1-B — type 2

$$\begin{vmatrix} x_1 & x & a \\ x & x & b \\ x & x & c \end{vmatrix} = 0$$

2. ii) $\begin{vmatrix} p & a & b+c \\ p & b & a+c \\ p & c & a+b \end{vmatrix}$

$$\Rightarrow p \begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix}$$

$C_2 = C_2 + C_3$

$$= p \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & b+c \\ 1 & a+b+c & b+c \end{vmatrix}$$

$$= p(a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & a+c \\ 1 & 1 & a+b \end{vmatrix}$$

$$= p(a+b+c) \times 0$$

$$= 0$$

v) $\begin{vmatrix} 1 & \cos 2\alpha & \sin\alpha \\ 1 & \cos 2\beta & \sin\beta \\ 1 & \cos 2\gamma & \sin\gamma \end{vmatrix}$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - \sin^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$R_1 = R_1 - R_2$

$R_2 = R_2 - R_3$

$\Rightarrow \begin{vmatrix} 0 & \cos 2\alpha - \cos 2\beta & \sin\alpha - \sin\beta \\ 0 & \cos 2\beta - \cos 2\gamma & \sin\beta - \sin\gamma \\ 1 & \cos 2\gamma & \sin\gamma \end{vmatrix}$

$\Rightarrow 1 \begin{vmatrix} \cos 2\alpha - \cos 2\beta & \sin\alpha - \sin\beta \\ \cos 2\beta - \cos 2\gamma & \sin\beta - \sin\gamma \end{vmatrix}$

$$= \begin{vmatrix} 1-2\sin^2\alpha - 1 + 2\sin^2\beta & \sin\alpha - \sin\beta \\ 1-2\sin^2\beta - 1 + 2\sin^2\gamma & \sin\beta - \sin\gamma \end{vmatrix}$$

$$= \begin{vmatrix} -2(\sin^2\alpha - \sin^2\beta) & \sin\alpha - \sin\beta \\ -2(\sin^2\beta - \sin^2\gamma) & \sin\beta - \sin\gamma \end{vmatrix}$$

$$= -2 \begin{vmatrix} (\sin\alpha + \sin\beta)\underline{(\sin\alpha - \sin\beta)} & \underline{\sin\alpha - \sin\beta} \\ (\sin\beta + \sin\gamma)\underline{(\sin\beta - \sin\gamma)} & \underline{\sin\beta - \sin\gamma} \end{vmatrix}$$

$$= -2(\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)\begin{vmatrix} \sin\alpha + \sin\beta & 1 \\ \sin\beta + \sin\gamma & 1 \end{vmatrix}$$

$$= -2(\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\alpha + \sin\beta - \sin\beta - \sin\gamma)$$

$$= -2(\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\alpha - \sin\gamma)$$

$$= -2(\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)\{-(\sin\gamma - \sin\alpha)\}$$

$$= 2(\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\gamma - \sin\alpha)$$

[Proved]

vi) $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$

$\log x + \log y = \log xy$

$\log x - \log y = \log \frac{x}{y}$

$= \begin{vmatrix} \log x & \log y & \log z \\ \log 2x - \log x & \log 2y - \log y & \log 2z - \log z \\ \log 3x - \log 2x & \log 3y - \log 2y & \log 3z - \log 2z \end{vmatrix}$

$R_2 = R_2 - R_1$

$R_3 = R_3 - R_2$

$= \begin{vmatrix} \log x & \log y & \log z \\ \log \frac{2x}{x} & \log \frac{2y}{y} & \log \frac{2z}{z} \\ \log \frac{3x}{2x} & \log \frac{3y}{2y} & \log \frac{3z}{2z} \end{vmatrix}$

$$= \begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log \frac{3}{2} & \log \frac{3}{2} & \log \frac{3}{2} \end{vmatrix}$$

$$= \log 2 \cdot \log \frac{3}{2} \begin{vmatrix} \log x & \log y & \log z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \log 2 \cdot \log \frac{3}{2} \times 0$$

$$= 0$$ [Proved]