

Representation of signed number

Binary number can be positive or negative.

Symbol '+' is used to represent positive number.

Symbol '-' is used to represent the negative number.

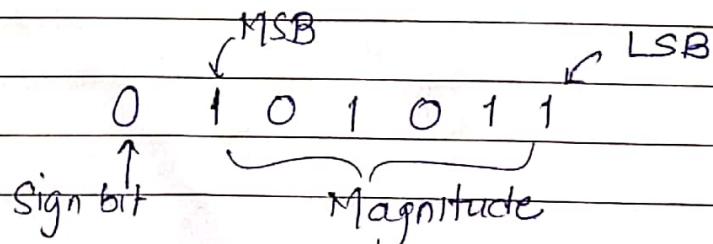
For representing signed number there are two methods.

- ① Sign magnitude form
- ② Complement form

Sign Magnitude form

To represent sign magnitude form there is two parts.

- ① Sign part
- ② Magnitude part



1 → Negative Number

0 → Positive Number

Q Represent (-42) using sign magnitude representation.

2 | 42

2 | 21

2 | 10

2 | 5

2 | 2

2 | 1

0 | 112

(42)₁₀

↑
signbit

$+42 \rightarrow 01101010$

$-42 \rightarrow 10101010$

↑
signbit

Q Represent (-124)₁₀ to its equivalent sign magnitude form

2 | 124

2 | 62

2 | 31

2 | 15

2 | 7

2 | 3

2 | 1

0 | 1

(124)₁₀ \rightarrow (1011100)₂

+124 \rightarrow (0111100)₂

-124 \rightarrow 1011100

Representation of signed number using 1's complement or 2's complement

The main advantage of using complement method is the reduction in hardware.

Secondly, they allow the representation of negative numbers.

for performing binary subtraction, use 1's complement & 2's complement.

1's complement of binary Number.

1's complement of binary number is obtained by complementing every bit of the number.

0 changed to 1

1 changed to 0

Ex: Find 1's complement of (1010010)

1 0 1 0 0 1 0 → 0 1 0 1 1 0 1

1's complement of this number is

0 1 0 1 1 0 1 0

$$\textcircled{1} \quad 10100110$$

$$\textcircled{2} \quad 110010$$

$$\textcircled{3} \quad (54)_{10}$$

$$\textcircled{4} \quad (136)_{10}$$

~~Q's complement method.~~

~~Q's complement = 1's complement + 1~~

Number is

0 0 1 0 1 1 0 1

Calculate Q's complement of this no.

~~Q's complement = 1's complement + 1~~

$$\begin{array}{r} \text{1's complement} = \\ + 1 1 1 1 1 1 1 \\ + 1 1 + 1 + 1 + 1 \\ \hline 0 0 0 0 1 1 0 1 0 0 1 1 \end{array}$$

① $(74)_{10}$

② $(236)_{10}$

Binary Subtraction using 1's complement

① Take 1's complement of 2nd number.

$$11000 - 10011 \rightarrow \text{10011} + 01101$$

② Add two numbers.

$$1101 - 011$$

③ If carry over is generated,
Add this carry with result.

④ If MSB = 0 result is positive & in its
true binary form.

⑤ If MSB = 1 result is negative & in its
1's complement form.

$$\begin{array}{r} 1011 - 011 \\ + 0110 - 0 \\ \hline 1000 \end{array}$$

$$\textcircled{1} \quad 110101 - 100101$$

1's complement = 011010

$$\begin{array}{r} 110101 \\ + 011010 \\ \hline 100111 \end{array}$$

↑ MSB

$$\begin{array}{r} 1111 \\ 010010111 \\ + 1 \\ \hline 110000 \end{array}$$

The difference is (10000)

$$\textcircled{2} \quad 101010 - 111001$$

1's complement of 111001 → 000110

$$\begin{array}{r} 101010 \\ + 000110 \\ \hline 011000 \end{array}$$

↑ MSB

The difference is -(110001)

1's complement → (-001110)

$$(1) \rightarrow 1101$$

$$(2) \rightarrow 0110$$

$$③ 1011.001 - 110.10$$

$$8 - 10110.100$$

1's complement of number is 1001.011

$$\begin{array}{r}
 & 1001.1 \\
 + & 1010 \\
 \hline
 1001001.100
 \end{array}$$

$$\begin{array}{r}
 0100.100 \\
 + 111 \\
 \hline
 010001001
 \end{array}$$

The difference is 1002.101

$$④ 10110.01 - 11010.10$$

1's complement of No. 11010.10

$$\begin{array}{r}
 000100101.01 \\
 + 110011001 \\
 \hline
 100101010.01
 \end{array}$$

The difference is -(11011.10)

1's complement = -(00100.00)

Q

$$\textcircled{1} \quad 128 - 8$$

$110\ldots 101\cdot 12 \rightarrow 1100$ (from sign bit)

$8 \rightarrow 1000$

i's complement of 8 $\rightarrow 0111$

1 1 1 . 1 1 carry

$$\begin{array}{r} 1100 \\ + 0111 \\ \hline \boxed{1} 0011 \end{array} \quad \begin{matrix} A \\ B \end{matrix}$$

Sum

0 0 1 . 0 0 1 0

1 1

0 0 1 1 1 0

+ 1

1010100 i's complement add

The difference is +4

$$\textcircled{2} \quad 8 - 12$$

$10\ldots 108 \rightarrow 1000$

$12 \rightarrow 1100$

i's complement of 12 $\rightarrow 100111$

10 . 1 0 1 0 0

. 0 1 0 0 0 1 1

+ 0 0 1 1

(01.1101) 1.01100 i's complement add

(100111) \rightarrow i's complement

i's complement of result $\rightarrow 0100$

The difference is $\rightarrow -(-4)_10$

Binary subtraction (2's complement)

1. Take 2's complement of second number.
2. Add two numbers, modulus 2ⁿ.
3. If carry over is generated ignore the carry.
4. If MSB = 1 Result is negative & in its 2's complement form.
5. If MSB = 0 Result is positive & in its true Binary form.

$$110011 - 101100 = 010111$$

(0011) 2⁵ and (0101) 2⁵

Binary Subtraction using 2's complement

$$\textcircled{1} \quad 10110 - 11010$$

1st Number $\rightarrow 10110$ just bba

2nd Number $\rightarrow 11010$ just bba
 1's complement of 2nd No $\rightarrow 00101$ bba
 $+ 1$

$$\begin{array}{r} 00101 \\ + 1 \\ \hline 00110 \end{array}$$

Addition

$$\begin{array}{r} 10110 \\ + 00110 \\ \hline 11100 \end{array}$$

MSB = 1 Result is negative & it's
 2's complement form

$$\begin{array}{r} \text{Result is } \rightarrow 1100, \\ 2's \text{ complement of Result } \rightarrow 0011 \\ + 1 \\ \hline 0100 \end{array}$$

The difference is (-100)

$$② \quad 110110 - 10110$$

1st Number \rightarrow 110110

2nd Number \rightarrow 010110

2's complement of 2nd No. \rightarrow 101001

$$\begin{array}{r} 110110 \\ + 101001 \\ \hline 101010 \end{array}$$

Addition

$$\begin{array}{r} 110110 \\ + 010110 \\ \hline 110000 \end{array}$$

\Rightarrow Ignore MSB
MSB = 1

Result is Negative & its 2's complement form

Result \rightarrow 011110 \leftarrow ①

$$\begin{array}{r} 0001011 \\ + 1100000 \\ \hline 1100000 \end{array}$$

$$\begin{array}{r} 1100000 \\ + 1100000 \\ \hline 0000000 \end{array}$$

$$③ \quad 1010.11 - 1001.01 \rightarrow 1.10$$

1.10 + 0.11 = 1.01

$$\begin{array}{r} 0000000 \\ + 1100000 \\ \hline 1100000 \end{array}$$

$$\begin{array}{r} 1100000 \\ + 1100000 \\ \hline 0000000 \end{array}$$

if 2's complement of 1.10 is taken then result will be

6 most significant bits

0000000 - 1100000 = 1000000

$$\begin{array}{r} 1000000 \\ + 1100000 \\ \hline 0100000 \end{array}$$

$$0100000$$

$$③ 10100.01 - 11011.10$$

$$④ (27)_{10} - (32)_{10}$$

2	27		2	32	
2	13	1	2	16	0
2	6	1	2	8	0
2	3	0	2	4	0
2	1	1	2	2	0
0	1	↑	2	1	0
			2	1	↑
			0	1	

$$⑤ (11011)_2 - (100000)_2$$

$$\textcircled{1} \rightarrow 011011$$

$$\textcircled{2} \rightarrow 110000$$

$$\begin{array}{r} 011111 \\ + 1 \\ \hline \end{array}$$

$$011111 + 1 = 100000$$

$$\begin{array}{r} 011011 \\ + 100000 \\ \hline 111011 \end{array}$$

MSB=1 Result is negative & is in its 2's complement form

2's complement $\rightarrow 000100$

$$\begin{array}{r} + 1 \\ \hline 000101 \end{array}$$

$$(-5)_{10}$$

Logic Minimization

Uncomplemented -

Complemented -

Complement

Complement mean negate the Number.

$$0 \rightarrow 1$$

$$1 \rightarrow 0$$

$$A \rightarrow \bar{A}$$

$$B \rightarrow \bar{B}$$

Uncomplemented - A, B, C

Complemented - $\bar{A}, \bar{B}, \bar{C}$

Standard / Canonical form

The canonical form are basic forms of expressing function in Boolean algebra.

In the canonical form every term consist of all the input variables in either complemented form or uncomplemented form.

$$Y = ABC + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$Y = (A+B+C)(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+C)$$

Minterm: A product term is minterm.

Each individual product term is called Minterm.

e.g.

$$ABC \quad A \quad 001 \quad 3BA \quad 0 \quad 0 \quad 0$$

$$\bar{A}BC \quad \bar{A} \quad 001 \quad 1BA \quad 1 \quad 0 \quad 0$$

$$AB\bar{C} \quad A \quad 010 \quad 3\bar{B}A \quad 0 \quad 1 \quad 0$$

$$\bar{A}\bar{B}C \quad \bar{A} \quad 010 \quad 2BA \quad 1 \quad 1 \quad 0$$

$$B+\bar{C}A \quad B \quad 001 \quad 3BA \quad 0 \quad 0 \quad 1$$

Maxterm:

Each individual sum term is called Maxterm.

Maxterm:

$$A+B+C \quad A \quad 001 \quad 2BA \quad 0 \quad 1 \quad 1$$

$$\bar{A}+B+C$$

$$A+\bar{B}+\bar{C}$$

$$\bar{A}+\bar{B}+\bar{C}$$

$$3AA + 2BA + 3\bar{B}A = 111$$

Minterm & Maxterms for two binary variables

Variable	Minterm	Maxterm
(0+1)(1+0)	Term 1: Represented by 001 Term 2: Represented by 010 Term 3: Represented by 100 Term 4: Represented by 110	Term 1: Represented by 001 Term 2: Represented by 010 Term 3: Represented by 100 Term 4: Represented by 110
0 0	$\bar{A}\bar{B}$	$\bar{A}+\bar{B}$

$$0 1 \quad AB \quad A \quad 010 \quad M_1$$

$$1 0 \quad A\bar{B} \quad \bar{A} \quad 100 \quad M_2$$

$$1 1 \quad \bar{A}\bar{B} \quad \bar{A} \quad 110 \quad M_3$$

Minterm - 0-Complement Maxterm - 0 Uncomplement
 1-Uncomplement 1-1 Complement

Minterm & Maxterm for three binary variable

Variable	Minterm	Maxterm		
A	B	C	Term Representation	Term Representation
0	0	0	$\bar{A}\bar{B}\bar{C}$	$m_0 \quad A+B+C$
0	0	1	$\bar{A}\bar{B}C$	$m_1 \quad A+B+\bar{C}$
0	1	0	$\bar{A}B\bar{C}$	$m_2 \quad A+\bar{B}+C$
0	1	1	$\bar{A}BC$	$m_3 \quad A+\bar{B}+C$
1	0	0	$A\bar{B}\bar{C}$	$m_4 \quad \bar{A}+B+C$
1	0	1	$A\bar{B}C$	$m_5 \quad \bar{A}+B+C$
1	1	0	$AB\bar{C}$	$m_6 \quad A+B+\bar{C}$
1	1	1	ABC	$m_7 \quad A+B+C$

0	0	0	$\bar{A}\bar{B}\bar{C}$	$m_0 \quad A+B+C$	M_0
0	0	1	$\bar{A}\bar{B}C$	$m_1 \quad A+B+\bar{C}$	M_1
0	1	0	$\bar{A}B\bar{C}$	$m_2 \quad A+\bar{B}+C$	M_2
0	1	1	$\bar{A}BC$	$m_3 \quad A+\bar{B}+C$	M_3
1	0	0	$A\bar{B}\bar{C}$	$m_4 \quad \bar{A}+B+C$	M_4
1	0	1	$A\bar{B}C$	$m_5 \quad \bar{A}+B+C$	M_5
1	1	0	$AB\bar{C}$	$m_6 \quad A+B+\bar{C}$	M_6
1	1	1	ABC	$m_7 \quad A+B+C$	M_7

Write a Minterm & Maxterm

$$\textcircled{Q} \quad F_1 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$$

Given $F_1 = 000 + 010 + 101$ is constraint

$$F_1 = m_0 + m_3 + m_5$$

$$\textcircled{Q} \quad F_1 = (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C}) \\ = (0+0+0) \cdot (0+1+0) \cdot (1+0+0) \\ = M_0 \cdot M_2 \cdot M_4 \quad M_1 \cdot M_3 \cdot M_5$$

$$\textcircled{Q} \quad F_1 = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C}) \\ = (1+0+0) \cdot (1+1+0) \cdot (1+0+1) \cdot (0+1+0) \\ = M_7 \cdot M_6 \cdot M_5 \cdot M_2 \cdot M_0 \cdot M_1 \cdot M_2 \cdot M_5$$

$$\textcircled{Q} \quad F_1 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C \\ = 001 + 011 + 100 + 110 \\ = m_1 + m_3 + m_4 + m_6$$

Boolean expression can be expressed in the two standard form

① Sum of Products (SOP)

② Product of Sums (POS)

Sum of Products Form (SOP)

The sum of product is a type of Boolean expression that consists of AND terms (product term) that are ORed together.

Every SOP expression consist of two or more AND (product) term that are ORed together.

Each product term has variable that are in normal or complemented form.

$$Y = ABC + A\bar{B}C + AB\bar{C} + B\bar{C}$$

Σ - symbolic representation of SOP.

$$Y = A\bar{B} + B\bar{C}D + A\bar{C}D$$

m_0, m_1, m_2 used for SOP.

Product of sum (POS)

The product of sum is a Boolean expression that consists of OR terms (sum terms) that are ANDed together.

Every sum term has variable that are in normal or complemented form

$$Y = (A+B+C)(A+\bar{B})(\bar{B}+C)$$

II - symbolic representation of Product of sum

Standard SOP & POS forms

Standard SOP form

If every product term contains all variables in complemented or uncomplemented form called standard SOP form

$$Y = ABC + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$$

Standard SOP form is also called standard minterm form.

Standard POS form

If every sum term contains all the variables in complemented or uncomplemented form then it is called standard POS form.

$$Y = (\bar{A} + B + C) \cdot (A + \bar{B} + \bar{C}) \cdot (A + B + \bar{C})$$

Standard POS form is also called maxterm standard form.

Conversion from Non standard SOP to

$$(A \oplus A) \oplus (B \oplus B) \oplus (C \oplus C) = Y$$

Convert the expression $Y = A + BC$
into standard SOP form

Find missing variable in each
product term

$$Y = A + BC$$

$(S + H + A) \cdot (S + B + A)$ \downarrow \rightarrow Variable A is missing

B & C missing.

Step 2

AND missing variable & their complement

$$Y = A(B + \bar{B})(C + \bar{C}) + BC(A + \bar{A})$$

Step 3

$$Y = (AB + A\bar{B})(C + \bar{C}) + ABC + \bar{A}BC$$

$$= ABC + A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$$

$$Y = ABC + A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

$$= 111 + 110 + 101 + 100 + 011$$

$$= m_7 + m_6 + m_5 + m_4 + m_3$$

$$Y = \sum (3, 4, 5, 6, 7)$$

$$\Sigma m(3, 4, 5, 6, 7)$$

Convert the expression in standard SOP form

$$Y = AB + BC + AC$$

$$Y = AB + BC + AC$$

C is missing A missing B missing

$$= AB(C + \bar{C}) + BC(A + \bar{A}) + AC(B + \bar{B})$$

$$= ABC + ABC + \bar{ABC} + \bar{ABC} + ABC + \bar{ABC}$$

$$= ABC + (ABC + \bar{ABC} + \bar{ABC})$$

$$= 00111 + 110 + 011 + 101$$

$$= m_7 + m_6 + m_3 + m_5$$

$$= m_3 + m_5 + m_6 + m_7$$

$$Y = \sum(m(3, 5, 6, 7))$$

Convert expression in standard SOP

$$F_1 = AB + BC$$

$$F_1 = AB + BC$$

A missing

$$= AB(C + \bar{C}) + BC(A + \bar{A})$$

$$= ABC + AB\bar{C} + \bar{ABC} + \bar{BC}$$

$$= (ABC) + (AB\bar{C} + \bar{ABC} + \bar{BC})$$

$$= 111 + 110 + 011$$

$$= (m_7 + m_6 + m_3)$$

$$F_1 = \sum(m(3, 6, 7))$$

$$(010) + (011) + (101) + (011) + (111) + (011)$$

$$F(PQR) = P\bar{Q} + MR + M\bar{R} + M\bar{P} + M\bar{P}\bar{R}$$

Conversion from non standard POS to POS

1. Determine the missing variable in each sum term.
2. OR each sum term with missing variable & its complement.
3. Use distributive law & expand the terms.
4. Simplify expression excluding the sum terms.

Express in standard POS form

$$Y = (A+B)(A+C)(B+\bar{C}) \quad T$$

Step 1

$$Y = (A+B)(A+C)(B+\bar{C})$$

\uparrow C is missing \uparrow B is missing \uparrow A is missing

Step 2

Y OR the sum term with missing variable

$$Y = [(A+B)+C\bar{C}] + [(A+C)+B\bar{B}] + [(B+\bar{C})+A\bar{A}]$$

$$(A+BC) = (A+B) + (A+C)$$

$$Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})$$

$$= (1+1+0)(110)(101)(110)(010)$$

$$= M_7 \cdot M_5 \cdot M_3 \cdot M_2 \cdot (0+0+0)(0+0+1)(0+1+0)(1+0+1)$$

$$Y = \prod (7, 5, 3, 2)$$

$$\checkmark M_0, M_1, M_2, M_5$$

(Q) Express the expression in standard POS form

$$F(A, B, C) = \sum m(0, 2, 5, 7)$$

$$F(A, B, C) = \sum m(0, 2, 5, 7)$$

Take complement of F :

$$F'(A, B, C) = \sum m(1, 3, 4, 6)$$

$$(1+0)(1+0+1+0) \cdot (m_1 + m_3 + m_4 + m_6)$$

(5) Take complement of F :

$$F'(A, B, C) = \prod M(1, 3, 4, 6)$$

$$(S+8+A')(S+8+A)(0+8+A)(0+8+A) = ?$$

$$= M_1 \cdot M_3 \cdot M_4 \cdot M_6$$

$$\cdot (001) \cdot (0+1+1) \cdot (1+0+0) \cdot (1+1+0)$$

$$= (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

$$(S, 2, 8, 10) M \prod - (0, 8, A)$$

(Q) Express in standard POS form

$$F_1 = (A+\bar{B})(B+C)$$

$$F_1 = [(A+\bar{B})+C\bar{C}] \quad [(B+C)+A\bar{A}]$$

$$A+\bar{B}C = (A+B)(A+C)$$

$$= (A+\bar{B}+C)(A+B+\bar{C})(B+C+A)(B+C+\bar{A})$$

$$= (0+1+0)(0+1+1)(0+0+0) + (0+0+0)$$

$$= M_2 \cdot M_3 \cdot M_0 \cdot M_4$$

$$= \prod M(0, 2, 3, 4)$$

c. Express given equation in standard POS form

$$F(A, B, C) = \prod M(0, 2, 5, 7)$$

$$F(A, B, C) = \prod M_0 + M_2 + M_5 + M_7$$

$$= M_0 + M_2 + M_5 + M_7$$

$$= (0+0+0) \cdot (0+1+0) \cdot (0+0+1) \cdot (1+1+1)$$

$$= (\bar{A}+B+C) (A+\bar{B}+C) (\bar{A}+B+\bar{C}) (\bar{A}+\bar{B}+\bar{C})$$

$$F = (A+B+C) (A+\bar{B}+C) (\bar{A}+B+\bar{C}) (\bar{A}+\bar{B}+\bar{C})$$

SOP form: $m(1, 3, 4, 6)$

$$F(A, B, C) = \prod M(0, 2, 5, 7)$$

$$\text{complement} = \sum m(1, 3, 4, 6)$$

$$(A+B+C)(\bar{A}+\bar{B}+\bar{C}) = 1$$

$$(A+B+C)(\bar{A}+\bar{B}+\bar{C}) + (\bar{A}+\bar{B}+\bar{C})(A+B+C) = 1$$

K-Map

Karnaugh map is a method used to simplify the algebraic expression in Boolean function without having complex theorem.

K-map is a graphical or pictorial method used to simplify Boolean expressions.

This method was invented by E.W.Veitch in 1952. In 1953 M. Karnaugh modified this graphical method of simplification. Hence it is called Karnaugh map.

A Kmaps comprises of boxes/squares called "cells".

Construction of K-map

1 variable Kmap consists of $2^1 = 2$ cells.

2 variable Kmap consists of $2^2 = 4$ cells

3 variable Kmap consists of $2^3 = 8$ cells

4 variable K-map consists of $2^4 = 16$ cells

Kmap for one variable

$2^1 = 2$ cells $\Rightarrow \bar{A} + A = Y$

A	0	1	\bar{A}	$\bar{A} + A = 1$	$\bar{A}A^T$	A^T
1	0	1	A	1	0	1

Regions = \bar{A} A $\bar{A}A^T$ A^T

1 0 \bar{A} A

Kmap for two variables

$2^2 = 4$ cells

A	B	00	01	$\bar{A}\bar{B}$	$\bar{A}B$
0	0	00	01	1	0
1	0	($\bar{A}H\bar{A}$) 11	($\bar{A}H'A$) $\bar{A}B$	$\bar{A}B$	$A\bar{B}$

$(1+0) \cdot (0+A) = A$

A	\bar{B}	B	$\bar{A}M$	$\bar{A}H$	$A\bar{B}$	A
0	0	1	(1, 00)	(0, 01)	$\bar{A}+B$	$A+\bar{B}$
1	0	1	(1, 10)	(0, 11)	$\bar{A}+B$	$A+\bar{B}$
2	1	0	(1, 11)	(0, 00)	\bar{A}	A
3	1	1	(1, 01)	(0, 10)		

$(1, 0)$ mts

Kmap for 3 variables

$2^3 = 8$ cells

A	$\bar{B}C$							
0	0	1	3	2	000	001	011	010
1	4	5	7	6	100	101	111	110

$\bar{A}BC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} = Y$

$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}B\bar{C}$	$\bar{A}B\bar{C}$
$A\bar{B}\bar{C}$	$A\bar{B}C$	$AB\bar{C}$	$AB\bar{C}$

$(3, 2, 6, 0)$ mts

A	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	$B\bar{C}$	$\bar{B}\bar{C}$	$\bar{B}\bar{C}$	$\bar{B}\bar{C}$
0	0	1	1	3	1	2	1
1	4	5	7	6	1	2	1

Q Plot the Boolean expression on K-map
 $Y = AB + \bar{A}\bar{B}$

$$Y = AB + \bar{A}\bar{B}$$

$$= 11 + 00 = m_1 + m_0$$

$$= \sum m(0,1)$$

A \ B	$\bar{A}\bar{B}$	B
\bar{A}	1 0	1
A	0 1	0 1

Q Plot the Boolean expression on K-map

$$Y = (A+B) \cdot (A+\bar{B})$$

$$Y = (A+B) \cdot (A+\bar{B})$$

$$= (1+0) (0+1)$$

$$= M_0 \cdot M_1$$

$$= \sum m(0,1)$$

$$= \sum m(2,3)$$

A \ B	$\bar{A}\bar{B}$	B
\bar{A}	0 0	0 1
A	1 0	1 1

Q Plot the Boolean expression on K-map

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

$$= 000 + 010 + 100 + 110$$

$$= m_0 + m_3 + m_5 + m_6$$

$$= \sum m(0,3,5,6)$$

A \ B	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
\bar{A}	1 0	1 1	0 1	1 1
A	0 1	0 1	1 0	1 1

C) Plot the Boolean expression on Kmap

$$\begin{aligned}
 Y &= (A + \bar{B} + C) \cdot (\bar{A} + B + C) \cdot (\bar{A} + B + \bar{C}) \cdot (\bar{A} + B + C) \\
 &= (0+1+0) \cdot (0+1+\emptyset) \cdot (0+0+0) \cdot (0+0+1) \\
 &= M_2 \cdot M_3 \cdot M_0 \cdot M_1 \\
 &= \overline{\text{IM}}(2, 3, 0, 1)
 \end{aligned}$$

A \ B	BC	$B\bar{C}$	$\bar{B}C$	$\bar{B}\bar{C}$	8
A	0 ⁰	0 ¹	0 ²	0 ³	
\bar{A}	1 ⁴	0 ⁵	1 ⁶	0 ⁷	6
	0110	1110	1010	0010	
	0111	1111	1011	0011	
	0101	1101	1001	0001	
	0000	0100	0010	0000	
	0001	0101	0011	0001	
	0010	0110	0000	0000	
	0011	0111	0001	0001	
	0100	1100	1000	0000	
	0101	1101	1001	0001	

Kmap for 4 variable

$(S+T+U+V) \oplus 2^4 = 16$ cells

$(1+D) AB \setminus CD$

0	1	3	2	M	M
4	5	7	6	(S+T+U)	M
12	13	15	14	(S+T+V)	M
8	9	11	10	58	59

0000	0001	10011	0010	A
0100	0101	1111	0110	
1100	1101	1111	1110	
1000	1001	1011	1010	

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}BCD$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
AB	$A\bar{B}\bar{C}\bar{D}$	$ABC\bar{D}$	$ABC\bar{D}$	$ABC\bar{D}$
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}CD$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
0	1	3	2	
4	5	7	6	
12	13	15	14	
8	9	11	10	

Q Plot the Boolean expression

$$Y = (A+B+C+D) (A+\bar{B}+C+\bar{D}), (\bar{A}+\bar{B}+C+\bar{D}) (A+C+B+\bar{D})$$

$$= (0+0+0+0) (0+1+0+1) (1+1+1+1) (1+0+0+1)$$

$$= M_0 \cdot M_5 \cdot M_{15} \cdot M_9$$

$$\Sigma m(0, 5, 9, 15)$$

	0	0	1	3	2
	4	0	5	7	6
	12	13	15	14	
	8	9	11	10	

Q Plot the Boolean expression

$$Y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$Y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + ABC\bar{D} + AB\bar{C}\bar{D}$$

$$= 0100 + 1010 + 1111 + 1100$$

$$= m_4 + m_{10} + m_{15} + m_{12}$$

$$\Sigma m(4, 10, 12, 15)$$

	0	1	3	2
	4	5	7	6
	12	13	15	14
	8	9	11	10

Gouping

Pair, Quad, Octet, Single

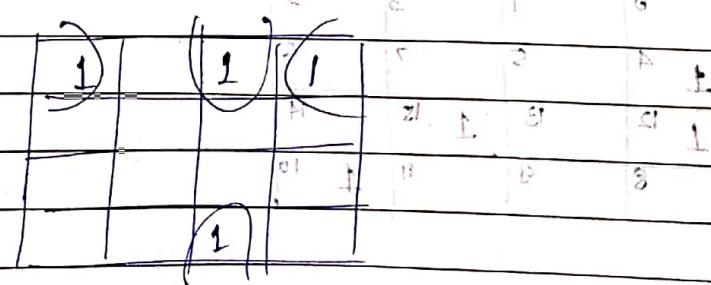
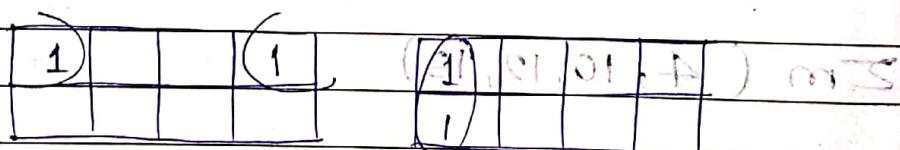
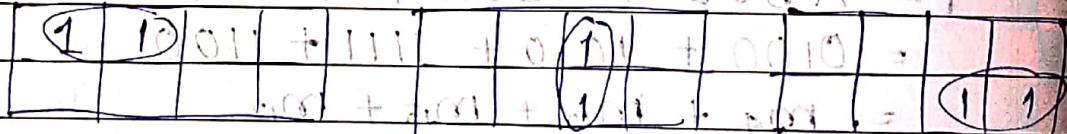
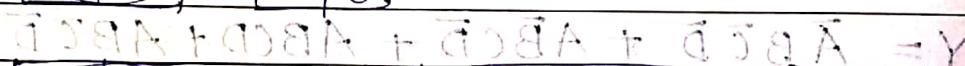
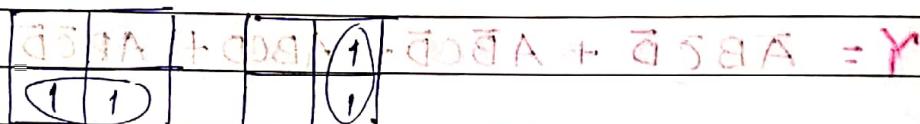
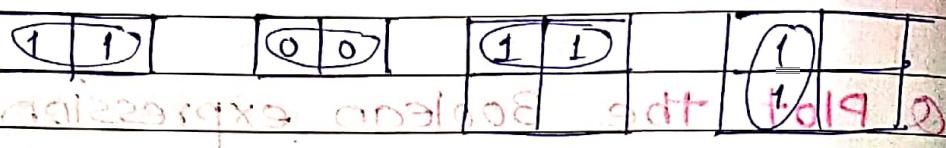
$$(1+0+0) \quad \underline{\text{cancel}} \quad (1+1+1) \quad (1+0+1+0) \quad (0+0+0+0) =$$

Single $\text{oM} + \text{eM} + \text{eM} = \text{oM}$

The single O ^{or} ~~and~~ is ~~an~~ ~~one~~ ~~one~~ Under
the single group.

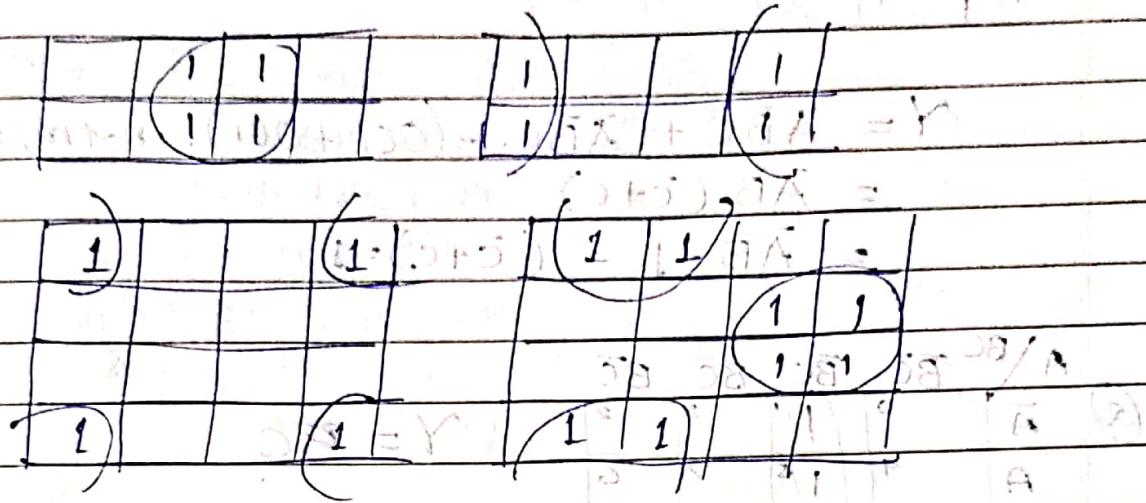
Pair

A group of two adjacent 1's or 0's is called as a pair



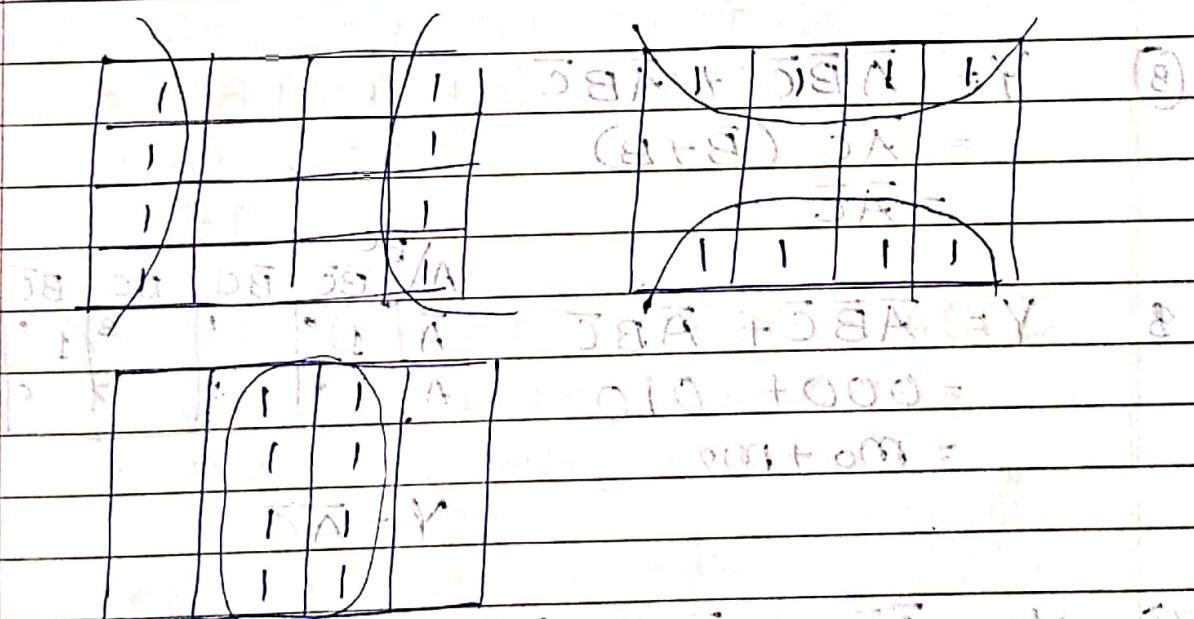
Quad

A group of four adjacent 1's or 0's is called as a quad.



Octate

A group of Eight adjacent 1's or 0's is called as a octate.



$$10000000 + 10000000 + 10000000 + 10000000 = 10000000 \quad (4)$$

$$(1+1+1+1)10000000 + (1+1+1+1)10000000 = 10000000$$

$$10000000 + 10000000 + 10000000 + 10000000 = 10000000$$

$$20000000 = 10000000 \quad (2,8,4)$$

A	B	C	Y
0	0	0	000
0	0	1	001
0	1	0	010
0	1	1	011
1	0	0	100
1	0	1	101
1	1	0	110
1	1	1	111

$$\begin{aligned}
 Y &= \bar{A}B + \bar{A}\bar{B}C + A\bar{B}C + ABC = (000+001) + m_1 + m_2 + (010+011) \\
 &= \bar{A}B(C+\bar{C}) + ABC = \bar{A}B + ABC
 \end{aligned}$$

A	B	C	Y
0	0	0	000
0	0	1	001
0	1	0	010
0	1	1	011
1	0	0	100
1	0	1	101
1	1	0	110
1	1	1	111

$$Y = \bar{A}BC + ABC$$

$$\begin{aligned}
 Y &= BC(\bar{A}+A) = BC \\
 &= \bar{B}C
 \end{aligned}$$

$$\begin{aligned}
 ⑤ \quad Y &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C \\
 &= \bar{A}\bar{C}(B+\bar{B}) \\
 &= \bar{A}\bar{C}
 \end{aligned}$$

$$\begin{aligned}
 ⑥ \quad Y &= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} \\
 &= 000 + 010 \\
 &= m_0 + m_2
 \end{aligned}$$

A	B	C	Y
0	0	0	000
0	0	1	010
0	1	0	100
0	1	1	101
1	0	0	110
1	0	1	111
1	1	0	100
1	1	1	101

$$Y = \bar{A}\bar{C}$$

$$\begin{aligned}
 ⑦ \quad Y &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} \\
 &= \bar{A}\bar{C}(\bar{B}+B) + \bar{B}C(\bar{A}+A) \\
 &= \bar{A}\bar{C} + \bar{B}C
 \end{aligned}$$

$$\begin{aligned}
 Y &= 001 + 011 + 001 + 101 \\
 &= m_1 + m_3 + m_4 + m_5 \\
 &= (1, 3, 5)
 \end{aligned}$$

A	B	C	Y
0	0	0	000
0	0	1	010
0	1	0	100
0	1	1	101
1	0	0	110
1	0	1	111
1	1	0	100
1	1	1	101

$$\begin{aligned}
 Y &= ① \bar{A}\bar{C} + ② \bar{B}C \\
 Y &= \bar{A}\bar{C} + \bar{B}C
 \end{aligned}$$

① $A \backslash BC$

\bar{A}	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
1	1 0	1 1	0 1	0 0
A	1	0	1	0

$$\begin{aligned}
 Y &= A\bar{B}\bar{C} + \bar{A}\bar{B}C = (000+001); m_0 + m_1 = (0,1) \\
 &= \bar{A}\bar{B} \cdot (\bar{C}+C) \\
 &= \bar{A}\bar{B} \cdot 1 \cdot (\bar{C}+C) = 1
 \end{aligned}$$

② $A \backslash BC$

\bar{A}	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
1	0	1	1	0
A	1	1	0	0

$$Y = A\bar{B}\bar{C} + A\bar{B}C$$

$$\begin{aligned}
 &= BC(\bar{A}+A); m_1 + m_3 = 1 \\
 &= BC \text{ (both 1 in m1 and m3)}
 \end{aligned}$$

③ $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$
 $= \bar{A}\bar{C} (B+B)$
 $= \bar{A}\bar{C}$

④ $Y = \bar{A}\bar{B}\bar{C} + \bar{A}'\bar{B}\bar{C}$
 $= 000 + 010$
 $= m_0 + m_2$

$A \backslash BC$

\bar{A}	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
1	0	1	3	1
A	1	0	1	0

$$Y = \bar{A}\bar{C}$$

⑤ $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}'\bar{B}\bar{C} + A\bar{B}\bar{C}$
 $= \bar{A}\bar{C}(\bar{B}+B) + BC(\bar{A}+A)$
 $= \bar{A}\bar{C} + BC$

$$\begin{aligned}
 Y &= 001 + 011 + 001 + 101 \\
 &= m_1 + m_3 + m_4 + m_5 \\
 &= (1, 3, 5)
 \end{aligned}$$

$A \backslash BC$

\bar{A}	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
1	0	1	1	3
A	1	0	1	0

$$\begin{aligned}
 Y &= 0\bar{A}C + 0\bar{B}C \\
 Y &= \bar{A}C + \bar{B}C
 \end{aligned}$$

$$\begin{aligned}
 ⑤ Y &= \bar{A}\bar{B}C + \bar{A}BC + ABC + A\bar{B}\bar{C} \\
 &= \bar{A}C(\bar{B}+B) + AB(C+\bar{C}) \\
 &= \bar{A}C + AB
 \end{aligned}$$

$$Y = \bar{A}\bar{B}C + \bar{A}BC + ABC + A\bar{B}\bar{C}$$

$$\begin{aligned}
 &= 001 + 011 + 111 + 110 \\
 &= m_1 + m_3 + m_7 + m_6 \\
 &= (1, 3, 6, 7)
 \end{aligned}$$

	$\bar{B}C$	$\bar{B}\bar{C}$	$B\bar{C}$	BC	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
\bar{A}	0	1	1	3	1	2	1	3
A	4	5	1	7	1	6	1	8

$$Y = \bar{A}C + AB$$

$$Y = \bar{A}\bar{B}A + \bar{A}B\bar{A} + \bar{A}BA + A\bar{B}A$$

$$\begin{aligned}
 ⑥ Y &= \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} \\
 &= \bar{A}B(\bar{C}\bar{D} + \bar{C}D + CD + C\bar{D}) \\
 &= \bar{A}B(\bar{C}(\bar{D}+D) + C(D+\bar{D})) \\
 &= \bar{A}B(\bar{C} + C) \\
 &= \bar{A}B
 \end{aligned}$$

$$\begin{aligned}
 Y &= \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} \\
 &= 0100 + 0101 + 0111 + 0110 \\
 &= m_4 + m_5 + m_7 + m_6
 \end{aligned}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	1	1	3	1	1	1	3
$\bar{A}B$	4	5	1	7	1	5	1	7
$A\bar{B}$	12	13	15	14	12	13	15	14
AB	8	9	11	10	8	9	11	10

$$Y = \bar{A}B$$

$$\begin{aligned}
 7) \quad Y &= \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} = Y \\
 &= 0100 + 1100 + 0010 + (1110) \text{ (A)} \\
 &= m_4 + m_{12} + m_6 + m_{14} \text{ (A)} \\
 &= (4, 6, 12, 14)
 \end{aligned}$$

	CD				Y			
AB	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	0	1	2	3
$\bar{A}\bar{B}$	1	4	5	7	1	6	3	8
$\bar{A}B$	12	3	13	15	14	8	38	28
$A\bar{B}$	8	9	10	11	10	11	12	13
AB	14	15	16	17	18	19	20	21

$$Y = B\bar{D}$$

$$\bar{B}A + \bar{D}A = Y$$

$$8) \quad \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

$$= 0000 + 0010 + 1000 + 1010$$

$$= m_2 + m_5 + m_8 + m_{10} \text{ (A)} \quad \bar{B}A + \bar{D}A = Y \quad (8)$$

$$= (0, 2, 8, 10) (\bar{C} + \bar{D} + \bar{A}3 + \bar{D}5) \text{ (A)}$$

	CD				Y			
AB	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	0	1	2	3
$\bar{A}\bar{B}$	1	4	5	7	1	2	(5+3)	8A
$\bar{A}B$	9	5	7	6				8A
$A\bar{B}$	12	13	15	14				
AB	1	8	9	10				

$$Y = \bar{A}\bar{B} + \bar{A}B + AB + A\bar{B} = Y$$

$$= 0110 + 1110 + 1010 + 0010 =$$

$$Y = \bar{B}D + \bar{A}B + \bar{A}D + AB + AD =$$

	CD				Y			
AB	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	0	1	2	3
$\bar{A}\bar{B}$	1	4	5	7	1	2	3	5
$\bar{A}B$	9	5	7	6				5
$A\bar{B}$	12	13	15	14				5
AB	8	9	11	10				5

$$Y = \bar{A}\bar{B} + BD + BC$$

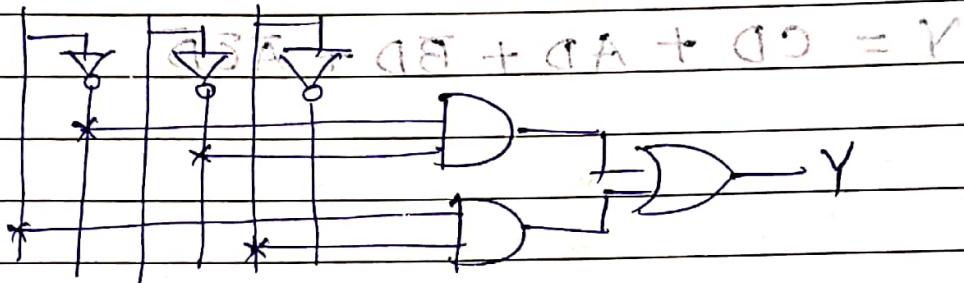
Q $f(A, B, C) = \sum m(1, 3, 5, 7)$

(21, f(A, B, C) = m₀, m₁, m₅, m₇)

A	BC	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC	$\bar{B}\bar{C}$
\bar{A}	1 0	1		1 3	2	1
A	4	1 5	1 1	7	6	0

$$Y = \bar{A}B + AC$$

① A B C



Q Minimize the equation using K-Map

$$Y = ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

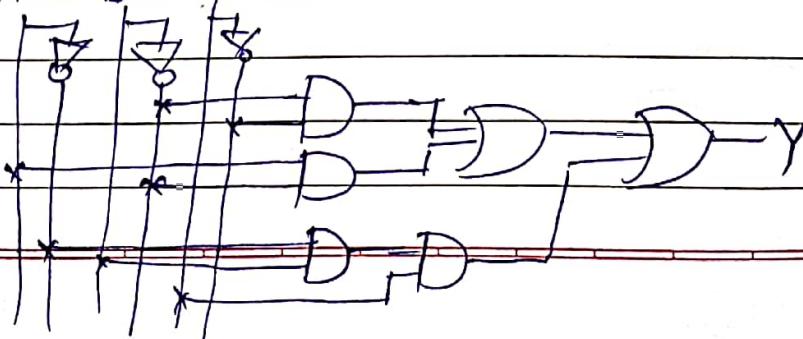
$$= 011 + 101 + 100 + 000$$

$$= m_3 + m_5 + m_4 + m_0$$

$$= \sum m(0, 3, 4, 5)$$

A	BC	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC	$\bar{B}\bar{C}$
\bar{A}	1 0	1	1	1 3	2	1
A	1 4	1 1		7	6	0

$$Y = \bar{B}\bar{C} + A\bar{B} + \bar{A}\bar{B}\bar{C}$$



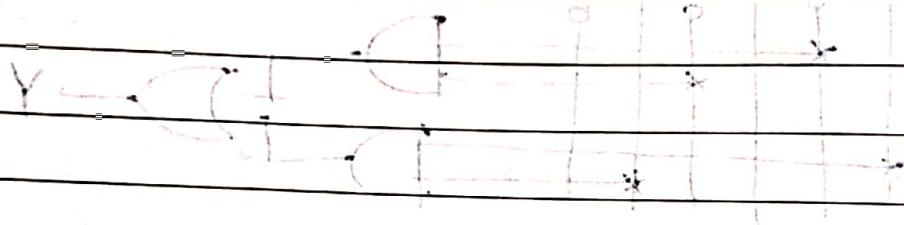
& Minimize KMap using K-Map

$$F(A, B, C, D) = \sum m(0, 1, 3, 4, 7, 9, 11, 13, 15)$$

	AB\CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	$\bar{C}\bar{D}$	
$\bar{A}\bar{B}$	1	1	1	1	0	0	1
$\bar{A}B$	1	1	1	1	0	0	1
$A\bar{B}$	0	1	13	1	14	1	1
AB	0	1	9	1	10	1	1

(1) (2) (3) 8 A = Y
→ 8 1

$$Y = CD + AD + BD + \bar{A}\bar{C}\bar{D}$$

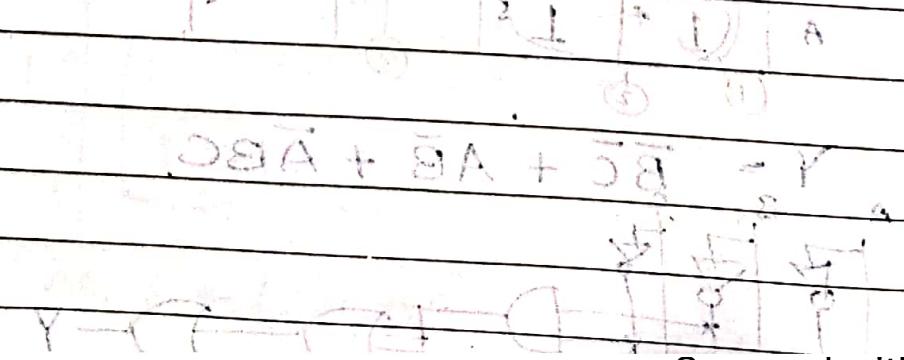


Q Minimize using K-Map

$$F(A, B, C, D) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$

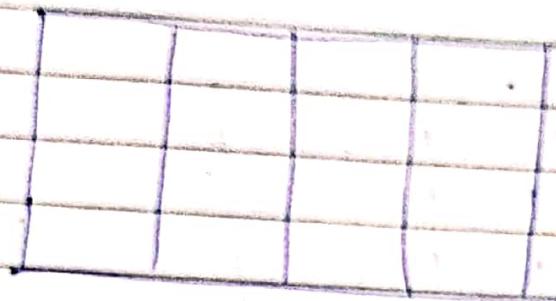
	$\bar{A}\bar{B}$	$\bar{A}B$	AB	AB	$\bar{A}\bar{B}$
3	000	+ 001	(1)	01	+ 110
4	1	1	(1)	1	1
5			(1)	(1)	m7
7			(1)	(1)	m5
9					m3
13					m1
14					m2
15					m0

$$Y = \bar{A}BC + A\bar{C}D + \bar{A}CD + ABC$$



Σ m	
Y	

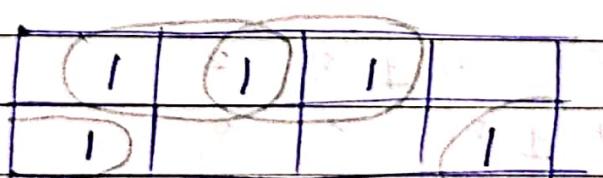
Q) $F(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13)$



4 quad

$$Y = \overline{C}\overline{D} + \overline{B}D + \overline{A}C + A\overline{C}$$

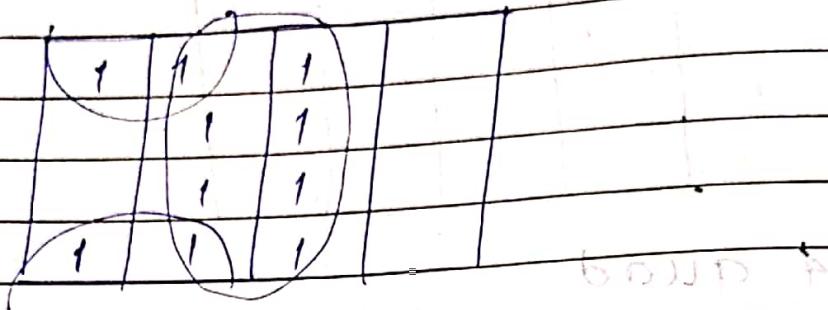
Q) $F(A, B, C) = \Sigma m(0, 1, 3, 4, 6)$ (A, B, C)



3 pairs

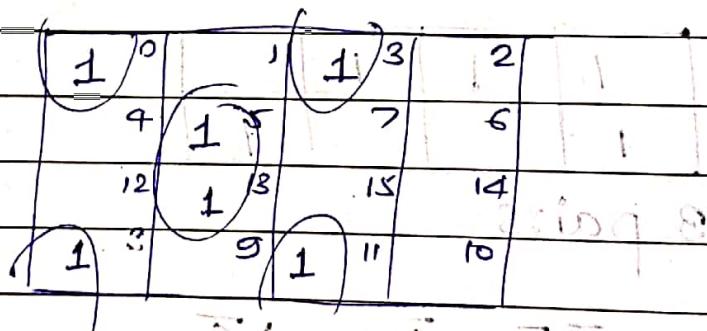
$$Y = \overline{B}\overline{C} + \overline{A}C + A\overline{C}$$

$\sum m(0, 1, 3, 5, 7, 8, 9, 11, 13, 15) = F(A, B, C, D)$



$$Y = B'C + DA + CB + CD = Y$$

$\sum m(0, 3, 5, 8, 11, 13) = F(A, B, C, D)$



$$BA + DA + CB = Y$$

K-Map using Don't care Condition

If the input combinations of the value of Boolean expression are not specified called as Don't care condition.
 'X' - for Don't care

$$F(A, B, C) = \sum m(1, 2, 4) + d(5, 6, 7)$$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	1	2	3
A	4	X	X	X

$$Y = A + \bar{B}C + B\bar{C}$$

$$\textcircled{2} \quad F(A, B, C) = \sum m(0, 1, 6, 7) + \sum d(3, 5)$$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	1	X	2
A	4	X	1	1

$$Y = C + \bar{A}\bar{B} + AB$$

=

$$\text{so } F(A, B, C) = \sum m(1, 2, 5, 7) + \sum d(0, 4, 6)$$

X	0	1	1	3	X	2
X	4	1	5	1	7	X

$$Y = A + \bar{B} + \bar{C}$$

$$\text{so } F(A, B, C) = A + \bar{B} + \bar{C} = (0, 8, 10)$$

0	1	2	3	4
0	1	2	3	4
0	X	X	X	A

$$SA + \bar{B}A + A = Y$$

$$\textcircled{Q} \quad F(A, B, C, D) = \sum m(0, 5, 9, 12, 13, 14, 15) + d(1, 2, 3, 4)$$

F, A, D

1	0	X	1	X	3	X	2
X	4	1	5	7	6		
1	12	1	13	1	15	1	4
8	1	9	0	1	3	3	10.

$$(2, 8) \in \{1, 12, 1, 13, 1, 15, 1\} \Rightarrow (0, 8, 10) \text{ } \textcircled{Q}$$

1	2	3	4
1	2	3	4
0	X	X	A

$$SA + \bar{B}A + D = Y$$

$$F(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + d(0, 2, 5)$$

X	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

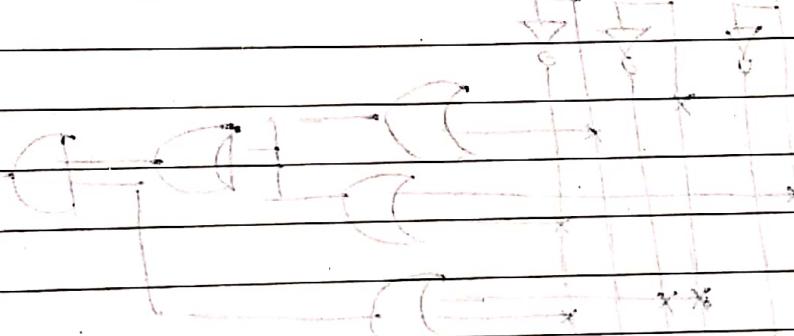
$(\bar{A} + B + \bar{C}) (\bar{B} + \bar{C} + D) (\bar{B} + \bar{C} + \bar{D})$
 $(\bar{A} + B + \bar{C}) (\bar{B} + \bar{C} + D) (\bar{B} + \bar{C} + \bar{D})$
 $(\bar{A} + B + \bar{C}) (\bar{B} + \bar{C} + \bar{D}) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + \bar{C}) (\bar{B} + \bar{C} + \bar{D}) (\bar{B} + C + D)$
 $(\bar{A} + B + \bar{C}) (\bar{B} + C + \bar{D}) (\bar{B} + C + D)$
 $(\bar{A} + B + C + \bar{D}) (\bar{B} + C + \bar{D}) (\bar{B} + C + D)$
 $(\bar{A} + B + C + \bar{D}) (\bar{B} + C + D) (\bar{B} + C + \bar{D})$
 $(\bar{A} + B + C + D) (\bar{B} + C + \bar{D}) (\bar{B} + C + D)$
 $(\bar{A} + B + C + D) (\bar{B} + C + D) (\bar{B} + C + \bar{D})$
 $(\bar{A} + B + C + D) (\bar{B} + C + D) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + D) (\bar{B} + C + D)$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + D) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + C + \bar{D}) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + C + D) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + D) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + \bar{D}) (\bar{B} + \bar{C} + D)$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + \bar{D}) (\bar{B} + \bar{C} + \bar{D})$
 $(\bar{A} + B + C + D) (\bar{B} + C + \bar{D}) (\bar{B} + \bar{C} + \bar{D})$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + D) (\bar{B} + \bar{C} + \bar{D})$
 $(\bar{A} + B + C + D) (\bar{B} + \bar{C} + \bar{D}) (\bar{B} + \bar{C} + \bar{D})$

$$F(A, B, C, D) = \bar{A} \bar{D} + C \bar{D}$$

$$\begin{array}{l}
 \bar{A} + \bar{B} \quad \bar{A} + B \quad \bar{A} + \bar{B} \quad \bar{A} + B \\
 | \quad | \quad | \quad | \\
 P \quad P \quad P \quad P \\
 \bar{A} \quad A \quad \bar{A} \quad A \\
 \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \quad \textcircled{4}
 \end{array}$$

$$(\bar{A} + \bar{B}) (\bar{A} + B) (\bar{A} + \bar{B}) = Y$$

$$F(A, B, C, D) = \sum m(1, 3, 4, 6, 8, 11, 15) + d(0, 5, 7)$$



Simplification of POS form using K-Map up

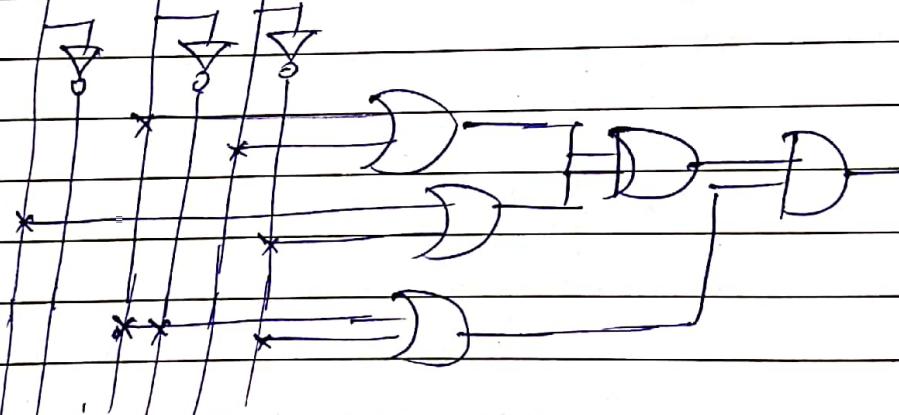
o Minimize the expression

$$\begin{aligned}
 Y &= (A+B+\bar{C}) (A+\bar{B}+\bar{C}) (\bar{A}+B+C) (\bar{A}+\bar{B}+\bar{C}) \\
 &\quad (A+B+C) (001) (011) (100) (111) (000) \\
 &= M_0 \cdot M_1 \cdot M_3 \cdot M_4 \cdot M_7 \cdot M_8 \cdot M_9
 \end{aligned}$$

	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A	(0)	(1)	(1)	(0)
\bar{A}	(0)	(0)	(1)	(0)
	0	1	2	3
	4	5	6	7

$$Y = (B+C) (A+\bar{C}) (\bar{B}+\bar{C})$$

(0,1,2,3) $\rightarrow A \cdot (B+1) \cdot (C+0,1,2) \Rightarrow (A, B, C) = (1, 1, 0)$



Date _____

Minimized Boolean Expression by Quine McCluskey Method or Tabular Method

Q) $f(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$

Minterm	A	B	C	D
m_0	0	0	0	0 ✓
m_1	0	0	1	0 ✓
m_2	0	0	1	1 ✓
m_3	0	1	1	0 ✓
m_7	0	1	1	1 ✓
m_8	1	0	0	0 ✓
m_{10}	1	0	1	0 ✓
m_{12}	1	1	0	0 ✓
m_{13}	1	1	0	1 ✓

Group	Minterm	A	B	C	D
0	$m_0 \checkmark$	0	0	0	0
1	$m_2 \checkmark$	0	0	1	0
	$m_8 \checkmark$	1	0	0	0
2	$m_3 \checkmark$	0	0	1	1
	$m_6 \checkmark$	0	1	1	0
	$m_{10} \checkmark$	1	0	1	0
	$m_{12} \checkmark$	1	1	0	0
3	$m_7 \checkmark$	0	1	1	1
	$m_{13} \checkmark$	1	1	0	1

Group:		Minteam	A	B	C	D
O	m ₂ m ₂		O	O	X	O
X	m ₂ m ₈		X	O	O	O
X	m ₂ m ₃		O	O	1	X
X	m ₂ m ₆		O	X	1	O
X	m ₂ m ₁₀		X	X	O	1
O	m ₈ m ₁₀	X	1	O	X	O
	m ₈ m ₁₀		1	X	O	O A

$$58A + 5A + 8B = Y \quad | \text{if}$$

$m_3 m_7$	0	x	1	1 ✓
$m_6 m_7$	0	1	1	x ✓
$m_6 m_7 \rightarrow$	0	1	1	x ✓

Group	Minterm	A	B	C	D
R	$\bar{A} \bar{B} \bar{C}$	+	+	+	+
S	$\bar{A} B \bar{C}$	+	+	+	+

Group

Prime Implicants	m_0	m_1	m_3	m_5	m_7	m_8	m_{10}	m_{12}	m_{13}
$A\bar{C}\bar{D}$ ($m_8 m_{12}$)	X						X	X	
$A\bar{B}\bar{C}$ ($m_2 m_{13}$)		X							(X)
$\bar{B}\bar{D}$ ($m_0 m_2 m_8 m_{10}$)			X				X	(X)	
$\bar{A}C\bar{D}$ ($m_0 m_3 m_6 m_7$)				X	(X)	(X)	(X)		

$$F = Y = \bar{B}\bar{D} + \bar{A}C + A\bar{B}\bar{C}$$

$$\Sigma m(0, 2, 3, 6, 7, 8, 10, 12, 13) m$$

AB	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1 ⁰	1 ¹	1 ³	1 ²
$\bar{A}B$	+	5	7	9
AB	1 ⁰	1 ¹	1 ⁵	1 ⁴
A \bar{B}	1 ⁸	X	1 ¹¹	1 ¹⁰

$$F = \bar{A}C + \bar{B}\bar{D} + A\bar{B}\bar{C}$$

$\bar{A}\bar{B}$	1	1	x	2
$\bar{A}B$	x	1	7	1
$A\bar{B}$	1	1	15	14
AB	1	1	1	10
$A\bar{B}$	8	6	11	

$$Y = B\bar{D} + \bar{A}\bar{C}D + B\bar{C}$$

$$F(A, B, C, D) = \sum m(1, 5, 6, 12, 13, 14) + d(2, 4)$$

DATA 10 X 9 80000 11

Binary Representation

	X	O	L	A	B	C	D	E
m ₁	X	L	O	O	O	O	O	O
m ₅	O	L	O	I	O	O	O	O
m ₆			O	I	I	O	O	O
m ₁₂	O	I	X	I	O	O	O	O
m ₁₃	I	I	X	I	O	O	O	O
m ₁₄	O	I	I	I	I	I	O	O
m ₂	X	O	O	O	I	O	O	O
m ₄		O	I	O	O	O	O	O

D C E B A **W** h i c h o n s t r u c t i o n s

Group X Minterm SA B C D

me x 0) I said o va

m_{12} 1 1 0 0 ✓

3 m_{13} 1 1 0 1 ✓

Digitized by srujanika@gmail.com

58 阿尼 + 45A

Group Minterm A B C D

1 $m_1 m_5$ 0 X 0 1 PI - $\bar{A}CD$

$m_2 m_6$ 0 X 0 0 PI - \bar{ACB}

$m_4 m_5$ 0 1 0 X ✓

$m_4 m_6$ 0 1 X 0 ✓

$m_4 m_{12}$ 1 X 1 0 0 ✓

2 $m_5 m_{13}$ 1 X 1 0 ✓

$m_6 m_{14}$ 1 X 1 1 D ✓

$m_{12} m_{13}$ 1 1 1 0 X ✓

$m_{12} m_{14}$ 0 + 0 1 X 0 ✓

Group Minterm A B C D

$m_4 m_5 m_{12} m_{13}$ AX 0 X \bar{BC}

1 $m_4 m_6 m_{12} m_{14}$ X 1 X 0 \bar{BD}

$m_4 m_{12} m_5 m_{13}$ CX 1 0 X \bar{BD}

$m_4 m_{12} m_6 m_{14}$ 0 X X 0 \bar{C}

PI	Minterm	m_1	m_5	m_6	m_{12}	m_{13}	m_{14}
$\bar{A}CD$	$m_1 m_5$	(X)	X				
\bar{ACD}	$m_2 m_6$			X			
\bar{BD}	$m_4 m_6 m_{12} m_{14}$			X	X	(X)	
\bar{BC}	$m_4 m_5 m_{12} m_{13}$			X	X	(X)	

$$Y = \bar{A}\bar{C}D + \bar{B}\bar{D} + \bar{B}\bar{C}$$

$$Y(A, B, C, D) = \sum m(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$$

Minterm Binary Representation

	A	B	C	D	
m_0	0	0	0	0	✓
m_1	0	0	0	1	✓
m_2	0	0	1	0	✓
m_3	0	0	1	1	✓
m_5	0	1	0	1	✓
m_7	0	1	1	1	✓
m_8	1	0	0	0	✓
m_9	1	0	0	1	✓
m_{11}	1	0	1	1	✓
m_{14}	1	1	1	0	✓

Group Minterm Representation

0 m_0 0 0 0 0 ✓

1 m_1 0 0 0 1 ✓

m_2 0 0 1 0 ✓

m_8 1 0 0 0 ✓

2 m_3 0 0 1 1 ✓

m_5 0 1 0 1 ✓

m_9 1 0 0 1 ✓

3 m_7 0 1 1 1 ✓

m_{11} 1 0 1 1 ✓

m_{14} 1 1 1 0 PI

ABCD

Group	Minterm	Representation
-------	---------	----------------

0	$m_0 m_1$	0 0 0 X ✓
	$m_0 m_2$	0 0 X 0 ✓
	$m_0 m_8$	X 0 0 0 ✓

1	$m_1 m_3$	0 0 X 1 ✓
	$m_1 m_5$	0 X 0 1 ✓
	$m_1 m_9$	X 0 0 1 ✓
	$m_2 m_5$	0 0 * * ✓
	$m_8 m_9$	1 0 0 X ✓

2	$m_3 m_7$	0 X 1 1 ✓
	$m_3 m_{11}$	X 0 1 1 ✓
	$m_5 m_7$	0 0 1 1 ✓
	$m_9 m_{11}$	1 0 X 1 ✓

Group	Minterm	Representation
-------	---------	----------------

0	$m_0 m_1 m_2 m_9$	0 0 0 X $\neg A \neg B$
	$m_0 m_1 m_5 m_9$	0 0 0 X $\neg A \neg C$
	$m_0 m_2 m_5 m_9$	0 0 0 X $\neg B \neg C$
	$m_0 m_8 m_9 m_{11}$	X 0 0 X $\neg B \neg D$

1	$m_1 m_3 m_5 m_7$	0 X X 1 $\neg A D$
	$m_1 m_9 m_9 m_{11}$	X 0 X 1 $\neg B D$
	$m_1 m_5 m_9 m_7$	0 X X 1 $\neg B D$
	$m_1 m_9 m_5 m_{11}$	X 0 X 1 $\neg A B$

2	0 1 1 0	0 0 1 1 $\neg A \neg B$
	1 1 1 0	0 0 1 1 $\neg A \neg B$

Group Minterm Representation

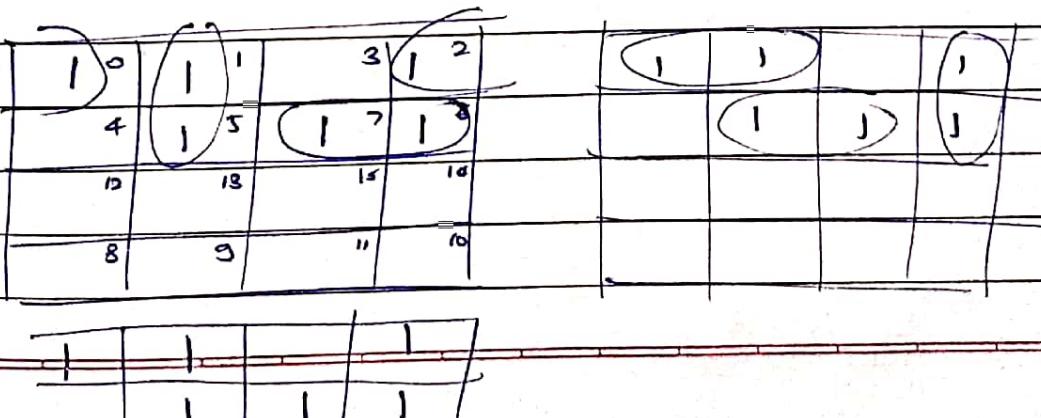
1	$m_0 m_1$	0 0 0 X $\bar{A} \bar{B} \bar{C}$
	$m_0 m_2$	0 0 X 0 $\bar{A} \bar{B} D$

2	$m_1 m_5$	0 X 0 1 $\bar{A} \bar{C} D$
	$m_2 m_6$	0 X 1 0 $\bar{A} C \bar{D}$

3	$m_5 m_7$	0 1 X 1 $\bar{A} B D$
	$m_6 m_7$	0 1 1 X $\bar{A} B C$

PI	Minterms	m_0	m_1	m_2	m_5	m_6	m_7
$\bar{A} \bar{B} \bar{C} \checkmark$	$m_0 m_1$	X	X				
$\bar{A} \bar{B} \bar{D}$	$m_0 m_2$	X		X			
$\bar{A} C \bar{D}$	$m_1 m_5$		X		X		
$\bar{A} C \bar{D} \checkmark$	$m_2 m_6$			X		X	
$\bar{A} B D \checkmark$	$m_5 m_7$				X		X
$\bar{A} B C$	$m_6 m_7$					X	X

$$Y = \bar{A} \bar{B} \bar{C} + \bar{A} C \bar{D} + \bar{A} B D$$

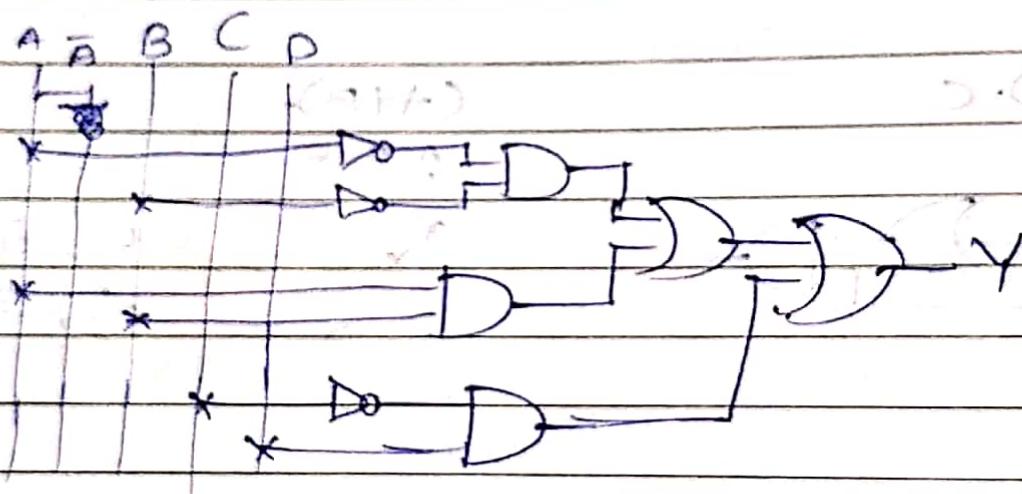


Minimize
Implementation boolean expression using K-Map
design using NAND gate only

$$F = \sum m(0, 5, 9, 12, 13, 14, 15) + d(1, 2, 3, 4)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	X	X	X
$\bar{A}B$	X	1	1	1
$A\bar{B}$	1	1	1	1
AB	1	1	1	1

$$Y = \bar{A}\bar{B} + AB + \bar{C}D$$



$$Y = \bar{A}\bar{B} + AB + \bar{C}D$$

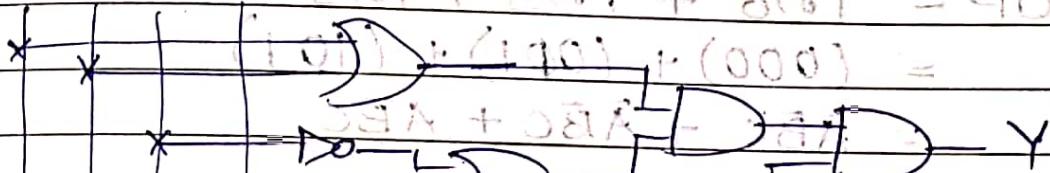
Simplify following logic function & realize using NOR gates

$$f(X, Y, Z) = \text{TTM}(12, 3, 7, 10, 11) + d(0, 15)$$

	$C+D$	$C+\bar{D}$	$(\bar{C}+D)$	$(\bar{C}+\bar{D})$	$(C+\bar{D})$	$(C+D)$
$A+B$	X	0	0	0	1	0
$A+\bar{B}$	M	4	5	0	1	0
$\bar{A}+\bar{B}$	M	12	13	X	15	1
$\bar{A}+B$	M	8	9	0	1	1

$$\begin{aligned} Y &= (A+B)(C\bar{D}) (B+\bar{C}) \\ &= (X+X)(\bar{Y}+\bar{Z})(X+\bar{Y}) \end{aligned}$$

$$w \times y \times z = \overline{(001 + 000)} + 001 = 002$$



$$\begin{aligned} &\overline{w} \cdot \overline{x} \cdot \overline{y} \cdot \overline{z} = \overline{w+x+y+z} = 002 \\ &(1 \times) \cdot (1 \times) \cdot (1 \times) \cdot (1 \times) = 002 \\ &w \cdot x \cdot y \cdot z = \overline{w+x+y+z} = 002 \\ &(001) \cdot (001) \cdot (001) \cdot (001) = 002 \\ &w \cdot x \cdot y \cdot z = \overline{w+x+y+z} = 002 \\ &(001) \cdot (001) \cdot (001) \cdot (001) = 002 \\ &w \cdot x \cdot y \cdot z = \overline{w+x+y+z} = 002 \\ &(001) \cdot (001) \cdot (001) \cdot (001) = 002 \end{aligned}$$

Write standard SOP & POS expression

A	B	C	function	Minterm	Maxterm
---	---	---	----------	---------	---------

0	0	0	1	m_0	M_8
---	---	---	---	-------	-------

0	0	1	0		M_1
---	---	---	---	--	-------

0	1	0	0		M_2
---	---	---	---	--	-------

0	1	1	1	m_3	M_7
---	---	---	---	-------	-------

1	0	0	0		M_4
---	---	---	---	--	-------

1	0	1	1	m_5	M_6
---	---	---	---	-------	-------

1	1	0	0		M_6
---	---	---	---	--	-------

1	1	1	0		M_7
---	---	---	---	--	-------

$$(S + A) (A + \bar{B}) (\bar{A} + \bar{C}) = Y$$

SOP form

$$(Y + X) (S + \bar{F}) (X + F) =$$

$$\begin{aligned} \text{SOP} &= m_0 + m_3 + m_5 \\ &= (000) + (001) + (101) \\ &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C. \end{aligned}$$

$$\text{POS} = M_1 \cdot M_2 \cdot M_4 \cdot M_6 \cdot M_7$$

$$= (001) \cdot (010) \cdot (100) \cdot (110) \cdot (111)$$

$$= (A + B + \bar{C}) (A + \bar{B} + C) (\bar{A} + B + C) (\bar{A} + \bar{B} + C) (\bar{A} + \bar{B} + \bar{C})$$