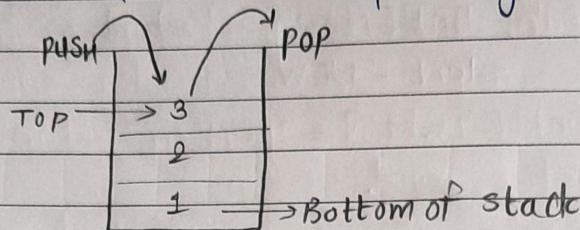


Unit 04 : Stack and Queue

* Stack :

- It's a collection of similar data elements where insertion & deletion operation takes place at only one end. Stack is also called "LIFO" (Last In First Out). It is linear D.S.

The insertion & deletion opn in case of stack are called as "PUSH" & "POP" respectively



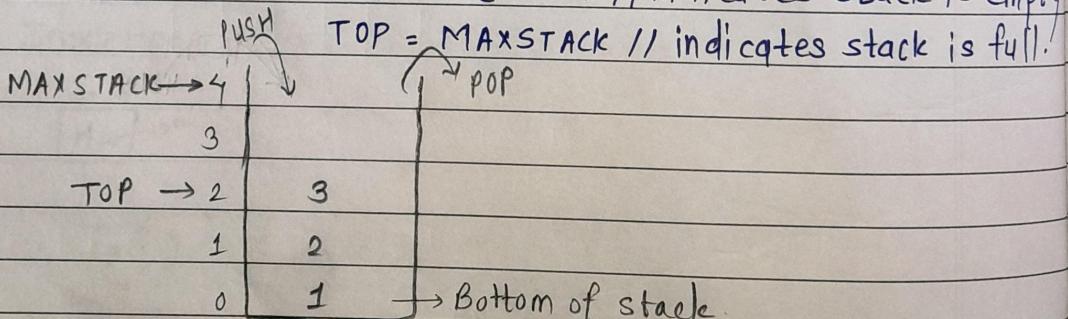
* Representation of stack in memory:

2 ways :- (i) Array representation (ii) Linked list representation

(i) Array representation:

In this, the pointer variable "TOP" which contains the locⁿ of topmost element of the stack, & the variable "MAXSTACK" which uses maximum number no. of elements can be held by the stack.

The condition :- $TOP = 0$ (NULL) // indicates stack is empty



In the diagram above, the stack contains 3 elements and the maximum elements the stack can hold is 5.

So, $MAXSTACK = 5$ & $TOP = 3$

* Operations on stack:

1. Push operation :

- The process of adding new element at the top of the stack is c/a 'push' opn. Pushing of new elements onto stack always increment top by 1.
- In case the array is full & there is no space for new inserⁿ i.e $\text{top} = \text{maxstack}$, then this condⁿ is c/a 'stack overflow'.

Algorithm :

PUSH (stack, top, maxstack, item)

Step 1: If $\text{top} = \text{maxstack}$ then

write overflow and exit.

Step 2 : $\text{top} = \text{top} + 1$

Step 3 : Set $\text{stack}[\text{top}] = \text{item}$

Step 4 : Exit

2. Pop operation :

- The process of deleting an element from the stack is c/a 'pop' opn. In case when stack doesn't contain any element i.e $\text{top} = \text{NULL}$, then this condⁿ is c/a 'stack underflow.'

Algorithm :

POP (top, item, stack)

Step 1: if $\text{top} = \text{NULL}$ then

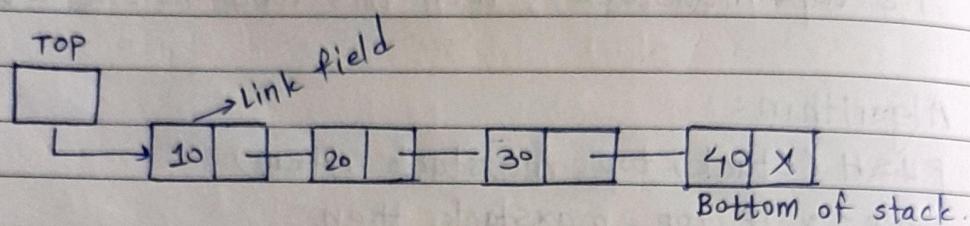
write underflow and exit

Step 2 : Set $\text{item} = \text{stack}[\text{top}]$

Step 3 : Set $\text{top} = \text{top} - 1$

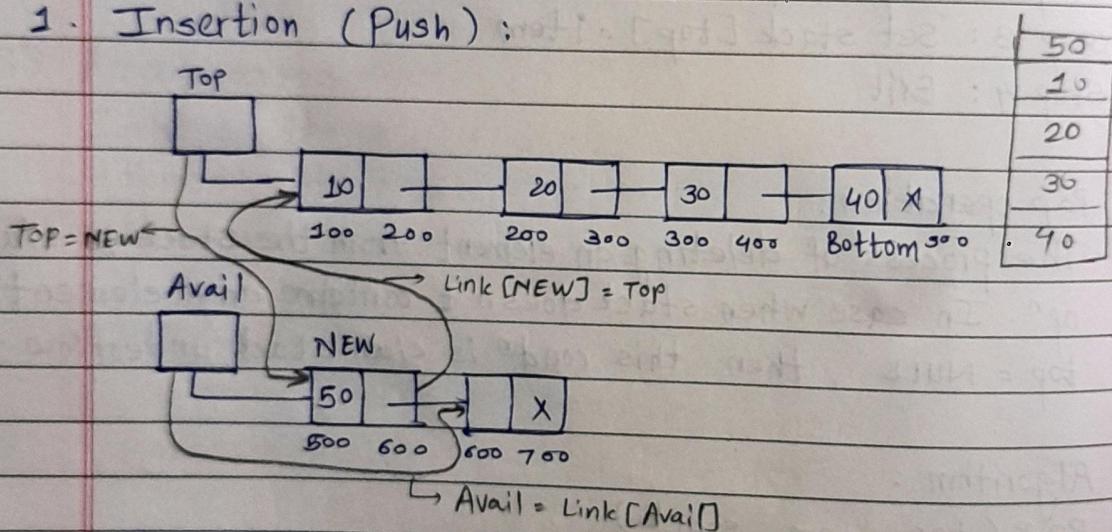
Step 4 : Exit

- Array representation of stack is very easy and convenient but it allow only representing fixed size stack.
- In some stack, the size of stack may vary during execution.
- So, the solution is to represent the stack using linked list.



* Operations on stack :

1. Insertion (Push) :



Algorithm :

`Push [TOP, NEW, Avail , Item]`

Step 1 : if `Avail = NULL`

write overflow and exit.

Step 2 : Set `Avail = NEW`

`Avail = Link[Avail]`

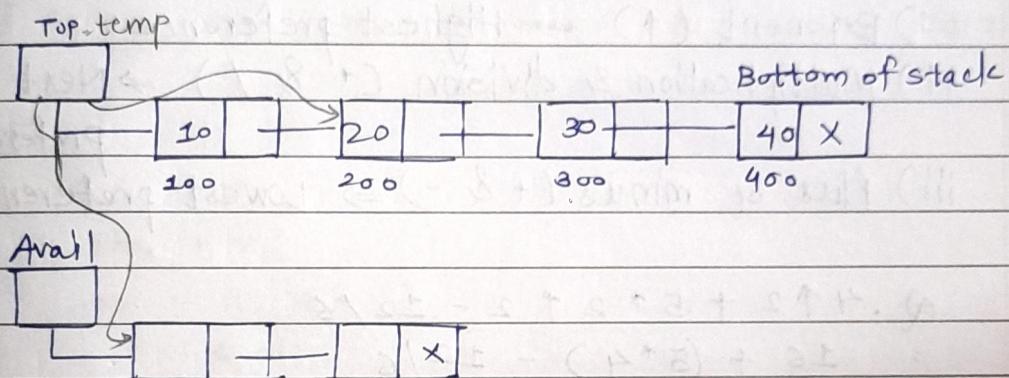
Step 3 : Set Link [NEW] = TOP

Step 4 : Set Info [NEW] = Item

Step 5 : Set Top = NEW

Step 6 : Exit

2. Deletion (Pop) :



Algorithm :

Pop [Top , Item , temp , Avail]

Step 1: if TOP = NULL
write underflow and exit

Step 2: set Item = Info [Top]

Step 3: set temp = Top

Step 4: Link [temp] = Top

Step 5: set Link [temp] = Avail

Avail = temp

24/11/23

- * conversion:- Infix to prefix & postfix \rightarrow push operators into the stack.
- Prefix to infix & postfix \rightarrow push operands into the stack from right to left.
- Postfix to infix & prefix \rightarrow same as prefix to infix & post
but from R \rightarrow L

* Polish notations:

- An arithmetic exp. consists of operators & operands.
- e.g. $(A + B) * C$
- The problem to evaluate the expression is to follow order of evaluation by using the preference rule.

- i) Exponent (\uparrow) \Rightarrow Highest preference.
- ii) Multiplication & division ($*$ & $/$) \Rightarrow Next highest preference.
- iii) Plus & minus ($+$ & $-$) \Rightarrow Lowest preference.

$$\begin{aligned} Q. \quad & 4 \uparrow 2 + 5 * 2 \uparrow 2 - 12 / 6 \\ &= 16 + (5 * 4) - 12 / 6 \\ &= 16 + 20 - 2 \\ &= 34 \end{aligned}$$

* Polish notations for arithmetic expression :

There are 3 notations :-

① Infix

\hookrightarrow The conventional way of writing an exp. is called "infix" notn. e.g. $A + B$, $C - D$ etc.

\hookrightarrow Syntax: $<\text{op}>$ operator $<\text{op}>$
 $\qquad\qquad\qquad$ operand

② Prefix

\hookrightarrow Always fix the operator before the operands.
e.g. $+AB$, $+CD$ etc.

\hookrightarrow Syntax: operator $<\text{op}>$ $<\text{op}>$

③ Postfix

\hookrightarrow Always fix the operator after the operands.
e.g. $AB+$, $CD-$

\hookrightarrow Syntax: $<\text{op}>$ $<\text{op}>$ operator

* Rules to convert infix exp. into prefix & postfix:

- 1) Parentheses must be solved first.
- 2) All the operators must be shifted only once according to their preferences.
- 3) The sub-exp. which has been converted into prefix & postfix is to be treated as single operand.
- 4) Once the conversion done, remove the parentheses.

e.g. Infix to prefix

$$A + (B * C)$$

$$\Rightarrow A + (* BC)$$

$$\Rightarrow + A (* BC)$$

$$\Rightarrow + A * BC$$

e.g. Infix to postfix

$$A + (B * C) \rightarrow \text{opr can be shifted only once} \checkmark$$

$$\Rightarrow A + (BC *)$$

$$\Rightarrow A (BC *) + \rightarrow + * \text{ not possible} \times$$

$$\Rightarrow ABC * +$$

* # Evaluation of postfix expression:

+ 8M Consider, the following exp., P: 5, 6, 2, +, *, 12, 4, /, -

To evaluate:-

- 1) Add ')' (closing paran.) at the end of exp. which indicates the terminating cond'.
- 2) Scan exp. from left \rightarrow Right.
- 3) While scanning iff operand encounters, then put it into stack.
- If operator encounters, then pop \rightarrow 2 operands from stack and perform op'.

Algorithm :

Postfix (val)

Step 1 : Add the closing parentheses at the end of the expression (right parentheses)

Step 2 : Scan the exp. from $L \rightarrow R$ & repeat st 3 & 4 for each element of expression until ')' is encountered.

Step 3 : If operand encounters, push it into stack

Step 4 : If operator encounters, then

- remove top 2 ele. from the stack.
- evaluate the exp.
- push the result onto stack

Step 5 : Set val to the top element of the stack

Step 6 : Exit

ans : 1) Add the right parentheses at the end of exp.

P : 5, 6, 2, +, *, 12, 4, /, -)

2) Scan the exp. until right parentheses is encountered.

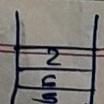
3) Push 5 onto the stack



4) Push 6 onto the stack



5) Push operand 2 onto the stack



6) Operator '+' is encountered, so pop 2 ele. from the stack & evaluate the result.

i.e $6 + 2 = 8$ & push 8 onto the stack

8
5

7) Operator '*' is encountered, so pop the elements from stack & evaluate it.

i.e $8 * 5 = 40$ & push 40 onto the stack

40
40

8) Push operand '12' onto the stack

12
40

9) Push operand 4 onto the stack

4
12
40

10) Operator '/' is encountered, so pop 2 ele. from the stack & evaluate it.

i.e $12 / 4 = 3$ & push 3 onto the stack

$4 / 12 \Rightarrow$ Not possible

3
40

11) Operator '-' is encountered, so pop the elements from the stack & evaluate it.

i.e $40 - 3 = 37$ & push 37 onto the stack

$3 - 40 \Rightarrow$ Not possible

37
37

12) As ')' encountered, stop the process.

Conversion of infix to postfix:

Assume the simple exp. The exp. may be parenthesized or unparenthesized. First we've to append the left parentheses at the end of infix exp. & initialize the stack = right parentheses
left?

Algorithm:

Step 1: Push open parentheses onto the stack and add close paren. at the end of expression.

Step 2: Scan the exp. from $L \rightarrow R$ & repeat step 3 to 6 until stack is empty.

Step 3: If operands encountered, then add it to exp.

Step 4: If open paren. is encountered, then push it onto stack.

Step 5: If an operator is encountered, then

i) Repeatedly pop the elements from stack & add it to the exp.,

Each operator which has same or high precedence than existing one.

ii) Add the operator to the stack.

Step 6: If right paren. is encountered, then

i) Repeatedly pop ele. from stack & add it to each operator in the highest precedence is pop first until the left paren. is encountered.

ii) Remove the left paren.

Step 7: Exit

Consider, the following exp:

$$Q: A + (B * C - CD / E \uparrow F) * G) * H$$

- Push the open paran. onto the stack & add the closing paran. at the end of exp.



- Scan from L → R

- Operand A is encountered, then add it to Q.

$$Q : A$$

- Operator + is encountered, so push it onto stack



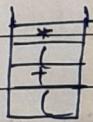
- Operator '(' is encountered, so push it onto the stack



- Operand B is encountered, so add to Q

$$Q : A B$$

- Operator * is encountered, so push it onto the stack



- Operand c is encountered, so add it to Q

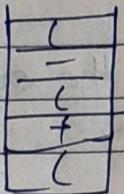
$$Q : A B C$$

- Operator - is encountered, but it's preference is less than that * (which have high preference) in stack, so first pop * operator, then push - onto the stack



$$Q : A B C *$$

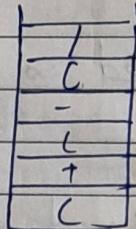
10) Operator '(' is encountered, so push it onto the stack



11) Operand @ D is encountered, so add it to Q.

$$Q : A \ B \ C \ * \ D$$

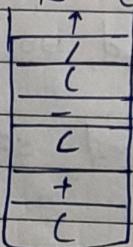
12) Operator / is encountered, so push it onto the stack.



13) Operand E is encountered, so add it to Q.

$$Q : + \ B \ C \ * \ D \ E$$

14) Operator ↑ is encountered, so add push it onto the stack



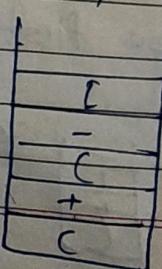
15) Operand F is encountered, so add it to q.

$$Q : A \ B \ C \ * \ D \ E \ F$$

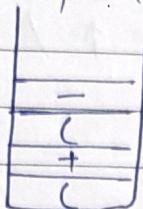
16) Operator ')' is encountered, so repeatedly pop the elements from stack until open paran. is encountered

$$Q : A \ B \ C \ * \ D \ E \ F \uparrow)$$

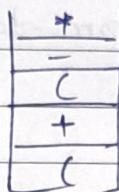
17) Operator -* is encou



~~mp~~ Remove open paran. from the stack and no need to add it to exp. Q



17) Operator * is encountered, so push it onto stack

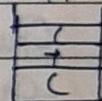


18) Operand G1 is encountered, so add it to Q.

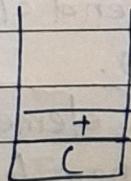
Q : A B C * D E F ↑ / G1

19) Operator ')' is encountered, so repeatedly pop elements until open paran. is encountered.

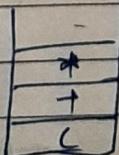
Q : A B C * D E F ↑ / G1 * -



Remove open paran. from stack & don't add to Q.



20) Operator * is encountered, so push it onto the stack.



21) Operand H is encountered, so add it to exp Q.

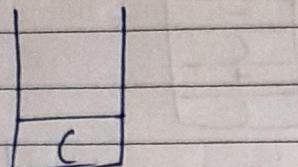
Q : A B C * D E F ↑ / G1 * - H

29-11-28

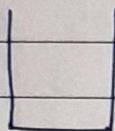
Page No.	
Date	

22) Operator ')' is encountered, so repeatedly pop ele. of stack until '(' encounter.

Q: A B C * D E F ↑ / G * - H * +



Remove open paran. from stack & don't add it to op.



23) Stack is empty, so stack stop the process.

Required expression is,

A B C * D E F ↑ / G * - H * +

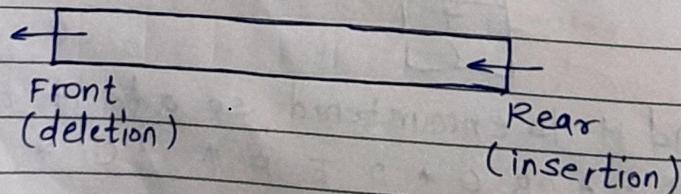
* Queue:

- Queue is the linear list of elements in which deletion is takes place at one end cla "Front" & insertion takes place at another end cla "Rear" end.

- It is also cla FIFO.

- The order in which the elements enter in a queue is the order in which they leave.

- The diff. with stack is that the insertion & deletion takes place only at one end.



Memory representation of queue:

There are 2 ways to represent the queue in memory:

i) Array representation

- A 1-D array, say $Q[1-N]$ can be used to represent a queue in this repⁿ. 2 pointers are associated with the — one is FRONT & another is REAR which indicates the 2 ends of queue. For the insertion of new element REAR will update & for deletion FRONT will be updated. There are 3 states of the queue with this repⁿ :-

- ① If the queue is empty, then $FRONT = REAR = -1$.

- ② If the queue is full, then $FRONT = 1$ & $REAR = N$ or $FRONT = REAR + 1$.

- ③ If the queue contain elements, then we can find no. of elements by using formula, $REAR - FRONT + 1$

Operations on queue:

1. Insertion: To insert an element.

2. Deletion : To delete the element.

1. Insertion :

Algo - Insert (Queue, Front, Rear, Item)

Step 1 : if $Front = 1$ and $Rear = N$

write : Queue is full & return.

Step 2 : if $Front = \text{NULL}$ then

set $Front = Rear = 1$

else if $Rear = N$ then

set $Rear = 1$

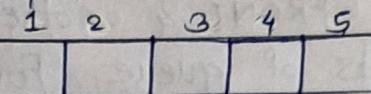
else

$R = R + 1$

Step 3: Set Queue[Rear] = Item

Step 4: Exit

e.g Consider, queue hold $N=5$ elements & consider initially queue is empty



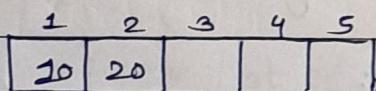
1) As Q is empty $\text{Front} = \text{Rear} = -1$

2) Insert 10 in Q.



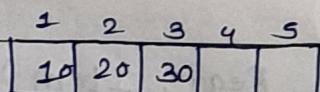
$\text{Front} = \text{Rear} = 1$

3) Insert 20



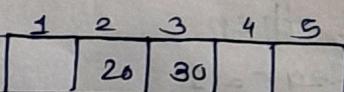
$\text{Front} = 1$, $\text{Rear} = 2$

4) Insert 30



$\text{Front} = 1$, $\text{Rear} = 3$

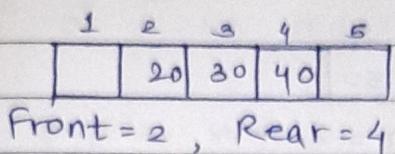
5) Delete 10 from Q.



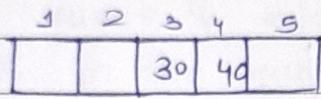
$\text{Front} = \text{Front} + 1 = 2$

$\text{Rear} = 3$

6) Insert 40

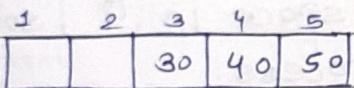


7) Delete 20



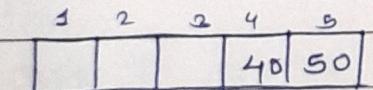
Front = 3, Rear = 4

8) Insert 50



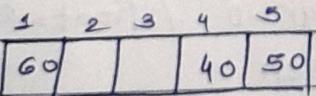
Front = 3, Rear = 5

9) Delete 30



Front = 2, Rear = 5

10) Insert 60



Rear = 1 // Once Q is full then reset Rear = 1

Front = 4

As Q is full i.e. Rear reach to the end of Q (R=N)

So, for the new insertion, reset Rear=1.

$$Q[Rear] = Q[1] = \text{Item} = 60$$

11) Insert 70

1	2	3	4	5
60	70		40	50

Front = 4, Rear = 2

12) Insert 80

1	2	3	4	5
60	70	80	40	50

Front = 4, Rear = 3

As Front = Rear + 1, Q is full.

As we don't have the space in Q, so stop processing and return from the process.

2. Deletion:

Algo-Delete (Q, Front, Rear, Item)

Step 1: if Front = NULL

 write: underflow and return.

Step 2: Set Item = Q[Front]

Step 3: if Front = Rear then

 set Front = Rear = 0 / Front = Rear = NULL

 else if Front = N then

 Set Front = 1 // Reset Front to 1.

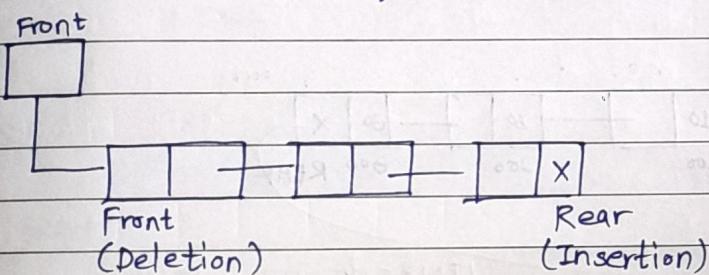
 else

 Front = Front + 1

Step 4: Exit.

* LL rep' of queue:

- LL rep' of queue requires 2 ptr. variables - Front & Rear.
- These pointer variables pointing to the nodes Front & Rear in LL.
- The info field of node holds the information or ele. of the queue & add' field holds the address of next ele. in the queue.



* Operations on queue used in linked list:

1. Insertion:

- In case of insertion algo. we need to check-
 - ① Whether there is a free space in `AVAIL` in list. if true, then remove 1st node from the `AVAIL` list.
 - ② Add item from Rear end.

Algorithm

`Insert(FRONT, REAR, AVAIL, ITEM, Q, LINK)`

Step 1: if `AVAIL = NULL` then

write : overflow and return

Step 2: Set `NEW = AVAIL`

`AVAIL = LINK[AVAIL]`

Step 3: Set INFO [NEW] = ITEM

LINK [NEW] = NULL

Step 4: if Set LINK [REAR] = NEW

REAR = NEW

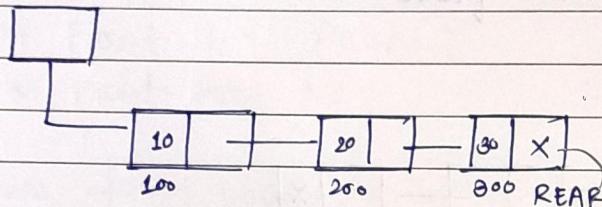
Step 5: if FRONT = NULL then

set FRONT = REAR = NEW // if queue is empty, then
insert new ele. as a start

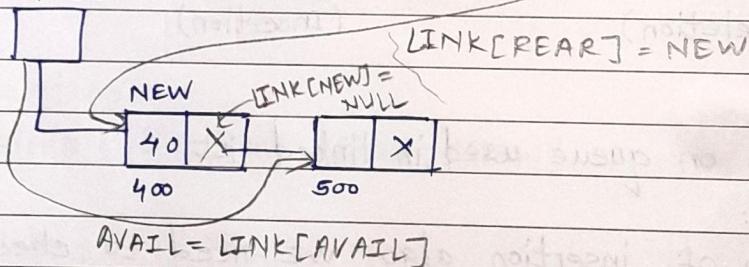
Step 6: Exit.

in the queue.

FRONT



AVAIL



2. Deletion:

- The deletion algo. delete the item from the LL. ↪
- This algo. is exactly same as we've seen in LL data structure except instead of 'start' we've to use 'Front' here and 'Rear' pointer indicates the end of list.

- For deletion, we've to check whether queue is empty or not.

i.e FRONT = NULL

then display the msg. 'underflow' & ^{exit} return from that process.

Algorithm:

Algo - delete (FRONT, REAR, PTR, INFO, ITEM)

Step 1: if FRONT = NULL then

write: underflow and exit.

Step 2: Set PTR = FRONT

Step 3: INFO [PTR] = ITEM // copy info. of first node

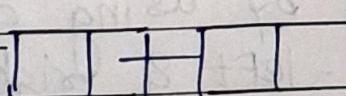
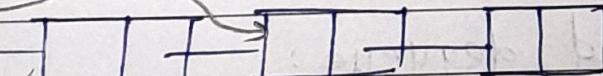
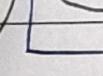
Step 4: FRONT = LINK [PTR] to delete.

Step 5: LINK [PTR] = AVAIL

AVAIL = PTR

Step 6: Exit

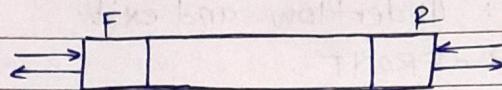
FRONT



01.12.28

* Dequeue (Double ended queue):

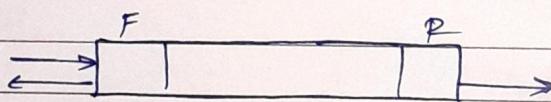
- Another type of queue is c/a "dequeue" or "deck"
- It is a linear list in which ele. can be inserted & deleted from both ends ; but not from the middle



* Types of dequeue:

1. Input restricted de.

- An input res. deque is a queue which allows insertion at only one end ; but allows deletion from both ends.



2. Output restricted deque:

- It's a deque which allows insertion from both ends ; but deletion at only one end.
- A deque is maintained by using circular queue or array with the pointers - left & right which points to the ends of the deque. It is a d.s which inherits the properties of stack & queue. This data struc. doesn't follow FIFO rule