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Adjustable Bipedal Gait Generation using Genetic Algorithm Optimized Fourier Series Formulation

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***Abstract* - This paper presents a method for optimally generating stable bipedal walking gaits, based on a Truncated Fourier Series Formulation with coefficients tuned by Genetic Algorithm. It also provides a way to adjust the stride-frequency, step-length or walking pattern in real-time. The proposed approach to gait synthesis is not limited by the robot kinematic structure and can be used to satisfy various motion assumptions. It is also easy to generate optimal gaits on terrains of different slopes or on stairs under different motion requirements. Dynamic simulation results show the validity and robustness of the approach. The gaits generated resulted in human-like motions optimized for stability, even walking speed and lower leg-strike velocity of the swing foot.**

Index Terms - Bipedal Locomotion, Fourier series, Genetic Algorithm, ZMP stability criterion

I. Introduction

It is a great dream to build a bipedal robot that has the agility or mobility similar to that of a human being. The difficulty of bipedal robot control is mainly due to the nonlinear dynamics involved, interaction with an unknown environment and under-actuated joint at the stance ankle [1]. Many algorithms have been proposed for gait synthesis in bipedal walking. To reduce the complexity of the analysis, some researchers adopted simplified dynamics model such as the inverted pendulum with certain assumptions on the robot's motion and structure [2], and some made use of the recorded human walking data as a demonstration [3]. Assumptions inevitably make the results from the ensuing analysis or simulation not so reflective of the actual robot dynamics when the derived algorithms are implemented on real robots, and limit the application to robots of various structures. Human walking data is a useful reference for gait analysis, but there still exists significant differences between the dynamics of robots and their human counterparts [1]. Besides, there also have been many control methods developed for the bipedal walking locomotion control. Examples of these include the trajectory planning using inverse kinematics with ZMP (Zero Moment Point) stability criterion [4], a biologically inspired approach using the idea of CPG (Central Pattern Generator) [3,5,6], the artificial intelligence based approach using Reinforcement Learning [1], and passive walking. CPG based approach is very attractive since it can result in closed loop trajectory-tracking [6]. However the relationship between the model properties and robot dynamics is not obvious and difficulties arise in searching for suitable parameters for the CPG model (Neuron Oscillator [5] or Van der Pol equation [6]). The Reinforcement Learning approach can be considered as a general motion generation approach for different robots but it takes time to online learn the correct motion and the time cost for the objective set is usually not predictable. Many ZMP methods are model-based with trajectory planning performed in the Cartesian space. Therefore, the generated motions result in a typical "bent-knee" posture to avoid Jacobian singularity. The bent-knee posture consumes more energy and the resulting motion is not as natural as the human gait [3].

In this paper, to clarify the inner feedback pathway and thus to mimic the CPG function, we present an approach based on a tailored form of Fourier Series Function (FSF), optimized using the Genetic Algorithm (GA) and assisted with the ZMP dynamic stability criterion. The approach is named as Genetic Algorithm Optimized Fourier Series Formulation (GAOFSF). The advantage of this GAOFSF approach can be summarized as follows:

1) It can take care of the time period (related to gait period) directly due to the properties of the Fourier series;

2) It can efficiently fulfil the multi-objective used to obtain an optimal motion in multi-aspect concurrently;

3) The step-length, walking rhythm (stride-frequency) and walking pattern are adjustable in real-time either with the assistance of a simple function or a record of the generated motion database;

4) It can be applied to various motion assumptions and robots of different sizes and inertia properties.

Section 2 describes the proposed GAOFSF gait generation approach. In Section 3, details of the objective functions, constraints, and optimization using GA are discussed. Section 4 presents the resulting motions when the approach was applied to two walking examples. Section 5 presents the simulation results based on the Yobotics! dynamic simulation software to confirm the validity of the approach.

II. The GAOFSF Approach

In the GAOFSF approach, a Truncated Fourier Series (TFS) formulation is used for the synthesis of the hip and knee joint angle trajectories. It differs from the former work [7]

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which represented joint trajectories of human walking by a full Fourier series function at the usage of parameters in Fourier series for real-time motion adjustment and the tailored format. In our approach, the cosine part of Fourier series could be omitted. In the presented approach, GA is used to search for the optimal values of the parameters in these formulations so as to achieve stable walking behaviour with desirable characteristics for a given biped. The overall scheme of bipedal walking control using GAOFSF is shown in Fig. 1.

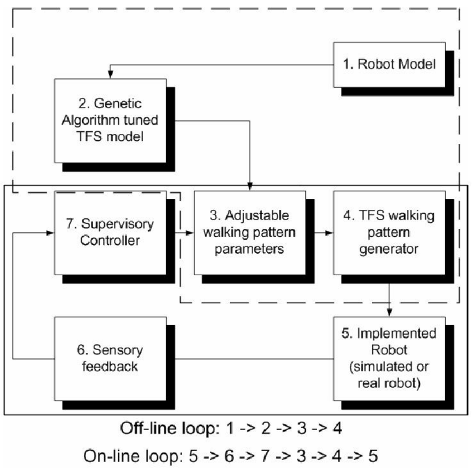

Fig. 1 Framework of the proposed GAOFSF approach.

The off-line part consists of blocks 1,2,3,4 which are responsible for the generation of walking pattern. The on-line loop consists of a supervisory controller which adjusts the walking motions through the adjustable walking pattern parameters. This paper discusses the off-line generation part.

The bipedal robot used to illustrate the application of the proposed approach is the seven-link planar robot (shown in Fig. 2) which is made up of a trunk and two legs. Each leg comprises a thigh, a shank and a foot. Each leg has three degrees of freedom: hip-pitch about the hip joint *H*, knee-pitch about the knee joint *K*, and ankle-pitch about the ankle joint *A*. *T* is the mid-point between the two hip joints.

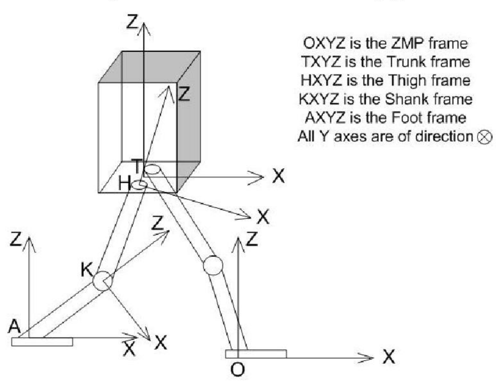

Fig. 2 Seven-link bipedal robot and its coordinate systems.

Gait synthesis for this robot involves the determination of the hip-pitch and knee-pitch angles for each of the legs. The ankle-pitch angles are set to be such that both feet are always maintained to be parallel to the ground.

As shown in Fig. 2, coordinate frames are attached to the trunk (*TXYZ*), the right thigh (*HXYZ*), the right shank (*KXYZ*) and the right foot (*AXYZ*). An additional coordinate frame (*OXYZ*) is attached to the stance ankle joint with *XY* plane always parallel to the ground. It is used as the reference frame for locating the ZMP. All the *Z* axes are longitudinal to the links. Referring to Fig. 2, define

Right Hip Angle, θ_{rh} = the angle of Axis *KH* from Axis *TZ* in the clockwise direction,

Right Knee Angle, θ_{rk} = the angle of Axis *KZ* from Axis *KH* in the clockwise direction.

The corresponding angles for the left leg, θ_{lh} and θ_{lk}, are similarly defined.

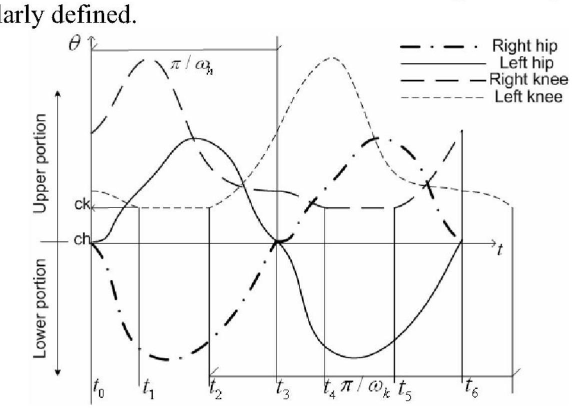

Fig. 3 Typical shapes of hip and knee trajectories

A natural human walking gait is cyclical. Referring to human gait analysis [8], typical shapes of trajectories for hip and knee angles in one cycle of locomotion can be represented by Fig. 3. The trajectories for both legs are identical in shape but are shifted in time relative to each other by half of the walking period. For example θ_{lh} for the left hip is identical to θ_{rh} for the right hip, except that θ_{lh} is time shifted by $(t_6-t_0)/2$ with respect to θ_{rh}. The gait period is given by $2\pi/\omega_h$ where ω_h is defined as the gait frequency in rad/sec.

It can also be noted that the joint angle trajectories have certain "offsets". The values of offsets influence the biped's posture during walking. For example c_h denotes the hip angle offset. This is the value of both hip joint angles when they are equal (when the two thighs cross each other). c_k denotes the knee angle offset. This is the value of the knee angle when the knee is locked during the support phase. t_1 and t_2 denote the start and end time, respectively, of the lock phase. If t_1 coincides with t_2, then there is no locking of the stance leg during walking.

Consider first the hip angle trajectories. They can be divided into an upper portion, θ_h^+, for which $\theta_h \geq c_h$, and a lower portion, θ_h^-, for which $\theta_h < c_h$. Therefore, referring to Fig. 3, for the two portions of the walking cycle, the hip joint angles for the two legs are given by

$$\begin{aligned} t &\in [t_0, t_3) \quad \theta_{rh} = \theta_h^-(t) \quad \theta_{lh} = \theta_h^+(t) \\ t &\in [t_3, t_6] \quad \theta_{rh} = \theta_h^+(t-t_3) \quad \theta_{lh} = \theta_h^-(t-t_3) \end{aligned} \tag{1}$$

where θ_{rh} and θ_{lh} are the right and the left hip joint angles, respectively.

Similarly, the right knee joint angle trajectory for different portions of the walking cycle is given by

$$\begin{aligned} &t \in [t_0, t_4) && \theta_{rk} = \theta_{k1}(t + t_6 - t_5) \\ &t \in [t_4, t_5) && \theta_{rk} = \theta_{k2} \\ &t \in [t_5, t_6] && \theta_{rk} = \theta_{k1}(t - t_5) \end{aligned} \tag{2}$$

where θ_{k1} is the knee joint trajectory from the beginning of swing phase, denoted by t_5 for the right knee in Fig. 3, to the instant in the support phase when the knee joint is locked, denoted by t_4 in Fig. 3. θ_{k2} is the locked angle for the knee joint.

Similarly, referring to Fig. 3, the joint angle for the left knee is given by

$$\begin{aligned} &t \in [t_0, t_s) && \theta_{lk} = \theta_{k1}(t + t_6 - t_2) \\ &t \in [t_s, t_2) && \theta_{lk} = \theta_{k2} \\ &t \in [t_2, t_6] && \theta_{lk} = \theta_{k1}(t - t_2) \end{aligned} \tag{3}$$

where t_1 is the instant when the stance knee is locked and t_2 is the instant when the walking phases of the two legs are switched.

From Fig. 3, it can be noted that all the upper and lower portions of the trajectories resemble part of a sinusoid. As such, a Fourier series representation for these curves is chosen which will not involve too many higher order terms. The general FSF is expressed as

$$f(t) = \frac{1}{2}a_0 + \sum_{i=1}^{\infty} a_i \sin(\frac{2\pi i}{T}t) + \sum_{i=1}^{\infty} b_i \cos(\frac{2\pi i}{T}t) \tag{4}$$

As discussed all the joint trajectories during a gait cycle can be divided into two portions. Each portion can be viewed as an odd function output according to the intersection with the angle axis. Therefore the sine series in the Fourier series function is simplified a Truncated Fourier Series (TFS) is used to model each portion as follows

$$f(t) = \sum_{i=1}^{n} a_i \sin i\omega t + c_f \qquad \omega = \frac{\pi}{T_s} \tag{5}$$

where a_i, n, and c_f are constants to be determined and ω is the fundamental frequency, defined as a walking stride-frequency here, determined by the period T_s which is half of T.

Using (1), (2), (3) and (5), and by inspection of the curves in Fig. 3, the TFS for the hip joints' trajectories are derived as

$$\theta_{rh}, \theta_{lh} = \begin{cases} \theta_h^+ = \sum_{i=1}^{n} R \cdot A_i \sin i\omega_h (t - t_h^+) + c_h \\ \theta_h^- = \sum_{i=1}^{n} R \cdot B_i \sin i\omega_h (t - t_h^-) + c_h \end{cases} \tag{6}$$

where $\omega_h = \pi/(t_3 - t_0) = \pi/(t_6 - t_3)$, which also determines the walking stride-frequency; A_i and B_i are constant coefficients, θ_h^+ and θ_h^- the upper and the lower portion, respectively of the hip joint trajectory, and t_h^+ and t_h^- are time-shift values according to (1). R is an amplitude scaling parameter used for changing the step-length. Initially, R is set to 1.

Similarly, the trajectories for the knee joints are represented as

$$\theta_{rk}, \theta_{lk} = \begin{cases} \theta_{k1} = \sum_{i=1}^{n} R \cdot C_i \sin i\omega_k (t - t_k) + c_k, \\ \theta_{k2} = c_k \geq 0 \end{cases} \tag{7}$$

where $\omega_k = \pi/((t_6 - t_2) + (t_1 - t_0))$ and C_i are constant coefficients and t_k is the time-shift.

Compared with other approximation approaches [9], the advantages of using this tailored FSF to synthesize the human-like walking gait for robots are as follows:

1) With only a few terms in the series, it can represent quite accurately the shapes of the required joint trajectories for human walking inspired bipedal robot gait. Note that the upper and the lower portions may not be symmetrical, but each of which is similar to half a sinusoid.

2) Each TFS used here is a simple expression that makes no mathematical assumption when considered as half of an odd function. The gait period is included directly. Other functions, such as the Spline, Gaussian or the full Fourier Series Function will require additional constraints to regulate the motion period.

3) Key parameters of the TFS can be easily adjusted on-line during walking to change the walking gait, either to change the desired pace, or in response to external perturbations.

III. Gait Optimization

A. Dynamic stability criterion—ZMP

The ZMP criterion [10,11] is used for evaluating the feasibility of the generated gait. Given a prescribed joint trajectories, if the ZMP is always within the area covered by the "footprint" of the robot, the robot will be able to execute the joints' trajectories without toppling. Maintaining ZMP to be within the footprint is thus used as a primary objective for the Genetic Algorithm (GA) (see next subsection).

In former works, ZMP is used for biped motions which are planned in Cartesian coordinates. Therefore, the singularity problem can arise as we mentioned in the beginning of this paper. In our method, the feasible gait description is directly given in Joint coordinate and thus the singularity problem is avoided. Furthermore, the human-like straight knee walking posture can be achieved.

The coordinates, (P_x, P_y), of the ZMP are computed as follows [11]

$$P_x = (\sum_{i=1}^{n} m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^{n} m_i\ddot{x}_i z_i - \sum_{i=1}^{n} I_{iy}\ddot{\Omega}_{iy}) / \sum_{i=1}^{n}(\ddot{z}_i + g)m_i \tag{8.1}$$

$$P_y = (\sum_{i=1}^{n} m_i(\ddot{z}_i + g)y_i - \sum_{i=1}^{n} m_i\ddot{y}_i z_i + \sum_{i=1}^{n} I_{ix}\ddot{\Omega}_{ix}) / \sum_{i=1}^{n}(\ddot{z}_i + g)m_i \tag{8.2}$$

where x_i, y_i, and z_i are the coordinates of the centroid of link I; m_i is mass; I_{ix} and Ω_{ix} are the centroidal moment of inertia and angle value, respectively, about X-axis; I_{iy} and Ω_{iy} are the corresponding parameters for the Y axis; and g is the gravitational constant. Only (8.1) is used here since we only consider the sagittal plane motion.

B. Optimization strategy—GA

In order to generate the desired joints' trajectories for stable walking, a suitable set of coefficients A_i, B_i, C_i and parameters c_h, c_k, t_1, t_2 (refer to Fig. 3) need to be obtained. t_1 is the instant when the knee of the stance foot starts to lock and t_2 is the instant when the walking phases of the two legs are switched. Genetic Algorithm (GA) [12] was used to search for the abovementioned coefficients.

To achieve a natural walking gait while maintaining stable walking, an objective function to be minimised is of the form

$$f_T = w_1 f_1 + w_2 f_2 + w_3 f_3 + w_4 f_4 \tag{9}$$

where f_1 = sum of the distances of the ZMP from the centre of the footprint over a walking cycle;

f_2 = standard deviation of the ZMP over a walking cycle;

f_3 = root mean square of the error of the speed of the trunk from the desired speed over a whole walking cycle;

f_4 = leg strike speed;

and w_1, w_2, w_3, and w_4 are the corresponding assigned weighting factors. Six motion constraints are also defined to ensure a valid and natural walking motion. These are:

- s_1 constrains the ZMP to be within the supporting footprint, $-0.1m<=ZMP(P_x)<=0.2m$ for flat terrain;
- s_2 constrains the swing height to be above a specified minimum value;
- s_3 constrains the trunk's speed to be always positive;
- s_4 constrains the swing foot's speed to be always positive except for the touching down motion;
- s_5 constrains the deviation of the step length from that required to within a small specified value;
- s_6 constrains the deviation of the foot's touch-down instant, t_d, from the planned phase switching time, t_2, for example, $|t_d - t_2| < 0.005\ s$.

Based on these constraints, a penalty function is defined as

$$P = \sum_{i=1}^{6} p_i s_i \tag{10}$$

where $p_i, i = 1,\ldots,6$, are the assigned penalty weighting factors. Using (9) and (10), the fitness function for the GA algorithm is established as follows

$$F = \begin{cases} 0, & \text{if } \sum_{i=1}^{n}|A_i| = 0 \text{ or } \sum_{i=1}^{n}|B_i| = 0 \text{ or } \sum_{i=1}^{n}|C_i| = 0 \\ \mathrm{C}_{\max} - f_T - P, & \text{otherwise} \end{cases} \tag{11}$$

The parameter C_{max} should be chosen that the fitness value for most, if not all, possible solutions are positive. Suitable values are chosen by trial and error, if the fitness value for a chromosome should work out to be negative, a small value is assigned to it a small probability of survival.

IV. Results

The aforementioned approach was applied on the planar bipedal walking robot, illustrated in Fig. 2, with the geometrical and inertial properties as shown in Table 1. All the links are modelled based on uniform mass distribution. This is to consider the sensitivity for the real-implementation. Fifth-order TFS i.e. with $n=5$, are used for the trajectories described in (6) and (7). To pursue a human-like walking with straight knee occurring at stance leg, we set $c_k=0$. Table 2 shows the parameter set for GA initialization and the coefficients for the objective and penalty functions. The proposed approach was applied to the two walking examples shown in Table 3.

TABLE I
GEOMETRICAL AND INERTIAL PROPERTIES OF THE BIPED

Link	Mass *(kg)*	Moment of inertia ($kg.m^2$) about *y* axis	Size (*m*) ($L\times W\times H$)or ($L\times d$)
Trunk	12	0.97	$0.2\times0.2\times0.45$
Thigh	6	0.0456	0.3×0.02
Shank	5	0.038	0.3×0.02
Foot	1	0.03012	$0.3\times0.1\times0.02$ [a]

[a] 2/3 of the length of the foot is before the ankle joint, and 1/3 of the length is behind the ankle joint.

TABLE II
PARAMETERS OF GA, OBJECTIVE AND PENALTY FUNCTIONS

Chromosome representation	real-valued GA
Initial population M	150
Generation number T	250
Crossover	heuristic crossover[12] arithmetic crossover[12] simple crossover[12]
Mutation	multi-non-uniform mutation[12]; non-uniform mutation[12] boundary mutation[12] uniform mutation[12]
Weights (objectives)	$w_i = [15\quad 50\quad 10\quad 40]$
Weights (penalty)	$p_i = [15\quad 20\quad 20\quad 30\quad 100\quad 70]$

TABLE III
TWO WALKING EXAMPLES

Flat Terrain Walking	Example 1	Example 2
Step length	0.33m	0.28m
Walking speed	0.45m/s	0.37m/s

The chromosome solutions in the format of [A_i, B_i, C_i, c_h, c_k, t_1, t_2], x_1 and x_2, are obtained for the two examples, respectively.

$$x_1 = [0.277\ \ -0.087\ \ 0.022\ \ -0.008\ \ -0.000\ \ -0.397\ \ -0.118$$
$$-0.024\ \ -0.017\ \ -0.006\ \ 0.457\ \ 0.200\ \ -0.038\ \ -0.077$$
$$-0.046\ \ -0.036\ \ 0.000\ \ 0.050\ \ 0.44]$$
$$x_2 = [0.238\ \ -0.057\ \ 0.012\ \ -0.004\ \ 0.0003\ \ -0.356\ \ -0.043$$
$$0.042\ \ 0.004\ \ -0.003\ \ 0.469\ \ 0.074\ \ -0.050\ \ 0.008$$
$$-0.017\ \ -0.021\ \ 0.000\ \ 0.065\ \ 0.47]$$

For the above two examples, ω_h = 4.28 rad/s and ω_h = 4.15 rad/s, respectively. The value of C_{max} used for the GA is 1400 and the fitness function values of the two solutions are 1287 and 1309, respectively, indicating good optimization outcomes. Furthermore, the effect of the 5th order coefficient is relatively small compared with the 1st order coefficient. Therefore, 5th order TFS is sufficient.

A. Example 1

For the gait generated for Example 1, Fig. 4 shows the ZMP trajectory for one walking cycle. Fig. 5 shows the position of the centroid of the trunk versus time. In both figures the reference coordinate frame is *OXYZ*.

Fig. 4 shows that the ZMP trajectory is always within the footprint, even though the specified step length is not small and the speed is not slow. From Fig. 5, it can be seen that a regular walking speed of 0.45 m/s has also been achieved. Fig. 6 to Fig. 9 further illustrate the motion obtained. From Fig. 7, which shows the difference of the vertical length projections, f_l and f_r, of the left and right legs, respectively, the instant of leg strike is observed at t_d=0.44s. This is equal to t_2 in the solution x_1 given earlier. From the resulted motion data, the leg strike velocity is v_x=0.15m/s, v_z=-0.2m/s. This value has been confirmed to be acceptable by the results of subsequent dynamic simulations. The actual step length of 0.32m is also very close to the target step length.

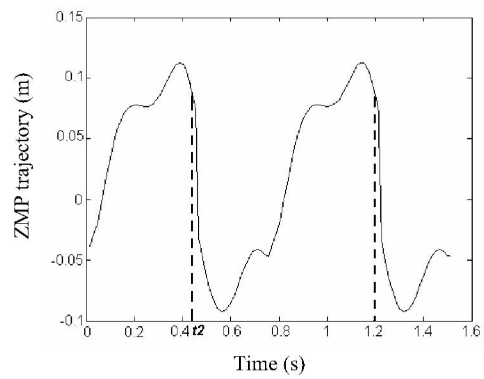

Fig. 4 ZMP Trajectory.
Dash lines indicate the switch of ZMP frame to new stance leg.

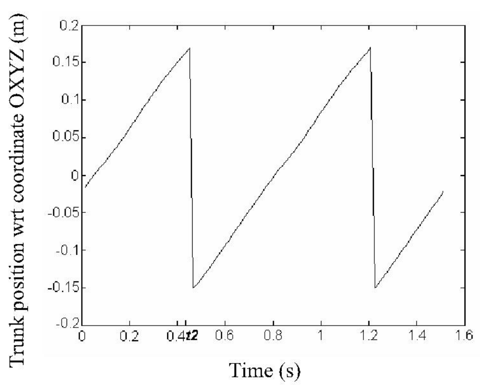

Fig. 5 Trunk position vs Time.

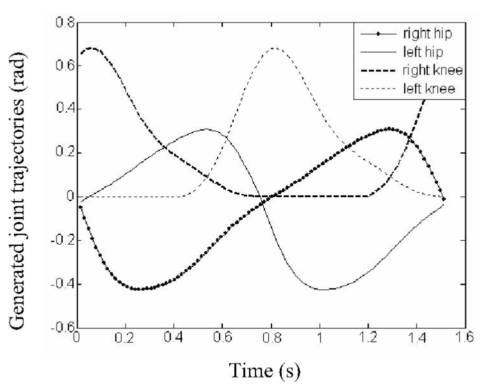

Fig. 6 Joints' trajectories.

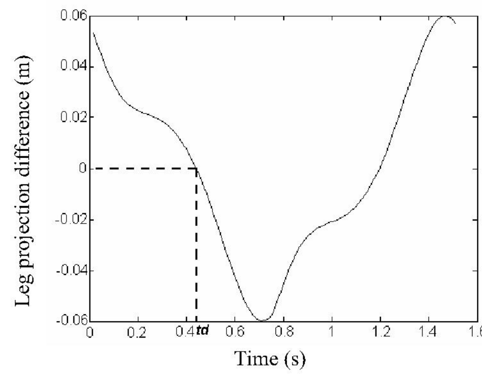

Fig. 7 f_l - f_r (m) versus time (refer to Fig. 8 with h_r = 0)

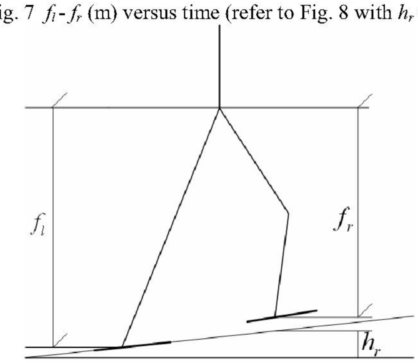

Fig. 8 Leg projection difference illustration.

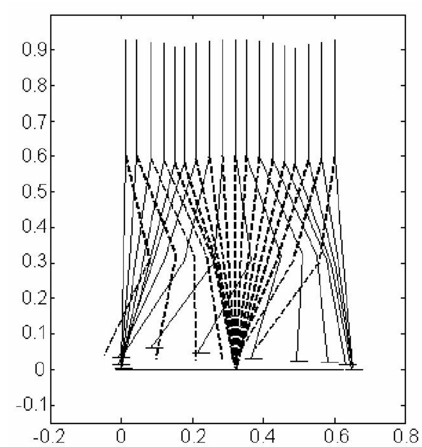

Fig. 9 Stick diagram of the motion.

B. *Example 2*

Similarly, the solution x_2 is applied. Fig. 10 shows the generated joints' trajectories. From the motion data, the ZMP trajectory is constrained within [-0.06m 0.06m] which is even tighter than that in Example 1. The leg strike velocity is v_x=0.12m/s and v_z=-0.224m/s. The step length is 0.27m and the walking speed is kept around 0.36m/s. The first leg strike instant is t_d=0.47s, which is equal to t_2 in the GA solution. The resulting walking pattern is shown in Fig. 11.

C. *General comments*

The results obtained for the two examples confirm that the proposed approach can be used to synthesise reliable trajectories for stable human-like walking gaits. This approach can also be used to generate gaits for walking on slopes and stairs [13].

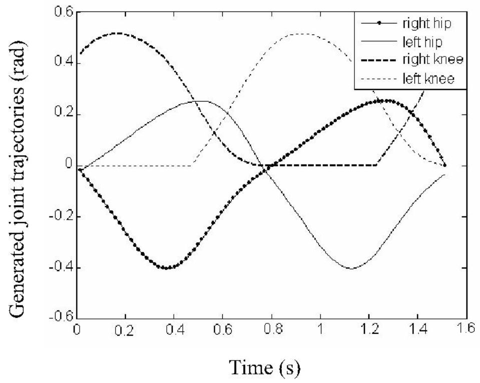

Fig. 10 Joints' trajectories

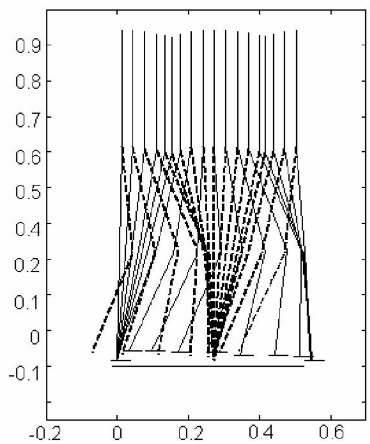

Fig. 11 Stick diagram of the planned motion.

V. Dynamic Simulations

The dynamic motions of the robot (with parameters shown in Table 1) using the trajectories generated in previous section were simulated in Yobotics! [14]. For all the joints, PD control was applied to track the generated joints' trajectories.

The resulting dynamic motions for Examples 1 and 2 are shown in Fig. 12 and Fig. 13, respectively. In both examples, the maximum tracking error was 3.4° for the hip joint and 2.6° for the knee joint. It was compared to 45°, and 40° of motion range for hip and knee respectively. The vertical force impact in the beginning of support exchange was about 130% of the robot weight and the required ground friction coefficient was less than 0.25. These results showed the feasibility of the generated walking pattern.

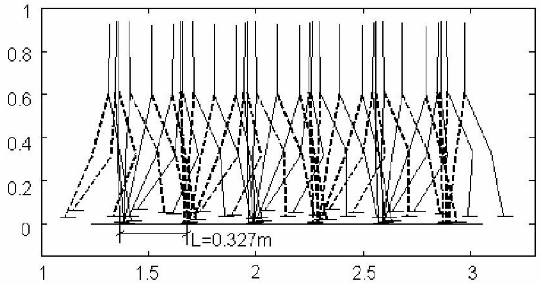

Fig. 12 Stick diagram of the dynamic motion of the robot for Example 1.

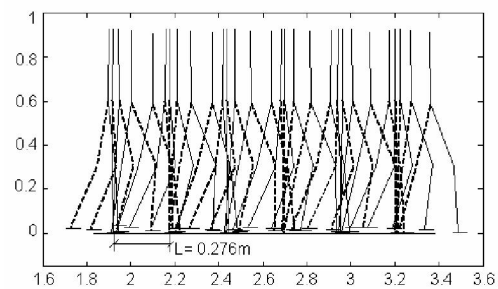

Fig. 13 Stick diagram of the dynamic motion of the robot for Example 2.

VI. Conclusion

This paper presents the GAOFSF approach and demonstrates that it is efficient and effective in generating optimal and reliable bipedal motion. It is not limited by either robot's geometrical and inertial properties or motion requirements. The proposed GAOFSF can also be applied to other walking situations including the 3D motions. Due to the inherent characteristics of TFS joint trajectory formulation, GAOFSF allows the walking rhythm, step-length and walking pattern to be online adjusted. The details of the online adjustments will be presented in our subsequent paper.

References

[1] C.M.Chew and G.A. Pratt, "Dynamic bipedal walking assisted by learning", *Robotica* 20, pp. 477-491, 2002.

[2] S. Kajita, O. Matsumoto and M. Saigo, "Real-time 3D walking pattern generation for a biped robot with telescopic legs", *Proc. of the IEEE Int. Conf. on Robotics & Automation, Seoul, Korea.* May 21-26, 2001.

[3] J.Nakanishi, J.Morimoto, G.Endo, G.Cheng, S.Schaal and M.Kawato, "Learning from demonstration and adaptation of biped locomotion", *Robotics & Autonomous Systems* 47, pp.79-91, 2004.

[4] S.Kagami, "A Fast Dynamically Equilibrated Walking Trajectory Generation Method of Humanoid Robot", *Autonomous Robots* 12, pp.71–82, 2002.

[5] H.Kimura and A.H.Cohen, "Biologically Inspired Adaptive Dynamic Walking of a Quadruped Robot", *Proc. of the 8th Int. Conf. on the Simulation of Adaptive Behaviour, Los Angeles,* pp. 201-210, Jul. 2004.

[6] T.Zielinska, "Biological Inspirations in Robotics: Motion Planning" *Proc. of 4th Asian Conf. on Industrial Automation and Robotics,* Bangkok, Thailand, 2005.

[7] "Biomechanics of motion", CISM COURSES AND LECTURES, *International centre for mechanical science.* No. 263, pp. 79-129. 1980.

[8] J.Perry MD, "Gait Analysis: Normal and Pathological Function" ISBN 1-55642-192-3, 1992.

[9] R.K.Rastogi, "Task oriented stable gait synthesis in biped locomotion", *Master thesis*, Indian Institute of Information Technology, Allahabad, May, 2005.

[10] M.Vukobratovic, B.Borovac, D.Surla and D.Stokic, "Biped Locomotion—Dynamics, Stability, Control and Application", *Springer*, 1990.

[11] P.Sardain and G.Bessonnet, "Forces Acting on a Biped Robot. Center of Pressure—Zero Moment Point", *IEEE Trans. on Systems, Man, & Cybernetics—part A:systems and humans,* vol. 34, No. 5, Sep. 2004.

[12] Z.Michalewicz, "Genetic Algorithms+Data Structures =Evolution Programs." *AI Series.Springer-Verlag,* 1994.

[13] L.Yang, C.M.Chew, T.Zielinska, & A. N. Poo, "Reliable and Adjustable Biped Gait Generation for Slopes Using a GA Optimized Fourier Series Formulation", *CISM courses and lectures, RoManSy 16, Robot Design, Dynamics, and Control, SpringerWien, NewYork*, vol. 487, 2006, pp.187-194.

[14] Users Guide for Yobotics!, *Yobotics, Inc*. 2000-2003.