21st Century Newton

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Abstract

Winner winner chicken dinner

Contents

1 Introduction

Quantifying the fragility of states based on human factors, such as **[** List here some example human factors **]** has been extensively studied by researchers and instutitions, such as in **[** cite some important works here. **]** However, these models does not consider the impact of environmental factors. For example, deterioation in natural environment may contribute on regional instability and violence **[** some references required here **]**. As environmental factors are important in determining a state or a region's sustainability, merely considering human factors is clearly insufficient.

2 Theoretical Analysis

In this part we propose theoretical framework for our analysis of the impact of climate change on *state fragility*.

2.1 Assumptions and Model Framework

Our hypothesis framework is illustrated in Figure 1. We propose two natural assumptions, based on which we derive the basic framework of our model.

1. The *state fragility*, a concept to estimate the sustainability of states, is dependent on and only on *human factors* and *environmental factors*.
2. The environmental factors and human factors interact with each other.

The assumptions are natural. Assumption 1 requires to quantify the fragility which considers both human factors and environmental factors. We propose a novel framework to quantify fragility by incorporating the human and environmental factors into a probabilistic framework.

Assumption 2 requires a more sophisticated analysis of the two factors, including their respective and joint effects on the fragility, and the interaction between them. We are especially interested in the effects of environmental factors, which include *direct* effect, which is the influence on fragility directly imposed by environmental factors; and *indirect* effect, which is the influence on fragility imposed by envirionmental factors indirectly through human factors. Two hypothesis model to explain the indirect effect is visualized in Figure 2(a) and 2(b).

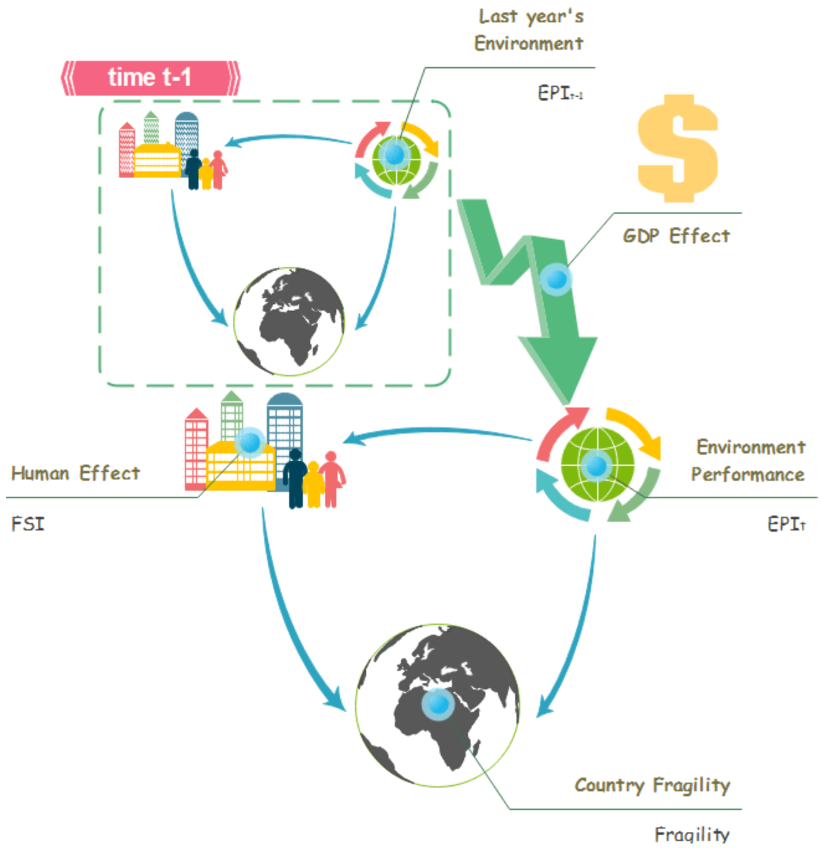
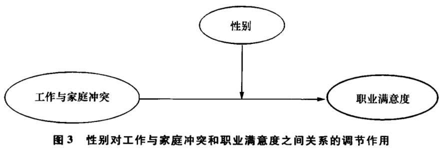


Figure 1: Hypothesis Model Illustration

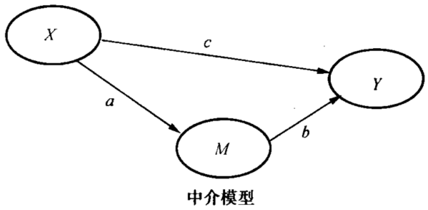
The direct effect of environmental factors is measured by the effect of environmental factors on fragility score, with an unbiased estimation of the effect obtained by propensity score matching **[** citation **]**. The indirect effect of environmental factors can be explained by two hypothesis models: the *moderator variable* model, as shown in Figure 2(a); and the *mediator variable* model, as shown in Figure 2(b). We verify these two hypothesis.

In the following of this section, we are dedicated to enrich our model by developing several key ingredients:

1. A novel fragility score measure incorporating both environmental and human factors;
2. The interaction pattern between human factors, envirionmental factors, and the fragility;
3. The temporal model of a state's environmental status.



(a) Moderator variable model



(b) Mediator variable model

Figure 2: Hypothesis models for indirect effect of environmental factors. E represents environmental factors, $\mathbf{H}$ represents human factors, and $\mathbf{F}$ represents state fragility. The fragility is jointly determined by E and $\mathbf{H}$, by two hypothesis approach.

The basic framework is sufficient to cover most of the requirements of the tasks. Furthermore, we discuss strengthes and weaknesses respectively in each part.

2.2 Representing the Two Factors

[notes on notations **]**

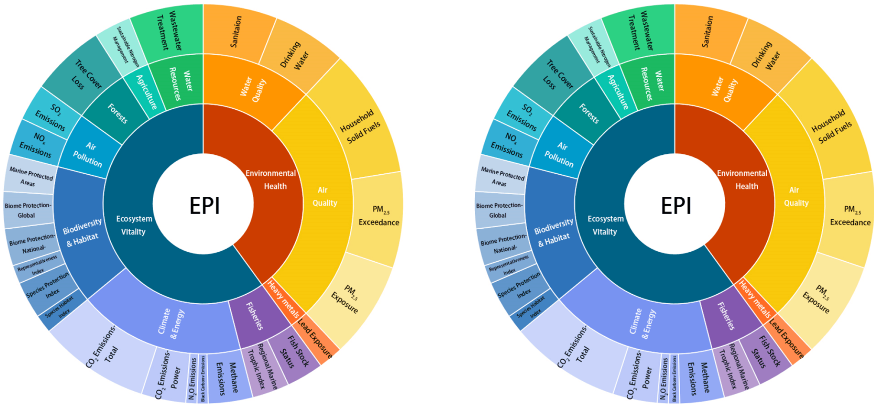
Notation	Description
$\mathbf{H}$	random variable of human factors
E	random variable of environmental factors
$\mathbf{F}$	binary random variable of fragility
$\mathrm{s_f}$	fragility score

Environmental Performance Index. It is an index to evaluate a state's environmental performance by **[** Who and citation **]**. It is composed of indicators in ecosystem vitality and environmental health.

2.3 Probabilistic Fragility Measure

[Need to emphasize that it is logistic regression. **]** In this part, we derive a novel fragile score, $\mathrm{s_f}$, which incorporates both environmental and human factors. The score $\mathrm{s_f}$ is based on probabilistic intuitions, and is therefore called the *probabilistic fragility score*, or the fragility score for convenience.

Without loss of generality, we refer to regions, sovereign states, and other concerned geographical entities as states.



(a) FSI (b) EPI

Figure 3: The two fragility indexes.

We assume that a state either fragile or stable, described by a binary random variable $\mathbf{F}$, where $\mathbf{F}=1$ if the considered state fragile, and $\mathbf{F}=0$ if it is stable. $\mathbf{H}$ and E are random variables describing the human and environmental factors of the state. For convenience, we further assume that E is binary, i.e. $\mathrm{E}=1$ when the state's environment is sustainable, and $\mathrm{E}=0$ when it is not.

Consider the probability of a state being fragile, given its human and environmental factors:

$$\mathbb{P}(\mathbf{F}=1|\,\mathrm{E}=e,\mathbf{H})\ \ (e=0,1) \tag{1}$$

The probability given in 1 quantifies the extent of fragility of the state, given certain environmental and human factors. It is higher when the state is more vulnerable. However, the conditional distribution is hard to estimate. We then factorize it into a more easily calculated form:

$$\mathbb{P}(\mathbf{F}=1|\,\mathrm{E}=e,\mathbf{H})=\frac{\mathbb{P}(\mathrm{E}=e,\mathbf{F}=1|\,\mathbf{H})}{\mathbb{P}(\mathrm{E}=e|\,\mathbf{H})}=\frac{\mathbb{P}(\mathbf{Z}=1|\,\mathbf{H})}{\mathbb{P}(\mathrm{E}=e|\,\mathbf{H})}\ \ (e=0,1) \tag{2}$$

In which we defined a new random variable $\mathbf{Z}=1$ if $\mathrm{E}=e,\mathbf{F}=1$ and $\mathbf{Z}=0$ otherwise. Eqn. 2 allows us to only estimate the conditional probability of two binary random variables given human factors $\mathbf{H}$.

For convenience of calculation, we assume linear relationships:

$$\begin{aligned}\log\frac{\mathbb{P}(\mathbf{Z}=1|\,\mathbf{H})}{\mathbb{P}(\mathbf{Z}=0|\,\mathbf{H})} &= \mathbf{W}_1\,\mathbf{H}+\mathbf{e}_1\\ \log\frac{\mathbb{P}(\mathrm{E}=e|\,\mathbf{H})}{\mathbb{P}(\mathrm{E}=1-e|\,\mathbf{H})} &= \mathbf{W}_2\,\mathbf{H}+\mathbf{e}_2\end{aligned} \tag{3}$$

Where $\mathbf{W}_i$ are parameters, $\mathbf{e}_i$ are Gaussian errors, $i=1,2$.

Using the linear assumption and the logistic form in Eqn 3, we obtain the estimate of the probabilities respectively:

$$\begin{aligned}\hat{p}_z &= \frac{\exp(\mathbf{W}_1\,\mathbf{H})}{1+\exp(\mathbf{W}_1\,\mathbf{H})}\\ \hat{p}_e &= \frac{\exp(\mathbf{W}_2\,\mathbf{H})}{1+\exp(\mathbf{W}_2\,\mathbf{H})}\end{aligned} \tag{4}$$

In order to make the estimated probability distribution resemble the true distribution, we estimate parameters $\mathbf{W}_1, \mathbf{W}_2$ by minimizing the cross entropy loss, which is equivalent to minimizing the KL divergence **[** reference needed **]** between the estimated distribution and the empirical distribution. **[** specific form omitted. **]**

Finally, probabilities in Eqn. 2 is replaced by the estimates given in Eqn. 4, yielding the *probabilistic fragility score*:

$$\mathrm{s_f} = \frac{\hat{p}_z}{\hat{p}_e} \tag{5}$$

Notes on the fragility score. The fragility score, $\mathrm{s_f}$, is derived based on probabilistic intuitions and linear assumptions. Higher $\mathrm{s_f}$ indicates higher risks of being fragile. However, the score $\mathrm{s_f}$ can be larger than one, and is, therefore, not in form of probability. However, it does not hurt its applicability: if the estimated $\hat{p}_z$ is larger than $\hat{p}_e$, we have even more reasons to believe that the considered state is fragile.

2.4 Modeling the Interaction between Variables

We then begin to model the relationship between the three variables: environmental factors E, human factors $\mathbf{H}$, and the fragility score $\mathrm{s_f}$.

Direct Effect of Environmental Factors

In order to measure the effect of environmental factors on fragility score, a naive approach would be to sample states of both sustainable environment and unsustainable environment, and compare their average fragility score. Formally, write s_f^0 as the average fragility score of the sustainable group, and s_f^1 as the average fragility score of the unsustainable group. One then compares the difference $s_f^* = s_f^0 - s_f^1$.

The above approach gives, however, biased estimation, because the apparent difference between these two groups may be depend on human factors that affected whether or not a state's environment is sustainable, instead of the environmental status per se. For example, the approach might compare scores of the states that are environmentally unsustainable and in political turmoil, with the scores of the states that are environmentally sustainably and politically stable. The difference of political status results in unbiased estimate of the effect of environmental factors.

To control for the differences of human factors between the sustainable group and the unsustainable group, we use the propensity score matching, a statistical technique that attempts to estimate unbiasly the effect of a variable, in this case, the environmental status.

Formally, the propensity score of a certain state is defined as the conditional probability of the environmental status given its human factors,

$$p = \mathbb{P}(\mathrm{E} = 1 | \, \mathbf{H}) \tag{6}$$

[environmental status? environmental factors? they are probably different. **]** In order to estimate the probability, we adopt similar procedures used in Section 2.3, using the score of logistic regression, $\hat{p}$, of human factors $\mathbf{H}$ against environmental status E. Then we match each of the unsustainable states to one sustainable state on propensity score, by using *Nearest Neighbor Matching*: each unsustainable state is matched to the sustainable state whose propensity score is the closest. As such, a new data set in which the sustainable group and the unsustainable group and their propensity scores are balanced, is obtained.

Based on the newly obtained data set, we calculate the adjusted score difference:

$$\hat{s}_f^* = \hat{s}_f^0 - \hat{s}_f^1. \tag{7}$$

Where $\hat{s}_f^0$, $\hat{s}_f^1$ are average scores of the sustainable and unsustainable group, drawn from the data set obtained by PSM.

Indirect Effect of Environmental Factors

In order to measure the indirect effect of environmental factors E, we propose two candidate models: the moderator variable model, and the mediator variable model.

[is the difference clearly explained? **]** The moderator variable model assumes that the environmental factors influence the fragility score, and the relationship is calibrated by the effect of human factors. In this case, the human factors $\mathbf{H}$ are called *the moderator variable*, as illustrated in Figure 2(a).**[** reference. **]**

The mediator variable model assumes that the environmental factors and human factors jointly influence the fragility score. Furthermore, the environmental factors influence the fragility score both directly, and indirectly through the human factors. The illustration of this model is as in Figure 2(b).**[** reference. **]**

Moderator variable model. The model is written as

$$\mathrm{s_f} = \mathbf{W}_1\,\mathbf{H} + W_2\,\mathrm{E} + \sum_{i=1}^{p} \beta_i h_i\,\mathrm{E} \tag{8}$$

where $\mathbf{H} = [h_1, \ldots, h_p]'$, and h_i is the ith component of $\mathbf{H}$, indicating a specific factor. $\mathbf{W}$ and β_i are parameters.

The term, $\sum_{i=1}^{p} \beta_i h_i$, represents the effect of each human factor on the relation between the environmental factor and the fragility score, since by taking partial derivative, we observe that

$$\frac{\partial s_f}{\partial\,\mathrm{E}} = W_2 + \sum_{i=1}^{p} \beta_i h_i$$

The derivative indicates that the effect of environmental factor E comes from both itself, described by W_2, and each human factor h_i, described by β_i. If the factor h_i has no effect on the relation, then β_i should be close to zero. Consider, therefore, the following statistical test:

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_p \tag{9}$$

The rejection of H_0 shows that the moderator effect of $\mathbf{H}$ is significant. Furthermore, we wish to specifically investigate the effect of each factor. Hence in the following test,

$$H_0 : \beta_i = 0 \tag{10}$$

if H_0 is rejected, we are confidence to say that the moderator effect of h_i is statistically significant.

If the moderator effect is indeed significant, then the extent of the effect of $\mathbf{H}$ is then quantified by $\beta = [\beta_1, \ldots, \beta_p]'$ and respective components.

Mediator variable model. **[** Human factors are multivariate. Maybe consider human factors only as FSI, a single variable. **]**

We first declare the following values:

- b_1: the coefficient of the linear regression model, in which E predicts $\mathrm{s_f}$.
- b_2: the coefficient of the linear regression model, in which $\mathbf{H}$ predicts $\mathrm{s_f}$.
- b_3: the coefficient of E in the linear regression model in which E and $\mathbf{H}$ jointly predicts $\mathrm{s_f}$.

- b_4: the coefficient of $\mathbf{H}$ in the linear regression model in which E and $\mathbf{H}$ jointly predicts $\mathrm{s_f}$.

First, we need to verify by statistical tests that b_1 and b_2 are significantly nonzero.

If $\mathbf{H}$ indeed acts as a mediator variable, by model assumption, b_4 is significantly nonzero. Furthermore, since E indirectly influences $\mathrm{s_f}$ through $\mathbf{H}$, the explaining power of E alone should be reduced once $\mathbf{H}$ is introduced. In this case, $b_3 < b_1$.

If all the above four conditions hold significantly, we are confident to say that human factors act as mediator variables, through which the environmental factors influence the fragility score.

2.5 Temporal Model

[hypothesis need to be specified here. **]** In order to better model climate change, we further consider time as a variable, and investigate how climate change would have influenced the fragility of states. We begin by formulating the following assumptions:

- The evolution of climate change is a Markov process, i.e. E_t is dependent on E_{t-1} and independent of $k < t - 1$.
- Human factors act as a moderator of EPI: it moderates the relation between E_{t-1} and E_t.
- Human factors are majorly embodied in economic status.

The first hypothesis is for convenience of modeling. The term ”moderator” in the second hypothesis is in consistence with the definition in Section 2.4. The third hypothesis is because we consider factors at the country level, therefore we reasonably assume that economic status is representative.

The idea of the hypothesis is simple: one may imagine that today’s environment depends on yesterday’s environment. Without invervention, the environment evolves all by itself. With the presence of human activity, the evolution of environment is ”moderated” by human factors.

We use EPI index as an indicator of environmental status and establish a temporal model of EPI to investigate climate change. Specifically, we choose Mauritius as an example state, and illustrate how and when it would reach a tipping point. The basic idea of our model is as illustrated in the formula below:

$$E_t = \beta_0 E_{t-1} + \beta_1 H_t + \beta_2 H_t \times E_{t-1} \tag{11}$$

In which we use the subscript t to denote the value at time t. The first term embodies the autoregressive property of the environmental factor: its present status depends on its previous status. THe second term formulates human factors. Since we assumed that economic status is representative, in experiments in Section 3.4, we would use GDP and GDP growth as indicators of human factors. The third term represents the mediator effect of human factors, derived from the second hypothesis.

One may notice that predicting E_t requires knowing H_t a priori, which is impossible. In order to approximate the evolution of climate change, we establish another temporal model to predict H_t, which is, in this case, GDP growth.

Modeling GDP growth We propose using ARIMA, a typical model widely used for time series forcasting, to model the time series of GDP growth rate. Generally, the model establishes that

$$Y_t = \sum_{i=1}^{p} a_i Y_{t-i} + \sum_{i=0}^{q} b_i \epsilon_{t-i} \tag{12}$$

$$Y_t = E_t - E_{t-k} \tag{13}$$

where p, q, k are hyperparameters, a_i, b_i are parameters to be estimated. The first sum in Eqn 12 is the autoregressive term, and the second sum is the moving average of white noises, where $\epsilon_{t-i} \sim \mathrm{N}(0, 1)$. Eqn. 13 performs differencing, where k is the order of differencing, to guarantee that the time series to be modeled is stationary.

Due to lack of space, the detailed description of this model is omitted. We encourage inerested readers to see detailed description in **[** where? **]**.

Modeling Government Invervention and Economic Boom Where the environment deteriorates, the government should step in to prevent the environment from becoming fragile. History of the development of many countries, such as UK and China, shows that fast economic development could be at the expense of environment performance. This is in consistence with the experiment results of our model, discussed in detail in Section 3.4.

The forcase of GDP growth is generally stable by our model, as shown in Figure 6(a). To model fast economic development at the cost of environment, we introduce the following two parameters:

- α: the government investment to neutralize environment deterioration;
- μ: the economic boom factor.

Specifically, the economic boom brings additional μ GDP growth every year. The government uses α of the annual GDP to ease the negative impact of fast economic development. It does not mean that the annual GDP or GDP growth rate is decreased by α;

the effect of government intervention is manifested in EPI. The specific implementation is described in Section 3.4.

In reality, the growth rate of GDP doesn't necessarily increase at a regular speed. In settings of fast economic development, for example, China, the GDP growth rate is jumped to a high level which is sustained for quite a long period of time, instead of growing steadily to a high level. However, our model setting is sufficient for stimulating the effect of high economic growth.**[** why? need a better reason. **]**

3 Experiments

3.1 Data Preparation

Dataset.

We prepared several representative datasets for the variables we need: E, $\mathbf{H}$, and $\mathbf{F}$.
Fragility Score Index.

Environmental Performance Index.

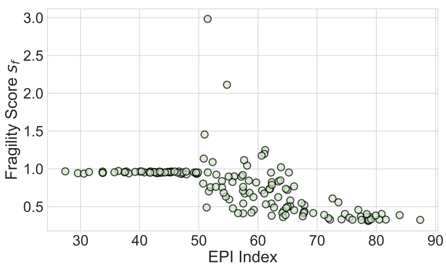
Other Indicators.

3.2 Calculation of Fragility Score

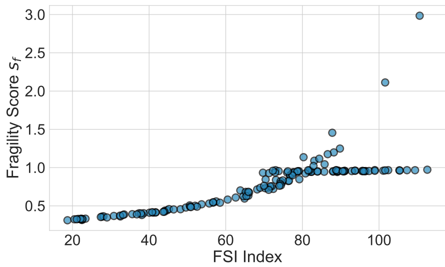
In order to calculate fragility score defined in Section 2.3, we need to identify, a priori, which states are fragile and which states are environmentally unstable. In order to achieve so, we determine that a state is fragile, i.e. $\mathbf{F} = 1$, if its FSI score is higher than a certain threshold F_0, and a state is environmentally fragile, i.e. $\mathrm{E} = 1$, if its EPI score is lower than a certain threshold E_0. The thresholds are chosen differently for each year, because the indexes of different years aren't necessarily calculated using the same methodology. Therefore, thresholds for each year are chosen to guarantee that the fragile states and environmentally fragile states occupy approximately 30% of the states, respectively. As such, each state's status of fragility and environmental fragility is approximated by HSI and EPI indexes. Furthermore, we use the 12 indicators used in the calculation of FSI, specified in **[** specify somewhere **]**, as components of human factors $\mathbf{H} = [h_1 \dots h_1 2]$.

Hence, all variables needed for the calculation of our fragility score, including

$\mathbf{H}, \mathrm{E}, \mathbf{F}$, are specified for each state. Logistic regression was run as described in Section 2.3 to obtain fragility score of each state. Finally, we visualize the relationship between the score and the two indexes in Figure 4. Figure 4(a) shows that states with



(a) EPI



(b) FSI

Figure 4: Relationship between Fragility Score $\mathrm{s_f}$ and FSI and EPI.

lower EPI indexes have higher scores in general, but the variance is high when the environment is worsened. Figure 4(b) shows that states with higher FSI obtain higher scores. The figures show that HSI majorly determines $\mathrm{s_f}$, while EPI acts as an adjustment.

Consistency Test. In order to show that $\mathrm{s_f}$ is a reasonable criterion of states' fragility, we need to make sure that a state which is completely better than another state, i.e. with higher EPI and lower FSI, obtains lower $\mathrm{s_f}$. In this case, we call that the two states are *inconsistent*. We therefore define the average reverse number order r:

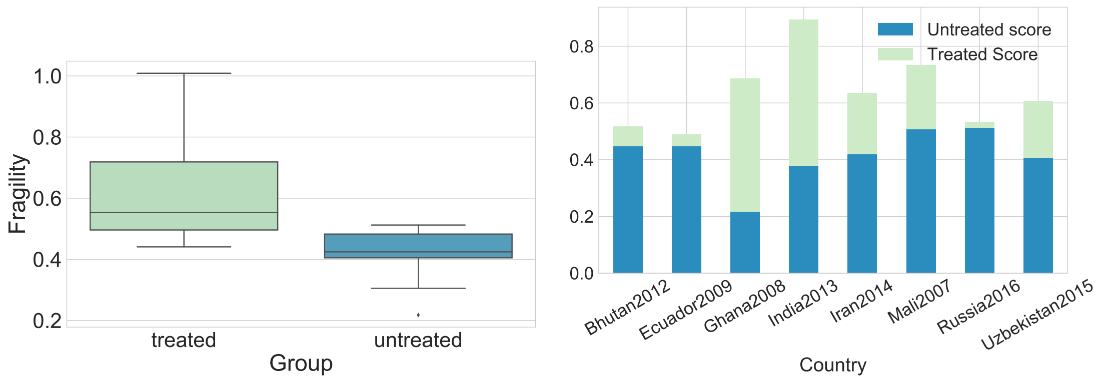
$$r \triangleq \frac{1}{2N} \sum_{k=1}^{N} \frac{r_k}{N} = \frac{1}{2N^2} \sum_{k=1}^{N} r_k \tag{14}$$

where for the kth state in the dataset, r_k is the number of other states that inconsistent with it.

The r_k calculated for the indexes in the year of 2017 is 0.01764, which is sufficiently low to bring confidence to the fragility score s_f.

3.3 Indirect Effect of Environmental Factors

By observing the relation between EPI index and fragility in Figure 4(a), we find that environmental factors exhibit different kinds of influence at different stages. When EPI is small, the change of fragility with respect to EPI is little. When EPI is higher than a certain threshold, $E_0 = 50$, the changes of EPI begins to induce changes of fragility score. It could be explained, since when a state's environmental performance is too low, it tends to become too unstable such that further deterioration would have minor effect.



(a) General Effect (b) Specific Countries

Figure 5: Direct Effect of Environmental Factors.

Therefore, we establish different models for states which are below and higher than the threshold, respectively, to explain the indirect effect of environmental factors in terms of moderator and mediator effects formulated in Section 2.4.

When the Index $\mathrm{EPI} > 50$

These states are comparably environmentally stable. For these states, change of environment significantly influence fragility.

Moderator Effect To examine the moderator effect, we establish a linear regression model:

$$\mathrm{s_f} \sim \mathrm{EPI} + \mathrm{F} \times \mathrm{EPI} + \mathrm{F} \tag{15}$$

where

$$S = \sum \mathrm{E_k} + \sum \mathrm{P_k} + \sum \mathrm{S_k} + \sum \mathrm{C_k} \tag{16}$$

is the sum of economic, political, social, and cohesion indicators used in FSI index, as in Table **??**[** somewhere **]. The second term is the effect of moderators.

We select significant variables from the regression model in Eqn. **??** by bidirectional step regression. The selected variables, their coefficients, p-value and level of significance are listed in Table 1.

We see that E1, E2, P3 appears both individually in Table 1 and also as factors of cross products. Therefore, EPI moderates the relations between these variables and fragility. There is no other variable that appear individually; therefore, we are confident that EPI act as a moderator between human factors and fragility.

Var.	Coef.	P-val.	Level
EPI	0.050534	7.44e-11	Super
E1	0.337434	0.003849	High
E2	-0.404308	0.002120	High
P3	0.363195	0.009289	High
EPI×E1	-0.004889	0.006607	High
EPI×E2	0.006458	0.002109	High
EPI×P3	-0.005600	0.011503	Medium
EPI×S2	-0.004610	0.000142	High

Table 1: Moderator Effect when $\mathrm{E} > 50$

Mediator Effect EPI's direct impact on fragility is already verified in Table 5 and Table 1, since predicting fragility using EPI alone would result in a even higher level of significance as in the model of Table 1.

We verify the role of human factors by establishing regression models in which EPI is used to predict each individual indicator of FSI. When EPI significantly predicts an indicator, we establish another regression model, in which EPI and the indicator jointly predict fragility.

We analyzed the indirect of environmental factors when EPI is high: both moderator and mediator effects take place. EPI moderates the relation of these following human factors:

- Economic Decline
- Uneven Economic Development
- Human Rights and Rule of Law
- Refugees and IDPs

Indeed, environmental performance does not have a clear and direct relationship with these variables. The moderator effect of environment on many economic indicators can be explained by Kuznets curve **[** reference **]**.

These following indicators are mediator variables, through which environmental factors indirectly exhibit influence on fragility:

- Public Services

- Demographic Pressures

Clearly, environmental factors directly influences a state's public services and demographic pressures. The influence then propagates to fragility.

When the Index $\mathrm{EPI} < 50$

These states are environmentally fragile, and fragility of these states is no longer sensitive to environmental fragility.

Mediator Effect. We establish a regression model, using EPI to predict fragility, for states with EPI< 50. However, the coefficient of EPI in this case is -0.0003162, with p-value being 0.225. EPI is insignificant, which is also verified in Figure 4(a). Since environment does not influence fragility directly, mediator effect is impossible.

Var.	Coef.	P-val.	Level
C1	6.174e-4	0.033153	Medium
C2	-1.334e-3	0.001708	High
C3	6.624e-3	0.001896	High
E1	9.62e-3	0.004577	High
E3	-4.759e-3	1.63e-14	Super
F1			

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$Fragile = 9.399{\times}10^{-1}{+}6.17{\times}10^{-4}C_1{-}1.33{\times}10^{-3}C_2{+}6.62{\times}10^{-3}C_3{+}9.62{\times}10^{-3}E_1{-}4.759{\times}10^{-3}E_3{+}$

Ł$EPI \times C_3, EPI \times F_3$łł

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3.4 Temporal Model

The temporal prescribed in Section 2.5 is implemented by these following steps: **Variables Preparation.** The following basic indicators are chosen as basis for our temporal model, and can be found on the data bank of the World Bank **[** specific? **]**.

- Gross Domestic Production (GDP) (constant 2010 US dollar)
- Gross Domestic Production Annual Growth Rate

We denote GDP at time t by G_t and its growth rate $R_t \triangleq \frac{\Delta G_t}{\Delta t} = \frac{G_t - G_{t-1}}{G_{t-1}}$. E_t is the EPI index in time t. Oftentimes variables are scaled, i.e first centralized by its mean, and normalized by its standard variance. The scaled version of these variables are written as $\overline{G_t}$, $\overline{R_t}$, and $\overline{E_t}$, while the scaling is performed according to the set of data at all time steps prior to and including t.

Forcasting GDP. The GDP Growth is modeled by ARIMA in Section 2.5. Furthermore, we choose three values of μ to represent different speed of economic development: $\mu = 0$ for stable, $\mu = 0.3$ for moderate, and $\mu = 0.5$ for fast. The prediction of GDP growth rate in these three settings is in Figure 6(a).

Predicting EPI. Now every variable is well prepared for the prediction of EPI index, as in Figure 6(b). The specifics are as below. The temporal equation is expressed as:

$$E_t = \beta_0 E_{t-1} + \beta_1 \log(G_t) + \beta_2 R_t + \beta_3 \overline{R_t} \times \overline{G_t} + \beta_4 \overline{R_t} \times \overline{R_t} \tag{17}$$

The parameters are determined by standard linear regression, with resutls listed in Figure 2. The signs of coefficients show the effects of variables:

Variable	E_{t-1}	$\log(G_t)$	R_t	$\overline{E_{t-1}} \times \overline{G_t}$	$\overline{E_{t-1}} \times \overline{R_t}$
Coefficient	0.35	-6.79	-0.21	-0.71	0.07

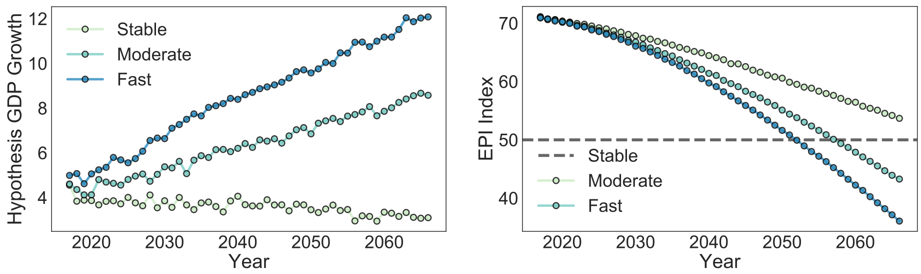
Table 2: Parameters for our Temporal Model.

- Good environmental performance tends to become better;
- Fast GDP Growth tends to damage environmental performance; however, when EPI is sufficiently high, the damage is neutralized.

[weakness: reflects the specific status of a country in a certain period. **]**

Trade-off between Development and Environment. From Figure 6(b), we see that in stable development setting, Mauritius does not reach a environmental fragile status within the range of 50 years.

However, faster economic development brings instablizing factors. Under moderate and fast development settings, where μ is 0.3 and 0.5 respectively, Mauritius would be environmentally fragile within 50 years. The faster the development, the sooner it becomes fragile.



(a) Future GDP Growth Rate (b) Future EPI Index

Figure 6: Future GDP and EPI Index. Each is predicted under three different development settings: fast economic growth, moderate economic growth, and stable economic growth.

We then consider another hyperparameter α proposed in Section 2.5, used to represent the investment of a country to control for environmental instability possibly brought by economic development.

Remember that α does not change the predicted value of G_t, R_t per se. Instead, when predicting EPI, the values of variables used in Eqn. 17 is adjusted: $R_t \leftarrow R_t - \alpha$, $G_t \leftarrow (R_t - \alpha)G_{t-1} + G_{t-1}$. Therefore, it is considered the annual governmental investment as percentage of GDP to improve environmental performance.

We visualized the minimum α, i.e. percentage of GDP as investment to protect environment, that is required to guaranteed that the state's EPI is larger than 50 in 50 years. The result is as in Figure 7.

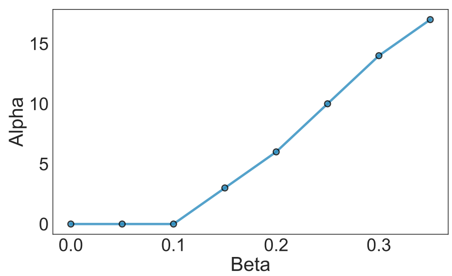


Figure 7: Trade-off between Development and Environment

Clearly, when the economic development is sufficiently large, the government need to invest a certain amount of money to ensure environmental stability.

We are interested in the cost of environmental protection. The investment is calculated as αG_t for each year t. For simplicity, we assume that the state gains $t\mu G_t$ for each year t for fast economic development, since theoretically, the economic development contributes accumulatively μ percent of GDP growth each year.

Therefore, annual cost of environmental protection is $(\alpha - t\mu)G_t$.

Figure 7 shows a trade-off between fast economic development and environmental protection. **[** need to show when the contribution is larger than the side effect. **]**

3.5 Regional Model

4 Related Work

5 Conclusion