Unit - V

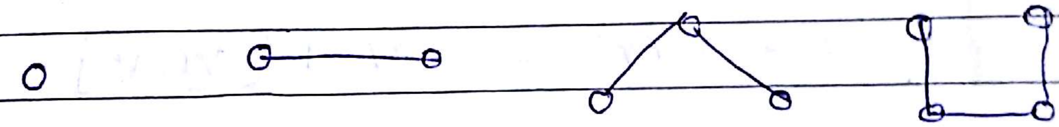
Trees

Q. Define Tree? What are diff. types of tree? Explain

→ Simplest types of Connected graph

Defination

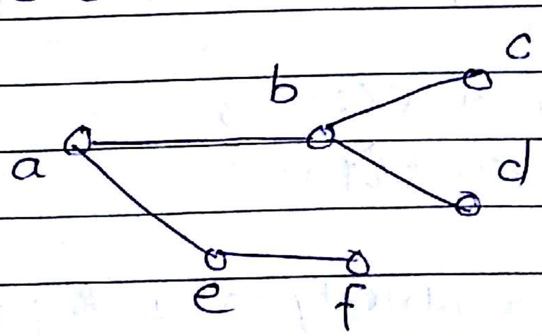
- A 'tree' is a simple connected graph without any circuit.
- i.e. tree is a connected acyclic graph.
- A Collection or set of an acyclic graphs is called a forest



All these graph are trees.

1] A vertex of degree 1 in a tree is called a leaf or a terminal node.

→ A vertex or a degree greater than one is called a branch node or internal node



c, d, f are leaves or terminal nodes
& a, b, e are branch node

Q. Define Eccentricity of a vertex

→ * Eccentricity of a vertex

The G be a connected graph G &
$v \in V(G)$.
The eccentricity of a vertex v is denoted
by $E(v)$ or $e(v)$ & defined as the farthest
distance from v to the vertex
from v in G

i.e $E(v) = \max d\{v_i, v\}$

Q. Define centre of a graph

→ * Centre of a graph

A vertex in a graph G with minimum
eccentricity is called a centre of G &
its eccentricity is called as radius of G
It is denoted by $r(G)$

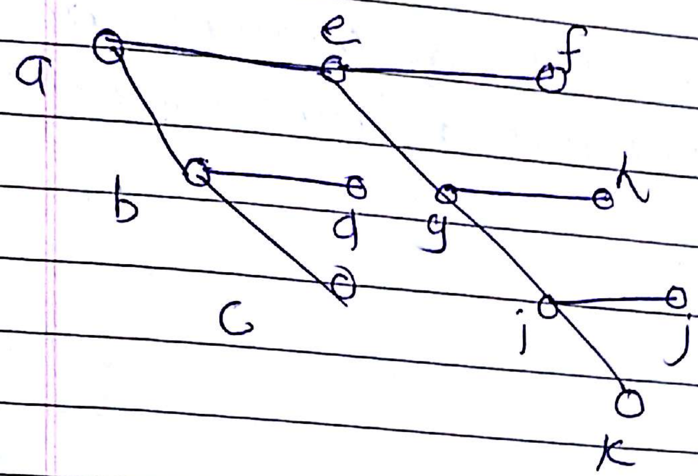
$e(a)=3$, $e(b)=2$, $e(c)=3$
$e(g)=3$, $e(d)=2$, $e(f)=3$

The smallest eccentricity is 2 for vertices
b & d
∴ b & d are the centre of G & $r(G)=2$

Q. What is mean by cut-vertex of a tree?

→ * Cut-vertex of a tree :-

- Vertex V whose removal from connected graph G, disconnects graph is called as cut vertex of G.



In graph G, a, b, e, g, i are cut vertices

Q. Define Rooted Tree

→ * Rooted Tree

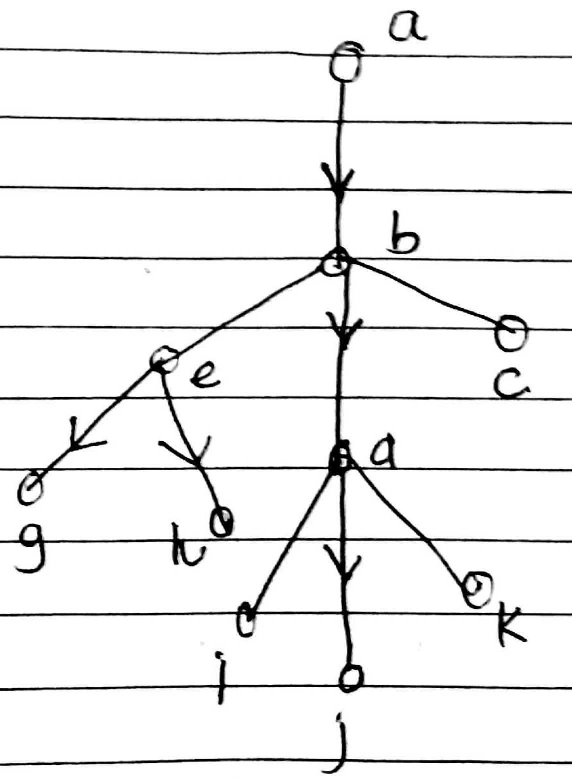
* A connected acyclic, directed graph is called a directed tree.

* A directed tree is called a rooted tree if there is exactly one vertex ~~oth~~ whose incoming degree is zero & the incoming degree of all other vertices are one

The vertex with incoming degree 0 is called the root of the rooted tree.

The vertex whose outgoing degree is zero is called leaf or terminal node.

A vertex whose outgoing & incoming degrees are non zero is called a branch node or an internal node.



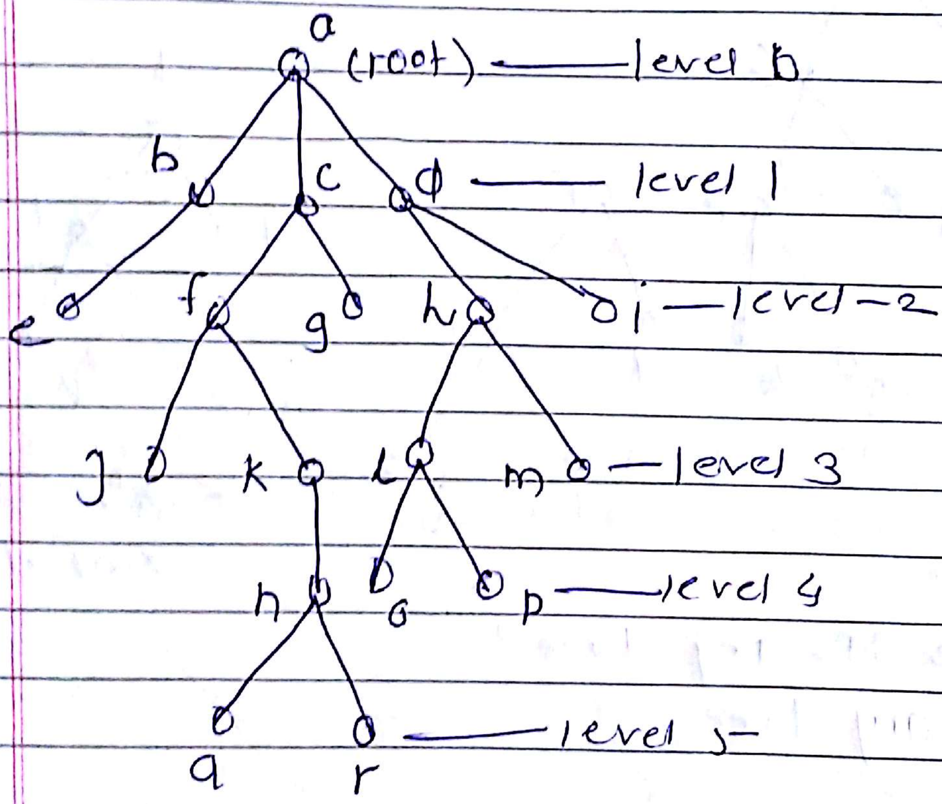
Rooted tree.

Q. Define Level & Height of a tree

→ Level and Height of a Tree

A vertex v in a rooted tree is said to be at level n if there is a path of length n from the root to the vertex v.

The hight of tree is the maximum of the levels of its vertices



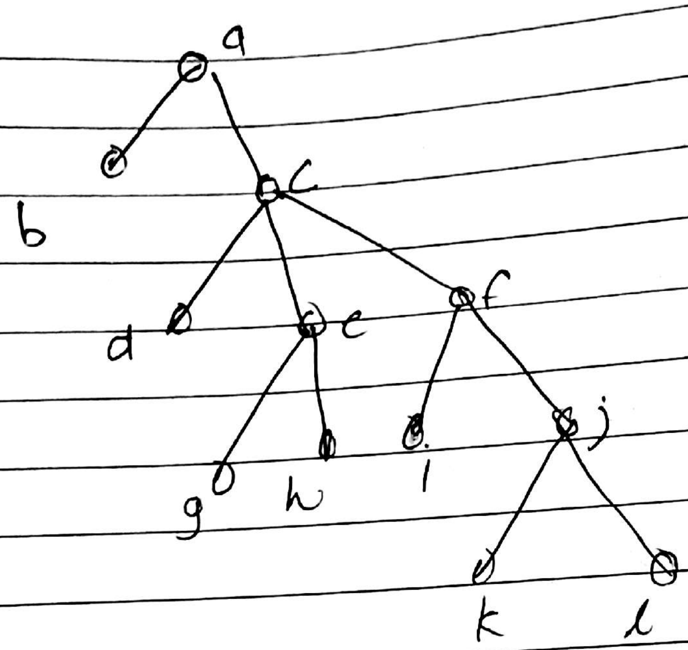
Height = 5

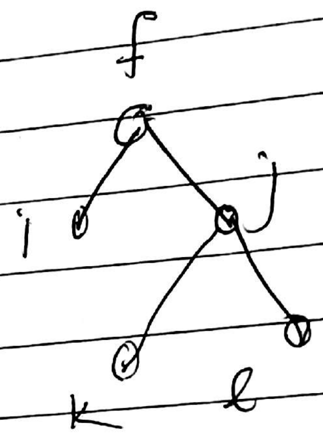
Q. Define Subtree

→ * Subtrees

Let T be a rooted tree & $x \in V(T)$

A vertex x together will all its descendants is called subtree of T rooted at x





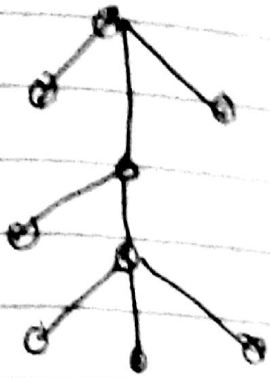
Subtree with root at f.

Q. Define M-ary tree?

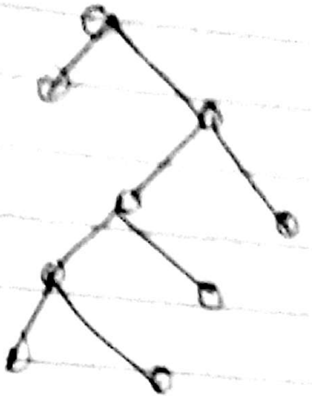
→ * m-ary tree

A rooted tree in which every interior node has at most m sons is called an m-ary tree.

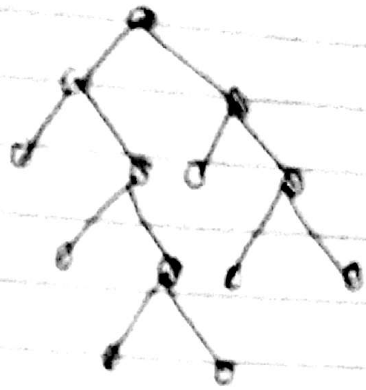
A m-ary tree is said to be regular m-ary tree or full m-ary tree if every branch node has exactly m sons.



3-ary tre



2-ary tree



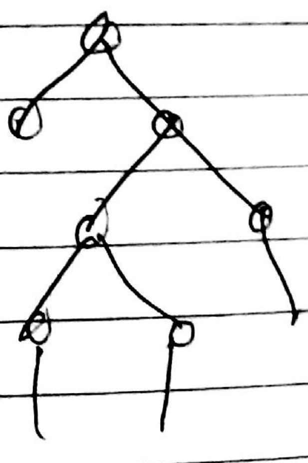
full 2-ary tree

- Q. Define binary tree? Explain in detail?

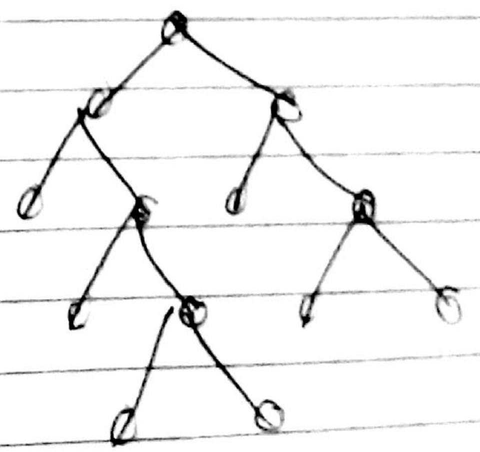
→* binary tree

An-m-ary tree is known as binary tree if every branch node has at most 2-sons.

* A binary tree is called as regular binary tree or full binary tree if every branch node has exactly 2 sons or zero sons



binary tree



full binary tree

Q. Explain binary tree traversal?

→ Binary tree traversal

- Traversing means visiting or processing all the nodes of a tree.

- A binary tree traversal is the visiting of each node of a tree only once according to some sequence.

- Two types of traversing binary trees

1] Depth-first traversal

A] Pre order traversal

B] Post order traversal

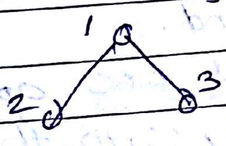
B] Post order traversal

c] In order traversal

1] Pre order traversal

Root → left → right

Root node is traversal first, followed by the left subtree & then right



⇒ 1 → 2 → 3

2] Post order traversal

left → right → root

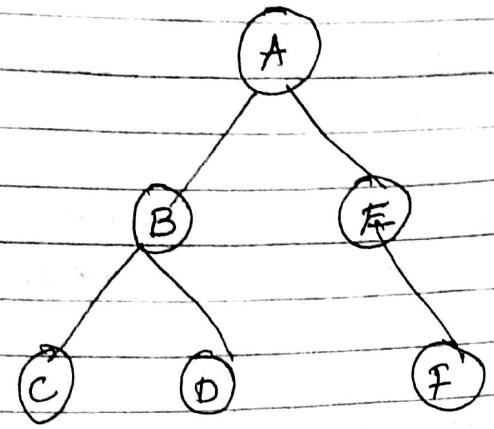
2 → 3 → 1

3] In order traversal

left → root → right

2 → 1 → 3

Consider following binary force



Preorder traversal

— A B C D E F

Post order traversal

— C D B F E A

In order traversal

— C B D A E F

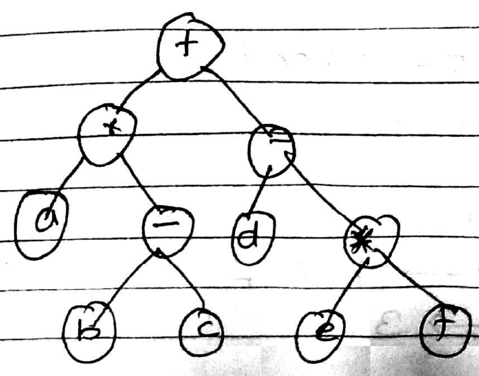
Q. What is binary expression Tree?

* Binary Expression Tree.

i] Each leaf node is an operand.

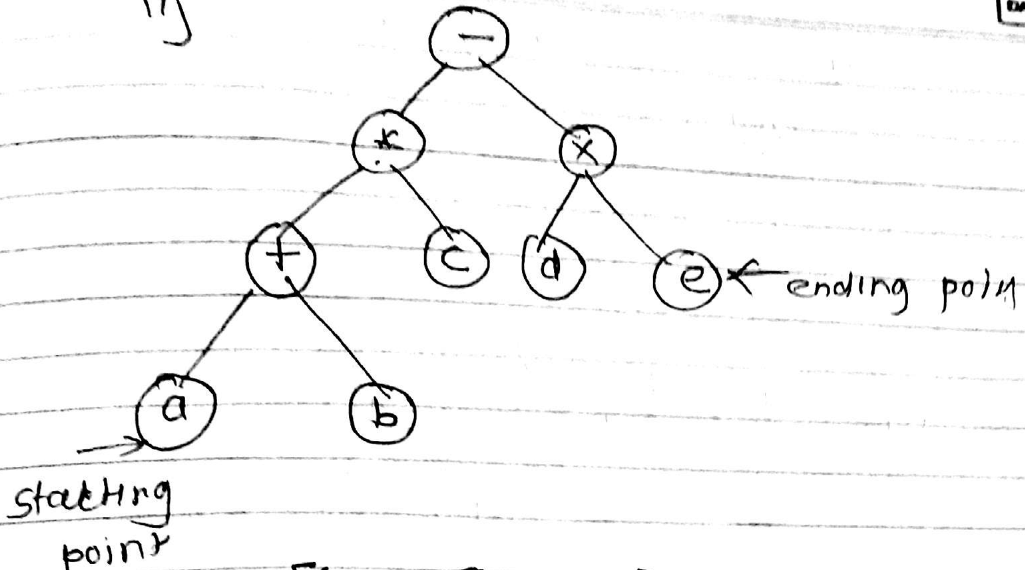
ii] The Eoot and Internal nodes are operators

iii] Subtrees are subexpressions, with the Eoot being an opeatar.



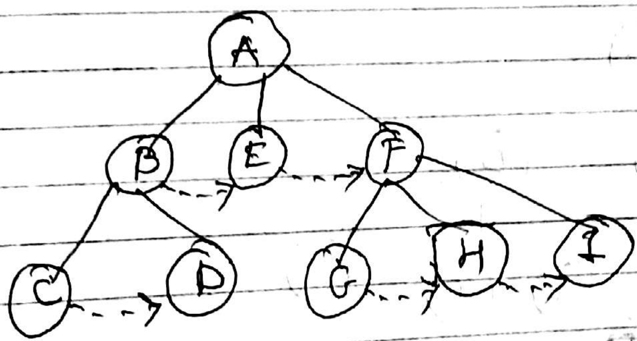
(a*(b-c)]+ [d-(exf)]

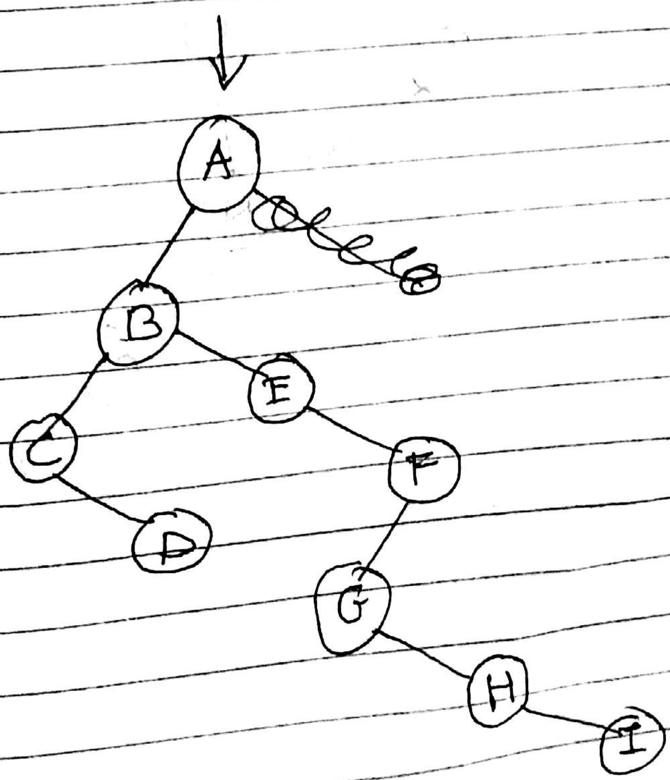
ii)



$[(a+b)*c] - [d \times e]$

* Conversion of General tree to binary tree





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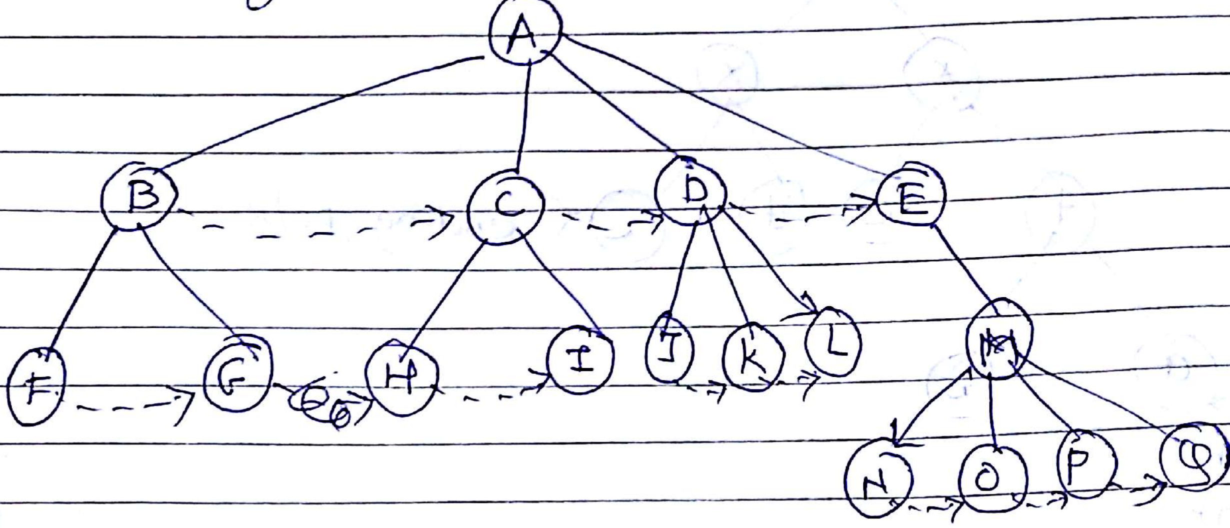
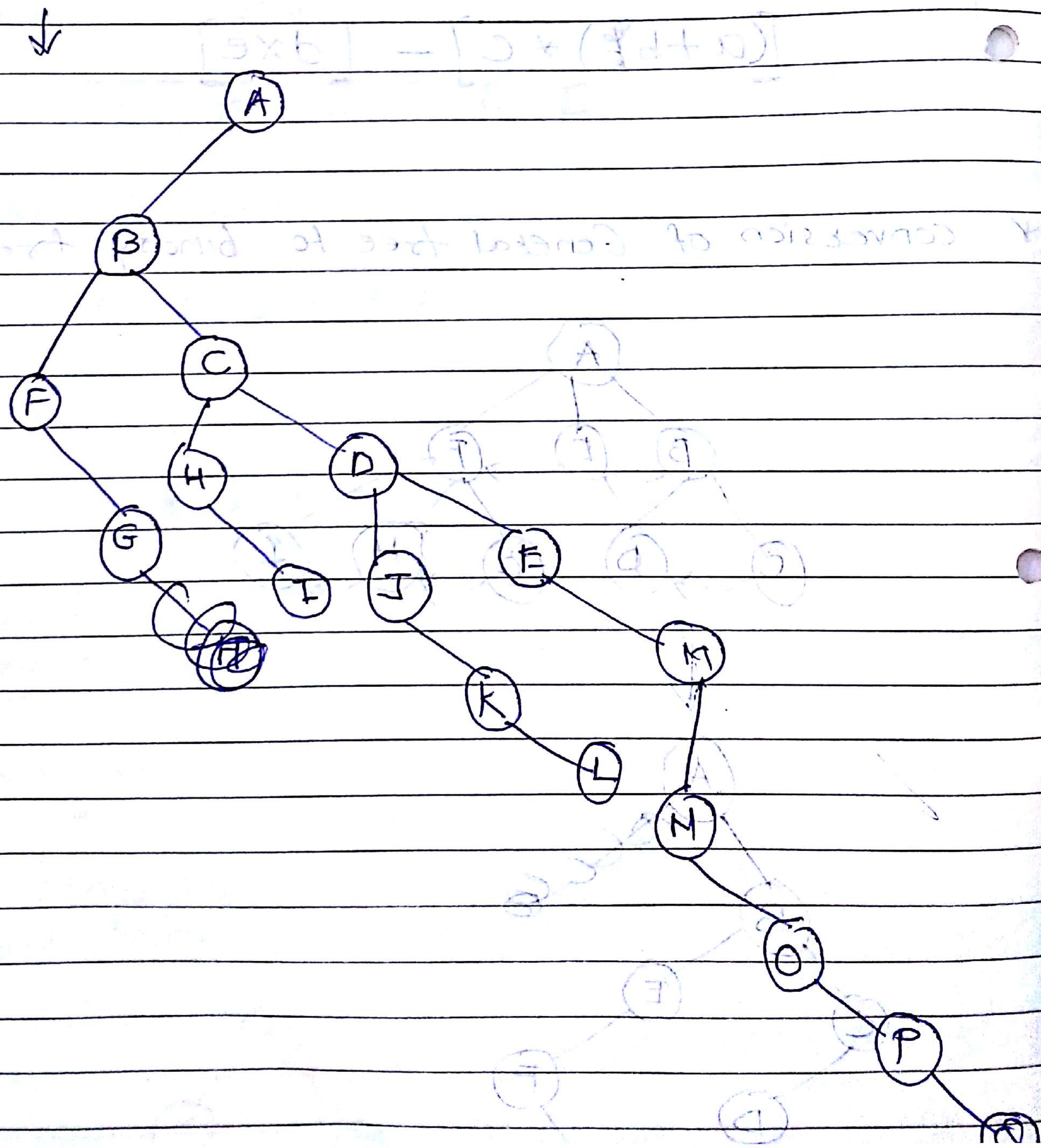
Convert the following tree into binary tree

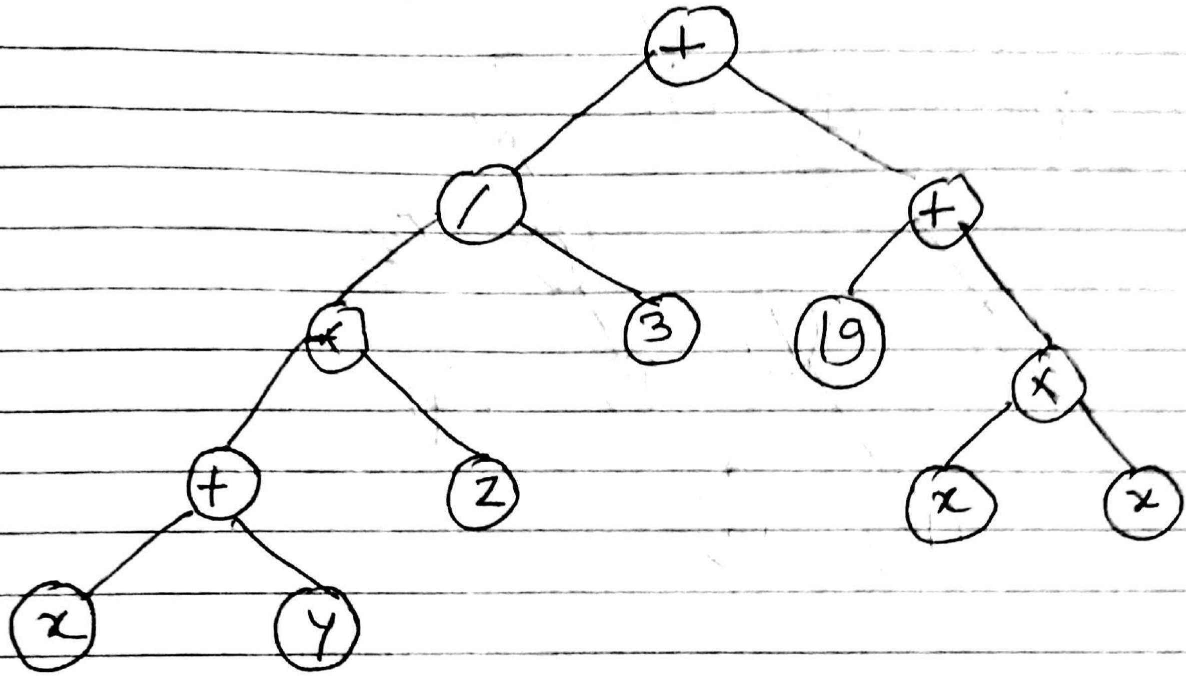




* Construct the labeled tree of the following algebraic expression

$$(((x+y)*z)/3)+(19+(x*x))$$

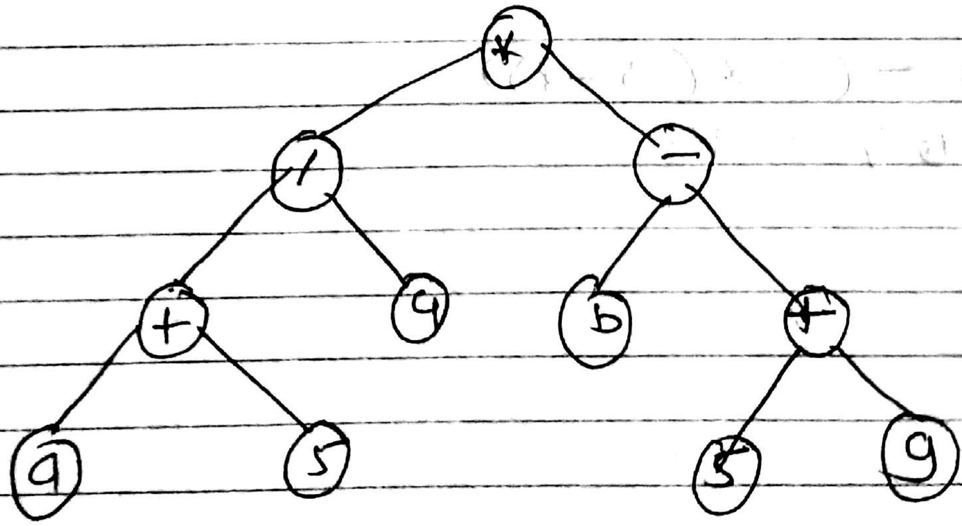
→



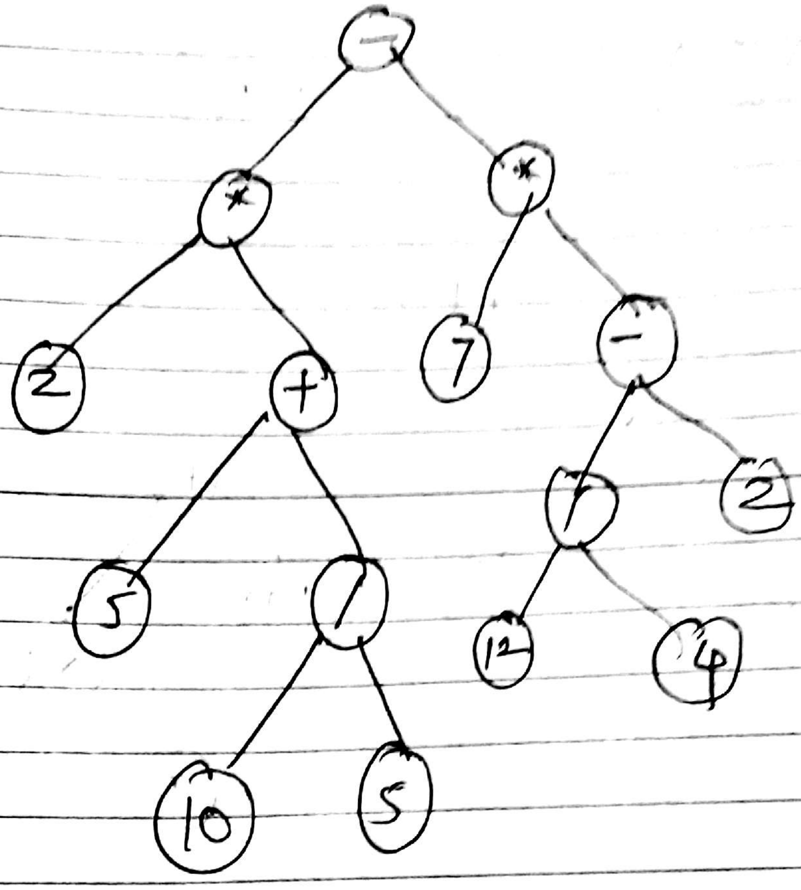
* Represent the expression

$$((a+5)/4)*(b-(5+9))$$

using binary tree



Write & evaluate the expression tree shown below



$(2)*((5)+(10/5))) - (((7)*((12/4)-(2))))$

$2 * (5+2) - (7 * (3-2))$

$14 - 7 = 14$

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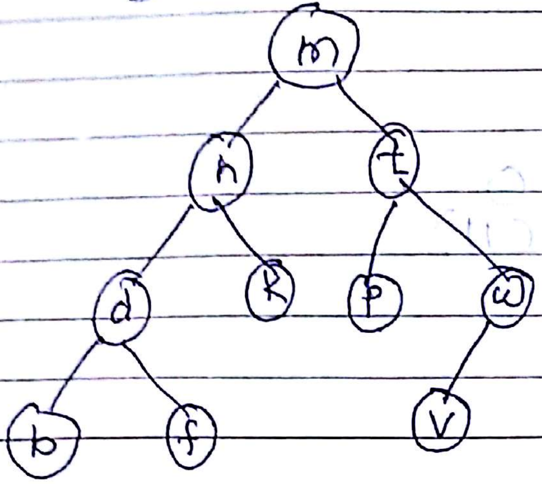
Find the preorder, postorder & inorder traversal of the following tree.



→

Preorder traversal

m h d b f k t p w v

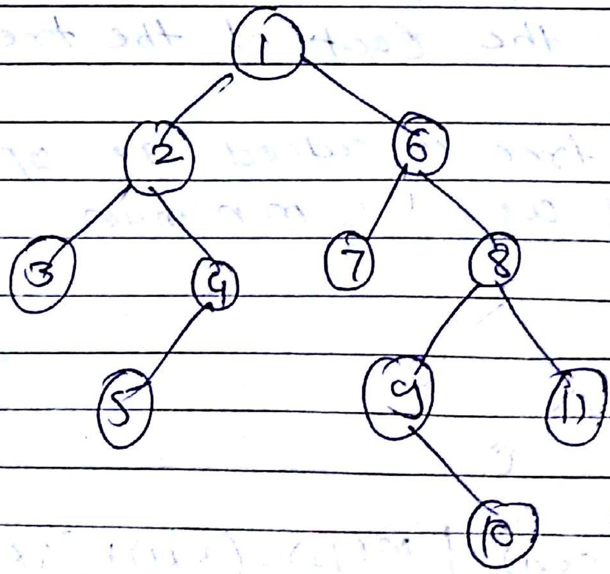
postorder traversal

b f d k h p v w t m

Inorder traversal

b d f h k m p t v w

ii]

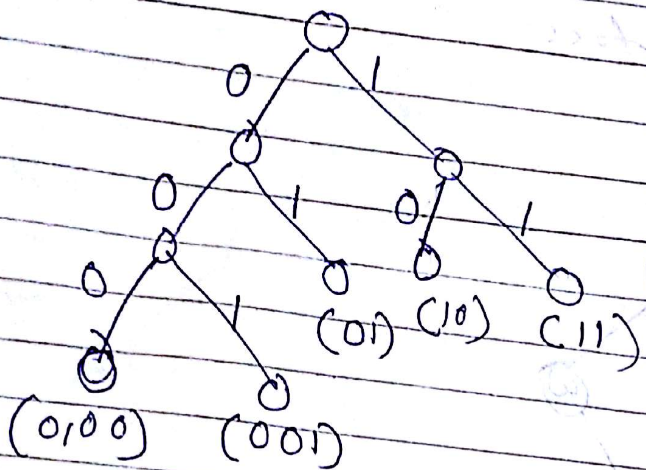


pre - 1 2 3 4 5 6 7 8 9 10 11

post - 3 5 4 2 7 10 9 11 8 6 1

In 3 2 5 4 1 7 6 9 10 8 11

Prefix Code



Q. Define optimal tree.

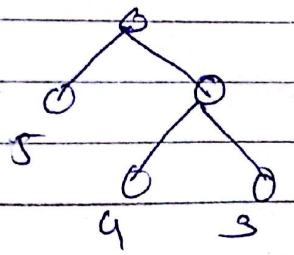
→ Optimal tree

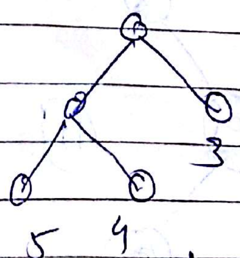
Let T be any full binary tree & the $w_1, w_2 \ldots w_t$ be the weights of the terminal vertices (leaves), Then the weight w of the binary tree is given by

$$W(T) = \sum_{i=1}^{t} w_i l_i$$

where l_i is the length of the path of the leaf i from the root of the tree.

The full binary tree is called an optimal tree if its weight is minimum.



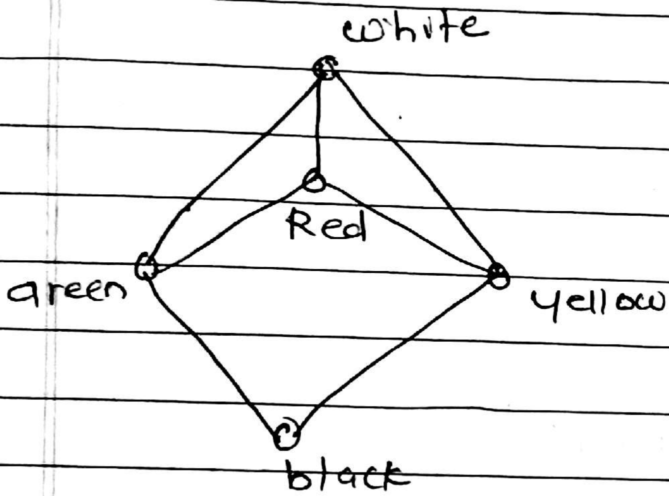


$W(T_1) = (3\times2) + (4\times2) + (5\times1)$	$W(T_2) = (3\times1) + (4\times2) + (5\times2)$
$= 6 + 8 + 5$	$= 3 + 8 + 10$
$= 19$	$= 21$

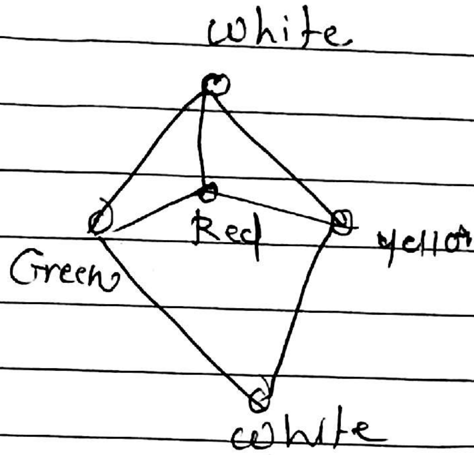
Q. Explain graph Coloring.

* Graph Coloring & Chromatic number

— A proper colouring of graph G is an assignment of colours to its vertices so that no two adjacent vertices have the same color.



(1)



(2)

As shown in fig. the number of colors used in first graph is 5 & in (2nd graph is four.

Our main interest is in proper coloring of graph which uses minimum number of colors

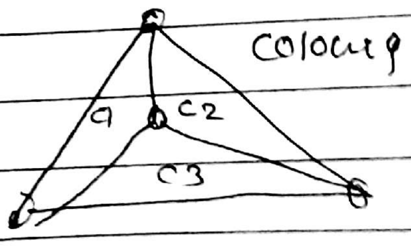
This number is called chromatic number

Chromatic number : minimum number of colors needed to produce a proper colouring of a graph G.

The chromatic number of graph G is denoted by $\chi(G)$. If graph G has chromatic number i.e. the $\chi(G)=k$ and graph is called k-chromatic.

1. null graph is 1 chromatic.
2. A complete graph K_n with n vertices is n chromatic because all its vertices are adjacent.

Q. Explain graph coloring?
A coloring of plannar graph



Q Define web graph?
* Web graph

A vertex A which represents the web page is connected to vertex B by direct edge from A to B if there is a link from web page A to web page B.
i.e if there is a direct edge joining vertex A to vertex B, there exists a hyperlink on webpage A referring to page B.

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Hence web graph describes the direct link betⁿ the pages of www.

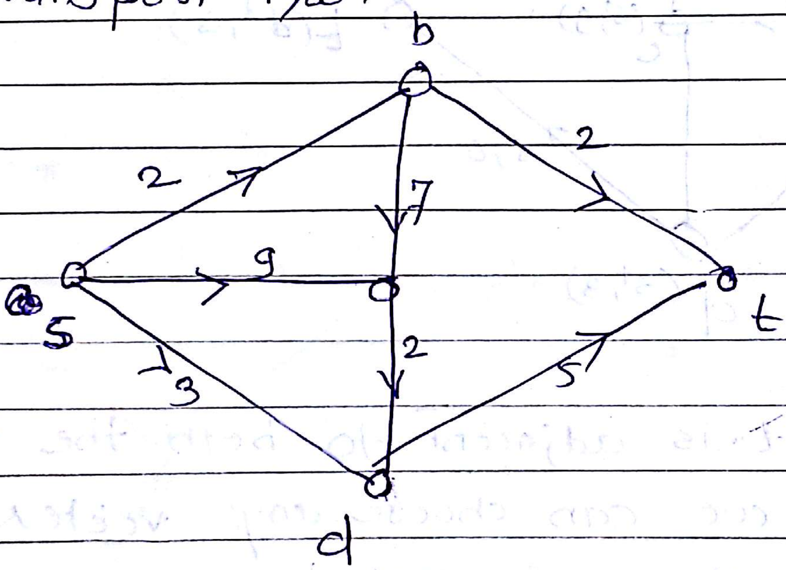
* Google map

- Google map is a web mapping services which provides the info. about geographical regions & sites, view of streets & route planning for travelling by foot, car or public transport

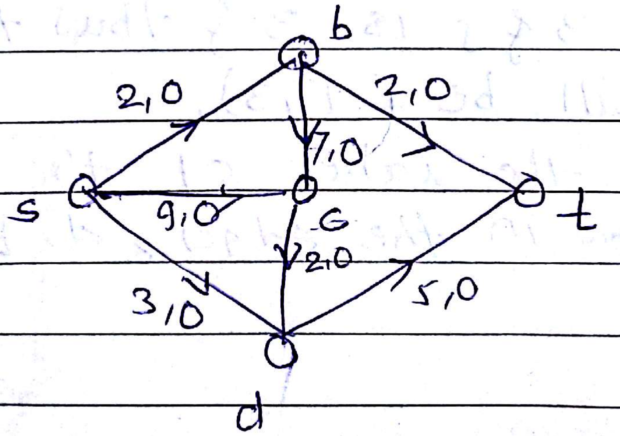
A problem space is a mathematical abstraction in a form of a tree.

- The root represents current state
- nodes reprent states of the game
- * edges represents moves
- * leaves represent final states

* Determine the maximum flow in the following transport n/w.



step 1: Assign the flow zero to each edge & the label $(-, \infty)$ to the source s.



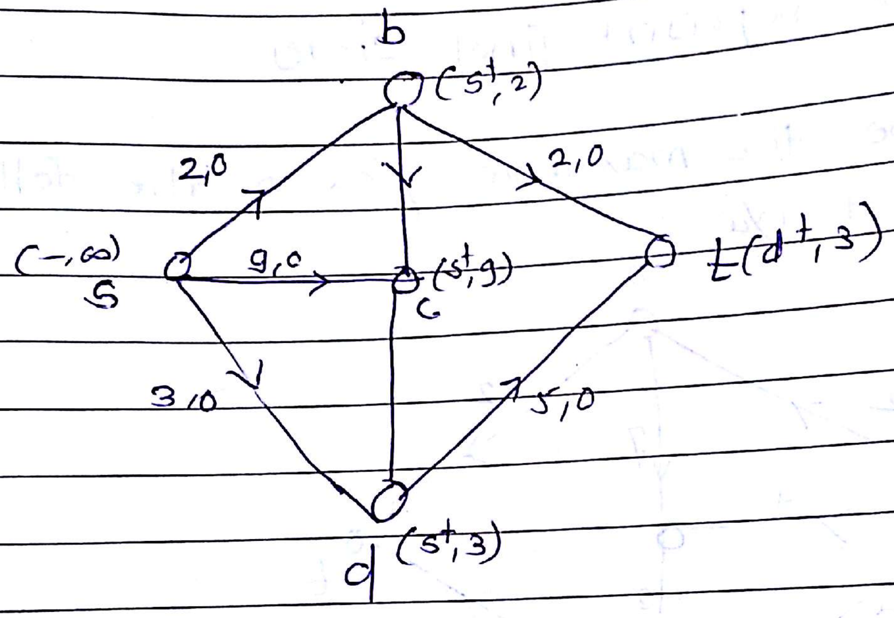
Step2:. The vertices b, c, d are adjacent to the source s.

∴ we label the vertices b, c, d

for the vertex b is $(s^+, 2)$

c is $(s^+, 9)$

d is $(s^+, 3)$



Now the sink t is adjacent to both the vertices b & d, so we can choose any vertex b or d for labelling of sink t.

we choose d

for the ~~ta~~ vertex d, the label is $(s^+, 3)$

minimum of 3 & 5 is 3 & thus the label of sink t will be $(d^+, 3)$,

According to the label of the sink, adjust the flow in the edges d, t & (s, d)

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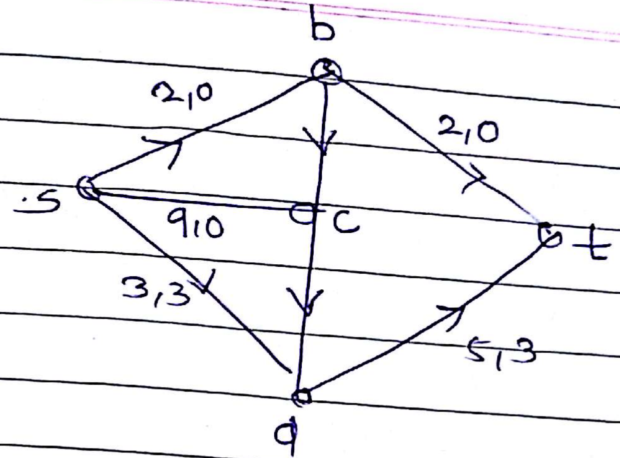
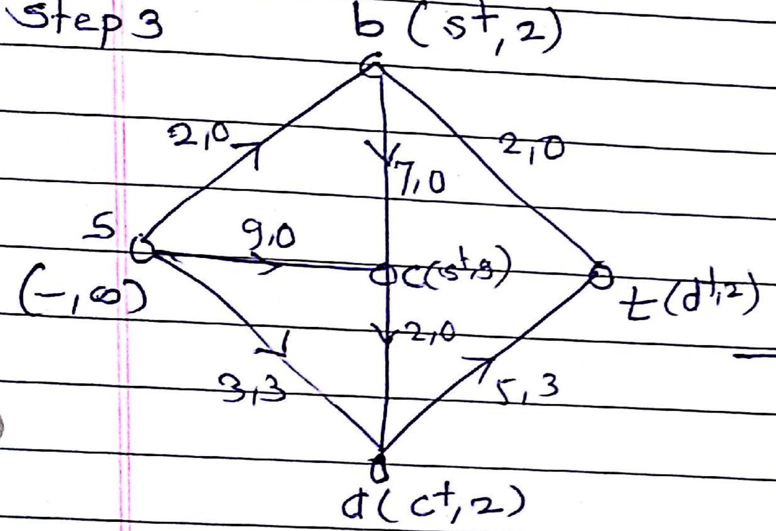
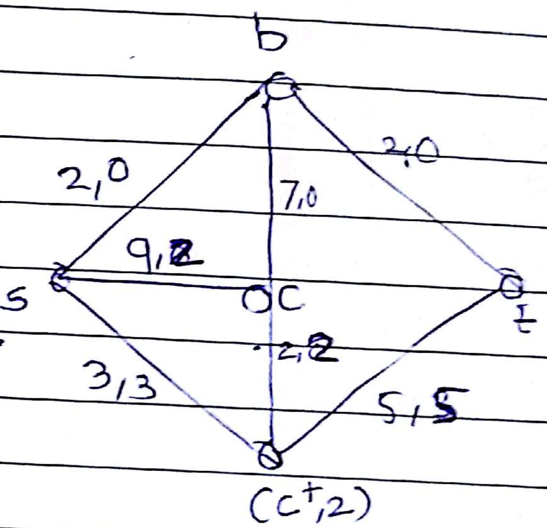


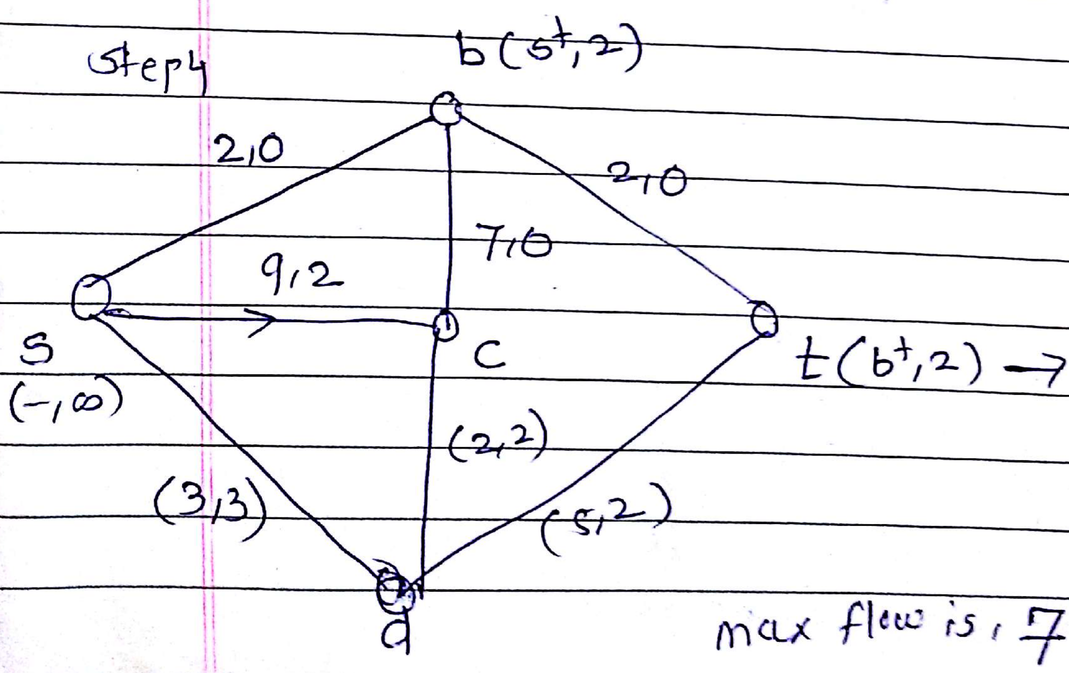
Repeat the step2. In each pass the new value of the flow is obtained.

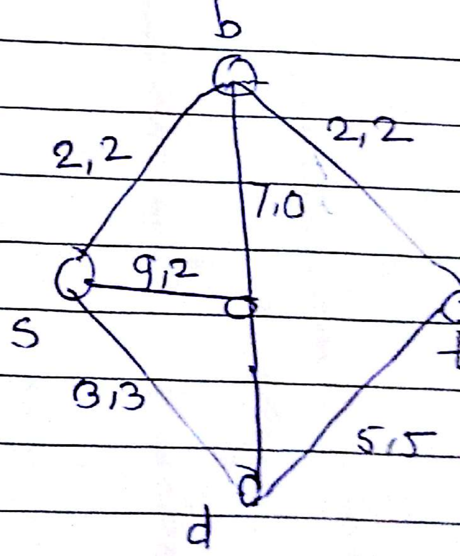
step 3





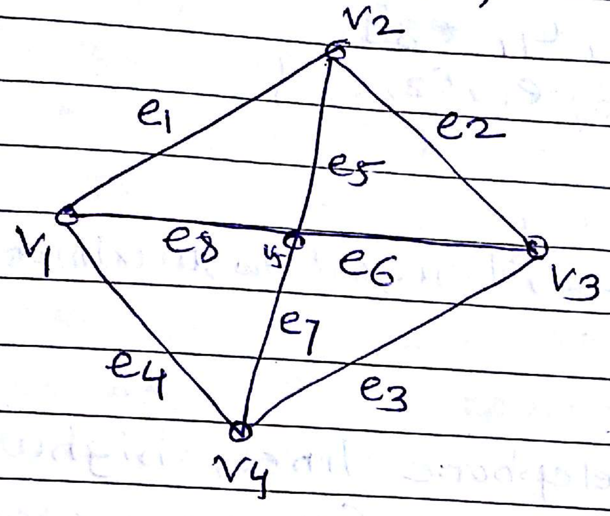
step4

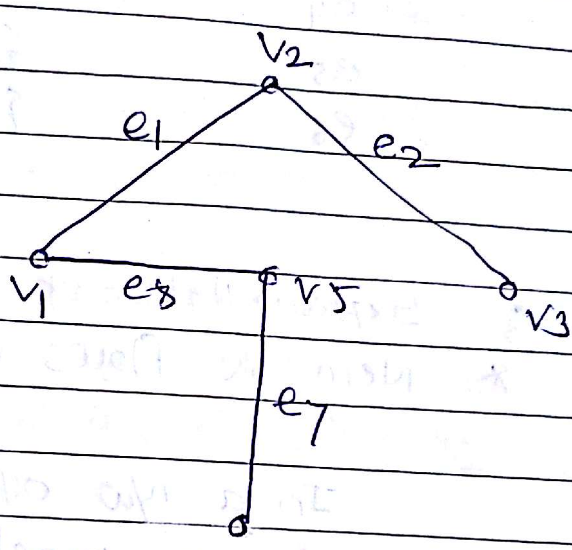




max flow is, 7

* Find the fundamental cutsets & fundamental circuits for the following graph w.r.t. given spanning tree.





→ Here the spanning tree has 4 branches e_1, e_2, e_7, e_8.

∴ there are 4 fundamental cutsets corresponding to each branch of T which are given below.

Branch	Corresponding Fundamental cutset
e_1	$\{e_1, e_5, e_6, e_3\}$
e_2	$\{e_2, e_3, e_6\}$
e_7	$\{e_7, e_3, e_4\}$
e_8	$\{e_8, e_3, e_4, e_5, e_6\}$

The chords of T are, e_3, e_4, e_5, e_6

∴ there are 4 fundamental circuit corresponding to each chord of T which are given below

Chords . corresponding fundamental circuits.

e_3 $\{e_1, e_2, e_7, e_8, e_3\}$

e_4 $\{e_4, e_7, e_8\}$

e_5 $\{e_5, e_1, e_8\}$

e_6 $\{e_6, e_1, e_2, e_8\}$

Q. Explain Network flows, transport n/w, maximum flow.

* Network flows :→

In a n/w of telephone lines, highways, railroads, pipelines of oils (gas or water) it is essential to know the maximum rate of flow that is possible from one station to another in the network.

This type of n/w is represented by a weighted directed graph in which the vertices are stations & edges are lines thr which the given commodity (i.e. water, gas, oil etc) flows

Transport N/w.

A weighted directed connected graph is said to be a transport n/w if the following conditions are satiesfied.

i] It is without loop

ii] There are one & only one vertex in a graph that has no incoming edge. It is Known as source.

iii] There are one & only one vertex in the graph that has no outgoing edge. It is known as sink.

iv] The weight of each edge is a non -ve real number. It is known as capacity of that edge. The capacity of the (i, j) edge is denoted by ω(i, j)

★ Maximum Flow:-

The maximum flow in a transport n/w is a flow that achieves the largest possible value.

Cut :- A cut in a transport n/w is a cutset of the undirected graph obtained from the transport n/w by ignoring the direction of the edges that separate the source from sink.

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Q. What is game tree? Explain in detail

→ * Game Tree.

* A directed graph whose nodes are positions in the game & edges are moves, is called a game tree.

The complete game tree is the game tree starting at the initial position & containing all possible moves from each position. The complete tree is the same tree as that obtained from extensive form game representation.

we classify game into three type

1. single player
   a] Travelling saleman problem
   b] Sliding puzzle
   c] Hamiltonian path
   d] Rubik's cude

2. Two player game
   a] chess
   b] Bandminton
   c] Tennis
   d] checkers

3. constraint satisfaction problems
   i] sudoku
   ii] Eight queen problem
   iii] Mathematical puzzy
   iv] Four queen problem

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Q. Define fundamental circuits & cutsets

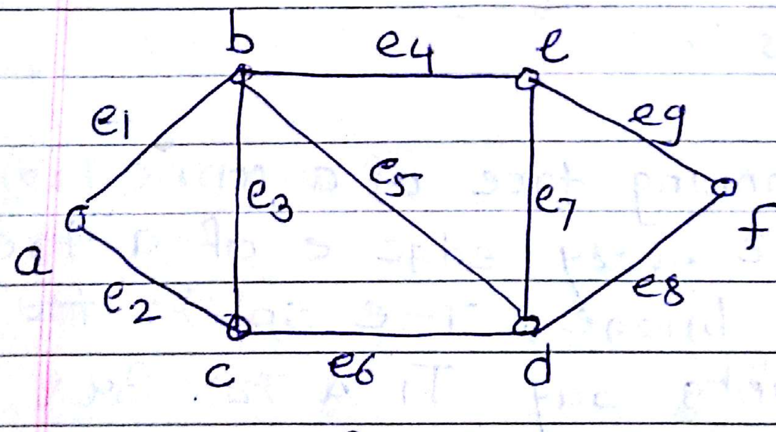
→ Fundamental Circuits and cutsets.

Let G be a connected graph & T be a spanning tree of G. An edge of a tree is called branch. An edge of G which is not in T is called chord of T. T+e contains a unique circuit called fundamental circuit of G with respect to T.

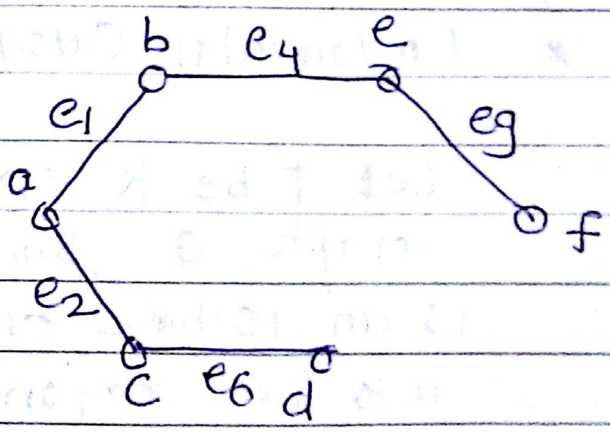
A fundamental circuit of a connected graph G is always with respect to a spanning tree of G

Therefore the different spanning trees will have different fundamental circuits.

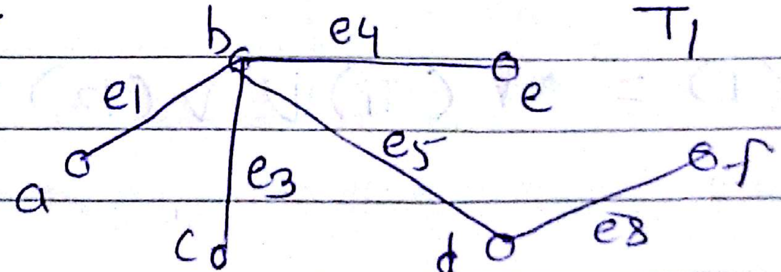
Consider the following graph G with two spanning trees T_1 and T_2



G



T_1



In graph G $V(G) = \{a, b, c, d, e, f\}$
edge set $E(G) = \{e_1, e_2, e_3 \dots e_9\}$

A] for spanning Tree T_1

branches of T_1 are e_1, e_2, e_4, e_6, e_9
Chords T_1 - e_3, e_5, e_4, e_8

Consider the following table of chords & corresponding fundamental circuits.

Chords	corresponding fundamental circuits
e_3	$\{e_1, e_2, e_3\}$
e_5	$\{e_1, e_2, e_6, e_5\}$
e_7	$\{e_1, e_2, e_4, e_6, e_7\}$
e_8	$\{e_1, e_2, e_4, e_6, e_9, e_8\}$

Q. Define fundamental cutsets?
* Fundamental Cutsets :

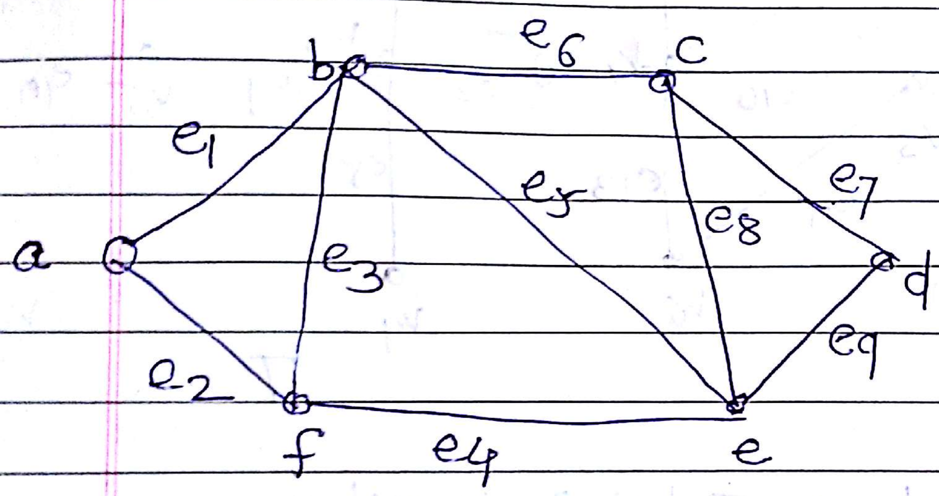
Let T be k spanning tree of a connected graph G. Since every edge e of a tree is an isthmus or bridge, T-e splits ~~to~~ into two components say T_1 & T_2. But

$V(T) = V(T_1) \cup V(T_2) = V(G)$

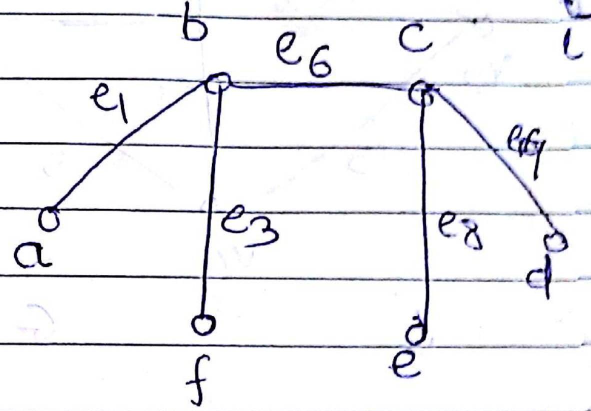
Scanned by CamScanner

The set E of edges of G which join a vertex in $V(T_1)$ to a vertex in $V(T_2)$ is a cutset of G

A cutset of G obtained in this manner is called a fundamental cutset of G with respect to T.



G



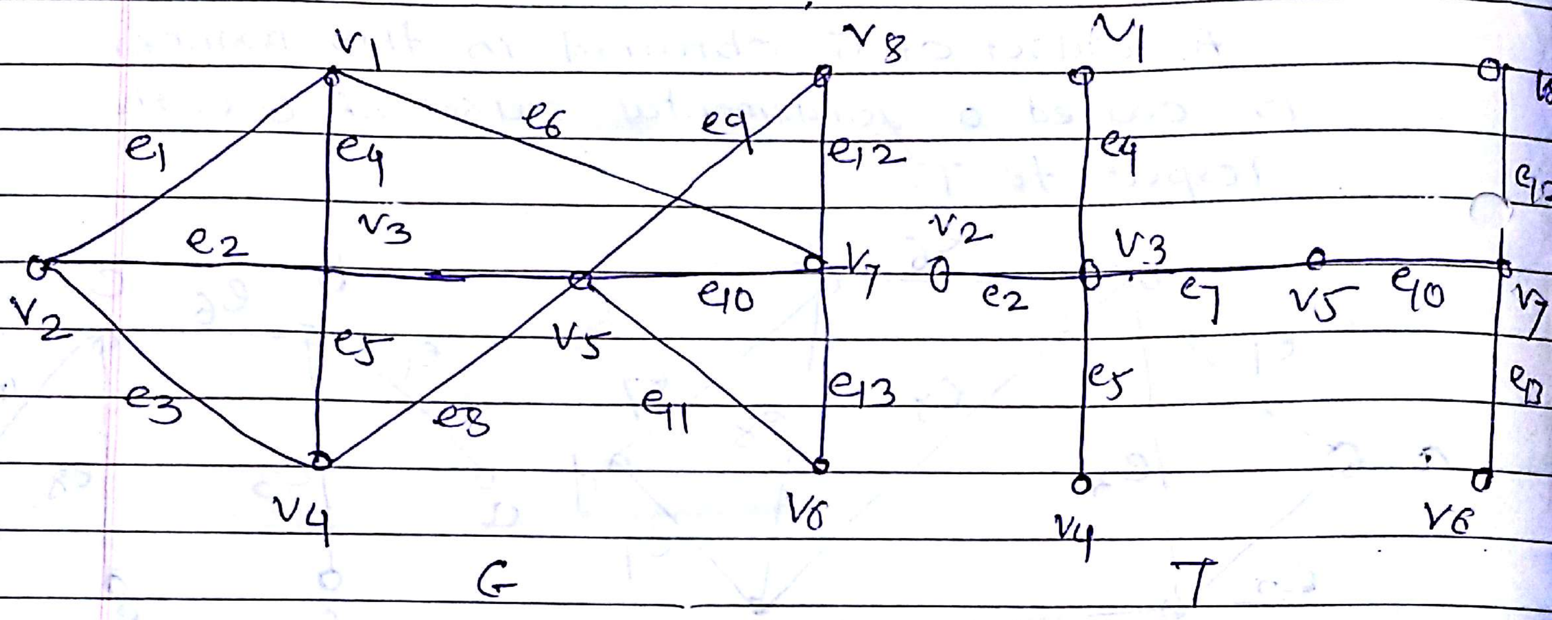
T

For spanning tree T_1

Branches of T_1 are e_1, e_3, e_6, e_7, e_8

Branches T_1	Corresponding fundamental cutset
e_1	$\{e_1, e_2\}$
e_3	$\{e_3, e_2, e_4\}$
e_6	$\{e_6, 65, e_4\}$
e_7	$\{e_7, e_9\}$
e_8	$\{e_8, \cancel{e_2}, e_4, e_9, e_5\}$

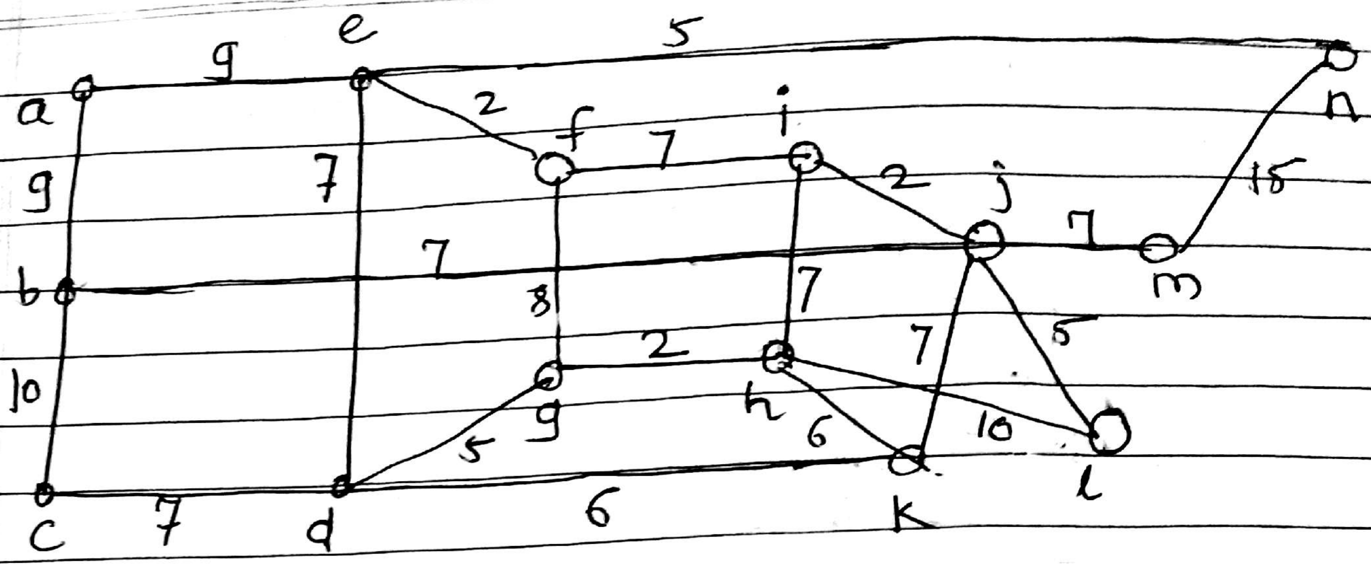
* Find the fundamental system of cutset for the graph G shown below w.r.t. the spanning tree T.



The spanning tree T has 7 branches
$\{e_2, e_4, e_5, e_7, e_{10}, e_{12}, e_{13}\}$

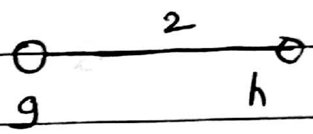
∴ ~~the~~ the seven fundamental cutsets of G w.r.t. T which are given below.

Branch	Fundamental cutset
e_2	$\{e_2, e_1, e_3\}$
e_4	$\{e_4, e_1, e_6\}$
e_5	$\{e_5, e_3, e_8\}$
e_7	$\{e_7, e_6, e_8\}$
e_{10}	$\{e_{10}, e_6, e_9, e_{11}\}$
e_{12}	$\{e_{12}, e_9\}$
e_{13}	$\{e_{13}, e_{11}\}$

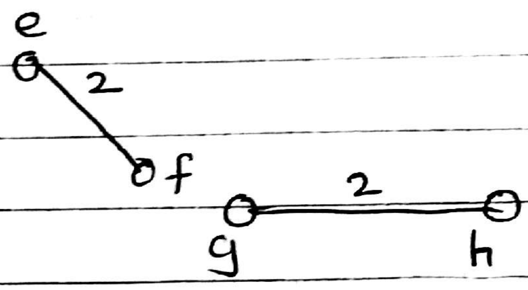


* kruskal's

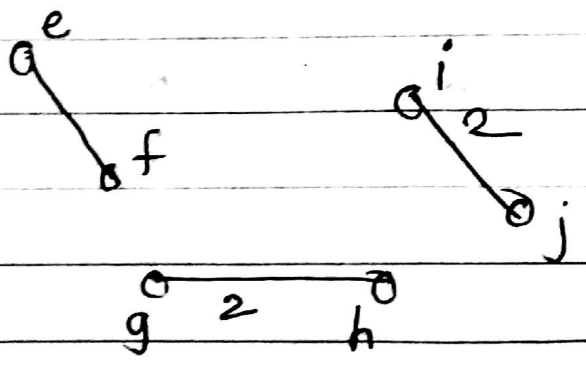
step1



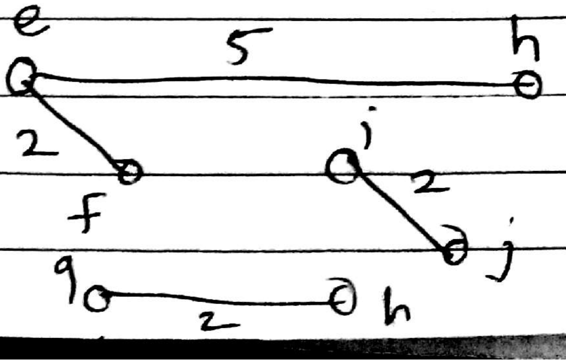
step 2



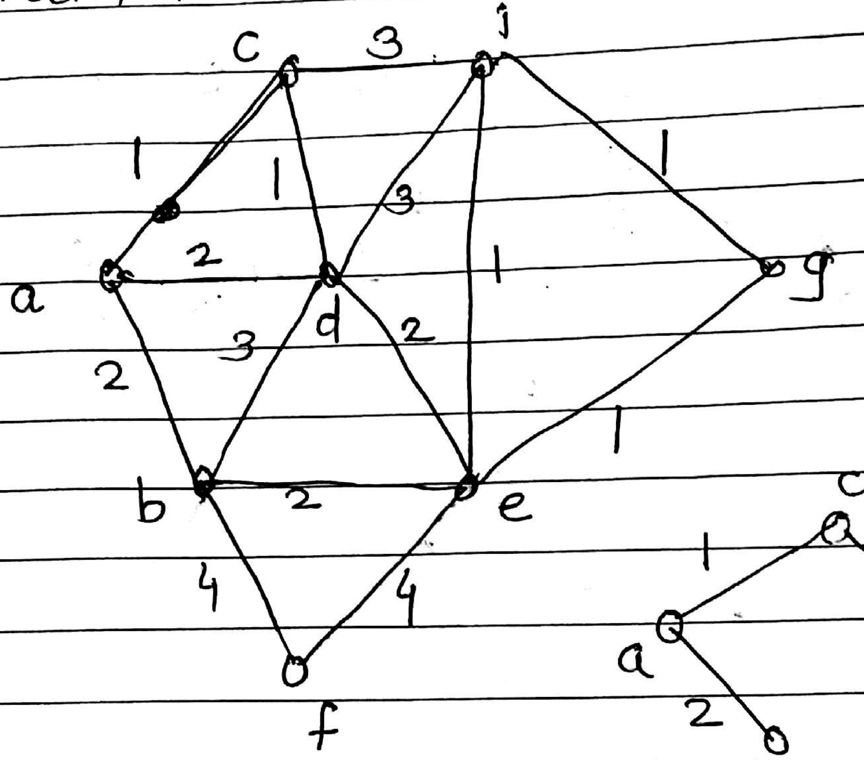
step3

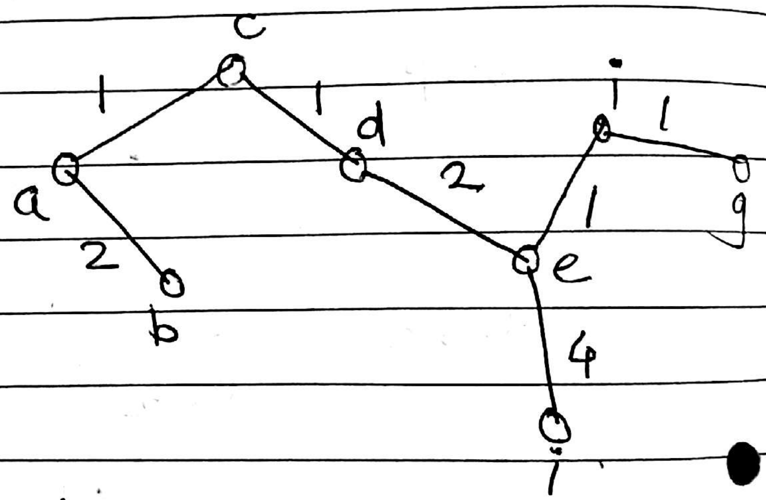


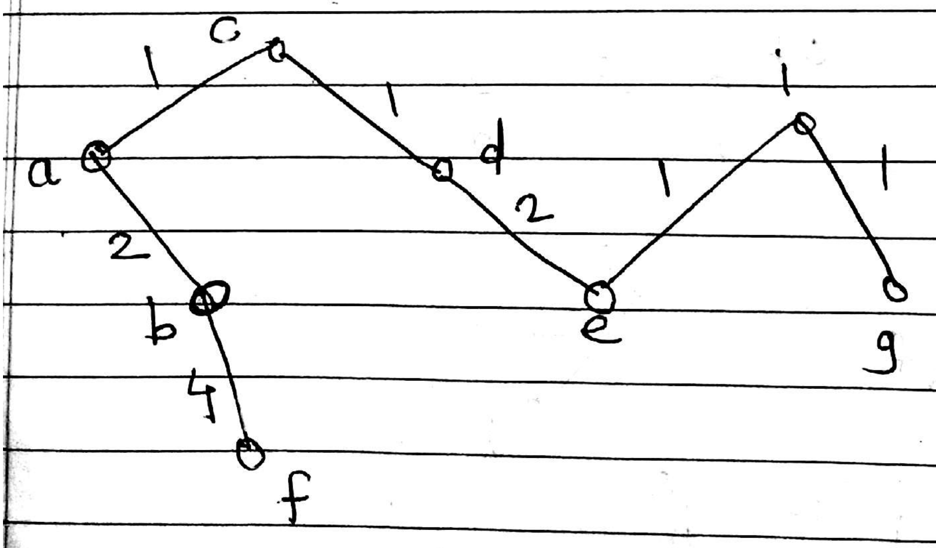
step4



Use prims algo to construct a minimal spanning tree starting from vertex a & vertex b verify that both trees have the same weight

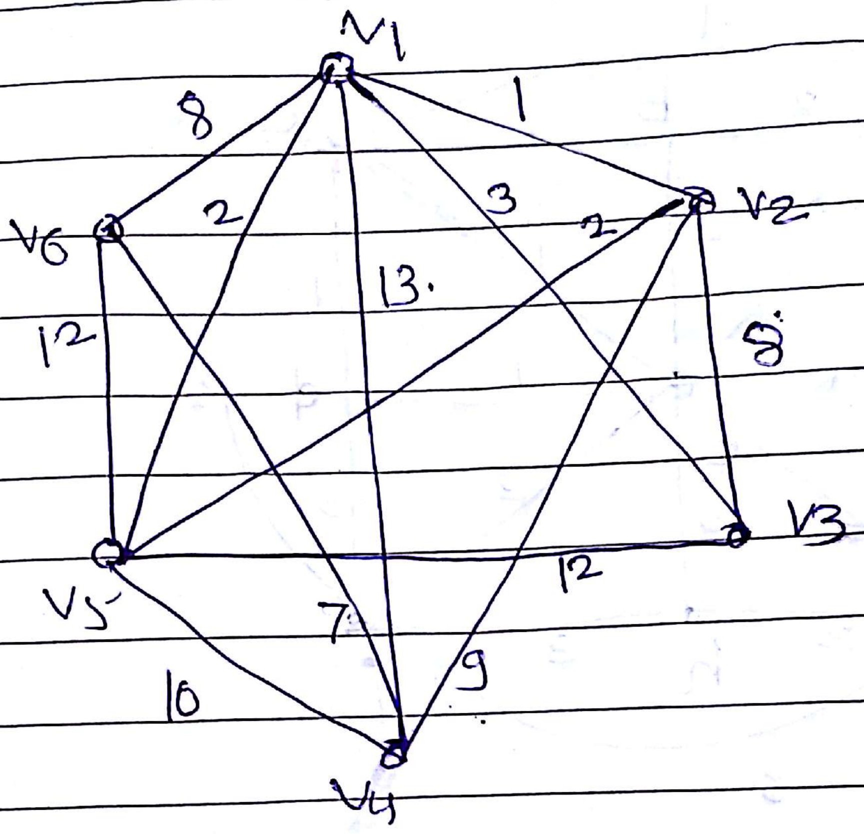


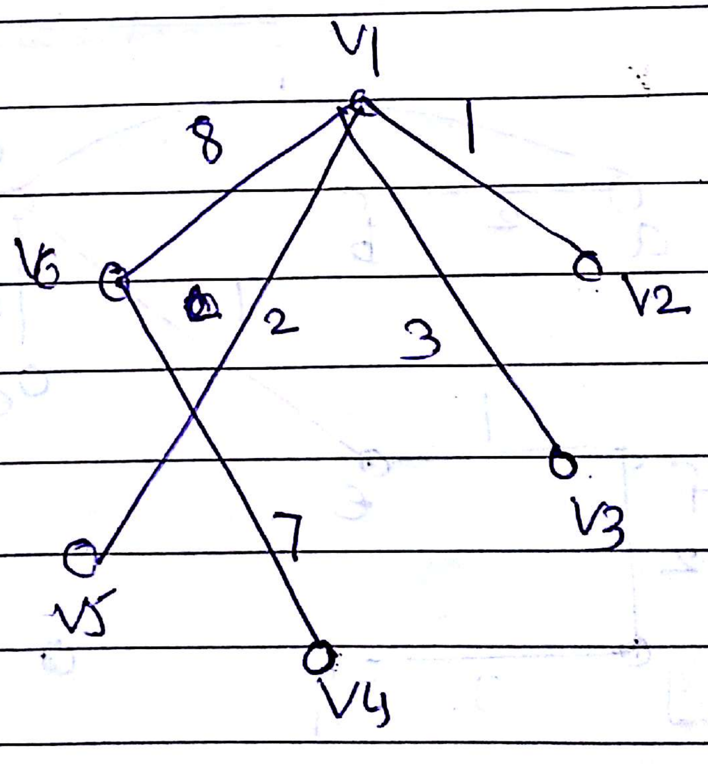




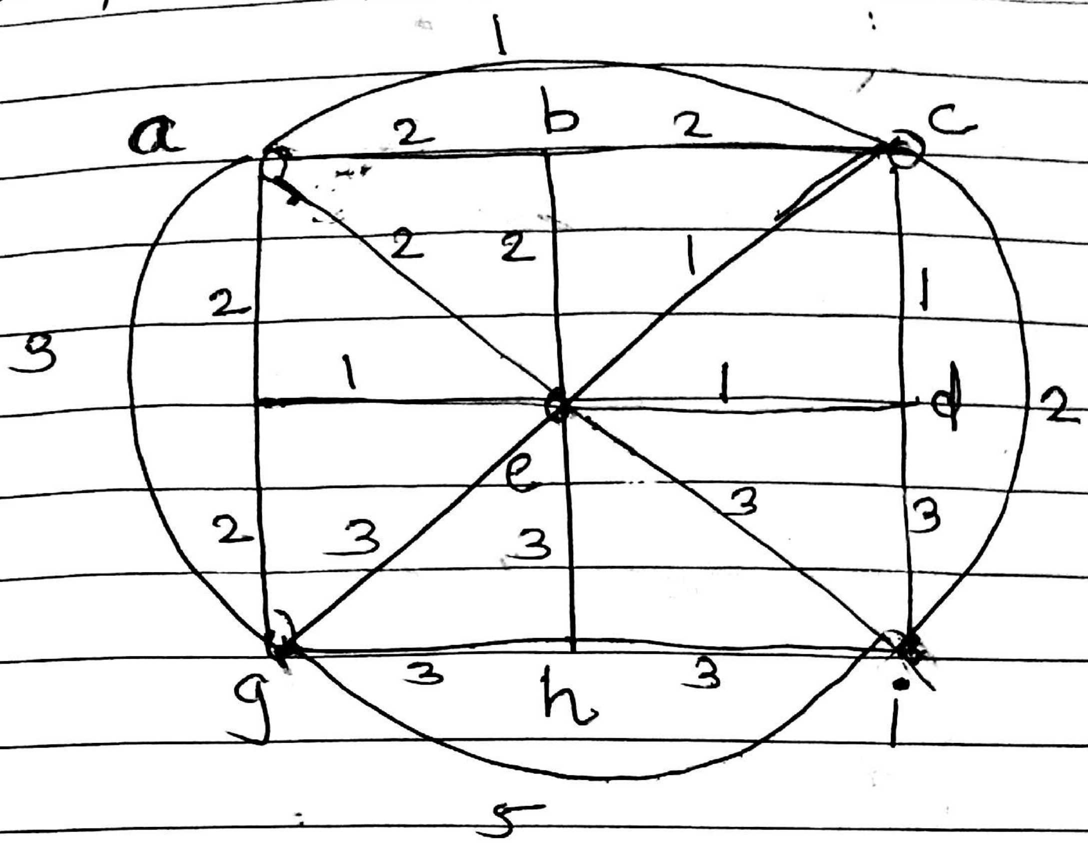
Total wt - 12

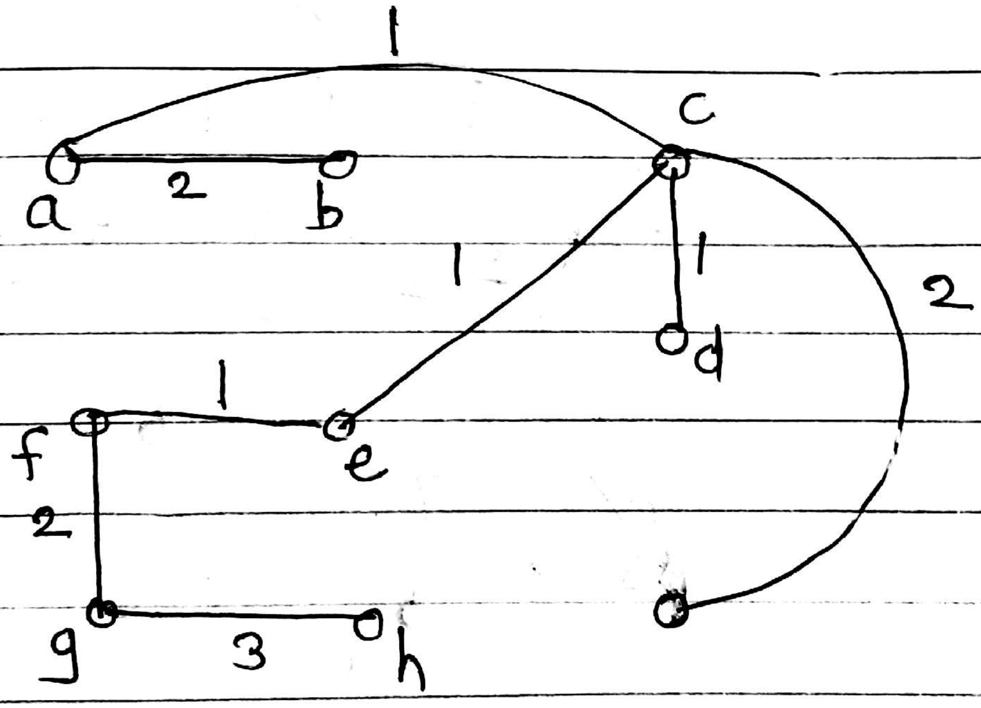
use kruskals to find minimum spanning tree



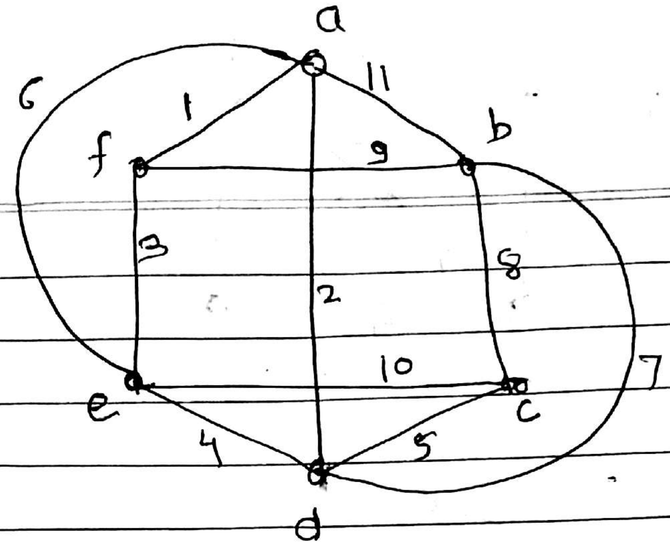


Use prim's algo.



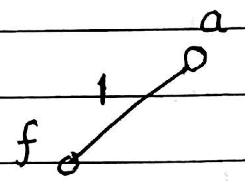


weight is 13



1]

2]



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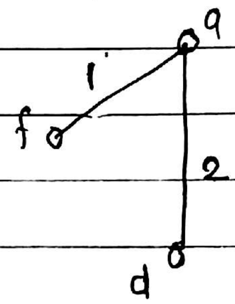

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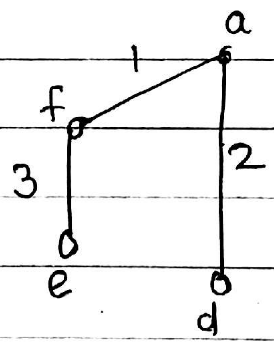

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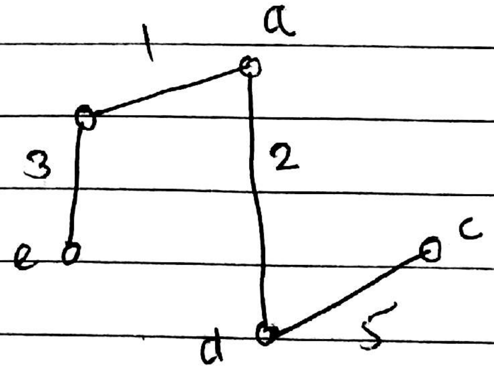

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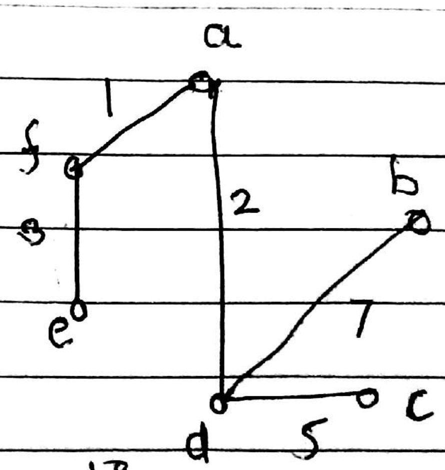
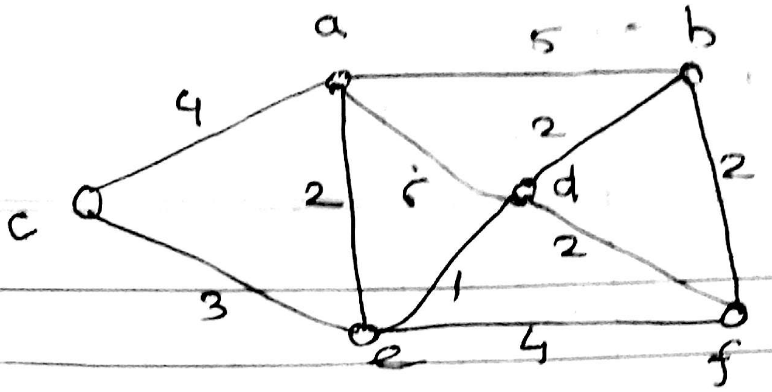
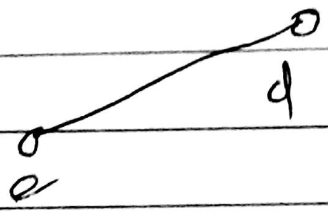


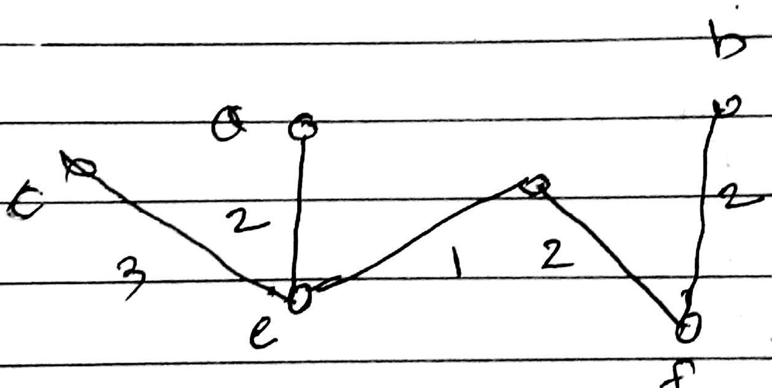
~~1+2+3+1~~ $3+1+2+7+5 = 18$

1

2]







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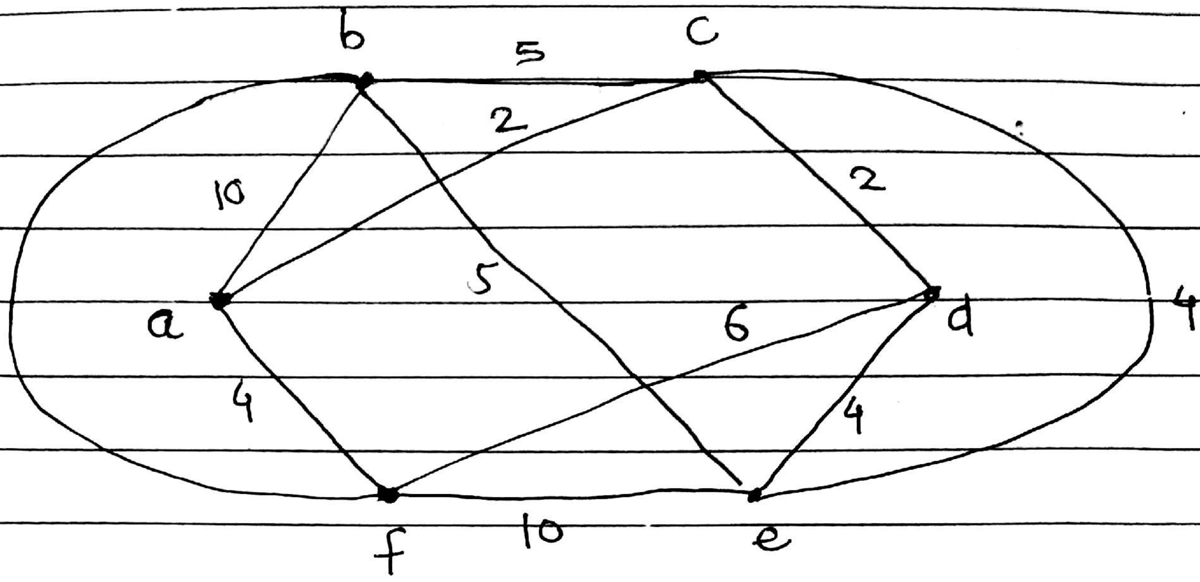
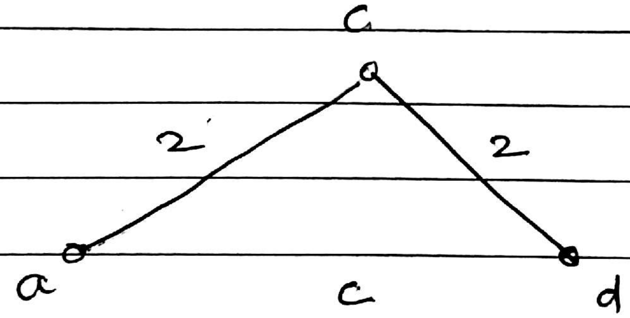
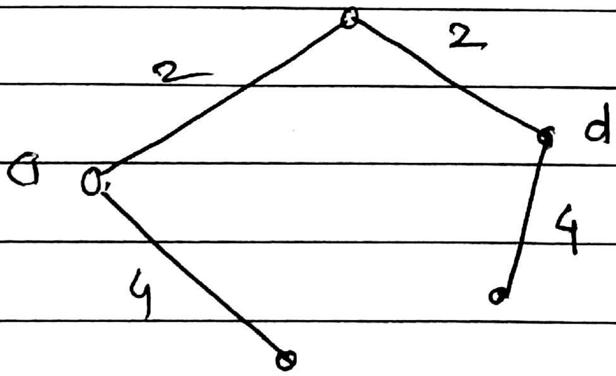
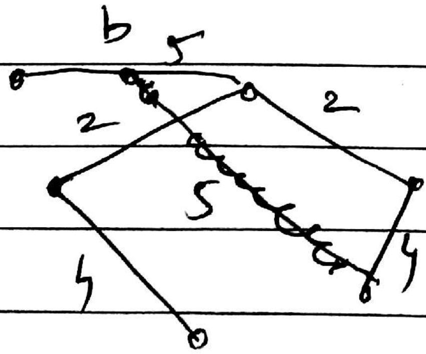








weight 11 17

2] suppose data items A, B, C, D, E, F, G occur in the following frequencies.

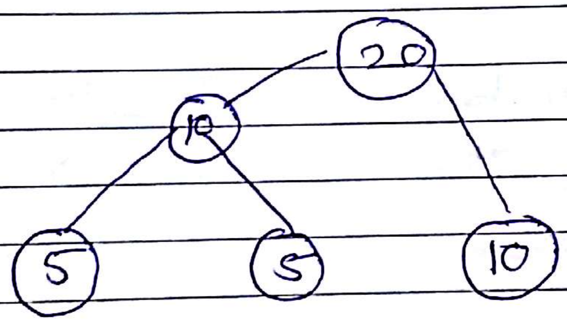
Data items	A	B	C	D	E	F	G
weight	10	30	5	15	20	15	05

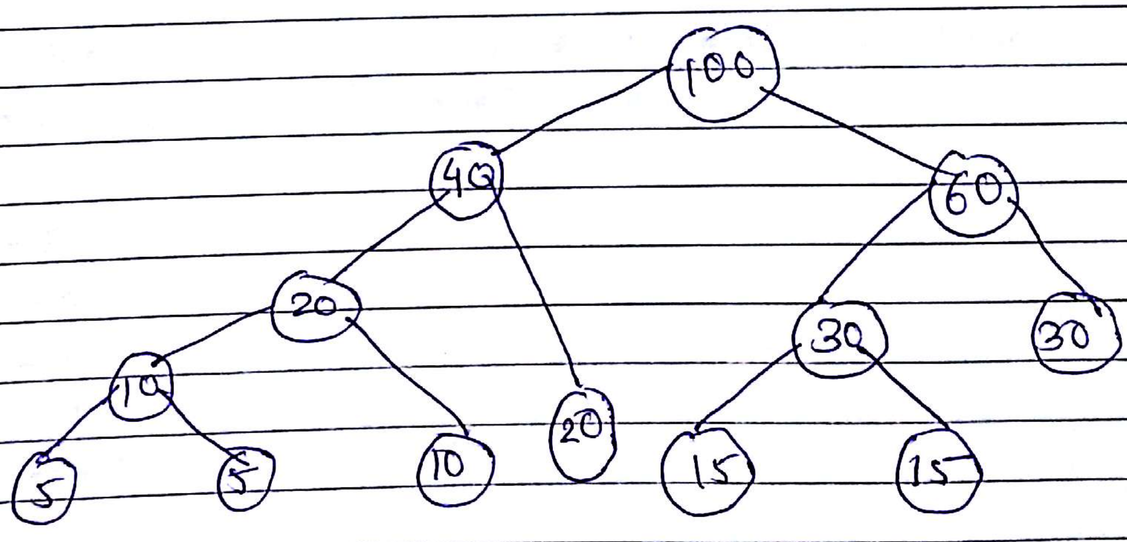
construct a Huffcode for the data.
what is the min. wt path length.

→

Sequence in increasing order

C	G	A	D	F	E	B
5	5	10	15	15	20	30
↑	↑					





min wt = $(10\times3)+(30\times2)+(5\times4)+(15\times3)+(20\times2)+(15\times3)+(5\times4) = 260$

min wt path = A → 3, B → 2, C → 4, D → 3, E → 2, F → 3, G → 4

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Q. Explain Huffman Algorithm to find an optimal Tree.

→ * Suppose $w_1, w_2 \dots w_t$ be the weights of the leaves & it is required to construct an optimal binary tree.

- 1. Arrange the weights in increasing order

2. Consider two leaves with the minimum weights w_1 & w_2. Replace these two leaves & their father by a leaf assign weight $w_1 + w_2$ to this new leaf

3. Repeat the step 2 for the weights $w_1, w_2, w_3 \dots w_t$ until no weight remain,

4. The tree obtained in this way is an optimal tree for given weights & stop

Eg.

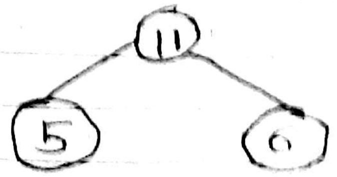
For the following set of weights construct optimal binary prefix code.

a - 5
b - 6
c - 6
d - 11
e - 20

1. Write in increasing order

5 6 6 11 20

2]



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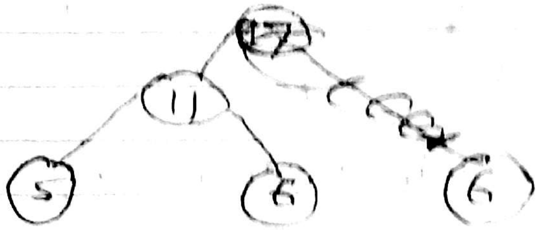

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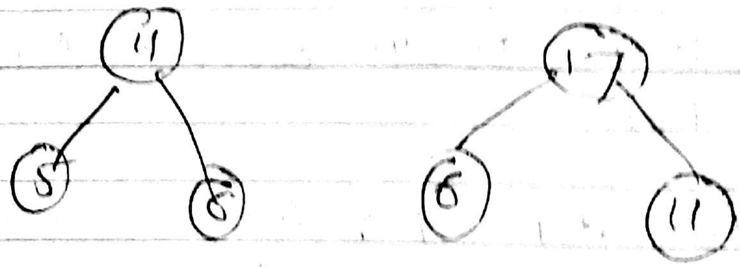

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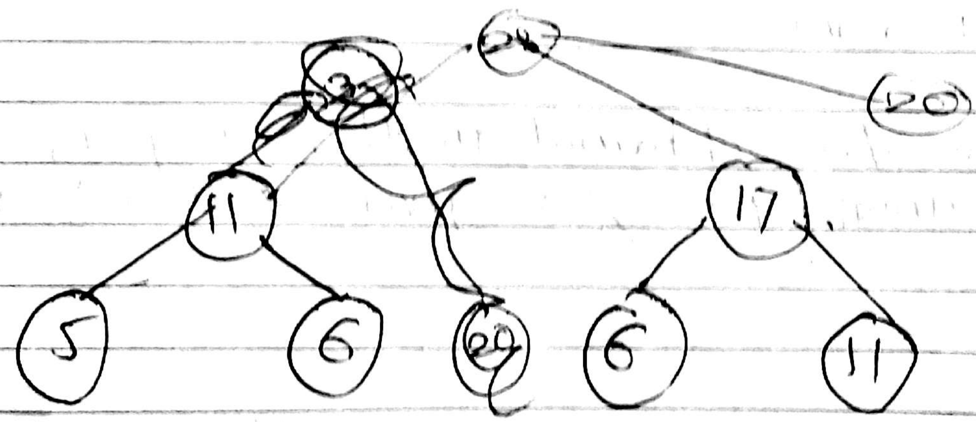
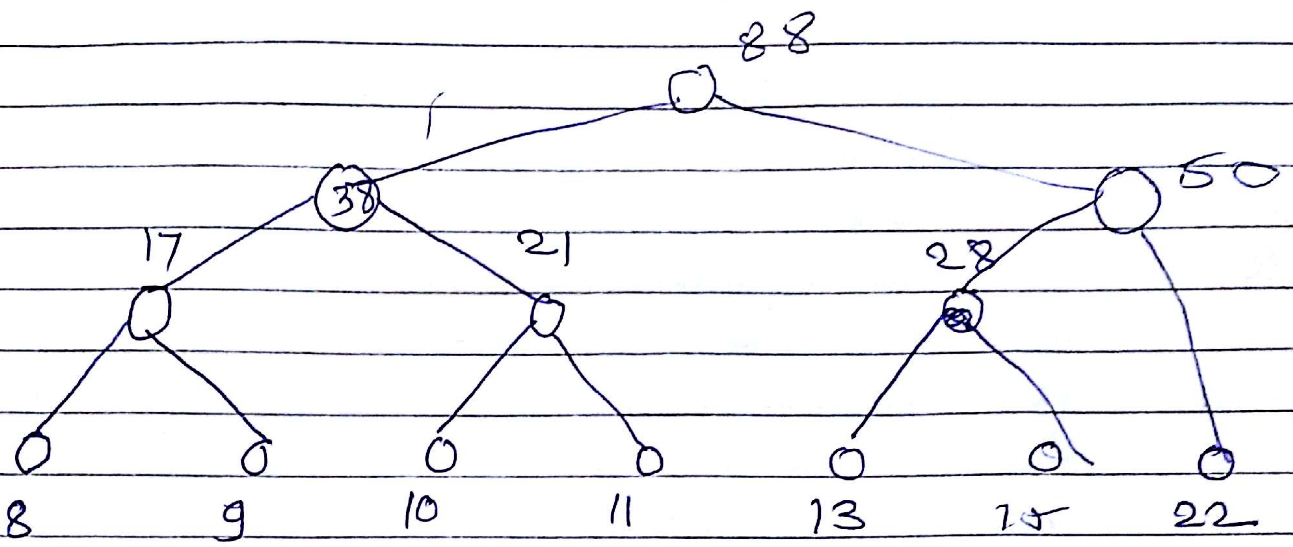


Construct an optimal tree for the weights
8 9 10 11 13 15 22
find the weight of the optimal tree



ii]