

Unit 1

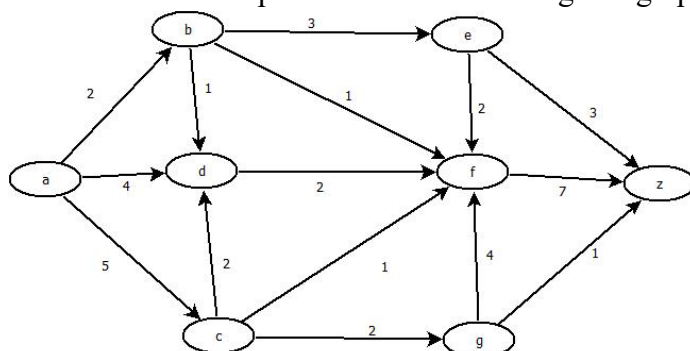
1. Prove by Induction $n \geq 0$
 $1 + a + a^2 + \dots + a^n = (1 - a^{n+1}) / (1 - a)$
2. Prove that $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$ is Tautology.
3. Obtain the conjunctive normal form of each of the following:
 - i) $P \wedge (P \rightarrow Q)$
 - ii) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$
 - iii) $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$
4. (i) Given that the value of $p \rightarrow q$ is false. Determine the value of $(\sim p \vee \sim q) \wedge q$.
(ii) Given that the value of $p \rightarrow q$ is true. Determine value of $\sim p \vee (p \leftrightarrow q)$.
5. Make use of developing a series of logical equivalences to show $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.
6. Make use of truth table to show $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Unit 2

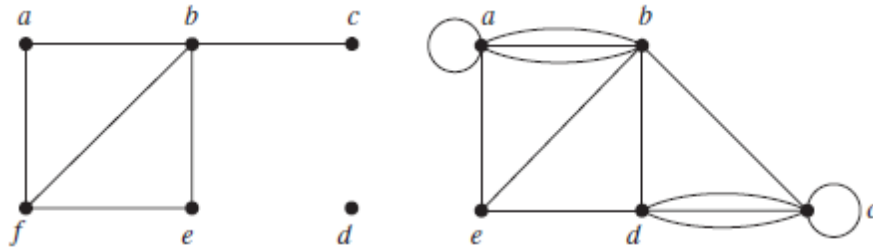
1. Prove Let $X = \{1, 2, \dots, 7\}$ and $R = \{(x, y) / x - y, \text{ is divisible by } 3\}$. Show that R is equivalence relation. Draw graph of R .
2. Draw Hasse diagram for $(\{1, 2, 3, 4\}, \leq)$
3. Let R be the relation on the set $A = \{a, b, c, d, e, f\}$ and $R = \{(a, c), (b, d), (c, a), (c, e), (d, b), (d, f), (e, c), (f, d)\}$. Find the transitive closure of R using Warshall's algorithm.
4. Construct the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$.
5. Let $A = \{a, b, c, d\}$ $B = \{1, 2, 3\}$. Determine whether the relation from A to B is function. Justify. If it is a function give the range.
 1. $R = \{(a, 1), (b, 2), (c, 1), (d, 2)\}$
 2. $R = \{(a, 1), (b, 2), (a, 2), (c, 1), (d, 2)\}$
6. If $x = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) \mid x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation. Draw the diagram of R
7. Construct the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 12, 24\}$.

Unit 3

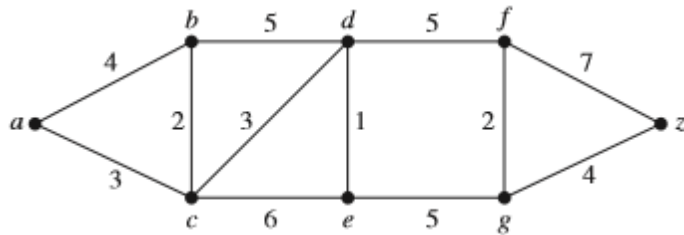
1. Find the shortest path between a to z for given graph using Dijkstra's algorithm



2. Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph G and H. Identify all isolated and pendant vertices.



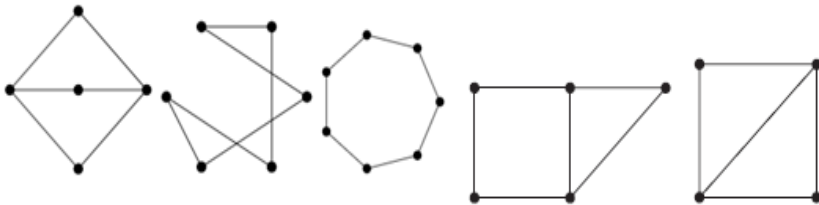
3. Find the length of a shortest path between a and z in the given weighted graph Using Dijkstra's Algorithm



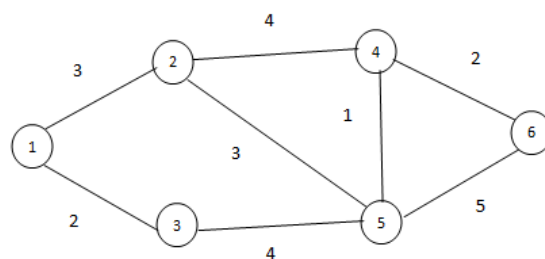
4. Define the following terms

1. Complete graph
2. Bipartite graph
3. Euler path and Circuit
4. Hamilton path and Circuit
5. Planar graph

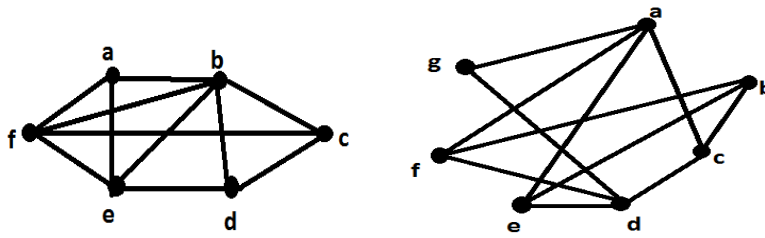
5. Justify your answer whether following graphs are bipartite graph or not?



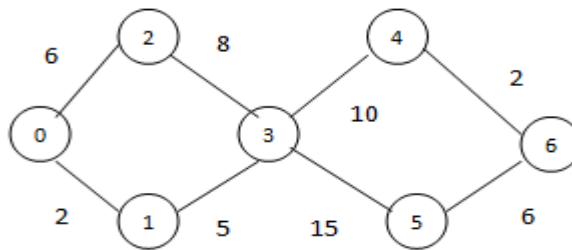
6. Construct following graph and find whether graph is planar graph or not? If it is planar graph then redraw it: i) $K_{3,3}$ ii) $K_{3,2}$ iii) $K_{4,4}$ iv) $K_{4,3}$ v) $K_{2,3}$ vi) $K_{3,2}$
7. Draw the following graph and write down the degree sequence of graph.
i) K_5 ii) K_4 iii) C_4 iv) C_5 v) W_4 vi) W_5 vii) $K_{3,2}$ viii) $K_{2,3}$ ix) Q_3 x) Q_2
8. Apply Dijkstra's algorithm and find the length of a shortest path between a and z in the given weighted graph.



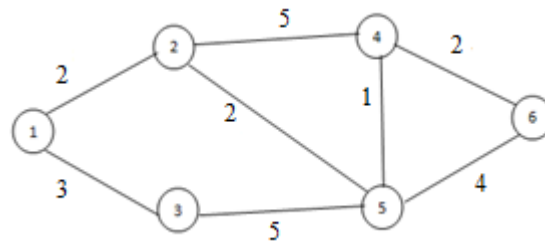
9. Partition of the set $V(G)$ into two disjoint sets $v_1(G)$ & $v_2(G)$ such that each edge of the graph has its one end in $v_1(G)$ & other end in $v_2(G)$. Write down whether it is bipartite graph or not?



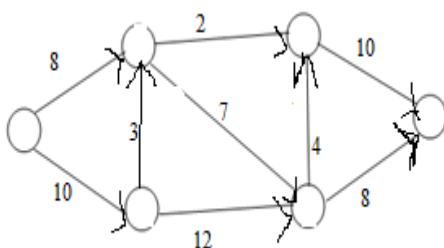
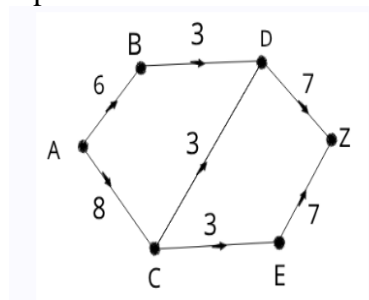
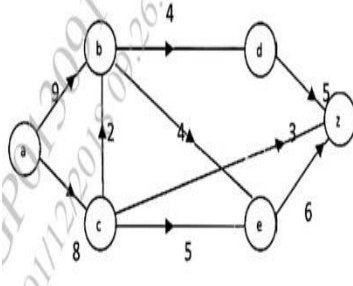
10. Apply Dijkstra's algorithm and find the length of a shortest path between 0 and 6 in the given weighted graph.



11. Apply Dijkstra's algorithm and find the length of a shortest path between a and z in the given weighted graph.



12. Determine the maximum flow in the transport network shown in following figure

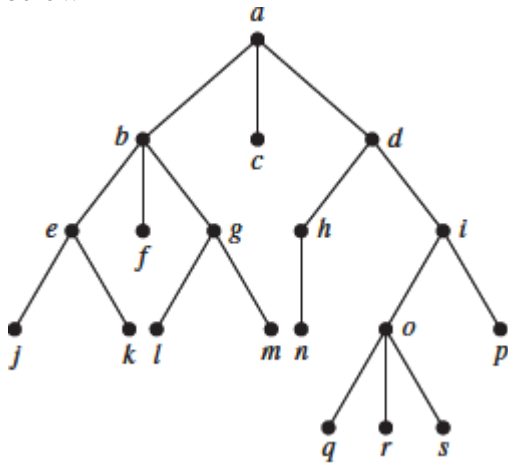


Unit 4

1. Explain following tree terminologies :

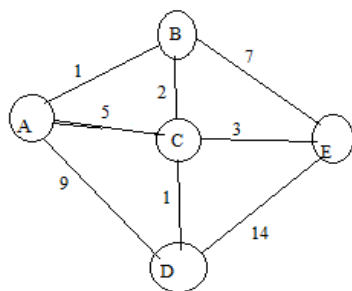
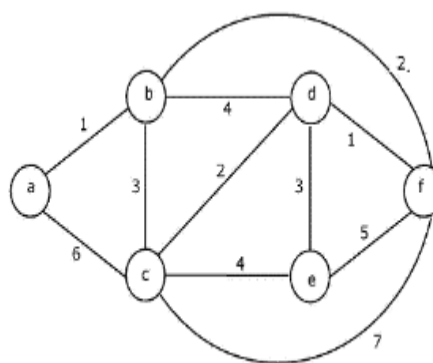
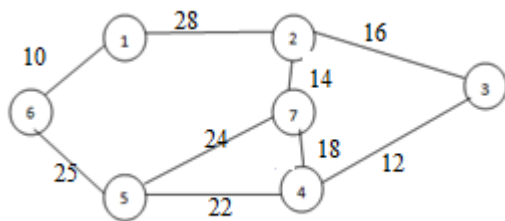
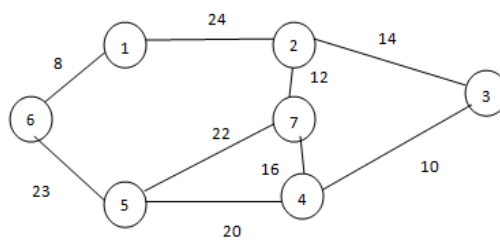
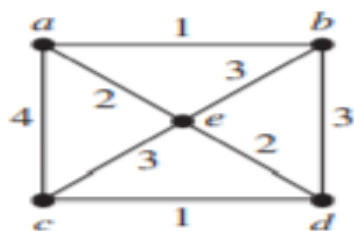
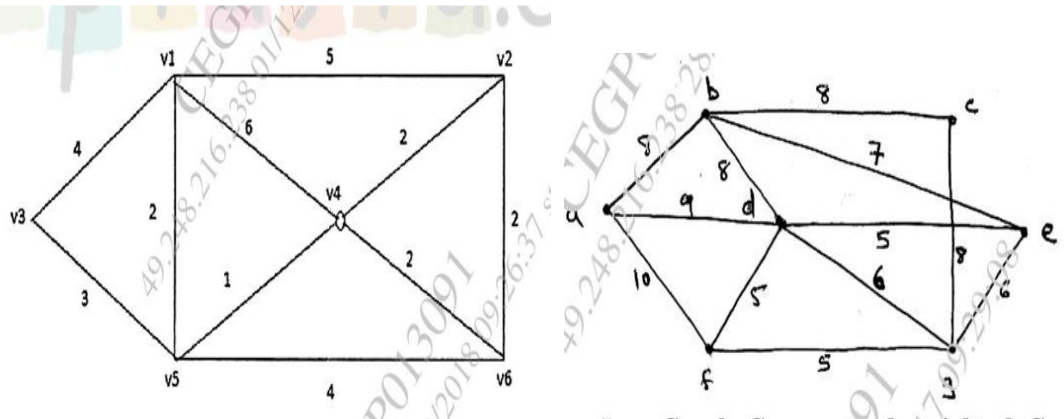
- Node
- Root
- Parent Node
- Leaf Node
- Children
- Internal node
- Degree of a node
- Degree of a tree
- Level of a tree
- Height of a tree
- Ancestors
- Descendants
- Sibling

2. Make use of concepts of tree and answer following questions with respect to tree shown below



- Which vertex is the root?
- Which vertices are internal?
- Which vertices are leaves?
- Which vertices are children of j ?
- Which vertex is the parent of h ?
- Which vertices are siblings of o ?
- Which vertices are ancestors of m ?
- Which vertices are descendants of b ?
- What is a level of the tree?
- What is a height of the tree?

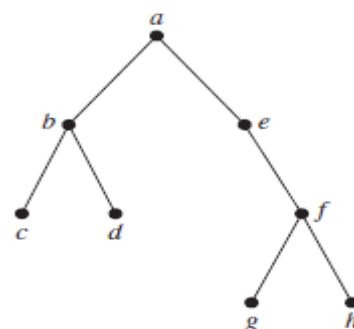
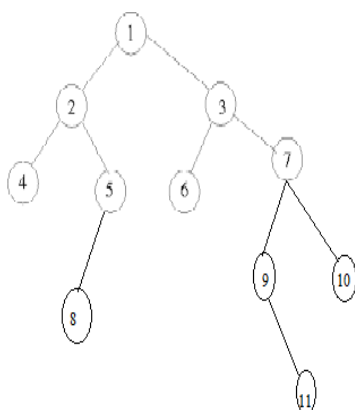
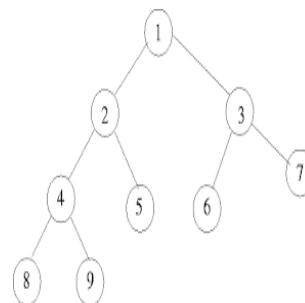
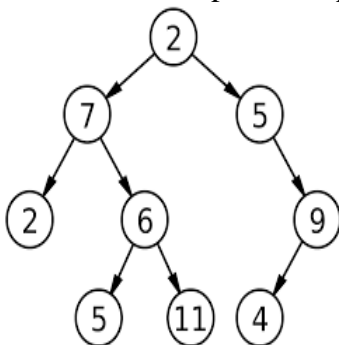
3. Construct the minimum spanning tree (MST) for the given graph using Prim's and Kruskal's Algorithm



4. Construct Huffman tree to encode the following symbols with the frequencies listed: A: 0.08, B: 0.10, C: 0.12, D: 0.15, E: 0.20, F: 0.35. What is the average number of bits used to encode a character?
5. Construct Huffman tree to encode the following symbols with the frequencies listed: A: 0.07, B: 0.09, C: 0.12, D: 0.22, E: 0.23, F: 0.27. What is the average number of bits used to encode a character?
- 6.

Suppose data items A, B, C, D, E, F, G occur in the following frequencies respectively 10, 30, 5, 15, 20, 15, 5. Construct a Huffman code for the data. What is the minimum weighted path length ? [6]

7. Construct a binary search tree for input data J, R, D, G, W, E, M, H, P, A, F, Q. List a root node, leaf nodes and interior nodes
8. shows the binary search tree that corresponds to the integer sequence {5, 3, 1, 9, 6, 4}. List a root node, leaf nodes and interior nodes
9. Construct a binary search tree for input data 200, 100, 300, 50, 150, 250, 400, 10, 75, 125, 175. List a root node, leaf nodes and interior nodes.
10. Build the preorder, postorder and inorder traversal of the binary tree as



Unit 5

Define :

- (i) Rings
- (ii) Integral domain
- (iii) Field.

1. Check whether $(Z, X5)$ is an abelian group where $Z = \{0,1,2,3,4\}$
2. Check whether $(Z, +6)$ is an abelian group where $Z = \{0,1,2,3,4,5\}$
3. Identify whether $(G, *)$ is an abelian group Consider G be the set of all non zero real numbers and let $a * b = ab / 4$
4. Identify whether $(G, *)$ is an abelian group Consider G be the set of all non zero real numbers and let $a * b = ab / 2$
5. Explain the steps to identify the algebraic structure $(Z, +, *)$ is a ring
6. Explain the steps to identify field with suitable example.
7. Explain the steps to identify integral domain with suitable example.
8. Explain the steps to identify ring with suitable example.
9. Explain homomorphism and normal subgroups.
10. Make use of $(2,6)$ encoding function e , where $e(00) = 011010$ $e(01) = 010101$ $e(10) = 101101$ $e(11) = 101100$ i) Find the minimum distance of e ii) How many errors will e detect?
11. Make use of $(2,5)$ encoding function e , where $e(00) = 01000$ $e(01) = 01010$ $e(10) = 00101$ $e(11) = 11110$ i) Find the minimum distance of e ii) How many errors will e detect?
12. 9. Make use of $(2,6)$ encoding function e , where $e(00) = 011000$ $e(01) = 010110$ $e(10) = 101011$ $e(11) = 101000$ i) Find the minimum distance of e ii) How many errors will e detect?
13. Make use of $(2,5)$ encoding function e , where $e(00) = 00000$ $e(01) = 01110$ $e(10) = 00111$ $e(11) = 11111$ i) Find the minimum distance of e ii) How many errors will e detect?