us improved our English greatly, which gave us a lot of help in applying for future education abroad. Zhigen got an offer from Cornell University in New York one year later and completed his Ph.D. degree in mathematics in 2009. He is currently an assistant professor in the Dept. of Statistics of Temple University in Pennsylvania. Zeng Lian received his doctoral degree in mathematics at Brigham Young University in Utah in 2008 and then was a postdoc at New York University from 2008 to 2011 and a visiting professor at University of Massachusetts Amherst from 2011 to 2012. He is currently a lecturer in Loughborough University in the United Kingdom.

Let us return to 2003 and continue my story. The failures during this period not only pushed us to improve ourselves but also increased my desire to compete for the highest award. Thus, I hoped to take part in the competition again the following year. Since Zhigen Zhao and Zeng Lian would graduate soon, I had to form a new team. This time, I invited Guoliang Wu, who ranked first among all the junior students in the Dept. of Mathematics. He also had the highest GRE score in my grade, which completely resolved the English problem in our team. More excitingly, he brought his friend Meng Wu, who was a smart student in his class and always proposed novel ideas for various questions.

Like a new iteration, we first participated in the Nankai University Mathematical Contest in Modeling in May 2003. The topic was constructing an evaluation scheme to rank top universities in China. The problem statement did not specify how we should evaluate a university, which made the evaluation criteria quite flexible. After careful investigations, we found that the most challenging thing was indeed how to convince our reviewers that our indices were reasonable and effective. To design a reliable scheme, we first collected historical data from several statistical year books and then restricted our discussion to only one aspect: the quality of the faculty resource, since few discussions were focusing on this index at that time. Instead of directly choosing criteria, we implemented a Delphi technique, selected 20 indices from the original 32 in the arithmetic average method, and then treated these indices in principal components analysis to find 8 principal components. Then we assigned the 20 indices to the 8 principal components according to orthogonality with some calculations, and finally constructed the evaluation system.

During this contest, we became more familiar with one another and much clearer about our individual responsibilities. I was good at analyzing a problem from different points of view and providing a comprehensive discussion; Guoliang Wu was an expert at solving problems, with a solid mathematics background; Meng Wu was really a smart researcher with inconceivable creativity and perceptive insight—he could always propose simple but efficient solutions. We also became friends during that period, which is one of the most brilliant moments in my memory.

Three months later, we took part in the CUMCM, in September 2003. This time, we chose a truck scheduling problem for an open-pit iron mine with a bunch of constraints. We almost immediately constructed a linear programming model to solve the problem, and used Lingo to get a numerical solution. In our solution, we applied the idea of partition according to geometrical characteristics to reduce the problem size. We then developed a small-scale exhaustive search algorithm, bounded by polynomial time, to find a local optimal partial assignment to the linear programming problem, and then solved the whole problem. With the help of fixed parameters, we could get a group of acceptable answers and select the optimal one.

However, such a smart design did not result in a good award, either. A faculty member in our department, also one of the judges for this contest, told us that the only reason we did not get the best score was because we had left out one of the subquestions in the statement. We could not believe that all three of us had ignored this subquestion! However, we did lose out on an award because of this small carelessness. This lesson was so profound that even now, when I want to start my work, I will check the instructions at least twice to avoid such avoidable problems.

With successes and failures, with training and competitions, we participated in the MCM in 2004. It was my sixth competition, and also my last chance to participate in a modeling competition. We chose to design and evaluate a fast QuickPass system for amusement parks to reduce visitors' waiting time in line. Actually, it is exactly the FastPass system in Disneyland. The question asked us to propose and test schemes for issuing QuickPasses to increase people's enjoyment of the amusement park. To provide a solid design, we discussed this problem for the entire first day of the competition. Later, we simulated several possible scenarios and summarized six indices to evaluate the system. In the next three days, our only work was to write a clear, solid, comprehensive, and complete report. Meng Wu took care of all programming aspects. He even learned how to use new software within two days. Guoliang Wu wrote the paper in English and made its appearance standard and professional. How lucky I was to have such strong partners! The competition is just a competition, but I will never forget the time spent with my teammates, the things I learned from them, and the support that I got from them. Obviously, this time, we double-checked the details of the questions and guaranteed the completeness of our report.

We won a Meritorious Winner Award that year, which was the best result in our university, and also the best result in our province of Tianjin.

Our university newspaper interviewed us and used almost a whole page to report our success (see **Figure 4**). We also won the Outstanding Scholarship of Nankai University (20 awarded out of nearly 3000 applicants) in 2004 because of this achievement. The MCM award also played an important role when I got the Award of Excellent Student majoring in Science and Technology, Tianjin (one of 10 awards in the Province of Tianjin, out of nearly 34,000 students). Even when I was on the job market after pursuing my Ph.D. degree, several interviewers were interested in my experience of modeling and asked me many related questions. To some degree, it was the foremost award on my resume. It influenced me a lot, for my major, for my career, and for my life.

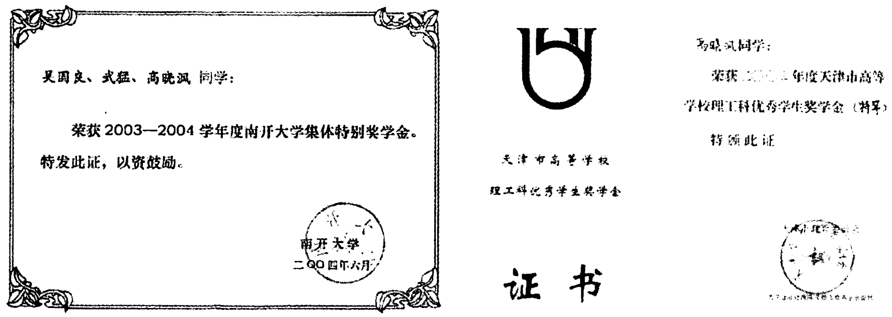

Figure 3. Xiaofeng Gao's certificates of Outstanding Scholarship of Nankai University and Excellent Student Majoring in Science and Technology, Tianjin Province.

I'm now an associate professor at a university in China. Instead of participating in COMAP's MCM, I advise students for this and other mathematical modeling competitions. But I always recall the memories of my experience and share our story with my students. I hope that more people can feel the fascination of a modeling contest, and I'm dedicating myself lifelong to its popularization.

Part II: Meng Wu's Experience

During the past decade, I have transformed from a player in modeling contests to an advisor. Now every time before the contest, I share my own experiences of the contest with my students, especially the beautiful and unforgettable memories during COMAP's MCM 2004.

"Teamwork!"—This is the first point that I emphasize when I teach my students. On our team we had Xiaofeng Gao, who was good at writing, and Guoliang Wu, who was proficient in English. Moreover, all three of us had a sound mathematical background as well as programming ability. Every time I think about it, I feel that the team formed quite accidentally, even though I cannot recall how the three of us came together. Our team


南开大学报 2004年4月16日 星期五 第3版

信息时代 健康生活

共青团"五四"评优工作展开

为校报工作提宝贵意见 南开学子关心母校发展

2004年国际大学生数学建模竞赛

我校学生成绩优异

耕耘收获 挥洒青春

大洋彼岸回音 致谢南开学子

《风雅南开》：一支难忘的歌

Figure 4. Newspaper reporting our success in COMAP's MCM in 2004.

Figure 5. Meng Wu.

was a relaxing and joyful one. In the gradually blurring memory, laughter accompanied us during the four days and nights, and soothed our tired nerves. This feeling has affected me ever since; and no matter how big the difficulties I have encountered, I have been able to face them with a positive attitude. And I often share this experience with my students: Mathematical modeling is an opportunity to refine yourself, and it is also a game. It is not the medal that is important, but to enjoy the journey during which all three teammates strive for a common goal, and enjoy the bits and pieces you harvest in this endeavor.

It is inevitable that difficult problems will be encountered. Some of them can be solved, while others cannot. But I was obviously the lucky one. I still remember that on the third morning, we formed our basic design idea, Xiaofeng started writing the report, and Guoliang was translating it into English. I had been admiring Xiaofeng's writing ability, and I heard that she had even published articles in newspapers already. Guoliang excelled in English, got high scores on the TOEFL exam, and was our class idol.

While looking at these two busy people, I felt that I needed to contribute. After discussion, I decided to write a program to simulate daily operation scenarios for the large amusement park. This program needed to simulate two separate scenarios, namely, the original plan of the park and our newly-designed one. To simulate as close to reality as possible, at almost every step of this process, I needed either to generate random numbers from suitable probabilistic distributions or to make judgments according to the situation. The multiple iterations that were involved increased the complexity of the program, and it took 20 minutes to run just once. However, I was totally frustrated after the first run of the program, which I had put so much effort into, because the number that represented the number of people turned out to be negative in the results! It was a daunting task to debug the program; I had to print out intermediate results to check every single step, and the 20-minute waiting time for each run was almost torturing me. Luckily, I persisted and the whole afternoon was not in vain. Finally, I finished the debugging. After that, every time I ran the program to generate various charts to be included in the report, I was filled with satisfaction and accomplishment; 20 minutes of waiting was no longer painful, but full of expectation. This is the most impressive memory of my experience—just as an old Chinese poem says, "the sudden encounters of the shade of a willow, bright flowers and a lovely village after endless mountains and rivers that leave doubt whether there is a path out" are most cherished.

Time flies, and it has almost been a decade since that modeling competition. That experience has long become a most precious treasure of my life. Each time I write my résumé, I proudly fill in "Meritorious Winner, Mathematical Contest in Modeling, 2004" in the most prominent part of the awards section. Three years ago, I was in a job interview after graduating from a Ph.D. program. After reading my résumé, an interviewing professor chatted with me about that modeling competition. It turned out that a team that he had advised also won a prize in the same year. It's really a small world! The interview went smoothly afterwards, and I received a job offer and became an assistant professor at Tianjin Normal University. After that, I naturally became an advisor for mathematical modeling competitions, and I sponsor teams to participate in every annual China Undergraduate Mathematical Contest in Modeling (CUMCM). At the same time every year, the past memories emerge, both vaguely and clearly.

Participating in COMAP's MCM brought me a lot in the past, so now I also hope that more and more young people could benefit from participating and obtain sweet memories and lifetime treasures from it.

Part III: Guoliang Wu's Experience

My first encounter with a mathematical contest in modeling was during my junior year in college. A friend of mine, Xiaofeng Gao, asked me if we could form a team and compete in the Mathematical Contest in Modeling. Frankly, I wasn't so confident at that time, because I had almost no prior experience in mathematical modeling, and little knowledge about the popular methods such as principal component analysis or programming skills. Nevertheless, I said yes, and I still feel lucky that I did so. We then brought my classmate Meng Wu into the team.

Admittedly, our team was not one of the commonly accepted ideal teams that have players with different backgrounds. Instead, all three of us were mathematics or applied mathematics majors. At that time, I was worried about lots of things: We lacked programming ability, we might fall into the same habitual pattern of thoughts without diversity, we did not have

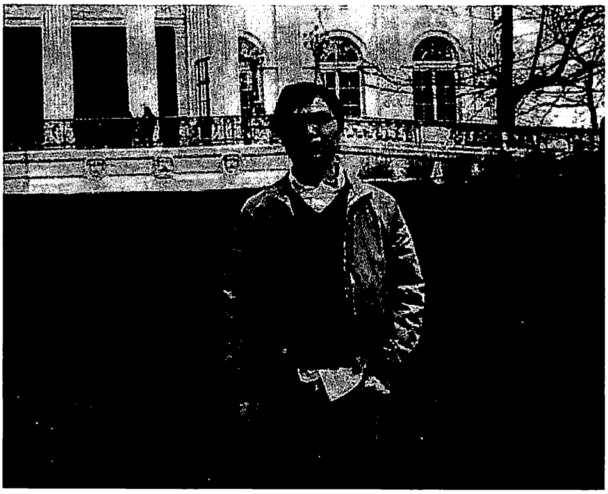

Figure 6. Guoliang Wu.

enough training (I still remember that we had to go to the library to get handbooks on the methods that might be useful before the contest), and so on.

But it turned out that what's more important is *teamwork*. We were familiar with each other, which made cooperation smooth. We also had our own respective advantages that complemented each other: Xiaofeng had a lot of experience with modeling contests and was a very good writer; Meng frequently came up with original and creative ideas and was also a joyful and easy-going person who was comfortable to work with; I was able to express ideas in clear English; and all of us had critical and analytical thinking skills.

Before COMAP's Mathematical Contest in Modeling, we had two opportunities to work together and train ourselves, once at the Nankai University Mathematical Contest in Modeling and the other at the China Undergraduate Mathematical Contest in Modeling (CUMCM).

The results of these two contests did not turn out quite satisfactory in terms of prizes. However, we did get some necessary training, understood the importance of teamwork, and had some rough idea of each person's responsibility.

Then, in February 2004, right after returning to school after the Chinese New Year, we participated in COMAP's MCM. We chose the problem of the QuickPass system for amusement parks. We knew that it was our last opportunity to participate in such a competition, yet we were not nervous. After almost 10 years, I can't recall the details of our report; but I clearly remember how we enjoyed the sleepless four days and nights. Our model itself was not complicated at all, and very little deep mathematical theory was applied. But the final outcome was surprisingly encouraging—we won a Meritorious Winner Award. The more I think about it, the more I feel that the result was due to our seamless teamwork. We were able to tackle a real-world problem with a simple mathematical model, implement it with simulations, and clearly present our ideas and model results in English.

The award was encouraging and exciting, and more prizes—the Award of Excellent Student Majoring in Science and Technology in Tianjin Province, and the Outstanding Scholarship of Nankai University—came after that more or less because of our winning in COMAP's MCM.

However, the real benefit to me was the experience itself of participating in the MCM. Every time I think about the days and nights that we were fighting together, I have more understanding of the importance of teamwork and friendship.

After graduation in 2004, I parted from my teammates and pursued a Ph.D. degree in stochastic optimization and partial differential equations at the University of California, Berkeley. Though my research topic was not directly linked to mathematical modeling, the idea behind it was related. Both are involved with applying mathematical theories to solve real-world problems. Even after nearly a decade, I am still directly benefiting from having participated in COMAP's MCM. Much like Meng's experience, when I was applying for jobs in investment banks a year ago, the interviewers were so interested in my experience of the MCM that I had to refresh my memory about our model and explain it to them. From my current colleagues at J.P. Morgan (who were also my interviewers then), I learned that that experience played an important role in my getting the job offer, since in my current position I also apply mathematical methods to develop models. The only difference is that the performance of the models could potentially affect the revenue of the firm.

I have benefited from the MCM during the last decade of my life. Now as we approach the 30th anniversary of the MCM/ICM contests, I am thrilled to share my experience with current college students. You can benefit from it more than you can imagine.

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Sustainable Water Management for Saudi Arabia in 2025 and Beyond

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The arid country of Saudi Arabia is depleting its water resources as agricultural and municipal demands for water overtax the capabilities of the country's primary source of water: its nonrenewable aquifers. Therefore, as the population of Saudi Arabia increases over the next 12 years, a new strategy is needed to govern the principle concerns of a water management plan: acquisition, storage, distribution, efficiency, cost, and sustainability.

We develop a cost-effective strategy designed to manage Saudi Arabia's water resources by the year 2025. Our plan balances both the water consumption needs of the general populace and the necessity of creating a long-term plan that can continue to meet water requirements well into the future. Specifically, our plan encompasses the following aspects of water usage in Saudi Arabia:

- **Fresh water acquisition:** We employ two large desalination plants, one on the west coast and the other on the east coast. These plants provide municipal water for the entire population of Saudi Arabia.
- **Storage capacity:** We utilize Gumbel extreme value theory to determine the frequency of extreme drought events in order to assist in informed decision-making regarding minimum storage levels in reservoirs and other storage mechanisms.
- **Water distribution:** We employ a minimal spanning tree model to connect the cities of Saudi Arabia with a water distribution pipeline network of minimum length. We also consider the location of the desalination plants to minimize the diameters of the pipes.
- **Agricultural efficiency:** We consider historical price, production, and water requirements for the most produced crops in Saudi Arabia. From

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this information, we produce a list of the most beneficial crops based on their profit-to-water-use ratio.

- **Cost and sustainability:** We consider the financial costs of each aspect of our plan. We seek to minimize total economic expenses without compromising the project's ability to meet the country's water needs for an extended period of time.

Nontechnical Water Plan Outline

Purpose and Objectives

Our team was tasked with the development of an effective, feasible, and cost-efficient water plan for Saudi Arabia's projected water needs in 2025. In general, the current water usage situation is characterized by significant depletion of nonrenewable groundwater aquifers to agricultural purposes. Additionally, expensive, modern methods of water harvesting, especially desalination from the Persian Gulf and the Red Sea, are expanding.

In terms of water, the United Nations classifies Saudi Arabia as an area of "absolute scarcity," with annual water supplies below 500 m^3 per person [United Nations 2012]. Under the U.N. model, such regions *will not be able to meet their water needs by 2025* by themselves, given current rates of water usage. Due to the extreme scarcity of precipitation in Saudi Arabia, we can consider the majority of its water resources—that is, its groundwater aquifers—as nonrenewable. In our analysis, we applied peak-theory principles in treating the Saudi water supply as nonrenewable and therefore reaching a peak before proceeding into decline. **We find that we have already reached this peak**, as depicted in **Figure 1**.

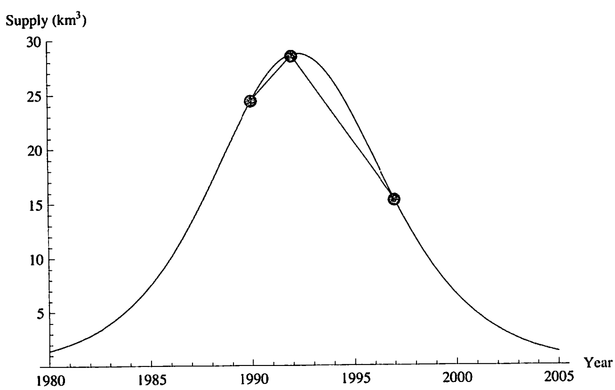

Figure 1. Water supply curve

In accordance with the direness of the water resource situation, we emphasize that **we value water efficiency and conservation significantly**

above cost, especially given the nation's relative wealth and longstanding governmental emphasis on water resource management. Thus, our goals are as follows, in order of importance:

1. **Achieve long-term water resource sustainability in 2025 and far beyond by limiting nonrenewable groundwater aquifer exploitation.**
2. **Develop an *optimal* water pipeline network to minimize transport costs for water stores on a city level and for desalination plants on a national level.**
3. **Maximize groundwater storage and transport *efficiency* by clearly quantifying and *minimizing* such issues as leakage or contamination in storage systems.**

As will be further discussed, our recommendation is to dedicate water resources generated from precipitation entirely to agricultural purposes, and desalination from seawater entirely to municipal and industrial purposes.

Sustainability by Limiting Agricultural Output

Our first objective revolves around limiting nonrenewable water usage with long-term interests in mind. In other words, **we seek to decrease the overdraw of nonrenewable water resources to zero**, where overdrawing generally refers to the effective annual "water deficit," (with the amount of harnessed precipitation as the input). Water in Saudi Arabia is dedicated to agriculture over any other purpose by an extremely large margin.

Thus, it is logical to focus on limiting agricultural use of water, in line with the current government strategy of making Saudi Arabia completely import-dependent for wheat by 2016. We impose the constraint of limiting agricultural water usage to the average precipitation in Saudi Arabia and analyze the water requirements and market price of the nine most heavily-produced crops. Our objective is a water-cost efficiency-ranking to define the ideal crops to grow while still maximizing economic productivity. From most to least economically profitable per water usage, we have:

watermelons, grapes, tomatoes, potatoes,
onions, barley, wheat, citrus, and sorghum.

The government could encourage the production of crops ranking high on this list—specifically watermelons, grapes, and tomatoes—by extending subsidies to farmers growing these crops, much like the policy of the 1990s when the nation was attempting to become completely self-sufficient for wheat.

Given the high variability in agricultural import prices, we do not attempt an analysis of the economic feasibility of high import-dependence for some of the lower-ranking crops. However, the Saudi Arabian government finds it feasible to be completely import-dependent for certain crops whose transport resource requirements are relatively small, specifically wheat.

A Pipeline Network for Desalinated Water

The second component of our water plan involves transport of desalinated water to the people of Saudi Arabia. One of the biggest issues with current water distribution is that there is no universal water supply network. This leads to intermittent water availability and unpredictable results in the event of a drought or other water shortage. Currently, many coastal cities are desalinating water, with 21 desalination plants supplying water to coastal cities and to some inland cities such as the capital, Riyadh. Current pipelines work well enough to prevent water shortages anywhere inland, but are not ideal. Riyadh receives piped water once every 2–3 days, but other major cities are allowed to go 7–9 days without piped water [Media Analytics Ltd. 2011].

Due to cost, we wish to minimize the amount of pipeline necessary. We model the network of cities using a spanning tree; minimizing the length of pipeline can then be accomplished via Prim's Algorithm. Taking population and location data from [Wikimapia 2009], [getamap.net n.d.] , [Gladstone 2013], [One World—Nations Online 2013], [GeoHive 2013]), we apply this algorithm to the 117 most populous cities—with the smallest having just over 4,000 people—generating the pipeline network depicted in **Figure 7**.

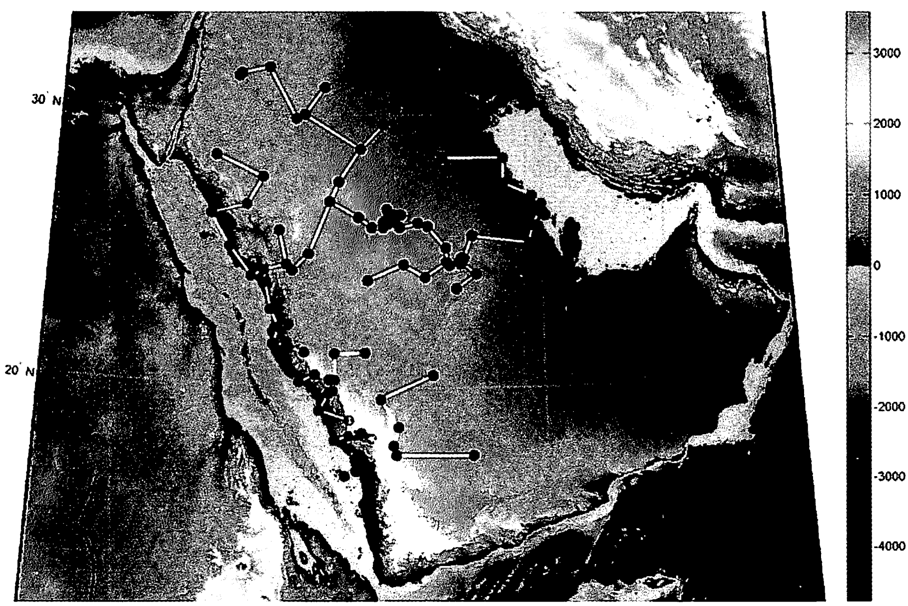

Figure 2. Connection plan for 117 major cities

The total length of this spanning tree is 8,258 km, substantially greater than most other pipelines. Such a pipeline network would provide direct water to over 97% of the population of Saudi Arabia.

Cost

The cost of such a pipeline network is proportional to pipe length, diameter, strength, and peak flow rate. The amount of water carried by each pipe should depend on the population served by that particular pipeline, along with the per capita rate of water consumption in Saudi Arabia. Additionally, the pipe to a city must must also pump water that passes through to other cities farther down the line; so pipes must be largest near desalination plants, but can be smaller farther away.

This cost does not include internal pipelines to distribute water within cities; however, we assume that most cities already possess adequate pipeline systems.

A Single Desalination Plant?

Our analysis indicates that if there is to be only one major desalination plant, it should be located in Rabigh (population 92,072) on the west coast by the Red Sea, 140 kilometers north of the major population center, Jeddah. The pipeline network (not counting the desalination plant itself) would cost approximately $18.1 billion dollars.

Two Desalination Plants

Employing a single desalination plant to serve the entire population would be problematical. In the event of breakdown or closure for maintenance or repairs, the entire country would be without water. Also, it might be more cost effective in terms of transport cost to have two desalination plants, one on each coast.

In our one-source network, we find the optimal pipeline to remove from the network so as to split the network into two separate trees. Our analysis suggests that desalination plants should be located in Alhart and Alkhirkhir. Alhart is a small suburb located by the larger city of Jeddah on the west coast, placing its desalination plant along the Red Sea; Alkhirkhir is a suburb of Ad Damman on the east coast by the Arabian Gulf, 50 km from Bahrain. **Figure 8** indicates the locations of Ad Damman and Alkhirkhiras with red squares. The pipe marked in red connects the two networks, and this pipe would carry water only in the event of closure of one of the desalination plants. Note the central location of each city within its own spanning tree.

The cost of this network is $12.9 billion, much lower than for the network with a single desalination plant; but that does not include the cost of a second desalination plant (which we consider further below). Nor does it include the costs of internal pipelines used to distribute water within cities;

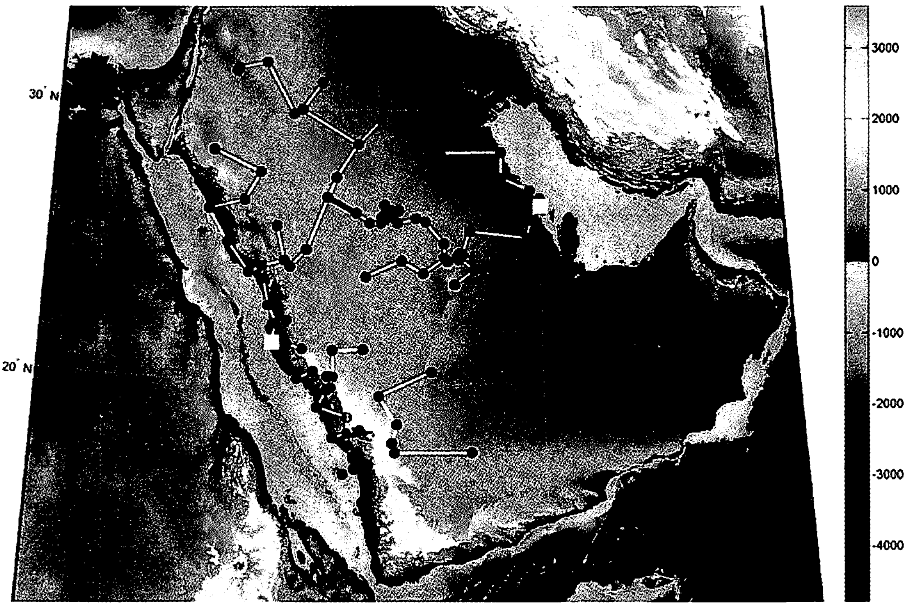

Figure 3. Two-source desalination network.

however, we assume that most cities already possess adequate pipeline systems.

Proposed Design Costs

In fact, the cost of desalination plants depends less on the number of plants than on their capacity. The cost is at most about $800 dollars / m^3 / day [American Water Works Association 2011]. Considering that municipal water consumption is 2.1 cubic kilometers per year [Gleick 2012], the total cost of desalination plants becomes $4.6 billion.

The total cost is thus $12.9 billion for the piping plus $4.6 for the plants, for a total of $17.5 billion; spread evenly over the years from now to 2025, the annual cost would be $1.5 billion.

Strengths and Weaknesses

Municipal Water

The design of the spanning tree guarantees minimum total distance, and it provides connection redundancy in case one plant must curtail service. We assume that length of pipe dictates cost much more than the diameter of pipe needed; this is usually a fair assumption but for extremely large pipes may be inaccurate. Pipe diameter calculations are based on current population data, but increases in population could eventually outstrip this design.

Agricultural Water

The agricultural water recommendations are based entirely on historical production and economic data. Assuming no change in production over the next few decades, the crop recommendations should be valid. Where the model could fall apart is when economic theory comes into play. By growing only the foods at the top of our list, their supply may outstrip the demand, profits would be reduced, and the order in the list of crops by profit-to-water-use ratio might change. So, the most effective way to use this information is as a guideline, nothing more.

Cost Analysis

We use historical data [American Water Works Association 2011] to predict the cost of the desalination plants, but the plants in our two-source design are much larger than any others ever built. We also neglect labor costs in the construction, which could be considerable over the 12-year construction time frame. We also neglect costs of pumping stations, maintenance costs, and so forth.

Technical Report on Design of a Water Plan for Saudi Arabia

Water Management in Saudi Arabia

The extreme aridity of the climate presents a significant challenge to the development of an effective and cost-efficient water policy, since the scant precipitation (approximately 60 mm per year [Food and Agriculture Organization 2013]) contributes to overall inadequacy and unreliability of renewable surface water resources. As a result, the current water supply policy is characterized by major investment in seawater desalination and distribution from the Persian Gulf and from the Red Sea; but the main source is nonrenewable groundwater from deep fossil aquifers.

The discovery of these significant groundwater sources in the 1960s spurred efforts to create an extensive irrigation network in the hopes that the country could become agriculturally self-sufficient for wheat, one of the country's most important agricultural products. While this goal was actually attained in the early 1990s, the project was abandoned by the early 2000s due to the alarmingly rapid decay in aquifer resources. Al-Ibrahim foresaw this problem in 1991, noting that given the depletion rate in the 1980s, the limited aquifer network would be depleted by the 2060s [Al-Ibrahim 1991]. In fact, the quick loss of groundwater resources to agriculture (accounting for around 85% of total water usage [Al-Ibrahim 1991]) has prompted the government to reverse course and a plan to make the nation 100% import-dependent for wheat by 2016 [Trompiz 2012].

The other primary water-harnessing strategy, desalination and subsequent transport, is also problematic due to high cost. For instance, the Shoaiba power and desalination plant to serve the Jeddah-Mecca metropolitan area cost of $2.8 billion [Dewar 2007].

The goal is to create a water plan for Saudi Arabia. This plan must be sustainable, cost effective, and efficient. We investigate the best way to retrieve, distribute, and manage water resources over long periods of time.

Major Assumptions

1. **Saudi Arabia does not have adequate water resource infrastructure.** Instead of recommending extensions to the current desalination and water pipeline network, we propose a new network from scratch. This is necessitated by the lack of specific, publicly-available hydrological infrastructure spatial data for Saudi Arabia. We recognize that Saudi Arabia is one of the most extensive practitioners of desalination and aquifer harvesting in the world, so we generalize the principles in our recommendations so that they can be easily extended from the existing system.

2. **The water available for use originates from one of three sources and is harvested in certain ways:**
   - Renewable surface water, stored and harvested using dams,
   - Nonrenewable groundwater, harvested by welling from aquifers, and
   - Seawater, harvested through desalination.

   This description of the fundamental human interaction with the hydrocosm is actually quite common in water resource management analysis [LoCascio 2008].

3. **Water is used for one of two purposes: agricultural or municipal.** Agricultural and municipal water uses comprise over 98% of total water usage [Abderrahman 2007].

4. **Demand for certain fundamentally water-intensive crops can be completely met by importation.** While we are not actually making such a drastic suggestion, our assumption is that supplies from other countries will be able to handle a large increase in demand for certain crop imports.

Water Plan Objectives

In 2007, the United Nations classified Saudi Arabia as an area of "absolute scarcity," with annual water supplies below 500 m^3 per person [United Nations 2012]. Under their model, such regions *will not be able to meet their water needs by 2025,* given current rates of water usage. Thus, we develop a water resources plan with the following objectives optimized, in order:

1. **Achieve long-term water resource *sustainability* in 2025 and far beyond by limiting nonrenewable groundwater aquifer exploitation.**
   We assume complete agricultural "outsourcing" of wheat by 2016 as planned by the Saudi government, which should assist in the achievement of this goal. However, we must account for increasing water demand given the rapidly-growing population (growing at 2.3% annually [United Nations Population Division 2013]) and economy (currently 6% GDP real growth rate [Central Intelligence Agency 2013]).

2. **Develop an *optimal* water pipeline network to *minimize* transport costs for water stores on a city level and for desalination plants on a national level.**
   The water pipeline network needs to span all of the cities so as to provide water for the entire population. The cost depends on both the length and the size of the pipes used. Specifically, the cost of a pipe is approximately $2 million per kilometer of length per meter of width [INGAA Foundation and ICF International 2009]. Therefore, the pipeline network should not only connect the cities of Saudi Arabia with as short a length of pipelines as possible but at the same time minimize the diameter of each pipe.

3. **Maximize groundwater welling storage and transport *efficiency* by clearly quantifying and *minimizing* such issues as leakage or contamination in storage systems.**
   A better information infrastructure must be developed to address potential problems; the information on metrics such as leakage rates and metering performance is frankly inadequate. Specifically, unaccounted-for water, or produced water that is "lost" before it reaches the consumer, ranges between 30% and 55%, with leakage rates between 30% and 50% [Abderrahman 2007].

We value water efficiency and conservation significantly above cost, given the direness of the water situation combined with the nation's relative wealth from oil resources. This is consistent with the Saudi government's history as one of the largest investors in water resource management as a percentage of total budget, with more than $100 billion invested from 1975 to 2000, or approximately 1.5% of GDP [LoCascio 2008].

Justification of a New Water Plan

The scarcity and unreliability of surface freshwater forces use of nonrenewable aquifers as the country's primary water source. In fact, Saudi Arabia ranks first in the world in overdrawing of natural freshwater resources [Frenken 2009]. As of 2005, Saudi Arabia produced 2.4 billion cubic meters of renewable freshwater per year, but was using of 21.4 billion cubic meters per year, with 17.3 billion cubic meters drawn from fossil aquifers. **Tables 1** and **2** provide publicly-available data on water usage and water supply over the past 30 years.

Table 1.
Water sources **supplies**, in billions of cubic meters [Abderrahman 2000].

Water Source	1990	1992	1997
Treated Wastewater	0.1	0.2	0.2
Desalination	0.5	0.8	0.8
Surface Water and Shallow Aquifers	2.1	2.1	2.1
Groundwater	24.5	28.6	15.4
Total	27.2	31.7	18.5

Table 2.
Water sources **usage**, in billions of cubic meters [Abderrahman 2000].

Year	Domestic and Industrial	Agricultural	Total
1980	0.5	1.9	2.4
1990	1.7	25.6	27.2
1992	1.9	29.8	31.7
1997	2.1	16.4	18.5
2000	2.9	11.2	14.1

One would expect that for these nonrenewable aquifers that the yearly supply would start out small, quickly increase until a maximum yearly supply is reached, then slowly decrease until the supply is fully depleted.

This kind of behavior has been studied before with the theory of "peak oil" [Deffyes 2001] The drilling of many oil wells often follows this same kind behavior of exponential growth, followed by leveling followed by exponential decay. This kind of curve has been called a Hubbert curve. While usually associated with oil, it can be extended to the usage of any kind of finite resource, such as coal, copper, and water.

Water from fossil aquifers that are not renewed from rainfall or other sources is most certainly finite, so its supply can be modeled by this kind of curve. There is little data about the fossil water supply. **Figures 4** and **5** plot data from **Tables 1** and **2** with a superimposed Hubbert curve.

There are few data points, so it is impossible to argue that future use will follow the indicated trend; but historical data of other limited resources

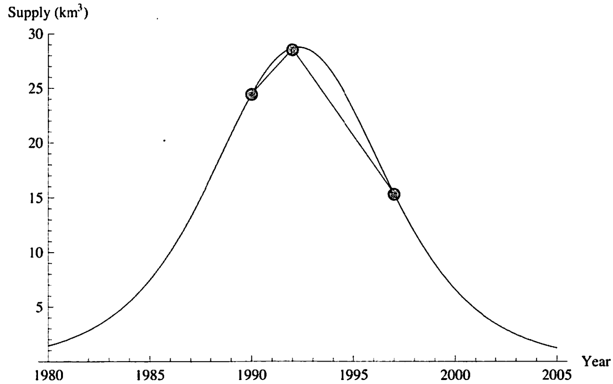

Figure 4. Fossil aquifer **supply** vs. year.

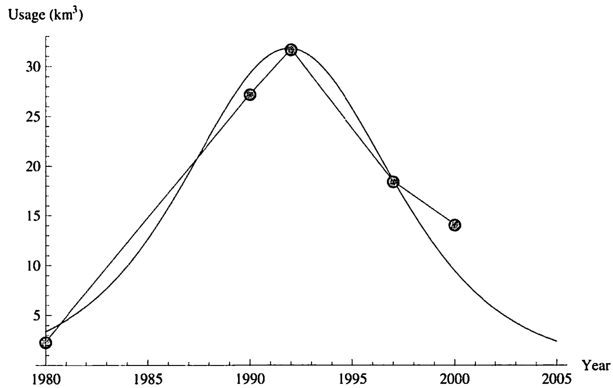

Figure 5 Fossil aquifer **usage** vs. year.

suggests that this will be the case and the fossil aquifers will soon run out of water to supply to the country. This impending shortage motivates our strong recommendation to move quickly to desalination as the major water source.

Our plan does not rely on the nonrenewable aquifers. We plan on desalination for municipal water and industrial water, and surface water for agriculture.

Design of a New Water Plan

In 2004, water usage in Saudi Arabia was 90% for agriculture, 9% for municipal water, and 1% for industry [Abderrahman 2006].

However, projected usage will change by the year 2025, when our plan will have been fully implemented. In 2020, municipal plus industrial water is projected to be 6.45 billion cubic meters per year [Presidency of Meteorology and Environment 2005, x]. Meanwhile, the government has plans that by 2016, 100% of food will be imported, essentially eliminating the need for agricultural water.

In terms of sources, in 2008 81% of water came from fossil aquifers, 11% from renewable ground and surface water, and 8% from desalination [Frenken 2009], [Saline Water Conservation Corporation 2013], [Rasooldeen 2010], [British Arabian Advisory Company 1979].

While it would be feasible to continue using fossil aquifers until they are depleted, it is impossible to predict when that would happen, which could lead to a water crisis. Ultimately, the goal needs to be to switch entirely to renewable water resources.

Current Distribution Inconsistencies

One of the biggest issues with current water distribution is that there is no universal supply network. This leads to intermittent water availability and unpredictable results in the event of a drought or other water shortage. Although 97% of the urban population has access to clean water, only 63% of the rural population does [Abderrahman 2000].

Currently, many coastal cities desalinate seawater, with 21 desalination plants across the country. Pipelines are in place and work well enough to prevent water shortages anywhere inland, but the situation is not ideal. Riyadh receives piped water once every 2–3 days, but other major cities go 7–9 days without piped water [Media Analytics Ltd. 2011]. Many smaller towns do not have piped water supplies, hence get their water by truck; doing this has low upfront costs but extremely high maintenance costs. An interconnected country-wide pipeline system is the best option to consistently and permanently deliver water to all of the population.

The biggest issue is how to connect all of the major cities with the least amount of pipeline. In terms of mathematical modeling, so as to connect all cities to the pipe network, the network should constitute a spanning tree for the cities. We note, though, that a minimal amount of pipeline would allow for no redundancy, such that water could be sent to the same city by a second pipeline if the first were to be out of service; however, our two-source solution below provides some redundancy.

Pipeline Map

The issue then becomes how to minimize cost. In a mathematical context, this is called a *minimum spanning tree*. A minimal spanning tree, by definition, has a shorter length than any other spanning tree that connects all of the vertices (cities). For a tree to be a minimal spanning tree, each possible (but unused) link between cities must be longer than each network link that would be placed in a closed cycle by the addition of the unused link [Jungnickel 1999]. Otherwise, if a possible link were shorter than a link currently employed in the minimal spanning tree, the length of the link to connect the cities in the cycle could be decreased by using the new, shorter connection rather than the longer one currently in use. In other words, the minimal spanning tree removes the longest link of any cycle so as to leave a path of minimum length with no cycles.

This property of the minimal spanning tree, that every link not in the network must be longer than the links that would be put in a cycle by its addition, can be employed in an algorithm to compute the minimal spanning tree for cities. We do this using *Prim's Algorithm* [Arney et al. 1997, 337–341]. Roughly, it follows this form:

1. Create a set of all points which need to be connected.
2. Create a set of all possible connections that could connect these points.
3. Add a connection of the shortest length that connects a point in the tree to a point not in the tree.
4. Repeat Step 3 until all points are connected in the tree.

We first use Prim's algorithm, as implemented in MATLAB, to find the minimal spanning tree to connect the 20 largest cities, which contain approximately 72% of the total population. The resulting pipeline network has a total length of 3,603 km and is shown in **Figure 6**.

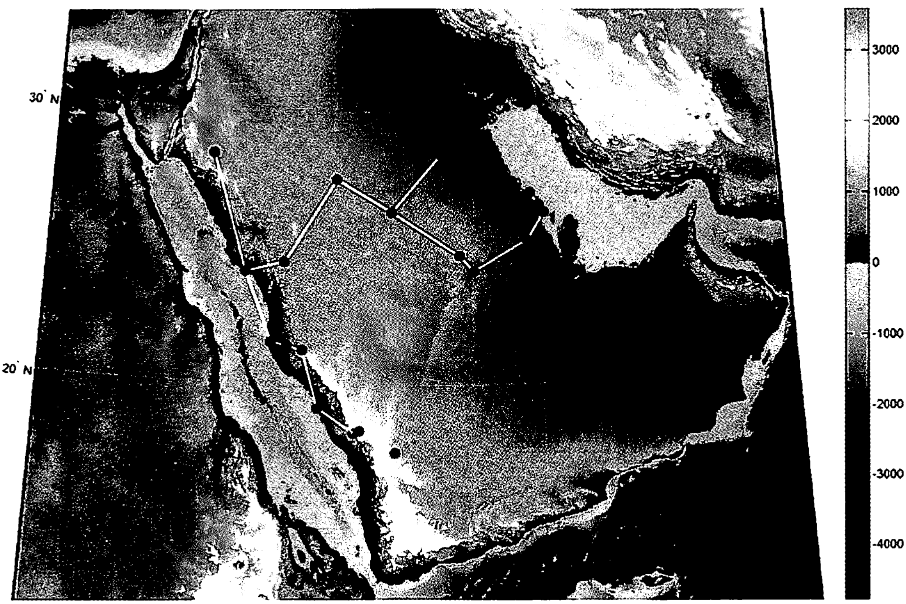

Figure 6. Connection plan for the 20 major cities

Next, we we do the same for the 117 most populous cities, arriving at the pipeline network of **Figure 7**. The total length of this spanning tree is

8,258 km, substantially longer than most other pipeline systems; it would provide water to 97% of the population.

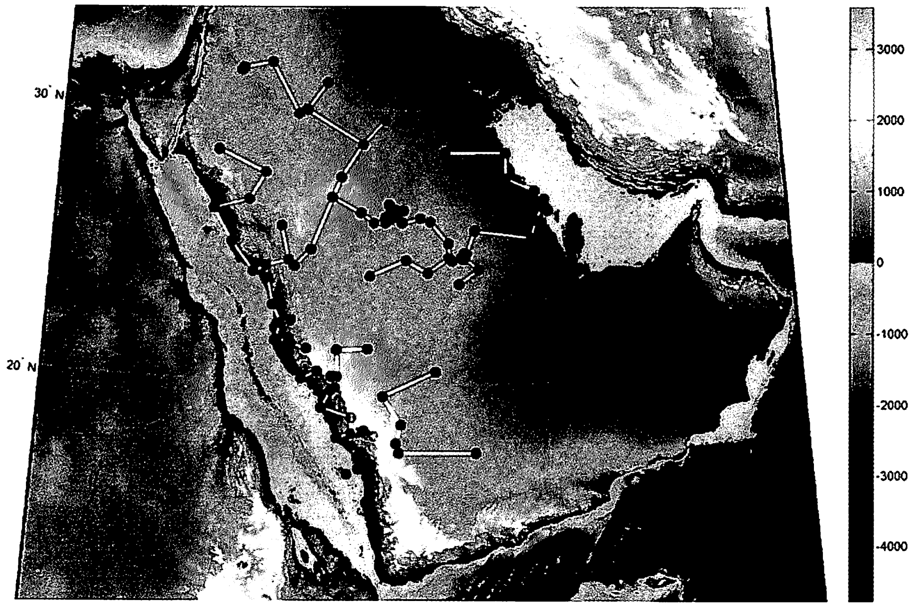

Figure 7. Connection plan for 117 major cities

Cost of the Proposed Design

The cost of a pipe is proportional to the following parameters:

- Length of pipe
- Diameter of pipe
- Strength of the pipe and peak flow rates

We neglect strength of the pipe, for the sake of simplicity and also because we do not plan to strain the pipes above recommended operating conditions. The cost of pipe is $2 million per kilometer of length per meter of width [INGAA Foundation and ICF International 2009]. From the minimal spanning tree, we already know the lengths of each pipeline segment. However, the size of each pipe depends on the rate at which the pipe must transfer water, which in turn depends on the population to be served. For instance, a pipe to the capital city of Riyadh must serve a very large population, so the pipe must be quite large.

We determine the relative size of each pipe from data for a pipeline that is currently being built to transport water into Riyadh. Its diameter is 1.83 m, to accommodate a flow of 990,000 m^3 / day [Saline Water Conservation Corporation 2013].

By assuming that the flow rate is proportional to the cross-sectional area of a circular pipe, we can determine the diameter needed for a specified flow rate.

The amount of water for each city depends on population and the per capita rate of water use. The total municipal water use is 2.1 cubic kilometers per year [Gleick 2012], or 5.75 million cubic meters per day, or 0.205 cubic meters per person per day. The daily water need of a city is this per capita value times the population of the city.

However, for a spanning tree network, a pipe to a given city must transport more than just the water required by that one city; the pipe must also pump water that passes through that city to other cities farther down the line.

Consequently, pipes must be largest near the desalination plants, and then pipe sizes can decrease farther away from the plants where there are fewer cities down the line that still need water. This means that the location of the desalination plant(s) must be chosen to minimize the total cost of the pipeline network by ensuring that as many pipes as possible can be made as small as possible.

One-Source Solution

MATLAB analysis indicates that if there is only one desalination plant, it should be located in Rabigh (population 92,072) on the west coast by the Red Sea, 140 km north of the major population center of Jeddah. The pipeline network (not counting the desalination plant itself) would cost $18.1 billion.

Employing a single desalination plant to serve the entire population would be problematical. In the event of breakdown or closure for maintenance or repairs, the entire country would be without water. Furthermore, pumping water across the entire country is rather inefficient: Cities on the east coast would receive desalinated water pumped all the way across the desert from the plant on the west coast.

Two-Source Solution

It might be more cost-effective to have two desalination plants, one on each coast. In this case, the pipe network splits into two separate spanning trees, each containing one desalination plant. To determine the locations of the desalination plants to obtain minimum cost, we examine each combination of coastal cities, along with each possible pipeline to remove from the overall spanning tree so as to split it into two separate trees. We again use MATLAB to analyze all the options, while ensuring that the two cities containing desalination plants are in separate trees, on opposite sides of the pipe that is removed from the original single-plant network.

Running the MATLAB simulation indicates that the pipe network indeed costs less when two desalination plants are employed. This result is to be expected, since each desalination plant serves a smaller network than before, so it does not need to desalinate and distribute as much water as when there is only one plant. As a result, the pipes near the desalination plants can be much smaller, since they do not need to distribute nearly as much water.

To minimize the total cost of the pipeline network for two desalination plants, the plants need to be located in Alhart and Alkhirkhir, a small suburb of Jeddah on the west coast, along the Red Sea. Alkhirkhir, meanwhile, is a suburb of Ad Damman on the east coast, on the Arabian Gulf, 50 km from Bahrain. **Figure 8** indicates the locations of Ad Damman and Alkhirkhir within the two-tree pipeline network.

The pipe marked in black connects the two networks and provides some redundancy; it would carry water only in the event that one desalination plant fails and the other is forced to provide water to the entire country. Note the central location of each city within its own spanning tree.

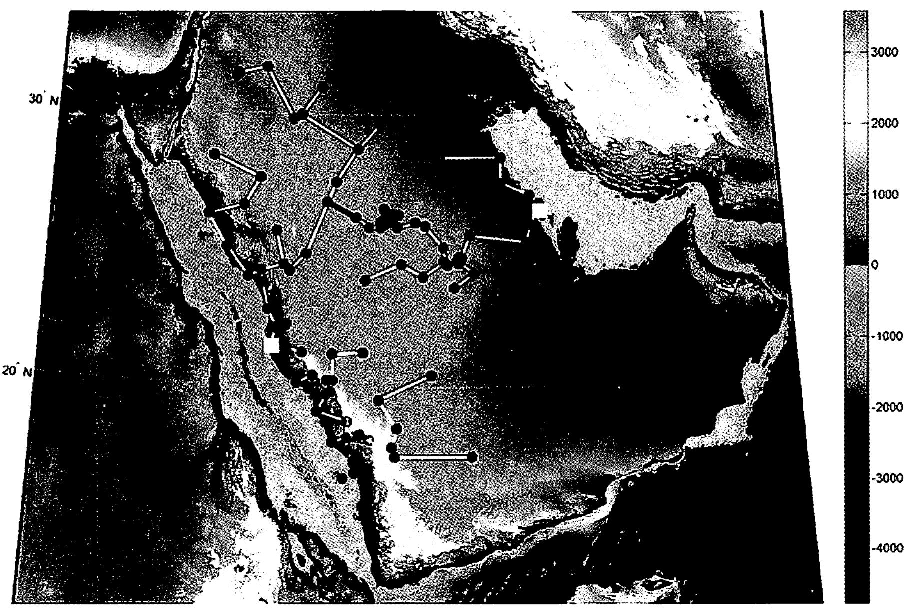

Figure 8. Two-source desalination network.

The total cost of the pipeline network for two desalination plants is approximately $12.9 billion, much lower than the cost of the network for a single desalination plant; however, the second network must also involve a second desalination plant, which raises the cost considerably. Also, this cost does not include the costs of internal pipelines used to distribute water within cities; however, we assume that most cities already possess adequate pipeline systems.

N-Source Solution

Our approach above for two desalination plants could be extended to consider a greater number of plants. Running the simulation again with different numbers of starting cities would produce an optimal design.

Agricultural Water

Recall that our first objective revolves around limiting nonrenewable water usage; **we seek to decrease the overdrawing rate of nonrenewable water resources to zero**. Agricultural water makes up the vast majority of the water that is used, with its main source being the fossil aquifers. However, in the southwestern part of the country, there are seasonal rivers, cultivated land, and some surface water.

It may be impossible to eliminate fossil aquifer usage without an even bigger overhaul of the country's water system. However, we have investigated how results could be achieved with smaller changes in farming practices, by using a simple optimization model.

We begin by creating an expression for profit:

$$\sum_{j=1}^{n} \left(\frac{\text{revenue}}{\text{hectare}}\right)_j x_j,$$

where the revenue per hectare is specific to the crop being grown, x_j is the number of hectares of each crop being grown, and n is the number of different crops.

We then create an equation for water use:

$$W(x) = \sum_{j=1}^{n} \left(\frac{\text{water}}{\text{hectare}}\right)_j x_j.$$

Finally, we formulate an equation for the amount of land used:

$$L(x) = \sum_{j=1}^{n} x_j.$$

We can then optimize either for maximum profit or for minimum water use. As expected, the results are to grow either the most profitable food or the most water-efficient food. These are not economically efficient strategies, for the supply of each crop would exceed the demand and reduce profit. Bringing in economic concepts of demand and necessity of different foods would improve these recommendations and make them more realistic.

Nevertheless, we determine the best crops to grow by their profit-to-water-use ratio, based on data in Al-Hazmi [n.d.], resulting in an ordered list of the best crop choices, sorted by decreasing revenue per unit of water usage:

watermelons, grapes, tomatoes, potatoes,
onions, barley,wheat, citrus, and sorghum

These crops are profitable but still minimize water usage. With government programs to encourage crops near the top of the list, water usage could be even further minimized. The government has already taken some steps in this direction, such as a decision to import all wheat by the year 2016; wheat is low on the list above, and thus should be replaced something better.

A more sustainable water usage plan could be attained by further increasing water supply. Similar to the municipal water distribution plan above, by piping desalinated water to farmlands, the issue of water restrictions could be reduced even more. Realistically, many farmers will continue to use fossil aquifers for water; but with the aquifer water supply on a downward trend, any steps in the right direction will be beneficial and prolong the supply.

Storage Considerations

We are interested in maintaining water resources in reservoirs and water tanks against events such as drought [Food and Agriculture Organization 2013]. The occurrence of events such as extreme drought can be modeled by the Gumbel extreme value distribution, whose cumulative distribution function is

$$F(x,\alpha,\beta) = \exp\left[-\exp\left(\frac{-(x-\alpha)}{\beta}\right)\right],$$

where α and β are parameters and x is the amount of rainfall.

This distribution function can be manipulated to generate the minimum rainfall as a function of time period length, that is,

$$x_T = \alpha - \beta \ln\left[-\ln\left(1-\frac{1}{T}\right)\right],$$

where T is the time period considered and x_T is the minimum rainfall during that time period. Using the method of moments estimation, it can be derived that:

$$\beta = \frac{\sigma}{\pi/6}, \qquad \alpha = \mu - \gamma\beta,$$

where

- μ is mean rainfall,
- σ is the standard deviation of rainfall, and

- $\gamma \approx 0.577$ is the Euler-Mascheroni constant.

Meteorologists characterize droughts (and floods) in terms of the *return period*: the average time between two events of the same size. **Figure 9** depicts, for 9 different locations in the country, annual rainfall vs. the return period. For example, the top curve, for a location that averages 200 mm / yr of rain, shows that on average every 100 months (8+ years) the amount of rain will be only 130 mm. Such considerations provide probabilistically-based information about extreme drought events, which can inform decisions regarding storage capacity [McCrodden et al. 2010].

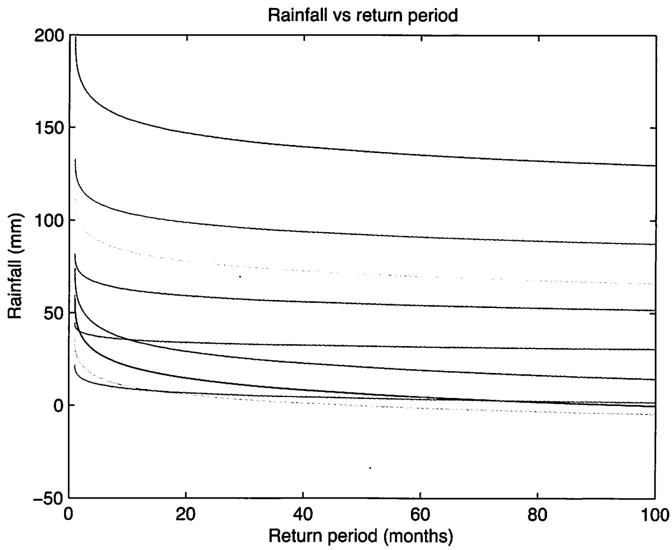

Figure 9. Rainfall vs. return period.

The rainfall data (represented by the different colors in the figure) is derived from nine rain-gauge stations around Saudi Arabia [Shahin 2007]. Since the graph indicates different amounts of rainfall in different locations, storage policy clearly must be determined for each city independently.

Costs of the Proposed Design

The cost of the pipeline is determined by four factors:

- Size of the desalination plant
- Length of pipe used
- Diameter of pipe used
- Number of desalination plants

In our pipeline network design, we already took into account the length of the pipe. After establishing the number of desalination plants, the sizes of the plants can be determined.

Ideal Design Cost

The total cost of the desalination plants depends less on the number of plants that need to be constructed and more on the total amount of water that the plants must process daily. Since cost is a function of the total amount of water that must be desalinated, the final cost of the desalination plants has more to do with a unit cost of dollars per cubic meter of water desalinated per day, as opposed to the number of plants constructed.

The unit cost for a large-scale desalination plant scales at about \$3 for a capacity of 1 gallon per day, or \$800 for 1 cubic meter per day [American Water Works Association 2011, 86, Figure 5-1]. Considering that the current rate of municipal water consumption in Saudi Arabia is 2.1 cubic kilometers per year [Gleick 2012], then the total cost of the desalination plants becomes

$$\frac{2.1 \text{ billion m}^3}{1 \text{ yr}} \times \frac{1 \text{ yr}}{365 \text{ d}} \times \frac{\$800}{\text{m}^3/\text{d}} \approx \$4.6 \text{ billion.}$$

We also examine the case of two large desalination plants at separate locations. This process could easily be extended to any other number of plants. With a cost of \$4.6 billion for the desalination plants and \$12.9 billion for the pipleline network, the total cost would be \$17.5 billion.

This is almost a final cost. It includes the price of labor for a desalination plant, but does not include the price of labor for the pipeline construction. It also does not include any maintenance costs. Assuming a linear distribution of costs from start to finish, the cost would be \$1.5 billion per year to complete this project by 2025.

A Plan on a Smaller Scale

We also examine the cost of a plan on a smaller scale. For example, supplying water to only the 20 most populous cities would use roughly half as much pipe but provide water to 72% of the population. **Figure 10** shows variation of number of people served vs. pipe length (and thus cost). The curve is approximately linear over the range from 4000 km to 8000 km, thus over the range from 20 cities to 117.

Summary

We design a water plan for Saudi Arabia to address water-supply issues that will otherwise occur in the coming years. Much of the water currently used comes from fossil aquifers; their water is not renewable, they are already in decline, and they will become depleted.

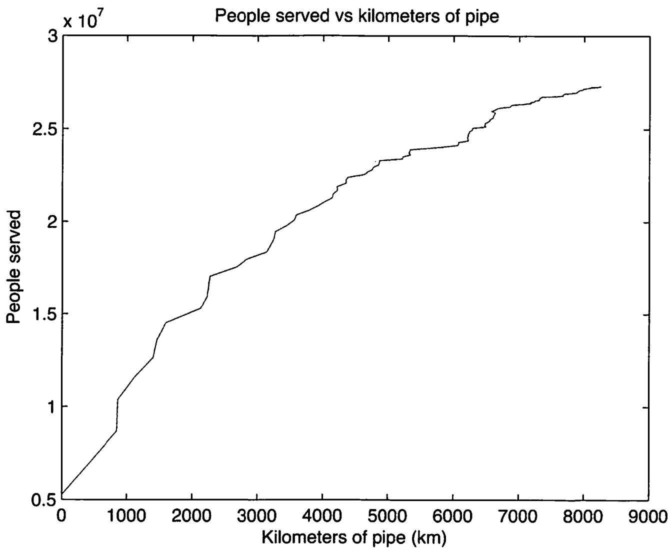

Figure 10. People served vs. kilometers of pipe.

Municipal Water

The first goal is to supply clean water to all residents. We propose a pipeline network: It reaches all cities, has minimal cost, and . is supplied entirely by coastal desalination plants. A network including all 117 cities would reach 97% of the population; a network to just the 20 most populous cities would reach 72% of the population, at about half the cost.

Agricultural Water

Most agricultural areas draw their water from the fossil aquifers, with a much smaller amount from surface water such as rivers and shallow wells. The ideal situation would be to constrain all agricultural water use to the surface water sources, which are replenished naturally. We determine a ranked list of crops that provide the best profit-to-water-use ratio. Governmental recommendations to farmers could thus reduce water use while maintaining profit levels.

Costs and Planning

The cost of materials for the ideal two-source desalination system is $17.5 billion (without the cost of the associated labor), or $ 1.5 billion per year over the years from 2014 through 2025. This cost could be offset by higher water tariffs; Saudi Arabia has some of the cheapest water worldwide, despite having one of the scarcest supplies. Furthermore, a more limited pipeline network could be built at about half the cost while serving 72% of the population.

Strengths and Weaknesses

Municipal Water

The design of the spanning tree guarantees minimum total distance, and it provides connection redundancy in case one plant must curtail service. We assume that length of pipe dictates cost much more than the diameter needed; this is usually a fair assumption but for extremely large pipes may be inaccurate. Pipe diameter calculations are based on current population data, but increases in population [Central Intelligence Agency 2013] could eventually outstrip this design.

Agricultural Water

The agricultural water recommendations are based entirely on historical production and economic data. Assuming no change in production over the next few decades, the crop recommendations should be valid. Where the model could fall apart is when economic theory comes into play. By growing only the foods at the top of our list, supply may outstrip the demand, and profits would then be reduced, perhaps producing redirection of effort (and water) to other crops.

Cost Analysis

We use historical data [American Water Works Association 2011] to predict the cost of the desalination plants, but the plants in our two-source design are much larger than any others ever built. We also neglect labor costs in the construction, which could be considerable over the 12-year construction time frame. We also neglect costs of pumping stations, maintenance costs, and so forth.

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Both 2013 Outstanding teams from the University of Colorado: on the left, Fiona Pigott, Christopher V. Aicher, and Tracy Babb of Team 22550, who were Outstanding on the Brownie Problem; in the middle, advisor Anne Dougherty; on the right, Gregory McQuie, Yueh-ya Hsu, and David Thomas, authors of this report.

Judges' Commentary: The Outstanding National Water Strategy Papers

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Problem Statement

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement, de-salinization, and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a nontechnical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the "best water strategy choice."

Countries: United States, China, Russia, Egypt, or Saudi Arabia

Introduction and Overview

This problem is similar to one used for the 2009 High School Mathematical Contest in Modeling (HiMCM)®. The problem was expanded for the 2013 Mathematical Contest in Modeling (MCM)® contest. The judges expected more sophisticated modeling, higher resolution, better analysis, and fuller interpretation by the MCM teams. The judges were not disappointed.

The problem was deliberately written to have a potentially overwhelming amount of detail, and to force the student teams to decide what simpli-

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fying assumptions could be made so the problem would be tractable in the time allowed yet provide useful insights. It was also written so that students with a background in only lower-division mathematics could attempt it.

To recognize the increasing international diversity of the student teams in the MCM, teams were allowed to choose among five possible countries. Reflecting the high participation rate from the People's Republic of China, by far the greatest number of teams chose to model China. Many fewer teams chose the United States, Egypt, or Saudi Arabia. This judge did not see a single paper that addressed Russia.

As has been the norm recently, there were required elements in the problem statement. Almost every paper included the required nontechnical position paper this year.

This commentary will discuss the various elements of the problem, with observations from the judging. It will then conclude with a summary.

Readers interested in a discussion of the mechanics of the judging process will find a very good report on the process in the accompanying commentary on the Brownie Pan Problem by Dr. Kelly Black [2013], on pp. 141–149 of this issue.

What is the "Best Water Strategy"?

Models are constructed to answer questions. Here, we are asked to identify the best water strategy. But what did a team mean by "best"? Was it to provide a certain level of water at least cost? Was it to have the most reliable supply? Was it to have the highest water quality? Was it to provide the greatest net economic benefit to the country? Many teams never stopped to define clearly the purpose of their models, but plunged immediately into modeling aspects of the problem.

The better teams considered carefully what they meant by "best" and included a discussion in their restatement of the problem.

Projected Water Needs

Predicting the water demand by 2025 was a required element of the problem. Student teams used a variety of methods. These included predicting the population at 2025 and then assuming a per-capita water use, and directly modeling the water usage for a prediction at 2025. Teams divided the countries in different ways, some by river basins and some by political entities.

Most teams that modeled the population and then multiplied by a per capita usage rate did not explore the possibility that the usage rate might change based on shifts to the underlying economy. On the other hand, some teams explicitly modeled industrial demand, agricultural demand, and domestic demand: These papers were stronger.

Prediction methods used included statistical methods (typically regression) and differential equation models (typically Gray's model). Almost every team reported a point estimate instead of an interval estimate (either a confidence interval for the mean demand or, better, a prediction interval for the actual demand). The judges ranked interval estimates much higher.

Teams varied in the sophistication of the predictors that they used to model demand. Some strictly analyzed the problem as a time series. Others included industrial development, climate change, loss of wetlands, and externally imposed changes to agricultural practices, such as prohibiting certain irrigation practices or the cultivation of water-intensive crops.

In addition to modeling uncertainty, the best teams included an assessment of the sensitivity of their demand models to changes in their assumptions.

Finally, many teams over-reported the precision of their results. It was not unusual to see 10 or 12 "significant" digits in the estimated demand, where the source data had only two or three significant digits. Many teams did not include the units for their answers, or used them inconsistently. These are poor practices.

Storage and Movement

Most teams modeled the movement of water as a network problem. Modeling choices included the selection of nodes, the representation of arcs, and the costs associated with the arcs.

Many teams divided the country into regions, and represented each region by a point, usually the regional capital. Arcs were constructed between these points to represent flow between regions. Most teams modeled the cost of construction and transport along these arcs as proportional to the geographic linear distance between the nodes. Almost every team neglected elevation differences, but the better teams at least acknowledged that they were ignoring those differences. Since the cost to lift water is very high compared to the cost to move it laterally, treating elevation was an important discriminator.

A few of the better papers modeled water distribution inside the regions as well.

The teams that modeled China usually acknowledged the South-to-North Water project and incorporated it directly or indirectly into their model. This included drawing cost data from the project.

Many teams incorporated environmental issues into the movement and storage options by acknowledging that some options resulted in human displacement, wetlands loss, and aquifer depletion. The better papers moved beyond subjective descriptions to quantitative models that were incorporated as elements when weighing the best national strategy.

The criteria for the problem that I as problem author developed before the contest included the comment that "teams that model wear-out or degradation of existing infrastructure and system reliability, and include consideration of those issues in their overall model, distinguish themselves if all other modeling parts are done very well." Few if any papers addressed this.

Desalination

Desalination was the second of the required topics. Many papers assumed it away, to their disadvantage. Modeling desalination involved infrastructure, power, cost of unit production, and then subsequent transportation, including lift, since by definition desalination starts at sea level. Most teams that addressed desalination considered cost, but few thoroughly treated transportation. Of note, one month after the contest date, Lockheed Martin filed a patent for a nanometer-thick desalination filter that claims to reduce the power needed for desalination by two orders of magnitude [Alexander 2013].

Conservation

Most teams modeled conservation. There were several approaches, primarily involving reduced use and recycling.

Reduced use models focused extensively on irrigation. Inefficient practices such as flooding of fields were chosen for elimination, and the resulting water savings calculated and subtracted from the overall demand.

A second class of reduced use focused on prohibiting crops that required significant amounts of water, choosing to import those, and focusing on crops that needed less water. The concept of "virtual water" was used to describe this, and the practice was noted as already in use in Saudi Arabia.

Water recycling by treatment was also discussed in many papers, affecting the cost, supply, and environmental aspects of the trade space.

Other topics addressed under conservation were the management of limited aquifers, reduced industrial use, and reduced domestic use.

Model Integration

Once teams had developed submodels for demand, storage and transport, desalination, and conservation, they had to integrate them in some manner to determine the "best" water strategy. There were several approaches used most frequently.

- One approach was to minimize cost, given a required supply. Students attempted to do this by linear programming, and in some cases by using the greedy algorithm or simulated annealing models.
- Many papers used the Analytic Hierarchy Process (AHP) to incorporate the different elements of the solution into one decision model. Teams used their own judgment to estimate the priorities among the elements. Since different teams provided different weights and inputs, solutions varied widely.
- Weaker papers did not have an explicit method to integrate their problem elements or to determine a "best" strategy. They found a feasible solution, and stopped.

Sensitivity Analysis and Model Testing

As in previous years, the judging criteria for this problem considered sensitivity analysis and model checking. Many papers neglected to consider these issues and were scored lower as a result.

Sensitivity analysis was appropriate for all elements of the models, and especially for the predicted demand, supply, and cost models. For AHP, sensitivity analysis would have involved varying the weights (or pairwise rankings) to explore what conditions would cause the alternative ranking to change.

Model testing took several forms. For prediction models, graphical methods for examining residuals of historical data were often used. Statistical tests of significance were used for regressions. Consistency checks were used for the AHP. The better papers used these methods and others to convince the reader that the models selected were appropriate.

Country Issues

In some countries (China, the United States, Russia), water scarcity is a problem only in parts of the country. Teams that elected to only deal with regions where there was scarcity were not penalized if that approach met their definition of "best."

Data were fully available for China and the United States, and to a lesser degree for Egypt, Russia, and Saudi Arabia. The judges recognized this when evaluating papers.

The overwhelming majority of papers chose to model China. Some papers initially advanced in the competition partly because they addressed countries that were almost completely neglected by most teams, so there would be some balance in the later judging rounds.

Communication

Papers were judged on the quality of the writing. Special attention was paid to the abstract and to the nontechnical letter.

The quality of writing, in general, is improving from year to year. This is notable in the papers that come from countries where English is not the primary language spoken. About half of the Outstanding papers this year were from teams where English was a second language, and that was a record.

The strongest abstracts included a definition of what the team meant by "best," the results of the model analysis, and an explicit, quantified description of the best strategy and its cost. The judges continue to be surprised by the number of papers where the abstract only describes what the team will attempt without describing what they found.

Similarly, many of the required nontechnical position papers omitted key details that a decision maker would need, such as particulars on the solution and the economic and noneconomic costs and benefits. A nontechnical letter does not mean that numbers are not included. Rather, it means that someone can read it meaningfully without having an education in advanced mathematics. Too many of the position papers omitted all details of the solution.

Papers that labeled figures and tables with informative captions were scored higher than those that did not.

The quality of citations was also a discriminator. Papers that cited their sources and provided complete references formatted according to a recognized standard were scored higher than those that did not.

Several of the very best papers were a joy to read. The explanations were clear and complete, and the phrasing was almost lyrical. The judges will continue to value outstanding writing.

Summary

The Outstanding teams modeled all the aspects of the problem described in the problem statement, included the standard contest discussions (assumptions, sensitivity analysis, strengths and weaknesses, etc.), had defensible cost models, explained the modeling choices made, and were well-written.

The judges were pleased by the student submissions. The topic allowed for a wide range of solutions, and the allowed choice of countries provided a diversity of solutions. The growth in the quality and number of submissions is very encouraging to those who work to promote the practice of good mathematical modeling.

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About the Author

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Judges' Commentary: The Frank Giordano Award for 2013

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Introduction

The Frank Giordano Award goes to a paper that demonstrates a very good example of the modeling process.

Having worked on the contest since its inception, Frank Giordano served as Contest Director for 20 years. As Frank says,

> It was my pleasure to work with talented and dedicated professionals to provide opportunities for students to realize their mathematical creativity and whet their appetites to learn additional mathematics. The enormous amount of positive feedback I have received from participants and faculty over the years indicates that the contest has made a huge impact on the lives of students and faculty, and also has had an impact on the mathematics curriculum and supporting laboratories worldwide. Thanks to all who have made this a rewarding and pleasant experience!

The Frank Giordano Award for 2013 goes to a team from Colorado College in Colorado Springs, CO. This solution paper, designated as Outstanding, is characterized by a high-quality application of the complete modeling process, including assumptions with clear justifications, a well-defined simulation, a case study, and sensitivity analysis. The paper showed originality

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and creativity in the modeling effort to solve the problem as given and was written clearly and concisely, making it a pleasure to read.

The Problem Statement

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement, desalinization, and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a non-technical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the "best water strategy choice."

The choice of countries was from the United States, China, Russia, Egypt, or Saudi Arabia.

The Colorado College team chose Saudi Arabia, and this dramatically impacted their modeling approaches. Their focus was on cost minimizing and production maximizing.

The Colorado College Paper

Executive Summary Sheet and Position Paper

This summary was well written and gave the reader a good idea of what to expect. It contained the appropriate specifics with regard to issues and was both concise and thorough. The team's position paper, written in an appropriate nontechnical manner, was longer than expected, but served as a true position paper, giving the government precise instructions and rationale.

Assumptions

After introducing each of three models used, specific assumptions were stated. For the most part, assumptions were reasonable. One assumption made in modeling the distribution problem was that the topography of Saudi Arabia is a plain two-dimensional surface. This was a common assumption made in the vast majority of submitted papers. Although the country has mountains, the assumption of flatness was probably less inappropriate for Saudi Arabia than for the other countries considered. Additionally, the team did recognize the impact of this assumption in their section on strengths and weaknesses of the model.

The Models and Methods

Considering Saudi Arabia's extensive coastline, the team began with a model for maximizing water production in desalination plants. After doing a review of the problem in general, of Saudi Arabia's efforts in this regard, and of previous attempts to model desalinated water production, they extended the Cobb-Douglas model to apply to reverse-osmosis plant production. They maximized the volume of water desalinated, subject to limitations on input factors and on the amount of electrical energy used. For their second model, the team minimized distribution costs of the water in attempting to serve the needs of agriculture, industry, and households. Whereas most teams created models to estimate future demand for water, this team relied on published studies about Saudi Arabia to estimate future needs. Although this technique was less sophisticated than most, for Saudi Arabia it seemed adequate. In minimizing distribution costs, they distinguished between the fixed costs associated with the pipes themselves and the laying of the pipes and the variable costs associated with pumping the water. Selected parameters were estimated using empirical studies in water distribution systems in the United States.

The third model focused on the maximization of production in wastewater treatment plants and once again used the Cobb-Douglas model. The team maximized the production level subject to constraints on the costs of electricity and other factors of production. The model would have been better if they had given additional rationale for the values used and expanded on factors associated with production.

Testing the Models

After determining the maximum amount of desalted water that could be produced each year, subject to limits on input factors and the amount of electrical energy used, the team tested the robustness of the result by doing sensitivity analysis, altering their inputs. To simulate the distribution of water, the team used consumption figures of the three main sectors: agriculture, industries, and households. They used hypothetical distances and did sensitivity analysis to validate that their model was not very sensitive to pipeline length and water consumption. The model would have been stronger if actual distances had been used. Due to a lack of specifics on the input information regarding other factors in wastewater treatment plants, such as labor, chemicals, and utilities, they were unable to properly estimate the power parameters in the Cobb Douglas model. They did demonstrate the impact of the government budget on production.

Recognizing Limitations of the Model

After each of the three models was presented and results demonstrated with sensitivity analysis, there was an analysis of the strengths and weaknesses of each model. They recognized limitations due to assumptions made and incomplete information available. The team also recognized the strengths of Saudi Arabia with regard to what the country has to offer in terms of natural resources and what they are presently doing regarding desalination.

References and Bibliography

The list of references was fairly thorough, and it was very good to see specific documentation of where those references were used in the paper.

Conclusion

The careful exposition in the development of the mathematical model made this paper one that the judges felt was worthy of the Outstanding designation. The team is to be congratulated on their analysis, their clarity, and using the mathematics that they knew to create and justify their own model for planning for the water needs of Saudi Arabia in the year 2025.

About the Authors

Rich West is a Mathematics Professor Emeritus from Francis Marion University in Florence, South Carolina, where he taught for 12 years. Prior to his time at Francis Marion, he served in the U.S. Army for 30 years, 14 of which were spent teaching at the U.S. Military Academy. He is currently working on a National Science Foundation Grant on freshman placement tests. He also serves as a Reading Consultant for AP Calculus and as a developmental editor for CLEP (College Level Equivalency Program) Calculus Exam. He has judged for both the MCM® and HiMCM® for over 10 years.

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ICM Modeling Forum

Results of the 2013 Interdisciplinary Contest in Modeling

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Introduction

A total of 957 teams from six countries spent a weekend working on an applied modeling problem involving the health of planet Earth in the 15th Interdisciplinary Contest in Modeling (ICM)®. This year's contest began on Thursday, January 31, and ended on Monday, February 4, 2013. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended interdisciplinary modeling problem concerning the state-changes associated with the health of planet Earth. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Five of the papers were judged to be Outstanding by the expert panel of judges.

COMAP's Interdisciplinary Contest in Modeling (ICM) involves students working in teams to model and analyze an open interdisciplinary problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders. The ICM is an example of COMAP's efforts in working towards these goals.

This year's problem was challenging in its demand for teams to utilize many aspects of science, mathematics, and analysis in their modeling and problem solving. The problem required teams to investigate the relationships of local

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and regional ecosystems to the global health of the planet. It required teams to understand concepts from the informational and environmental or social sciences to build network and statistical models to track the potential changes in Earth's global health. In order to accomplish their tasks, the students had to consider many difficult and complex disciplinary and interdisciplinary issues. The problem also included the requirements of the ICM to perform thorough analysis, research, creativity, and effective communication. All members of the 957 competing teams are to be congratulated for their excellent work and dedication to interdisciplinary modeling and problem solving.

Next year, we will continue the network science theme for the contest problem. Teams preparing for the 2014 contest should consider reviewing interdisciplinary topics in the areas of network science and social network analysis and assemble teams accordingly.

A Brief History of the ICM

Over the 15 years of the ICM, the contest has grown appreciably. In 1999, judging the papers of 40 ICM teams seemed like a challenge; but this year's judges were confident that we found five papers to receive the Outstanding classifications. As always, a panel of expert judges read the papers, judged their attributes, debated their merits, and decided on the rankings reported in this article. Looking at the range of topics over these 15 years (provided in **Table 1**), the contest shows its interdisciplinarity with problems involving elements from chemistry, physics, biology, engineering, information science, medicine, business, and network science. The problems also show a balance of public (government) and private (business) issues.

Table 1.

Participating teams and topics in the first 15 years of the ICM.

Year	Number of teams	Topic
1999	40	Controlling the spread of ground pollution
2000	70	Controlling elephant populations
2001	83	Controlling zebra mussel populations
2002	106	Preserving the habitat of the scrub lizard
2003	146	Designing an airport screening system
2004	143	Designing information technology security for a campus
2005	164	Harvesting and managing exhaustible resources
2006	224	Modeling HIV / AIDS infections and finances
2007	273	Designing a viable kidney exchange network
2008	380	Measuring utility in health care networks
2009	374	Balancing a water-based ecosystem affected by fish farming
2010	356	Controlling ocean debris
2011	735	Measuring the impact of electric vehicles
2012	1329	Identifying criminals in a conspiracy network
2013	957	Network modeling of Planet Earth's health

The Problem Statement: Network Modeling of Earth's Health

Background

Society is interested in developing and using models to forecast the biological/environmental health conditions of our planet. Many scientific studies have concluded there is growing stress on Earth's environmental and biological systems, but there are very few global models to test those claims.

The UN-backed *Millennium Ecosystem Assessment Synthesis Report* [2005] found that nearly two-thirds of Earth's life-supporting ecosystems, including clean water, pure air, and stable climate, are being degraded by unsustainable use. Humans are often blamed for much of this damage. Soaring demands for food, fresh water, fuel, and timber have contributed to dramatic environmental changes, from deforestation to air, land, and water pollution.

Despite considerable research on local habitats and regional factors, current models do not adequately inform decision makers on global issues of public and government concern in important high impact areas involving the overall health of the planet. Many models ignore complex global factors and are unable to determine the long-range impacts of potential policies. While scientists realize that the complex relationships and cross-effects in the myriad environmental and biological systems are the driving forces in many of the issues of concern in Earth's biosphere, current models often ignore these relationships or limit the systems' connections. The system complexities manifest in multiple interactions, feedback loops, emergent behaviors, and impending state changes or tipping points.

A recent *Nature* article [Barnosky 2012] by 22 internationally-known scientists outlines the need for scientific models and the importance of predicting potential state changes of the planetary health systems. The article provides two specific quantitative modeling challenges for better predictive models:

1. We need to improve bio-forecasting through global models that embrace the complexity of Earth's interrelated systems and include the effects of local conditions on the global system and vice versa.

2. Models are needed to identify factors that could produce unhealthy global state-shifts and to show how to use effective ecosystem management to prevent or limit these impending state changes.

The resulting research question is whether we can build global models, using local or regional components of the Earth's health, that predict potential state changes and help decision makers design policies based on potential impact on Earth's health. Although many warning signs are appearing, no one knows if Planet Earth is truly nearing a global tipping point or if such an extreme state is inevitable.

The *Nature* article and many others point out several important elements at work in the Earth's ecosystem (e.g., local factors, global impacts, multi-dimensional factors and relationships, varying time and spatial scales). There are also many factors that can be included in such a model—human population, resource and habitat stress, habitat transformation, energy consumption, climate change, land use patterns, pollution, atmospheric chemistry, ocean chemistry, bio diversity, and political patterns such as social unrest and economic instability. Paleontologists have studied and modeled ecosystem behavior and response during previous cataclysmic state shifts and thus historic-based qualitative and quantitative information can provide background for future predictive models. However, it should be noted that human effects have increased significantly in our current biosphere situation.

Requirements:

You are members of the International Coalition of Modelers (ICM) which will soon be hosting a workshop entitled "Networks and Health of Planet Earth"; and your research leader has asked you to perform modeling and analysis in advance of the workshop. The leader requires your team to do the following:

Requirement 1:

Build a dynamic global network model of some aspect of Earth's health (you develop the measure) by identifying local elements of this condition (network nodes) and appropriately connecting them (network links) to track relationship and attribute effects.

Since the dynamic nature of these effects is important, this network model must include a dynamic time element that allows the model to predict future states of this health measure. For example, your nodes could be nations, continents, oceans, habitats, or any combination of these or other elements which together constitute a global model. Your links could represent nodal or environmental influences or the flow or propagation of physical elements (such as pollution) over time. Your health measure could be any element of Earth's condition to include demographic, biological, environmental, social, political, physical, or chemical conditions.

Be sure to define all the elements of your model and explain the scientific bases for your modeling decisions on network measures, nodal entities, and link properties.

Determine a methodology to set any parameters and explain how you could test your model if sufficient data were available.

What kinds of data could be used to validate or verify the efficacy of your model? (If you do not have the necessary data to determine parameters or perform verification, do not throw out the model. Your supervisor realizes that, at this stage, good creative ideas and theories are as important as verified data-based models.)

Make sure to include the human element in your model and explain where human behavior and government policies could affect the results of your model.

Requirement 2:

Run your model to see how it predicts future Earth health. You may need to estimate parameters that you would normally determine from data. Again, this is just to test and understand the elements of your model, not to use it for prediction or decision making. What kinds of factors will your model produce? Could it predict state change or tipping points in Earth's condition? Could it provide warning about global consequences of changing local conditions? Could it inform decision makers on important policies? Do you take into account the human elements in your measures and network properties?

Requirement 3:

One of the powerful elements of using network modeling is the ability to analyze the network structure. Can network properties help identify critical nodes or relationships in your model? If so, perform such analysis. How sensitive is your model to missing links or changing relationships? Does your model use feedback loops or take into account uncertainties? What are the data collection issues? Does your model react to various human policies and therefore could it help inform planning?

Requirement 4:

Write a 20-page report (summary sheet does not count in the 20 pages) that explains your model and its potential. Be sure to detail the strengths and weaknesses of the model. Your supervisor will use your report as a major theme in the upcoming workshop and if it is appropriate and insightful to planetary health modeling will ask you to present at the upcoming workshop. Good luck in your network modeling work!

Potentially Helpful References

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Watson, Robert T., and A. Hamid Zakri (co-chairs). 2005. *Ecosystems and Human Well-Being: Synthesis.* Millennium Ecosystem Assessment Board. Washington, DC: Island Press. `http://www.unep.org/maweb/documents/document.356.aspx.pdf`.

The Results

The 957 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each paper was then read preliminarily by triage judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in late March. The judges classified the 957 submitted papers as follows:

	Outstanding	Finalist	Meritorious	Honorable Mention	Successful Participant	Total
earth's Health	5	7	121	380	444	957

Outstanding Teams

Institution and Advisor	Team Members
"Two-layered Coupled Network Model of Earth's Health"	
Northwestern Polytechnical University	YiXiao Shu
Xi'an, China	YuHui Wang
LuLu Pan	Chao Chen
"Two-tier Communication Network Model of Global Health"	
Beijing University of Posts and Telecommunications	Xu Yao
Beijing, China	Yingnan Xiao
Xinchao Zhao	Chao Wang

'Measuring Earth's Health by CO2: A Technology Diffusion Network Approach"
Peking University
Beijing, China
Shu Lin Zhou
Ruo Fei Zhao
Xin Qi Zhao
Ming He Hu

"You Pollute, I Polluted!"
Zhejiang University
Hangzhou, China
Yuan Ying
Changyou Zhu
Weicheng Bai
Wenqing Yang

"Saving the Green with the Greens"
Rensselaer Polytechnic Institute
Troy, NY
Mark Holmes
Diogo Moitinho de Almeida
Eric Shapiro
Amanda Knight

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Director. Additional awards were presented to the team from SSE, Beijing University of Posts and Telecommunications, by the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors
Chris Arney, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Joseph Myers, Computing Sciences Division, Army Research Office, Research Triangle Park, NC

Associate Directors
Rodney Sturdivant, Dept. of Statistics, Ohio State University, Columbus, OH
Tina Hartley, Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY

Judges
Amanda Beecher, Dept of Mathematics, Ramapo College, Mahwah, NJ
Rachelle DeCoste, Dept of Mathematics, Wheaton College, Norton, MA
Kayla de la Haye, RAND Corporation, Santa Monica, CA
John Kobza, Dept. of Industrial Engineering, Texas Tech University, Lubbock, TX
Kari Murad, Science Department, College of St. Rose, Albany, NY

Kathleen Snook, COMAP Consultant, Bedford, MA
Robert Ulman, Network Sciences Division, Army Research Office, Research Triangle Park, NC
Jie Wang, Computer Science Dept., University of Massachusetts, Lowell, Lowell, MA

Triage Judges

Jerrod Adams, Chris Arney, Jocelyn Bell, Kevin Blaine, Alex Chaney, Peter Charbonneau, Nicholas Clark, Gabe Costa, Michelle Craddock, Kevin Cummiskey, Chris Eastburg, Michael Findlay, Hilary Fletcher, James Gatewood, Andy Glen, Paul Goethals, Tina Hartley, Alex Heidenberg, Heather Jackson, John Jackson, Phil LaCasse, Andrew Lee, Doug McInvale, Timothy Povich, Jarrod Shingleton, James Starling, Rodney Sturdivant, Andrew Swedberg, Johan Thiel, Chris Weld, Brian Winkel, and Shaw Yoshitani.
—all of Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY;
Kathryn Coronges and Luke Gerdes of the Dept of Behavioral Sciences, U.S. Military Academy, West Point, NY;
Rob Nowicki, Jon Roginski, and Keith DeGregory, U.S. Army;
Amanda Beecher, Dept. of Mathematics, Ramapo College of New Jersey, Mahwah, NJ;
Elizabeth Russell, Dept. of Mathematics, Western New England College, Springfield, MA;
Csilla Szabo, Dept. of Mathematics, Rensselaer Polytechnic Institute, Troy, NY;
Sheila Miller, Dept. of Mathematics, NY City College of Technology, Brooklyn, NY; and
Robert Wooster, Dept of Mathematics, College of Wooster, Wooster, OH.

Acknowledgments

We thank:

- the Institute for Operations Research and the Management Sciences (INFORMS) for its support in judging and providing prizes for a winning team, and
- all the ICM judges for their valuable and unflagging efforts.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the team papers here is the result of undergraduates working on a problem over a weekend. Editing (and usually substantial cutting) has taken place; minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. The student authors have proofed the results. Please peruse these students' efforts in that context.

To the potential ICM advisor:
It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

The complete roster of participating teams and results has become too long to reproduce in the *Journal*. It can now be found at the COMAP Website:

`http://www.comap.com/undergraduate/contests/mcm/contests/2013/results/2013_ICM_Results.pdf`

About the Author

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM)®. In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

Saving the Green with the Greens

Diogo Moitinho de Almeida
Eric Shapiro
Amanda Knight

Rensselaer Polytechnic Institute
Troy, NY

Advisor: Mark Holmes

Summary

Problem Clarification

With environmental doom impending, governments across the globe are trying to find the best way to combat this fate. Unfortunately, they do not have access to models for the decision making that they wish to achieve, due to a lack of understanding of the direct human element in the situation. We base our model entirely on human relationships and influences—something that policy makers have the most control over.

Model Design

Our model directly predicts human results (measured in 2013 U.S. dollars) with economic variables that are easily influenced by legal policy. We further improve our initial design into a network model by incorporating geographic proximity, diplomatic relations, and clustering data. All parameters are derived entirely from a data-driven approach.

Results

Our design, shown in **Figure 1**, gives us excellent prediction accuracy, stable solutions based on multiple forms of sensitivity analysis, and easily interpretable results. Our network model allows us to use the famous PageRank measure to determine the most influential nations. Additionally, running simulations on individual countries, implementing optimal policy, and measuring the global effects of each on the total economic loss due to the environment, shows exactly which countries are most influential to the

The UMAP Journal 34 (2) (2013) 211–228.

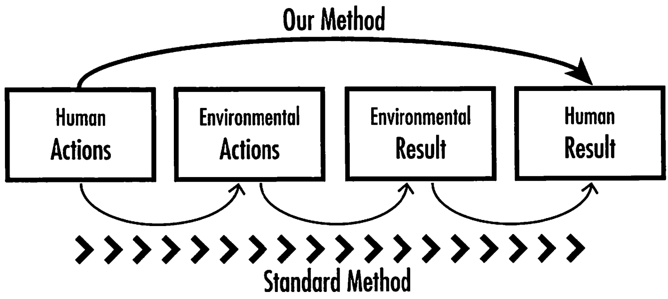

Figure 1. Comparison of our approach with that of others.

fate of Earth's health, and the necessary conditions on which to stabilize the world's rising environmental damage toll.

Introduction

Clarification of the Problem

Despite the best efforts of the scientific community, many aspects of environmental science are difficult to model, and it is especially difficult to see the cause-and-effect dynamics of human action, due to the inherent randomness of the relationship between anthropogenic factors and environmental response [Pindyck 2007]. Given the difficulties, we seek to create a new model that looks to the human aspect of ecological damage, for better understanding of how human reactions can alter the fate of the planet.

Model Design

Standard approaches to environmental modeling involve measuring intermediate environmental variables as the link between human actions towards Earth's environment and the Earth's response.

While there are merits to that approach, there are also drawbacks, such as ignoring the potential for human reaction to the environment [Chakravorty et al. 1997]. Because most models do not focus on this relationship, that approach has not given policy makers a very clear picture of what needs to happen on their part.

We propose a model that takes a data-driven approach to directly predict human results from human actions using machine learning techniques. Using this methodology allows us to avoid the pitfalls of the standard method by using human action as our independent variable. The advantage is that we fit less of the noise of environmental variables that behave indistinguishably from randomness, and we also have the possibility of capturing latent variables that may not normally be measured by environmental studies.

Earth Damage Score (EDS)

To develop a meaningful measure for the environmental harm experienced by any particular country, we define the yearly Earth Damage Score according to the economic loss by that country, measured in 2013 U.S. dollars, as induced by natural disasters and carbon dioxide damage, using data from the World Bank and from Maplecroft. We use this measure because it represents environmental effects in a format that is easily digestible and directly relevant to policy makers.

We design our models to predict the yearly percentage increase in each country's Earth Damage Score, allowing the models to take compounded effects into account. That is necessary for environmental predictions, because the effects of past actions persevere even as new measures are taken.

As an example, **Figure 2** shows the EDS history for the U.S. and China.

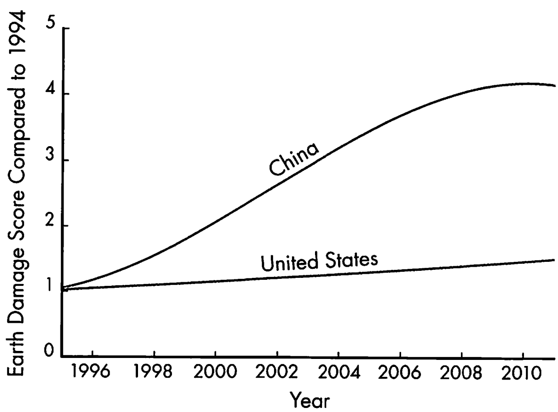

Figure 2. EDS history for the U.S. and for China.

We use root-mean-square error as a metric for the inaccuracies of our predicted data. This commonly used measure for error is obtained by squaring the data, then calculating the arithmetic mean of the difference between the actual and predicted data and, finally, taking the square root. This is ideal because it ensures that small errors in differing directions do not cancel each other and that the unit of measurement is kept meaningful.

Human Actions

We use certain economic variables as inputs into our model because they are capable of being influenced directly by legal policy and because they are indicated by evidence to influence environmental factors.

Population and Change in Population

Not only does a rise in population inherently increase agricultural and industrial growth, it adds more bodies to shelter and transport, and it contributes more waste [Goodland 1992].

Agricultural Growth

Agricultural practices today result in deforestation, the release of pesticides, soil degradation, and water pollution from runoff. They are also a tremendous consumer of energy and a large contributing factor to the emission of carbon dioxide and other toxic chemicals into the atmosphere [Trautmann et al. 2012].

Industrial Growth

While industrial development is necessary to move away from our consumption of fossil fuels and toward a more sustainable source of energy, it nonetheless continues to contribute to the addition of toxic chemicals to the environment and carbon dioxide emissions [Grubb et al. 2004].

GDP Growth

Correlations have been found between economic growth and the production rate of pollutants by a country [Grubb et al. 2004; Friedlingstein et al. 2010].

Literacy Rate

Literacy rate is a good indicator of poverty within a country and it is correlated to other factors such as agriculture and industrial growth [Ahluwalia 1976]. It combines a number of factors that are subject to human influence into one easily obtained number.

Data Preparation

All of our economic variables are from the list of world development indicators in the World Bank's Databank. We use the first derivative of EDS with respect to time to run our simulation and obtain future predictions, while controlling the second derivative directly to represent policy changes in each country.

Assumptions

- **Environmental damage is influenced by human actions.** Our model measures the effect of human action on economic loss due to environmental factors. This is widely perceived to be true, but the assumption is necessary for the creation of our model.
- **Human action can be controlled, at least in part by legal policy.** This is also perceived to be true, and necessary for predicting the possible outcomes of policy change.
- **Countries that are near each other share similar environmental effects.** Assuming that environmental effects spread beyond borders allows us to model the interdependencies of the environment.
- **Policy changes are likely to propagate across diplomatic links.** This assumption allows our model to capture the ripple effect on the spreading of ideology between nations.
- **Each country's environmental invariants behave approximately as constant throughout the time frame of the analysis.** Our model doesn't take environmental variables into account, and thus the model captures national invariants such as size, latitude, and longitude.
- **Our calculated Earth Damage Score is a good proxy for the true economic loss due to environmental damage.** Solving for any country's true economic loss due to the environment is an incredibly complex problem, and is difficult to measure (it is hard to truly know how something such as biodiversity loss will cost); therefore we must assume that our measure approximates the true loss well.

Our Model

Tikhonov Regularization

For a simplistic model, we used Tikhonov regularization, a machine learning algorithm and a form of linear regression that includes a regularization matrix to prevent overfitting of the data [Hoerl and Kennard 1970]. This algorithm is appropriate for the model because it is especially well-suited to problems with limited data and a relatively small Vapnik–Chervonenkis dimension [Vapnik 2000]. Using other models in this situation would lead to much worse prediction rates for future applications. We have:

$$\min_{\mathbf{w}} ||A\mathbf{w}-\mathbf{b}||^2 + ||\Gamma\mathbf{w}||^2, \hat{w} = \left(A^T A + \Gamma^T\Gamma\right)^{-1} A^T\mathbf{b},$$

where

- the economic variables are the features of the algorithm represented by the A matrix,
- the changes in EDS are represented by the $\mathbf{b}$ vector, and
- the identity matrix is used as the regularization matrix Γ.
- one binary variable for each country is added, as an additional feature to allow the algorithm to find optimal constant values for each country, allowing the algorithm to take environmental and geographical invariants into account.
- $\mathbf{w}$ is the vector of weights for each feature, with the weights of each binary country variable equal to that country's constant value.
- $\hat{w}$ is the set of weights that minimizes the objective function, and is what is used for future prediction tasks.

As a comparison, we also implemented a similar model with environmental variables (carbon dioxide emissions, electric power consumption, water pollution, livestock production and forest area) instead of economic variables, so as to have a baseline for model accuracy. We have two more baselines: a simulated model where the EDS is predicted with a normally distributed random variable, and another naïve model that predicts the arithmetic mean regardless of input variables. As we can see from **Table 1**, the economic model does significantly better than either the random model or the constant model, but it is 35% less accurate than the environmental model.

Table 1.
Comparison of models.

Model	Root-Mean-Square Error (percent per year)
Random	2.86
Constant (at mean)	2.39
Economic	1.55
Environmental	1.50

Geographic Network Model

Our simplistic model treats each country as an independent entity and isn't designed to incorporate more sophisticated parameters such as geographical proximity. We improve our model by creating a global network and using a weighted modification of a k-nearest neighbor algorithm [Coomans and Massart 1982]. Each node in our network represents a country, with size proportional to the number of countries nearby (this measure is called *degree centrality* (**Figure 3**), which corresponds to the influence of a country over its neighbors). Its color and relative position within the graph correspond to the modularity class to which it belongs, a commonly-used measure that intuitively gives an indication of the influential community to which a country belongs.

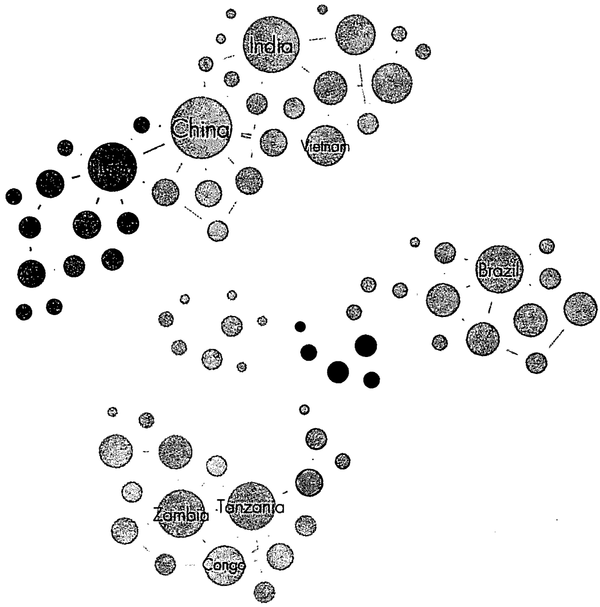

Figure 3. Geographic network of countries. Countries are considered adjacent if they border each other; node size corresponds to the number of bordering countries, and color represents modularity class (intuitively, the community to which a country belongs).

The graph's adjacency matrix is generated by assigning links if countries border each other geographically or are otherwise determined to be in extremely close geographical proximity.

Our methodology is first to find the estimates of the simple model, and then have the final predicted value to be a weighted average of all nodes of distance two or less apart, using a standard weighting scheme of one divided by the distance of the node plus one.

$$\mathrm{EDS}_{i,\mathrm{final}} \frac{\sum_{||i,j|| \leq 2} \frac{\mathrm{EDS}_{j,\mathrm{initial}}}{||i,j||+1}}{\sum_{||i,j|| \leq 2} \frac{1}{||i,j||+1}}.$$

Applying this modification to our simplistic model results in a 25% decrease in root-mean-square-error for predicting EDS.

Hybrid Network Model

Policies of a country can easily influence those of its allies [Hartigan and Wong 1979]. This fact leads us to model another dynamic: the diplomatic relationships among countries. By designing an adjacency matrix with weights corresponding to the perceived strength of the diplomatic relations and applying the same algorithm as for the geographic network model, we obtain a model capable of simulating the "ripple effect," demonstrating social influence [Cialdini 2001].

We create the adjacency matrix by weighting each link's strength by the number of times that each pair of countries are in common political and economic intergovernmental organizations. This matrix is then normalized so that the maximum value of a link's weight cannot exceed 1. We assume that this value is a good measure of the diplomatic relationship between nations. We apply an unsupervised learning algorithm, k-means clustering with three means, to find relationships between any pair of countries with similar behavior [Hartigan and Wong 1979]. These similarities are also represented as edges in the modified adjacency matrix.

Figure 4 graphs the 88 EDS curves, one for each country in our data, with the color of the curve corresponding to the cluster that the country belongs to. We further demonstrate the clustering by plotting the data points of each country in a two-dimensional representation (**Figure 5**) using a technique called randomized principal component analysis [Rokhlin 2009].

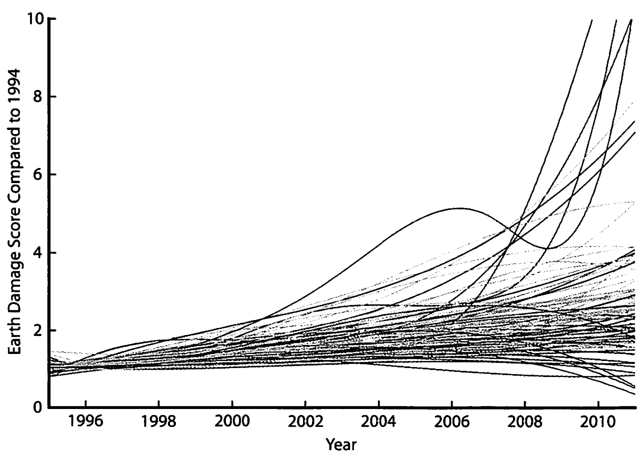

Figure 4. EDS curves for countries.

To recapture the influence of geographic proximity in our model, we combine our two previous networks by assigning them parameters so as to create a realistically-scaled linear combination of link weights. The minimum error is obtained when the scaling parameter for the diplomatic network is roughly 9 times that of the geographic network (**Figure 6**). Us-

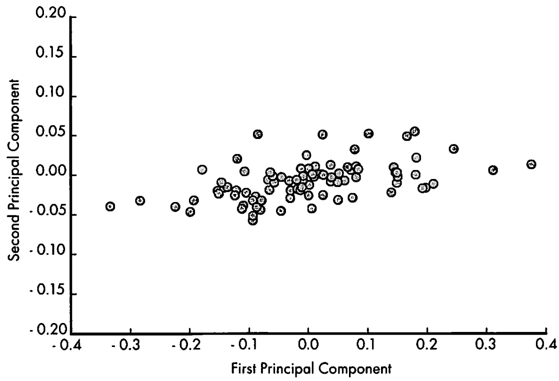

Figure 5. Graph of second principal component vs. first principal component, illustrating clustering of countries (by color).

ing these parameters, we obtain the optimal combination of geographic and diplomatic relationships between countries—our hybrid network (**Figure 7**).

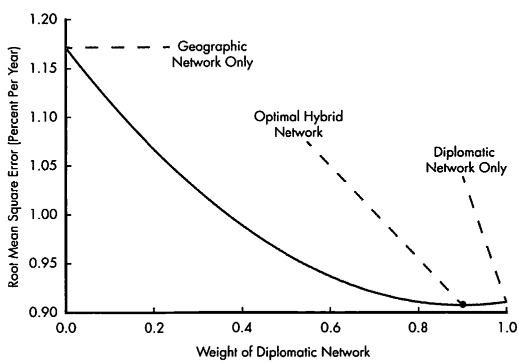

Figure 6. Comparison of root-mean-square error for the different networks.

This allows us to obtain an optimal RMS error 23% lower than with the geographical network model alone (**Figure 8**).

Measuring Centrality with PageRank

As a measure of a country's importance in this hybrid model, we can no longer simply use degree centrality, because nodes with links of lesser weight are less influential than those with greater weight. It thus is necessary to adopt a new metric for the influence of a node.

A common choice is *betweenness centrality*, which measures how many times a node acts as a bridge. This is not a good criterion for us, because there are a number of countries (such as Kyrgyzstan) that interact with

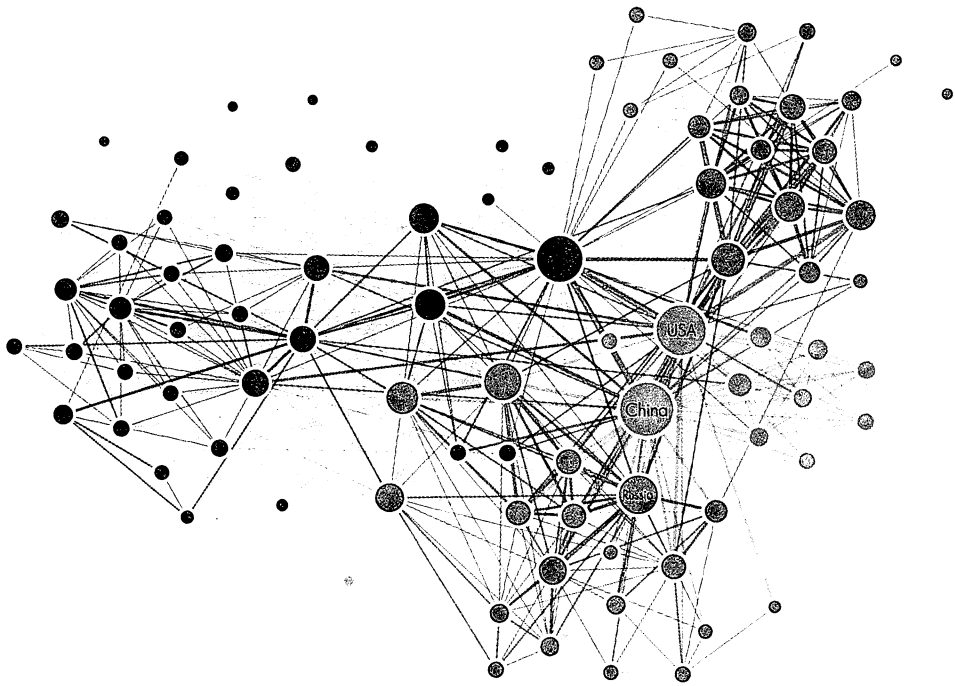

Figure 7. Hybrid network. Countries are considered adjacent if they are geographically close or share diplomatic links. Node size corresponds to the PageRank of each country, our measure of diplomatic influence. Color represents modularity class (intuitively, the community to which a country belongs).

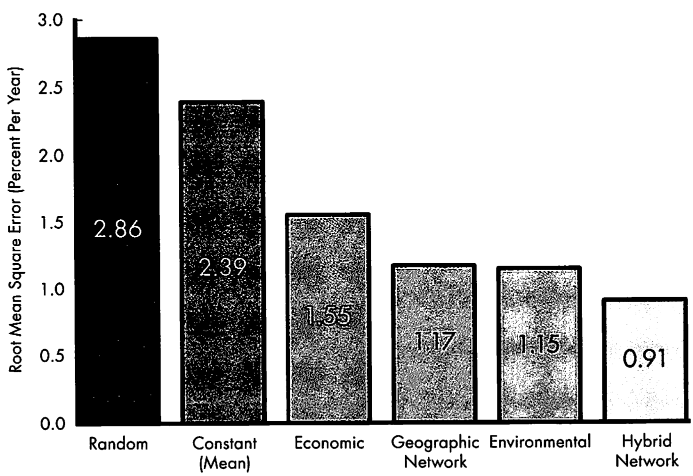

Figure 8. Comparison of models.

several distinct groups yet have very little influential power.

Another choice is *closeness centrality*, which is essentially a measure of average path length between a given node and any other node in the network. Again, this is a bad choice; although it gives some sense of how quickly policy changes could propagate through the network, it still neglects the weights of each connection and thus does not provide a very good idea of a country's influence.

Instead, we use the PageRank link analysis algorithm to determine our metric of influential power [Page et al. 1999]; it is represented in **Figure 7** by the size of each node. Color and location again represent modularity class.

The Biggest Influences

As another method of identifying key nodes in our graph, we find which countries could make the largest overall effect on average EDS. To calculate this for each country, we solve for the direction of the second derivative of economic variables, which we assume policy could influence, that would minimize average EDS, and find the difference between implementing a predicted optimal policy versus not implementing any policy. Not implementing a policy is equivalent to setting the second derivative of the economic variables to be zero. The results of this analysis are shown in **Figure 9**.

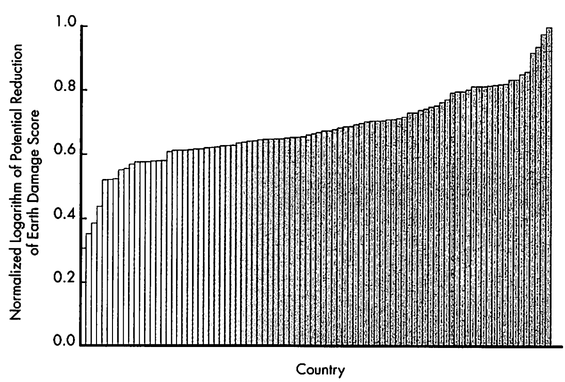

Figure 9. Potential reduction of EDS for the countries.

Our model predicts that the countries that could make the largest difference in the world's EDS are China, the United States, and to a lesser extent India. The difference that these "top 3" could make is several orders of magnitude larger than that possible by the rest of the countries (**Figure 10**).

Our model predicts that

- implementing optimal policy in any two of the top three countries would be enough stabilize the EDS score for many generations, and

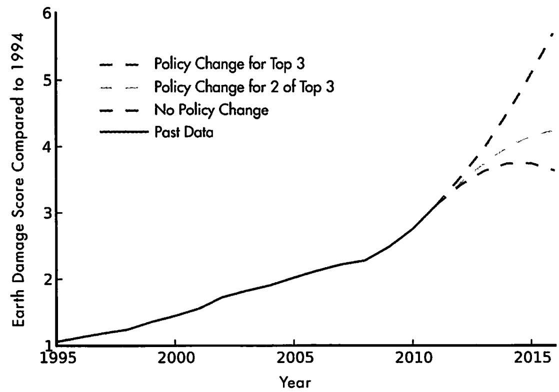

Figure 10. Potential reduction of EDS by the U.S., China, and India.

- implementing optimal policy in all three of China, the U.S., and India could bring the entire world's EDS down in slightly less than five years.

Furthermore, this analysis gives us a human interpretable direction of the optimal second derivative of our economic variables. Based on the data, optimal policy greatly emphasizes a decrease in the growth rate of the country's population. This knowledge can be used to further inform policy makers of the most effective policies to reduce EDS.

Sensitivity Analysis

A certain amount of confidence in environmental models is necessary for policy-making decisions. Without such confidence, it is difficult to determine the best policy and its potential benefits; but currently policy makers do not have this level of confidence [Pindyck 2007]. To increase confidence in our results, we perform multiple forms of sensitivity analysis.

Cross Validation

We use 10-fold cross validation, a common standard for measuring the robustness of algorithms [Kohavi 1995]. The method involves randomly partitioning the data into tenths; for each tenth, the remaining nine are used to predict its EDS. For our data, this method shows a cross-validation error less than 6% higher than training-set error, demonstrating robustness of the model [Krogh and Vedelsby 1995].

Gaussian Noise

As another form of testing the stability of our predictions, we add to our input variables Gaussian noise with increasing standard deviation, so as to check if our predictions are stable with perturbations in our data. For low amounts of noise, we achieve remarkably comparable accuracy to that of the model with correct data; and for standard deviations of the noise less than 0.8, we obtain comparable accuracy to the model that takes environmental variables into account (**Figure 11**).

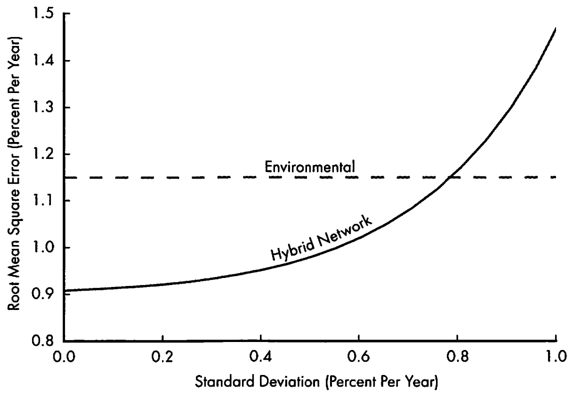

Figure 11. Comparison of the hybrid network model to the model with environmental variables, based on the effect of Gaussian noise (with specified standard deviation) in the hybrid network.

Test-Set Future Prediction

As a third form of verifying model accuracy, we partition the last three years of our data into a test set and train the model (including clustering and hyperparameter optimization) on only the earlier data (1995–2008). We then attempt to predict the EDS of our test set data given its economic variables. This prediction is within 5% of the true EDS, showing our model to provide excellent accuracy for medium length time intervals (less than 15 years) [Schapire 1999].

Strengths

Human-Relevant

Our independent and dependent variables are directly relevant to people without needing to pass through complex calculations to see potential impact.

Flexible Model

Our model was designed to be extremely flexible with the types of data that it can use for prediction. This design choice was made in hope that if it's used in the future, we can provide even better predictions.

Stable Predictions

We verify with a variety of methods that our model has very stable solutions, allowing us to make predictions with confidence, which is especially important in this field.

Simplicity and Human Interpretability

By using simple but powerful models, we retain most of the accuracy of a complex model, with the added benefit that every parameter of the algorithm is interpretable by humans, thus empowering decision-making ability.

Weaknesses

Data Limitations

We use an entirely data-driven model to attain maximum accuracy. Unfortunately, we can't take countries into account whose data is too incomplete, even with state-of-the-art data-imputation techniques. This causes a selection bias within our model; most notably, a majority of countries in the European Union are missing.

Domain Expertise

Like most machine learning algorithms, our model can't easily use the knowledge of an expert in environmental science to enhance its predictive capabilities. We justify the usage of machine learning because most models today have an entirely different focus from ours, and thus wouldn't significantly aid our model.

Long-Term Predictions

Our model suffers from the same flaw as all other environmental models: a lack of understanding of the Earth's reactions to the "state shifts" that we are so very concerned with, e.g., global climate change. When the Earth crosses these "critical transitions," it tends to very abruptly override the very trends all models are based on [Barnosky et al. 2012]. We certainly do not know what will happen to the human factors when the Earth crosses this threshold.

Recommendations

We have recommendations for policy makers:

- **Record More Data** Recording data for your country will only help inform you and all of your fellow citizens.
- **Tailor Your Models** By including economic and other social measures in creating ecological predictions, you not only add an important dimension to the problem itself, but it is easier to understand how we humans can influence our environmental future.
- **Make a Change** Our research indicates that the best use of policy is population management. Our model has predicted this to be good for both the environment and the economy, and others have made similar claims [Bongaarts 1992]. The highest compliment that we as a team could receive is for our model to be used with even more data to help inform and make a positive change in policy.

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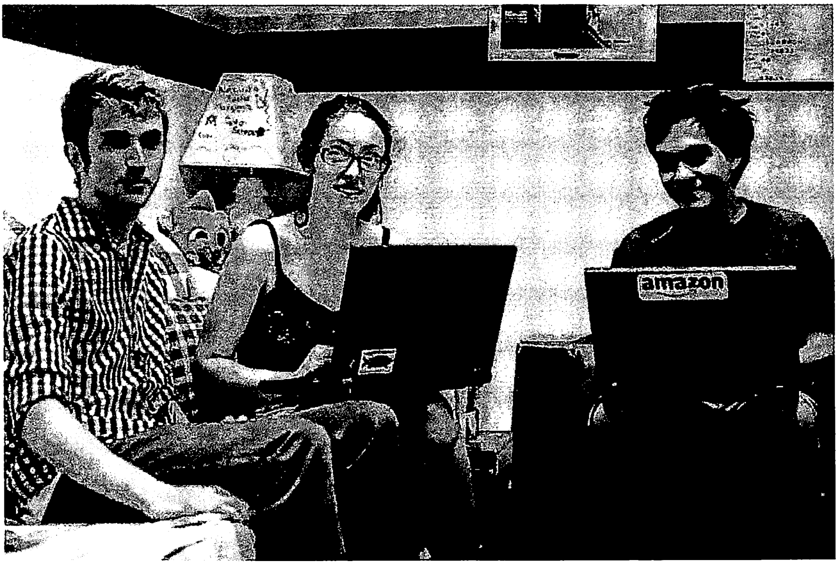

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Judges' Commentary: Modeling Earth's Health

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Introduction

The general topic area for this year's Interdisciplinary Contest in Modeling (ICM)® was network science. This year's problem was an extremely challenging one that resulted in a decline in submitted reports for ICM from 1329 teams in 2012 to 957 teams this year. It is announced in this issue that network science and / or social network analysis will continue to be the topical area for next year's problem as well. So, for teams who want to prepare early for next year's contest, prepare by studying network modeling and assembling a team with that subject in mind.

The ICM continues to be an opportunity for teams of students to tackle challenging, real-world problems that require a wide breadth of understanding in multiple academic subjects. These elements of study are often included in the definition of network science—an emerging subject that blends structures, processes, data and applications from mathematics, computer science, operations research, sociology, information science, and

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several other fields. ICM problems are often open-ended, challenging interdisciplinary problems. This year's problem required significant innovation and creativity to build a viable model of Earth's health.

The complexity of this ICM problem and the short four-day time limit of the contest require effective communication and coordination of effort among team members. One of the challenging issues for ICM teams is how to best organize and collaborate to use each team member's skills and talents to tackle the complex nature of these problems. Teams that resolve this organizational challenge and cooperate well often submit solutions that can rise to the higher level of ICM awards.

This Year's Earth's Health Problem

There have been recent concerns about the growing stress on Earth's environmental and biological systems, but there are very few global models to test these concerns and what can be done though governmental policies to mitigate the stress. Therefore, society has been interested in developing and using models to forecast the biological / environmental health conditions of our planet.

The UN-backed Millennium Ecosystem Assessment Synthesis Report found that nearly two-thirds of Earth's life-supporting ecosystems, including clean water, pure air, and stable climate, are being degraded by unsustainable use. Soaring demands for food, fresh water, fuel, and timber have contributed to dramatic environmental changes, from deforestation to air, land, and water pollution.

Many studies conclude that current models do not adequately inform decision makers on global issues of public concern in these important high impact areas. While scientists realize that the complex relationships and cross-effects in the myriad environmental and biological systems are the driving forces in many of the issues of concern in Earth's biosphere, current models often ignore these network relationships or limit the systems' connections. The network-based complexities manifest in multiple interactions, feedback loops, emergent behaviors, and impending state changes or tipping points.

Although many warning signs are appearing, no one knows if Planet Earth is truly nearing a global tipping point. There are many factors that can be included in such a model—human population, resource and habitat stress, habitat transformation, energy consumption, climate change, land use patterns, pollution, atmospheric chemistry, ocean chemistry, bio diversity, and political patterns such as social unrest and economic instability. This year's ICM problem required the following:

Requirement 1:

The team had to build a dynamic global network model of some aspect of Earth's health by identifying local elements of this condition and appropriately connecting them. The model should have included a dynamic time element that allowed the model to predict future states. Possible models that were suggested were network nodes as nations, continents, oceans, or habitats; and network links could represent environmental influences or the flow or propagation of physical elements. Additional elements of this requirement were data availability to validate or verify the efficacy of your model and inclusion of a human element in the model to explain how human behavior and government policies could affect the model.

Requirement 2:

Next was the requirement to run the model to see how it predicts future Earth health and to test the elements of the model—not to use it for prediction or decision-making. Teams had to consider if the model could predict state-change or tipping points and help inform decision makers on important policies.

Requirement 3:

The third requirement was to analyze the model's network structure. This was included to see how network science could be utilized in a deeper, more theoretical, understanding of a global network.

Judges' Criteria

The panel of judges was impressed by the modeling in the submissions of many teams and fascinated by the variety of creative approaches that teams used to address the issues and questions that were posed by the problem. Many papers were rich in modeling methodology and some in modeling creativity.

To ensure that the individual judges assessed submissions on the same criteria, a judging rubric and guide was developed. The general framework used to evaluate submissions is described below.

Executive Summary

It was important that teams succinctly and clearly explained the highlights of their submissions. The executive summary should contain brief descriptions of both the modeling approach and the bottom-line results. The main sections of the report provide a more detailed explanation of the contents of the executive summary. One mark of an Outstanding paper was a summary with a well-connected and concise description of the approach used, the results obtained, and any recommendations.

Modeling

A well-defined measure of Earth's health was needed to initiate and build a viable model. Many teams used a pollution measure, such as carbon emission, CO2, heavy-metal pollution, air quality, or ocean water quality; others used a biology-based measure such as antibiotic health, food webs, or species extinctions. Other areas used to construct planet health measures were based on political, economic or financial data.

Some teams incorporated a game-theory framework in which the model pitted two competitors —Planet Earth against human destroyers, human technology benefits against technology's destructive forces, or the Earth's resilience against pollutants caused by humans. In many ways, these models resemble predator-prey models, with Earth filling the role of prey.

Still other teams used the conceptual framework of network connections and measured health by determining the probability of future effects within the context of currently-known data. For some teams, health was a scalar value, while others established a multidimensional vector with components such as land use and forest coverage, human population, economy, technological progress, political stability, and resource utilization. Still others used a multilevel approach with aggregation of measures at regional or local ecosystems, countries, continents, oceans, and earth levels.

The resulting mathematics included differential equation models resembling SIS/SIR epidemic models, neural networks, and discrete dynamical systems. Some teams used the explicit structures of networks or graphs to determine classic local or global network metrics, properties, or node clusters; in this case, critical assumptions like the directionality of influence and connection within the network lead to viable network models. Other teams ignored any network structure and performed classical data mining and element classification and discrimination; in this case, teams often found prioritization and ordering easier but their model offered little predictive capability.

No matter the modeling framework, the assumptions needed for these models and the careful and appropriate development of these models were important in evaluating the quality of the solutions. The better submissions explicitly discussed why key assumptions were made and how these assumptions affected model development. Stronger submissions presented a balanced mix of mathematics and prose rather than a series of equations and parameter values without explanation. One major discriminator was the use or misuse of arbitrary parameters without any explanation or analysis. Establishing and explaining parameter values in models are at least as significant as making and validating assumptions.

Probably the most challenging aspect of this ICM problem was the determination of the possibility or the effects of tipping points. The problem asked teams to assess if their model could predict state change or tipping points in Earth's condition in order to provide warning about global consequences of changing local conditions. Some teams ran simulations or performed data analysis to find threshold values or state change conditions. Other models included feedback loops that changed as model parameters or outputs changed. The judges recognized the difficulty in performing this analysis and rewarded strong tipping-point models.

Science

Teams often used environmental factors as proxies or indicators of planet heath. These highly sophisticated and complex dynamic processes are one possible type to model Earth's health. Some teams did effective background research and analysis of this aspect of the problem, and included elements of scientific analysis in their model, or described how their model could accommodate such capability had appropriate data been available.

No matter what modeling was performed by the teams, the interdisciplinary nature of this problem was fully revealed in its requirements and in the background investigation performed by the teams. The ICM students were exposed to many scientific fields in the context of their background research and the team reports required proper documentation of the team's research sources.

Data/Validity/Sensitivity

Once the model was created, the use of test data and checks on the accuracy and robustness of the solution helped some teams to build confidence in their modeling approach. Sensitivity analysis to determine the effects of assumptions and changing data and errors can be empowering to the modelers; this is especially true for highly-structured and powerful data-rich models like networks. Some network structures are highly robust and flexible, while others are fragile and highly sensitive to data. While this sensitivity analysis is a challenging element of network modeling, it was important to address this issue in the report, especially as it related to the effects of government policy making. This was another challenging element of the problem—how to use the model to inform and guide potential policy decisions. Teams that did this well quickly rose to the top of the judges' evaluations.

Strengths/Weaknesses

Discussion of the strengths and weaknesses of the models is where students demonstrate their understanding of what they have created. The utility of a model fades quickly if team members do not understand the limitations or constraints of their assumptions or the implications of their methodology. Networks are complex structures; therefore, the strengths and weakness are often hidden from direct view or full control of the modeler. Some of the better reports presented these elements despite these challenges.

Communication/Visuals/Charts

To clearly explain solutions, teams used multiple modes of expression including diagrams and graphs, and clearly-written English. A report that could not be understood did not progress to the final rounds of judging. Judges were often well-informed through the amazing array of powerful charts and graphs that explained both models and results. **Figures 1–3** show just a brief glimpse the richness of this kind of graphical analysis and reporting.

Recommendations

Teams were specifically asked to discuss how governmental policies could affect the results of their model and determine if there were impending state changes or tipping points in Earth's condition. It was important to include a recommendation of how modeling could inform decision makers on deciding on important policies as related to humans' effects on planet Earth. The ability of teams to evaluate the results of their analysis to make recommendations was important in identifying strong submissions.

Discussion of the Outstanding Papers

Many different approaches were used by ICM teams to model various aspects of the problem. As a result, the submissions this year were diverse and interesting to read. Overall, the basic modeling was often sound, creative, and powerful. Those papers that did not reach final judging generally suffered from two shortcomings:

- Some lacked clear explanation of the structure of their model. They provided some details but not a complete description of their model and its purpose.
- Others failed to connect their mathematical models to the aspects and basic elements of the science of their health measurement.

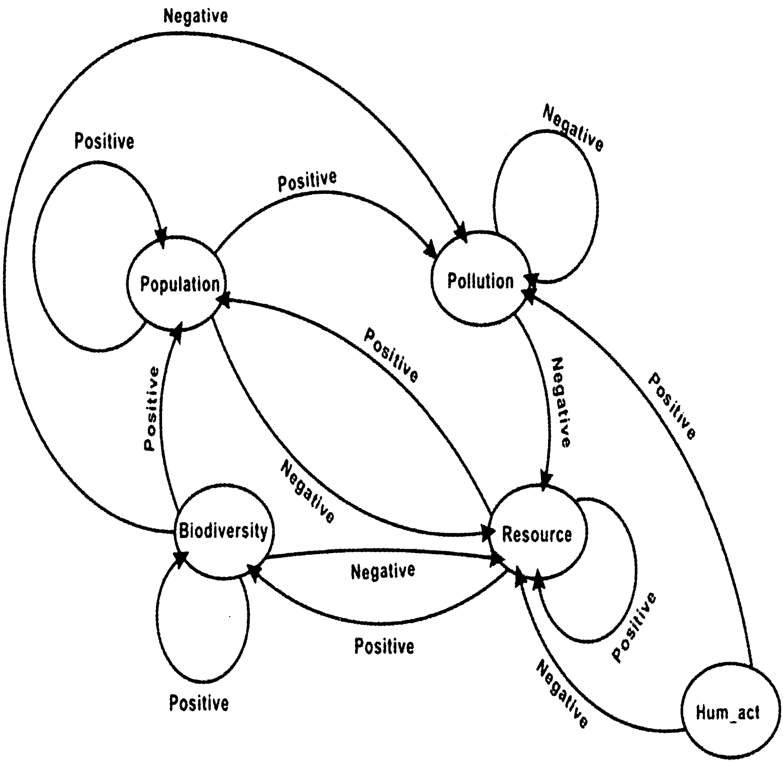

Figure 1. Some teams provided simple but informative graphic schematics to show the relationships and connections of the network framework of their models. This graphic is from Team 17802 from Sun Yat-sen University.

In general, incomplete communication was the most significant discriminator in determining which papers reached the final judging stage.

Although the outstanding papers used different methodologies, they all addressed the problem in a comprehensive way by embracing the complexity of the Earth's health issues that they had chosen. These papers were generally well-written and presented explanations of their modeling procedures. In several Outstanding papers, a unique or innovative approach distinguished them from the rest of the finalists. Others were noteworthy for either the thoroughness of their modeling or the power of their communicated results. Summaries of the five Outstanding papers follow.

Northwestern Polytechnical University: "Two-layered Coupled Network Model of Earth's Health"

The team's approach was to develop a two-layered coupled network model based on the Lotka-Volterra equations and reaction-diffusion. They

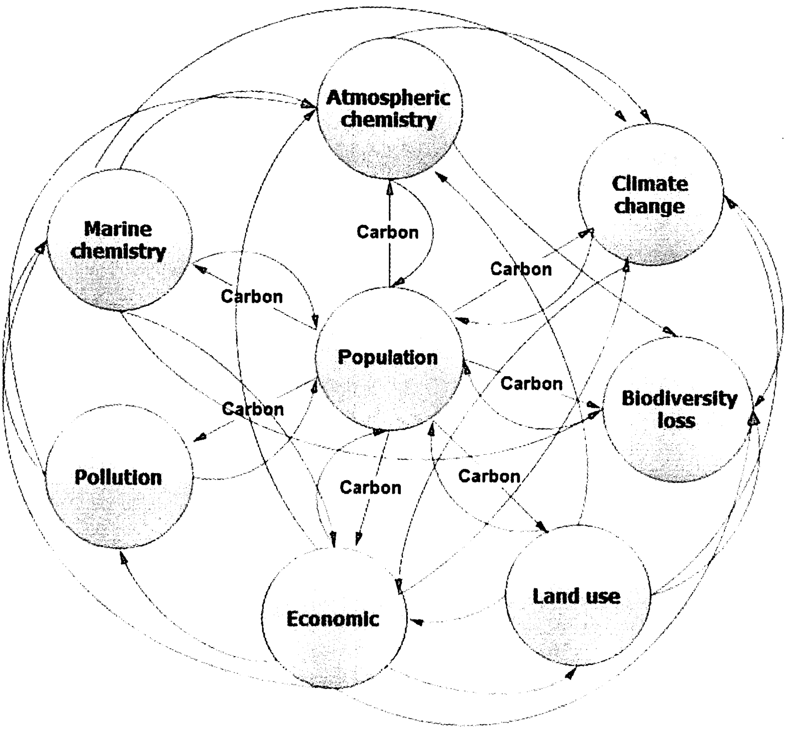

Figure 2. Another simple and informative graphic schematic. The graphic is from Team 19289 from Harbin Engineering University.

used their model to analyze, evaluate, and predict the biological and environmental health condition of Earth. The top-level nodes of their two-layer network are various ecoregions, selected through global geography and climate distribution. The subnetwork nodes for each region are seven elements of Earth's condition—population density, water, forests, air quality, biodiversity, climate change, and energy consumption.

The team developed a system of time-dependent differential equations to describe the interaction of all elements. The regional effects are then propagated among the neighboring regions in the top-layer network to analyze and forecast properties and structures of the global network. The team used data from a Yale University study to set the parameters using least squares. This complex dynamic model predicts the future trend of every element in each region, thus producing a measure of Earth's health. The team addressed the possibilities of tipping points by isolating their regional element equations and performing simulations. While identifying the possibility of tipping points, the team did not try to simulate all the conditions and possibilities, due to time and data constraints. According

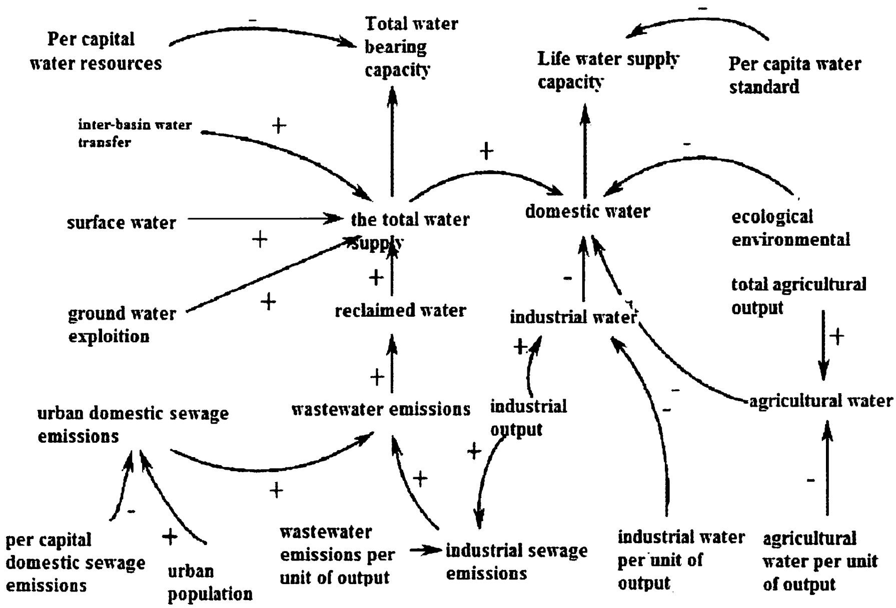

Figure 3. Some reports contained more elaborate conceptual flow diagrams, like this one from Team 17629 from Harbin Institute of Technology.

to their results, population density has the most effect on the remaining elements.

Of course, the idea of the team's work was to make suggestions that could be provided for government and decision makers based on changing parameter values and tipping points. The judges were impressed by the team's analysis of their network structure. They searched for importance of elements in the model by determining centrality measures of every element via network topology analysis. Their work demonstrated that forests were the most central structurally-based point of the network, while population density was the central process-based element. In their sensitivity analysis, the network model was converted to a stochastic model to test various components. In performing the analysis and synthesis of their network, the team was motivated to propose a new topology for a network with weights and a structurally and process-based conception of "influence degree" to evaluate influence capacity of the nodes.

The modeling and analysis performed by this team from Northwestern Polytechnical University were both innovative and powerful. The judges were impressed by their inclusion of so many perspectives and their ability to cope with and employ the complexity of their network model.

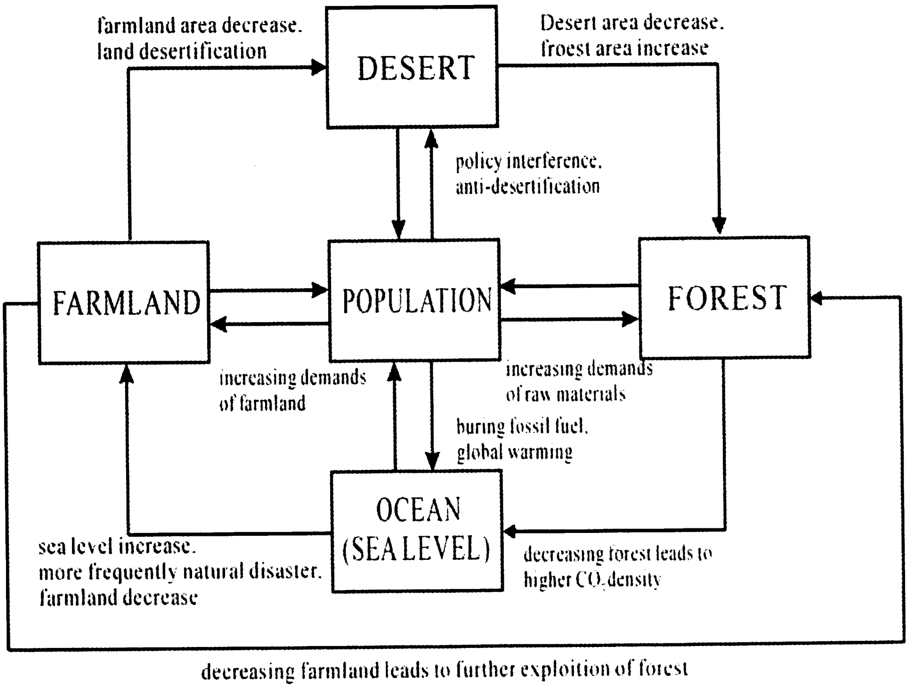

Figure 4. Other graphics showed detailed flow from various levels and elements in their model, like this one from the report submitted by Team 17785 from Harbin Institute of Technology.

Beijing University of Posts and Telecommunications: "Two-tier Communication Network Model of Global Health"

The team presented a two-tier network model as a communication network to study environment degradation caused by population change and air pollution. The top-tier network models the global environment, while the second-tier network (also referred to as the node tier network) models local conditions of smaller regions. The team followed the protocol design paradigm in communication networks to study the inner workings of the model using the standard machine learning technique. In particular, the team used the Bayesian Belief Network (BBN) to model interactions of the factors that could cause environment degradation with respect to the inherent trait of nodal area. They devised a nodal health measure to study local health conditions of the network following the analytical hierarchy process (AHP) approach, and used it to predict the tipping point at the node tier.

To study the global tier, the team investigated the difference of the influences of a node's active state and inactive state on signal transmission. They used the percolation ratio of the active nodes as the global health measurement. Using network theory they were able to obtain a global tipping point.

Finally, the team discussed what datasets were needed to validate their model. Toward this end, they applied the methodology of parameter estimation. And to the judges' approval, the team also discussed in detail how policies and models could impact each other.

The judges were impressed by the team's clear chain of thoughts in model constructions and analysis. The team presented an executive summary and a thoughtful background introduction. The team realized that it was difficult to validate their model because of the lack of any useful datasets. The judges liked the way the team handled this deficiency: The team did not hide this fact; instead, they stated this difficulty and explained what datasets they would need to validate their models and to analyze sensitivity and uncertainty. They provided justifications to each assumption, concept, design, and equation that they developed, and cited their sources. The judges also appreciated their theoretical approach and the creative thinking of using communication protocols, the BBN model, and percolation theory to study the behavior of the network. The models presented and analysis performed by this team were theoretically strong, and the judges were impressed by their clear report and rigorous solution.

Peking University: "Measuring Earth's Health by CO2: A Technology Diffusion Network Approach"

The team measured the Earth's health condition using carbon dioxide concentration and devised three interacting models to predict the future CO2 concentration level.

- The first model, called the Technology Diffusion Model, was the core model, where the nodes represented individual countries with CO2 emission-reduction technologies as measured through the Technology Index, and the links represented the diffusion of technology between countries.

- The second model, called the CO2 Regression Model, used datasets of economic growth and structural changes in energy consumption from the last two decades to predict the CO2 emission level.

- The third model, called the CO2 Absorption Model, was used to investigate the carbon cycle in terms of the nature's ability to absorb CO2. The team noted that the global temperature is proportional to the CO2 level, and an abnormally elevated temperature makes plants sick and hinders photosynthesis. When plants can no longer absorb CO2, no more organic matter will be produced, and the network model will reach its tipping point.

This dynamic global modeling approach allowed the team to study the Earth's complex ecosystem with a focus on the CO2 emission reduction technology. The team also provided a robust sensitivity analysis and showed how their model reacts when parameters were adjusted according to the changes of policies. Using their model, they were able to show that countries with more advanced economic and technological levels tend to have more influence on the global outcomes.

The judges were impressed by this unique solution. Unlike the previous two reports where two-layered network models were presented, this report provided a single-layered network model, where each equation was defined and analyzed. The report provided adequate explanations to the assumptions they made, and the team was able to validate their model.

Zhejiang University: "You Pollute, I Pollute!"

The approach taken by this team was to focus on ocean pollution as a marker for the Earth's health, and then subsequently to model the flow of pollution through eight regions, or nodes, of the global coastal water system. The team took a staged approach to modeling this system.

- First, they developed a Pollution Diffusion Model, adapted from the Princeton Ocean Model, where they defined eight key coastal regions as nodes in this network, and incorporated information on local pollution inputs, ocean currents, degradation of pollution through natural purification, and geographic distance between nodes, as components of their model. The goal was to analyze the performance of one region (node) and the interaction between nodes to ultimately evaluate how pollutants spread through the system and impacted the pollution levels of coastal regions across the globe.
- The second stage incorporated a Control System Model, which used a finite difference method to identify a solution for their Pollution Diffusion Model to predict future ocean health. A block diagram was used to represent the relationships in the system; and by varying the combinations of inputs in this system, the team simulated the outcomes of different scenarios, describing conditions of stability and drastic change—or tipping points. Some scenarios were based on different patterns of industrial developmental and pollution emissions in global regions, recognizing the importance of human and government factors and how these pollution inputs interact with their system. The team also considered a scenario of an unexpected major pollution event, such as a major oil spill. This gave them insight into the usefulness of their model for predicting outcomes of various real-world scenarios, environmental disasters, and different policy options. Sensitivity analyses were performed by testing the response of the system when eliminating particular nodes.
- The final stage of the modeling process introduced a Social Network Analysis Model to identify critical nodes in the ocean pollution system. The team appropriately selected a closeness centrality measure to identify "Influencing" and "Influenced" centrality scores for each coastal region, allowing them to rank the importance of nodes in terms of their

pollution input, and their susceptibility to being impacted by pollution diffusion in the system. They integrated the information from these models to describe how to make policy decisions that would impact water pollution and ultimately Earth's health.

The judges were impressed by many aspects of this paper, in particular the team's effort to address all of the requirements of the problem. They identified a creative marker of Earth's Health, ocean pollution, and did a good job of defining the diffusion of pollution through a networked system of key geographic regions. The team had a clear goal of predicting global and local levels of pollution in coastal waters, and used network science in an interesting way to understand the diffusion process, and how nodes both influenced, and were influenced by, this network.

The judges were also happy to see that the team evaluated the sensitivity of their model, as well as how their model performed under different scenarios with varying pollution inputs.

An innovative aspect of this paper was that the team considered the human and government effects in their models. The judges were also impressed by how the team linked these results to policy decisions, and discussed the usefulness of their model in informing policy change.

Finally, the paper was recognized for clearly outlining reasonable model assumptions, strengths and weaknesses of the model, and potential improvements. The potential usefulness of the model for predicting the Earth's Health and as a tool for policy change was made clear because the team outlined how data could be used to tune their model. What stood out the most was how the team used simulations and network analysis to show how changes in pollution inputs and in the structure of the network would have real implications on ocean pollution.

Rensselaer Polytechnic Institute: "Saving the Green with the Greens"

This group focused on economic systems between nations to predict Earth's Health, arguing that this information would be especially relevant to decision makers and could inform policy. Their paper presented a careful review of previous approaches to modeling environmental outcomes and provided a convincing argument as to why their Earth Damage Score (EDS), based on economic variables, was viable and useful. The team's strategy was data-driven by incorporating a range of national economic indicators from the World Bank. They used data on economic losses due to environmental factors to define EDS for nations over time; they also incorporated data on population, agricultural growth, GDP growth, and literacy rates into their models. The models were developed using innovative, incremental stages, starting with a linear regression approach. The judges found their comparison of simple economic models predicting the EDS to models that make use of environmental variables (e.g., carbon dioxide emissions, water pollution, etc.) to be very revealing. The team concluded that although their economic model was somewhat less accurate than environmental models, it performed significantly better.

This initial model was enhanced by incorporating a Geographic Network Model, where the nodes are countries, and links between them are defined based on countries' geographic proximity. The value of this model was increased by incorporating information on diplomatic relations and cross-country collaboration to further weight the ties between nodes. This rich network data is used to identify important aspects of the network structure: communities of nations who are closely connected in this system, countries who are the most central and prominent in this network, and countries with the most significant impact on the Earth Damage Score, by using a PageRank algorithm. The team used simulations to understand how key policy changes in some of the prominent nodes could have impacts on the global EDS. They identified key policy avenues—such as population growth—that are important triggers that might lead to overall system behaviors.

Their paper concludes with a summary of the strengths and weaknesses of the model and recommendations for further development of the model to improve its ability to predict global outcomes. The judges concluded that this economic model was a creative and useful approach that would be effective for motivating and informing decision-making and policy changes. The judges were also impressed by how the system of nations was defined using a complex network perspective. In particular, the team adopted a "multiplex" network approach, where links between nations were defined by more than one dimension—geographic distance as well diplomatic and social relationships. The paper also incorporated useful network statistics and presented a nice discussion of how different measures of network centrality were useful for their modeling approach. Finally, the paper was rated positively for having clearly identified and rationalized the assumptions of their model. This team effectively made use of graphics and tables to describe their model and its results.

Conclusion

Among the 957 papers, there were many strong submissions that made judging both exciting and a challenge. It was very gratifying to see so many students with the ability to combine modeling, science and effective communication skills in order to understand such a complex problem and build a viable model for its analysis. We look forward to next year's ICM, which will involve another problem in network science.

Recommendations for Future Participants

- **Answer the problem.** Weak papers sometimes do not address a significant part of the problem. Outstanding teams often cover all the bases and then go beyond.
- **Time management is critical.** Every year there are submissions that do an outstanding job on one aspect of the problem, then "run out of gas" and are unable to complete their solution. Outstanding teams have a plan and adjust as needed to submit a complete solution.
- **Coordinate your plan.** It is obvious in many weak papers that the work and writing was spilt between group members, then pieced together into the final report. For example, the output from one model doesn't match the input for the next model or a section appears in the paper that does not fit with the rest of the report. The more your team can coordinate the efforts of its members, the stronger your final submission will be.
- **The model itself is not the solution.** Some weak papers present a strong model, then stop. Outstanding teams use their models to understand the problem and recommend or produce a solution.
- **Explain what you are doing and why.** Weak teams tend to use too many equations and too few words. Problem approaches appear out of nowhere. Outstanding teams explain what they are doing and why.

About the Authors

Chris Arney graduated from West Point and served as an intelligence officer in the U.S. Army. His academic studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his 30-year military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris then moved to RTP (Research Triangle Park), NC, where he served for various durations as chair of the Mathematical Sciences Division, of the Network Sciences Division, and of the Information Sciences Directorate of the Army Research Office. Chris has authored 22 books, written more than 120 technical articles, and given more than 250 presentations and 40 workshops. His technical interests include mathematical modeling, cooperative systems, pursuit-evasion modeling, robotics, artificial intelligence, military operations modeling, and network science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula. He is the founding director of COMAP's Interdisciplinary Contest in Modeling (ICM). In August 2009, he rejoined the faculty at West Point as the Network Science Chair and Professor of Mathematics.

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