1 Deriving the full-headwind maximum speed from the given constraints

We assume the cyclist sustains a fixed maximum mechanical power output $P_{\max}$ (maximum sustainable mechanical power), and that aerodynamic drag is the only resistance in this subsection (rolling resistance is added later). The aerodynamic drag parameter is denoted by k (a constant depending on ρ, C_d, and A_f), and the cyclist's ground velocity vector is $\mathbf{v}$ with magnitude $v = |\mathbf{v}|$ (the cyclist's ground speed). Let $\mathbf{w}$ be the wind velocity vector with magnitude $W = |\mathbf{w}|$ (the wind speed), and define the relative air velocity of the cyclist by

$$\mathbf{u} = \mathbf{v} - \mathbf{w}, \qquad |\mathbf{u}| \text{ is the relative air speed.}$$

Under these definitions the maximum power requirement can be written as

$$P_{\max} = k|\mathbf{u}|\,(\mathbf{u} \cdot \mathbf{v}). \tag{1}$$

For a *pure tailwind or headwind*, the wind is colinear with the cyclist's motion, so $\mathbf{u}$ is parallel to $\mathbf{v}$ and therefore $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,v$. Hence the power reduces to

$$P_{\max} = k|\mathbf{u}|^2\,v. \tag{2}$$

Let θ be the angle between the wind direction and the cyclist's direction of travel, with $\theta = 0$ for a tailwind and $\theta = \pi$ for a headwind. Then the colinear special cases for the relative air speed are

$$|\mathbf{u}| = \begin{cases} v - W, & \theta = 0 \quad \text{(tailwind)}, \\ v, & W = 0 \quad \text{(no wind)}, \\ v + W, & \theta = \pi \quad \text{(headwind)}. \end{cases}$$

Step 1: No wind (top speed $v_0 = 20$ mph)

Let v_0 denote the cyclist's top speed in still air (no wind); the constraint gives $v_0 = 20$ mph. When $W = 0$, we have $|\mathbf{u}| = v_0$, so Eq. (2) gives

$$P_{\max} = k v_0^3. \tag{3}$$

Step 2: Full tailwind (top speed $v_1 = 25$ mph) determines W

Let v_1 denote the cyclist's top speed with a full tailwind; the constraint gives $v_1 = 25$ mph. For a full tailwind, $|\mathbf{u}| = v_1 - W$, so Eq. (2) gives

$$P_{\max} = k(v_1 - W)^2\,v_1. \tag{4}$$

Equating Eqs. (3) and (4) and cancelling k yields

$$v_0^3 = v_1(v_1 - W)^2. \tag{5}$$

Solving Eq. (5) for the wind speed W gives

$$W = v_1 - \sqrt{\frac{v_0^3}{v_1}}. \tag{6}$$

Substituting $v_0 = 20$ mph and $v_1 = 25$ mph gives

$$W = 25 - \sqrt{\frac{20^3}{25}} = 25 - \sqrt{320} = 25 - 8\sqrt{5} \approx 7.11 \text{ mph}.$$

Step 3: Full headwind maximum speed

Let v_h denote the cyclist's maximum speed in a full headwind. For a full headwind, $|\mathbf{u}| = v_h + W$, so Eq. (2) gives

$$P_{\max} = k(v_h + W)^2\, v_h. \tag{7}$$

Equating Eq. (3) with (7) and cancelling k yields the headwind top-speed condition

$$v_h(v_h + W)^2 = v_0^3. \tag{8}$$

Expanding Eq. (8) makes the cubic form explicit:

$$v_h^3 + 2Wv_h^2 + W^2 v_h - v_0^3 = 0. \tag{9}$$

Substituting $v_0 = 20$ mph (so $v_0^3 = 8000$) and $W \approx 7.11$ mph gives

$$v_h(v_h + 7.11)^2 = 8000,$$

whose positive real solution (found numerically) is

$$v_h \approx 15.56 \text{ mph}. \tag{10}$$

Therefore, the cyclist's maximum speed in a full headwind is approximately 15.6 mph under the wind-only model.