

Quantitative Industrial Ecology

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Abstract—The scope and scale of industrialization apparently have progressed to a stage where the base of the engineering sciences must now be expanded beyond the realm of distinct industrial and agricultural technologies to the realm of industrial ecosystems, in which discrete technologies are characterized as design and innovation parameters in organized networks of economic production and consumption processes. The presumed objective of industrial ecosystem design and innovation is to efficiently and economically deploy natural and industrial resources with minimal risks to the integrity of significant natural processes of the biosphere. Mathematical tools for such an expansion derive from the premise that, for given intervals of time, industrial and natural ecosystems and their interactions can be characterized by hierarchies of bounded networks of biotic and abiotic resource acquisition, conversion, and transfer processes. The hierarchies illustrated in this paper range from modular networks of production technologies in manufacturing and refining to food chains and sequential products to biological communities and industrial production networks to product life cycles and beyond.

The ecology of each resource conversion process within a bounded network of processes is characterized mathematically in terms of the principles of material and energy balance, with energy as one of many biotic and abiotic enabling factors of resource acquisition, conversion, and transfer that are ecologically quantified and monetarily priced. Design equations are developed that map the ecologies of bounded networks of ecological processes into the ecology of the network at its boundary, thereby providing operational procedures for mathematically defining hierarchical network structures. At the lowest level of ecological organization, all constituent network processes are observable first-order natural processes or engineered technologies.

The economics of a given industrial enterprise are evaluated as an explicit mathematical function of its network of production technologies, its ecological organization, and its ecological boundary prices; thereby providing procedures for the coordinated design, management, and accounting of its technological, ecological, and economic dimensions at hierarchical levels of organization from factory floors and farms to corporate levels and beyond. Sustainable industrialization is shown to be achievable through on-line, risk-control pricing mechanisms.

Index Terms—Agro-ecosystems, ecological economics, ecological engineering, ecological modeling, economic production model, ecosystem ecology, environmental management, environmental policy formulation, industrial ecology, mathematical ecology, resource management, strategic planning, sustainable industrialization, sustainable technologies.

I. INTRODUCTION

HUMAN populations and the technological scope and scale of industrialization continue to expand at rates far beyond historical precedents. Until recently, the impacts of the engineered technologies in agriculture, industry, transportation, and urbanization on biotic processes, including those of human populations, have been local and largely manageable by reengineering and retrofitting, without excessive or irreversible impacts on regional economies and their natural environment. Regional and global impacts are now being identified even though more than three fourths of the planet is covered with water, ice, extreme climates, or terrain that is, as yet, outside the realm of intense technological or socioeconomic exploitation. Also, many of the populated regions of the world are, as yet, underdeveloped, compared to the more technologically advanced regions.

As industrialization continues, it must do so with increased preconstruction understanding of both its ecological and its economic dimensions at local, regional, and global scales of ecological organization as well as at enterprise, corporate, and industry levels of economic organization. The risks of disrupting the homeostatic and functional stability of large-scale natural processes of the biosphere and its abiotic reservoirs, such as the various layers of the atmosphere, apparently can increase geometrically under the compounding factors of geometric growth in human population, industrialization, and urbanization [5], [8], [20]–[22].

Improved ecological perspectives in the design and deployment of processing technologies certainly can help reduce some of the risks [4], as can increased emphasis on low-carbon energy fuels and life-cycle design of products [19]. Also, technological processing of residuals can mitigate or even eliminate some of this growing ecological risk [1], [2]. Thus, neither developed nor underdeveloped nations need adopt or expand high-risk technologies to reap the economic benefits of industrialization and the declines in population growth rates believed to be associated with technological development and improved economic security, even though there is little agreement on the causal relationships [10].

While scientific discoveries and technological innovations are necessary, they are not always sufficient, even with technically successful local and regional demonstrations, to guarantee economically attractive products or their national and global adaptations and adoptions [6]. Economic acceptance and geographic dispersion of technological innovations are strongly conditioned by the demographics and cultural traditions of population groups around the world [17] and how they relate systemically to existing technologies of production, consumption, and distribution. What is needed, among other

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things, are analytical and computational tools for assessing alternative technologies in the context of alternative historically and culturally conditioned ecologies and economies.

New and improved materials, products and production technologies, genetic engineering, and advanced information technologies and systems are being addressed in public and private engineering research and development programs as well as in government laboratories and regulatory programs around the world. Yet, useful analytical tools for quantitatively relating the economic dimensions of industrial enterprises and economies to their ecological dimensions remains a major weakness [5]. Engineers, in particular, need quantitative ecological design tools for assessing both ecological and monetary measures of industrial ecosystem performance for alternative processing technologies, network organizations, and price structures. Operations managers need tools for system-wide control of processing residuals, for handling logistics of materials, and for controlling the quality and cost of products. Ecologists need improved quantitative paradigms to better characterize natural ecosystems as operating components of industrial production systems and as constituents of the planet's evolving industrialized biosphere. Corporate managers need new ecologically based financial accounting formats to provide economists, sociologists, politicians, and others with new quantitative information on the relationships between ecological and economic dimensions of industrialized ecosystems, so that policy formulation and analysis efforts can be better informed. Yet, the subject of quantitative industrial ecology, for the most part, has fallen between the disciplinary chairs of academia and the agencies of government, and it commands little attention from within the business world.

The purpose of this paper is to present a defining mathematical theory of industrial ecology as a quantitative engineering discipline for analyzing, designing, and managing networks of technological processes at hierarchical levels of industrial and economic organization. The discipline is presented as a mathematical foundation for deploying modern information technologies and software packages as tools for evaluating the basic ecological and economic dimensions of economic production and consumption systems. Such tools are essential to identify alternative processing technologies, network organizations, and price structures in comprehensive ecological design and operations management.

The authors characterize industrial ecosystems as hierarchical networks of biotic and abiotic conversions on the material states of resources and their spatial locations. In principle, the network hierarchies can extend from first-order engineered processes in factories and refineries and quasinatural processes on farms and managed landscapes to the multiorder processes that characterize industrial enterprises, political economies, and the biosphere. The ecology of each process in the hierarchy is derived from the ecology of a bounded network of lower order, antecedent processes, beginning with observable first-order natural processes and engineered technologies. At enterprise levels of economic organization, monetary measures of ecological performance are defined in terms of the network ecology and prices assigned to ecological variables at network boundaries by economic markets and other pricing

mechanisms. These relationships, called the ecological design equations, provide a mathematical framework for evaluating alternative feed stocks, products, production technologies, systems of production, and product life cycles to assess the extent to which they are both ecologically and economically feasible in the context of given ecological and economic environments. They provide a mathematical foundation for coordinated, system-wide operations management and management accounting. They also provide the foundations for synthesizing on-line, risk-control pricing mechanisms in political economies essential to ecologically sustainable industrialization.

II. ECOLOGICAL PROCESSES AND NETWORKS

Mass and energy have long been recognized as fundamental aspects of the universe. At any given point in time, mass and energy may combine in various physical forms, generally referred to here as abiotic materials, objects, or systems. Scientific discoveries of the last century or so in astronomy, physics, and chemistry have built a substantial body of theory and natural laws on how abiotic material objects and energy are formed and interact. These range from fundamental particles and forms of energy postulated to have existed at the time of the big bang to the objects, energy, celestial motions, and the lifecycles of cosmic dust clouds, stars, quasars, galaxies, black holes, comets, and satellites. Theories and natural laws pertaining to the interactions among the various forms of mass and energy provide the foundations for analytical tools to characterize the natural abiotic processes of the earth, generally identified as inorganic chemistry, meteorology, geology, hydrology, and physical limnology and oceanography. But, abiotic processes represent only a part of the material-energy dynamics on, or near, the surface of the earth.

It is postulated that about three billion years ago, abiotic processes lead to the emergence of self-reproducing, carbon-based macromolecules, their subsequent organization into multimolecules and eventually to the living and evolving populations of cellular organisms [7], [14]. Living cellular and multicellular populations added an entirely new class of organized materials and processes on the planet earth. These new classes of transformations on the material states of resources are called biotic processes, and they add to the earth an entirely new class of variables with interrelationships among themselves and with their abiotic environment, generally identified as the atmosphere, hydrosphere, and geosphere. The new class of interrelationships include, in particular, the utilization of chemical and radiant energy and structured materials to aggregate resources in support of living, growing, and evolving populations of organisms. It also caused the release of products and residuals on scales and durations sufficient to redistribute and convert abiotic materials and objects on the earth into an interacting biosphere of biotic and abiotic processes with many entirely new characteristics.

A. Ecological Processes

For given intervals of ecological time, a given region of the biosphere can be characterized mathematically in terms of the equations of material and energy balance at a closed analytical

boundary circumscribing the region. Let the column vector \mathbf{y}_r , called the material *subjects* of resource conversion, represent the resource and residual flows across the process boundary. Let the column vectors \mathbf{y}_c and \mathbf{y}_o , called the material *objects* of resource conversion, represent the time rates at which material objects accumulate within the boundary and the time rates at which material objects are transferred across the boundary, respectively. Let the *ecological state* of the process be defined as vector ψ , satisfying the differential equation of ecological state

$$\dot{\psi} = \mathbf{B}\psi + \mathbf{y}_c, \quad \text{for } \psi \geq \psi_m \quad (1)$$

where \mathbf{B} is called the characteristic matrix of the process and ψ_m its minimum functional and sustainable ecological state.

The equations of resource conversion for the process are taken as

$$\mathbf{y}_r = \mathbf{C}\dot{\psi} + \mathbf{K}\mathbf{y}_o, \quad \text{for } \dot{\psi} \geq 0 \quad (2)$$

where the rectangular matrices \mathbf{C} and \mathbf{K} are called the coefficients of resource conversion for ecological growth and for ecological production, respectively.

From (1) and (2), the equations of resource conversion can also be written as

$$\mathbf{y}_r = \mathbf{C}\mathbf{B}\psi + \mathbf{C}\mathbf{y}_c + \mathbf{K}\mathbf{y}_o. \quad (3)$$

The vectors \mathbf{y}_c and \mathbf{y}_o can be characterized as the flow rates of ecological capital (*eco-capital*) and ecological products, respectively. The vectors $\mathbf{C}\mathbf{y}_c$ and $\mathbf{K}\mathbf{y}_o$ identify the flows of resources and residuals required for eco-capital formation and for product formation, respectively. The vector $\mathbf{B}\psi$ represents the time rate at which the ecological state of abiotic processes decreases as a result of functional failures or the time rate at which the ecological state of biotic processes increases as a result of self-reproduction, depending on whether the eigenvalues of \mathbf{B} are negative or positive. The vector $\mathbf{C}\mathbf{B}\psi$ identifies the resources and residuals for the replacement of failed abiotic objects of ecological state or for biotic self-reproduction, as the case may be.

Flow rates for the objects and subjects of resource conversion are always represented by positive real numbers, with residuals representing outflows from the defining boundary. The ecological state ψ is represented by positive real numbers, but the accumulation rate $\dot{\psi}$ may assume both positive and negative values. When resources are formulated as inflows and products as outflows, the process is identified as a production process if $\mathbf{y}_o \neq 0$ and as a consumption process if $\mathbf{y}_o \equiv 0$. When products are formulated as inflows and resources as outflows, the process is identified as a reduction process. The mathematics is the same for both orientations.

The objects of ecological states for abiotic industrial production processes can be quantified as inventories of process-specific production equipment, such as computers, milling machines, and robots, or function-specific real estate, such as factories and refineries, depending upon the level of ecological organization. In such systems, ψ_m represents the minimum accumulation of technological components required to sustain the process, thereby establishing a lower bound on the scope

and scale of the defining analytical boundary. Whatever the scale, the characteristic matrix \mathbf{B} typically has small negative values characteristic of low-component replacement rates. Further, since the form and substance of abiotic products typically differ fundamentally from abiotic components of ecological state, the ecological state of the process must be established before production begins. Thus, the equations of resource conversion for abiotic production processes can be subdivided into an eco-capital formation phase and a production phase, as characterized by the first and last term, respectively, in (2). When the product vector \mathbf{y}_o is of first order, the coefficient matrix \mathbf{K} reduces to a column vector. The coefficients of resource conversion for first-order processes are generally observable from field measurements, but they are not observable for multiorder processes.

The objects of ecological state for biotic processes in agricultural and natural resource production can be quantified in terms of species-specific biomass or populations. In this instance, the coefficients of the matrix \mathbf{B} represent the reproduction parameters of the population ψ . For example, when a population of animals is subdivided into juveniles, females, and males, the entries of the coefficients matrix \mathbf{B} identify birth rates, sex ratios, and survival rates. In biotic processes, ψ_m represents the minimum population of biological organisms that can be sustained, both genetically and nutritionally, in a healthy living environment, thereby also establishing a lower bound on the scope and scale of the defining analytical boundary for the process. The characteristic matrix \mathbf{B} has one or more positive eigenvalues determined, in part, by the quality of the living environment. When the products of biotic process are identical in form and substance to the objects of ecological states, as in live stock production, for example, the process is similar to natural predation. In contrast, biotic processes, such as dairy and egg production, are ecologically akin to natural parasitism. The equations of resource conversion for predatory biotic processes represent a special case in which \mathbf{K} is an empty set and $\mathbf{y}_o \equiv \mathbf{y}_c$.

The enabling factors of resource conversion are quantified in terms of energy, species-specific populations of biological organisms, and skill-specific work forces with access to information and automation technologies, such as computers and automated process machines, telecommunication networks, and database systems. With such information technologies, entire networks of technological processes can be mobilized as enabling factors of resource conversion. The temporal rates at which enabling factors are deployed in a given resource conversion process are represented by row vector \mathbf{e} , called the *enabling power* of resource conversion. Deployment rates for abiotic energy, called abiotic enabling power, are typically measured in watts or in kilowatt-hours per day. Deployment rates for general-purpose machines, such as computers, robots, programmable processing machines, and communication and transportation networks, are identified as machine power and quantified in terms of process-specific machines per hour or network-specific access per hour, each with explicit external abiotic power requirements. Deployment rates for biological organisms, here identified as biotic power, can be quantified in terms of soil-specific hectares of variety-specific green plants

per year, populations of breed-specific animals per year, and populations of expertise-specific persons per hour or day, each complete with its implicit internal biotic source of energy.

The vector of enabling power of resource conversion \mathbf{e} , for a given ecological process, is expressed as the sum of two vectors: the power for ecological growth \mathbf{e}_c and the power of ecological production \mathbf{e}_p , which are expressed as

$$\mathbf{e} = \mathbf{e}_c + \mathbf{e}_p = \dot{\psi}' G(\dot{\psi}) + \mathbf{y}_o' \mathbf{F}(\mathbf{y}_o) \quad (4)$$

where $G(\dot{\psi})$ and $\mathbf{F}(\mathbf{y}_o)$ are rectangular matrices with rows $\mathbf{g}_i(\dot{\psi}_i)$ and $\mathbf{f}_i(\mathbf{y}_{oi})$, $i = 1, 2, \dots$ representing the enabling power deployed per object of conversion.

Enabling power per object of conversion is here defined as factor-specific *time costs* of resource conversion, as distinct from monetary costs of resource conversion. Abiotic energy time costs are measured in kilowatts per object of conversion. Biotic and machine time costs are measured in hectares of variety-specific plants/year per object, breed-specific animals/day per object, expertise-specific persons/hour per object, or network-specific access and process-specific machines/hour per object of conversion, as appropriate.

Clearly, the form and substance of the objects of conversion \mathbf{y}_c and \mathbf{y}_o depend on genetic and learned ecological information encoded in biological organisms, the creative, manipulative, and managerial skills of humans, and the technological information engineers encode into computers, robots, telecommunication networks, processing machines, and factory layouts, for example. In this respect, both the enabling factors of resource conversion and the products of resource conversion can be, and sometimes are, characterized as carriers of ecological and technological information.

For first-order resource conversion processes, (4) reduces to a vector equation with two vectors of power, each of which is directly observable from field measurements

$$\mathbf{e}_c = \mathbf{e}_{co}(\psi) + \dot{\psi} \mathbf{g}_m(\dot{\psi}) = \mathbf{e}_{co} + \mathbf{g}_1 \dot{\psi} + \mathbf{g}_2 \dot{\psi}^2 + \dots \quad (4g)$$

$$\mathbf{e}_p = \mathbf{e}_{po} + \mathbf{y}_o \mathbf{f}_m(\mathbf{y}_o) = \mathbf{e}_{po} + \mathbf{f}_1 \mathbf{y}_o + \mathbf{f}_2 \mathbf{y}_o^2 + \dots \quad (4p)$$

where row vector $\mathbf{e}_{co}(\psi)$ represents the state-dependent, sustaining power of ecological growth and \mathbf{e}_{po} represents the maintenance power of production. Row vectors $\dot{\psi} \mathbf{g}_m(\dot{\psi})$ and $\mathbf{y}_o \mathbf{f}_m(\mathbf{y}_o)$ are called the marginal power for ecological growth and replacement and for production, respectively. Row vectors $\mathbf{g}_m(\dot{\psi})$ and $\mathbf{f}_m(\mathbf{y}_o)$ represent the marginal time cost for ecological growth and replacement and for production, respectively. Vectors \mathbf{g}_1 and \mathbf{g}_2 and vectors \mathbf{f}_1 and \mathbf{f}_2 are appropriately referred to as first- and second-order marginal time costs. As shown later, the enabling power for multiorder conversion processes is computed from the enabling power for precursor networks of first-order observable processes.

Together, the equations of ecological state given in (1), the equations of resource conversion given in (2), and the enabling power of resource conversion given in (4) are said to characterize the ecology of the process at a given point in time. Since the mathematical forms of these equations are process independent, all the information required to uniquely characterize the ecology of a given process, at a given point in time, is implicit in tables of numbers; namely, a) the

coefficients of resource conversion \mathbf{C} and \mathbf{K} , b) the vector of ecological state ψ and the characteristic matrix \mathbf{B} , and c) the matrices of time costs $\mathbf{G}(\psi)$ and $\mathbf{F}(\mathbf{y}_o)$.

B. The Economics of Ecological Processes

Let the variables in the equations characterizing the ecology of a given resource conversion process be represented by the vector $\mathbf{v}' = [\mathbf{y}_o' \mathbf{y}_c' \mathbf{e}' \dot{\psi}']$, identified as the ecological *boundary variables* of the process. Let a vector $\mathbf{p}'_v = [\mathbf{p}'_o \mathbf{p}'_r \mathbf{p}'_e \mathbf{p}'_\psi]$, called the ecological *boundary prices*, represent monetary values assigned to the corresponding boundary variables at a given point in time (however determined). Numerically, the vectors \mathbf{v}' and \mathbf{p}'_v , respectively, represent the ecological and the economic dimensions of the ecological process, and the scalar product $\mathbf{p}'_v \mathbf{v} = \mathbf{v}' \mathbf{p}_v$ defines the *ecological economics* of the process with four scalar components

$$\mathbf{p}'_v \mathbf{v} = \mathbf{p}'_o \mathbf{y}_o + \mathbf{p}'_r \mathbf{y}_r + \mathbf{p}'_e \mathbf{e}' + \mathbf{p}'_\psi \dot{\psi} \quad (5)$$

where $\mathbf{p}'_o \mathbf{y}_o$ represents the temporal rate of economic revenue from the objects of production, $\mathbf{p}'_r \mathbf{y}_r$ and $\mathbf{p}'_e \mathbf{e}' = \mathbf{e} \mathbf{p}_e$, respectively, represent the temporal rates of economic expenditure for resources and residuals and for enabling factors of resource acquisition, conversion, and transport, and $\mathbf{p}'_\psi \dot{\psi}$ represents the temporal rate of capital investment in the ecological state of the process. Economic returns are represented here by positive numbers, with expenditures and investments represented as negative numbers. For a given ecological state $\dot{\psi} = 0$, the ecological economics in (5) are defined as the *ecological cash flow* of the process.

The concept of economic value added to resources by a given process is defined in terms of ecological cash flow and the capital invested in developing and maintaining the ecological state of the process. For engineered processes where all components of the state vector ψ are dedicated to the production of a single product, the gross capital invested (Ci) is computed as

$$\text{Ci} = \int_{t_o}^{t_1} \mathbf{p}'_\psi \dot{\psi} dt \quad (6)$$

where $\dot{\psi}$ is defined in (1) and $(t_1 - t_o)$, is the time period of investment. The capital investment is amortized over time to the stream of products according a specific set of extra-ecological accounting rules defined by political economies [11]. In “flexible” industrial and agricultural operations, the ecological state for a given process may be deployed sequentially in the production of a variety of different products, with the duration of deployment depending on the product variety [9]. In this instance, the components of ecological state are characterized as enabling factors of resource conversion with machine-specific time costs entered as components of the enabling vector \mathbf{e} . The corresponding amortized capital costs are entered as components of the price vector \mathbf{p}_ψ , along with other price factors. Thus, amortized capital investments in engineered processes can be allocated among multiple objects of production on the basis of processing time.

Other monetary measures of ecological performance for engineered processes, such as return on investment and return

on capital, derive from the components of ecological economics defined in (5), the concept of value added, and various other extra-ecological factors of the political economies, such as tax and interest rates. However, economic valuations of ecological state vectors is not operationally possible for self-reproducing natural biotic processes that, in fact, have evolved over evolutionary time at no financial costs to humans. In principle, such natural biotic processes can be sustained as components of industrialized ecosystems by pricing the natural resources harvested from them in direct relationship to the temporal rate of harvest and the associated risks of modifying ecological state of the processes, as discussed later.

The boundary vector \mathbf{v} of a given process includes only those ecological variables of resource acquisition, conversion, and transfer that are numerically significant. However, for economic coherence, all numerically significant ecological variables must be priced as components of the vector \mathbf{p}_v . Boundary variables that provide ecological linkages between processes are potentially subject to constraint and priced by interenterprise markets, as discussed later. Boundary variables that are not so constrained are generally not numerically significant, as is often the case for fresh air, usable water, and ubiquitous solar radiation. However, products and residuals dispersed into the atmosphere, hydrosphere, and geosphere are numerically significant when they induce risks to other process. Principles of risk-control pricing for harvested natural resources and for products and residuals dispersed from engineered processes into the natural environment are developed later as principles of sustainable industrialization.

C. Networks of Ecological Processes

Sets of ecological processes, as defined by multiple closed analytical boundaries within the biosphere, are operationally constrained through, among other things, the exchange of abiotic and biotic materials (products, resources, and residuals) according to the principles of interprocess material continuity. Material resource flows into a given process j are typically products of precursor resource conversion processes, with time costs of resource conversion and transfer allocated to them. The time costs associated with incoming resources are reallocated to the products and residuals of the process j , according to the principles of energy balance at the boundary of process j . As objects of production are transferred from one process to another, the associated time costs accumulate into what are here called *ecological time costs*, abbreviated as *eco-costs*. Row vectors of eco-costs \mathbf{x}_{ci} and \mathbf{x}_{oi} are mathematically defined for all objects of ecological state, \mathbf{y}_{ci} and all objects of production, \mathbf{y}_{oi} , according to matrix equations

$$\mathbf{X}_c = \mathbf{G}(\dot{\psi}) - \mathbf{C}'\mathbf{X}_r \quad (7c)$$

$$\mathbf{X}_o = \mathbf{F}(\mathbf{y}_o) - \mathbf{K}'\mathbf{X}_r \quad (7o)$$

where \mathbf{X}_c and \mathbf{X}_o are rectangular matrices with columns corresponding to enabling factors of resource conversion and rows corresponding to eco-costs for objects of ecological state and objects of production, respectively. The rows of the rectangular matrix \mathbf{X}_r represent the eco-costs of the resources and residuals at the boundary of the process.

It is easy to show that for the definitions in (7), the scaler product of material flows and the eco-costs at the boundaries of ecological processes represents the power of resource conversion, as given in (4). Thus

$$\dot{\psi}'\mathbf{X}_c + \mathbf{y}_o'\mathbf{X}_o + \mathbf{y}_r'\mathbf{X}_r = \dot{\psi}'\mathbf{G}(\dot{\psi}) + \mathbf{y}_o'\mathbf{F}(\mathbf{y}_o) = e. \quad (8)$$

From the principles of energy balance *between* processes, it is also easy to show that the equations characterizing the relationships among interprocess eco-costs are always mathematically orthogonal to the equations of interprocess material continuity; from which it follows that the equations of material continuity are sufficient to uniquely characterize the *organization* of ecological networks. However, unlike electrical voltages, for example, eco-costs are not operationally defined in terms of field measurements. Thus, it is not possible to characterize ecological networks in terms of observations on ecological costs and ecological material flows, as analogs to electrical voltages and electrical currents. Rather, networks of ecological processes must be characterized in terms of observations on ecological flows, conversion power, and network organization. This is basically how and why the theory of industrial ecology presented here differs from the network approach to systems ecology pioneered by Odum [13].

The fundamental mathematical problem of natural and industrial ecology, as well as production economics, is that of computing the ecology of bounded networks of processes from the ecologies of the network processes and the network organization. Then, and only then, can a bounded network of resource conversion processes be represented as an equivalent single process, perhaps with multiple products, i.e., as multi-order processes. In general, the ecology of multiorder process cannot be determined directly from field measurements. It must be computed from the organization of networks of first-order processes that are, in fact, observable and/or subject to engineering design. First-order ecological processes subject to engineering design are generally known as technologies.

The problem of computing the coefficients of resource conversion for multiorder processes from engineered technologies stands as a fundamental theoretical and practical limitation on all economic input–output models of the Loentief type [12]. Also, it stands as the source of unresolved issues and procedures in operations management and management accounting [9]. In the paragraphs following, the authors show how the ecologies of a given network of natural processes and engineered technologies are mathematically mapped into the ecology of the network at its boundary, with a corresponding definition of ecosystem economics.

III. BOUNDARY REPRESENTATIONS OF ECOLOGICAL NETWORKS

A. Industrial Ecosystems

By definition, an industrial ecosystem is a bounded network of interrelated natural and engineered resource acquisition, conversion, and transfer processes. Consider a bounded network of j processes, and let the equations of material continuity between the processes be given in the standard

format

$$\mathbf{y}_o = \mathbf{A}\mathbf{y}_{rn} + \mathbf{y}_h \quad (9)$$

where the vector \mathbf{y}_o represents the set of all product flows within the network boundary, the vector \mathbf{y}_{rn} represents the set of resource and residual (r&r) flows within the network boundary, and the vector \mathbf{y}_h , of the same order as \mathbf{y}_o , represents products harvested from the bounded network, i.e., the flow of products across the network boundary. The matrix \mathbf{A} , with entries $(0, \pm 1)$, uniquely defines the organization of the bounded network.

Let the equations of ecological state and the equations of resource conversion for the j network processes be compiled in the standard formats

$$\dot{\psi} = \mathbf{B}\psi + \mathbf{y}_c \text{ (equations of ecological state)} \quad (10s)$$

$$\mathbf{y}_{rn} = \mathbf{C}_n\dot{\psi} + \mathbf{K}_n\mathbf{y}_o \text{ (r\&r flows within the network)} \quad (10n)$$

$$\mathbf{y}_{rb} = \mathbf{C}_b\dot{\psi} + \mathbf{K}_b\mathbf{y}_o \text{ (r\&r flows across the boundary)} \quad (10b)$$

where ψ represents the direct sum of the state vectors for the j processes, \mathbf{y}_{rn} represents the set of all resource and residual (r&r) flows within the network boundary, and \mathbf{y}_{rb} represents the set of all resource and residual (r&r) flows across the network boundary.

Let the enabling power for the j network processes be compiled in the standard format

$$\mathbf{e} = \Sigma\mathbf{E}_o + \dot{\psi}'\mathbf{G} + \mathbf{y}'_o\mathbf{F} \quad (11)$$

where Σ is the unit row vector, the rows of \mathbf{E}_o represent the sustaining power for the j network processes and the rows of \mathbf{G} and \mathbf{F} represent the first-order marginal time costs of resource conversion and transfer for the j network processes.

From (9) and (10n), the *gross network production* \mathbf{y}_o for the bounded network is

$$\mathbf{y}_o = \mathbf{N}\dot{\psi} + \mathbf{M}\mathbf{y}_h \quad (12)$$

where, for $\mathbf{A}\mathbf{K}_n \neq \mathbf{I}$

$$\mathbf{M} = (\mathbf{I} - \mathbf{A}\mathbf{K}_n)^{-1} \text{ and } \mathbf{N} = \mathbf{M}\mathbf{A}\mathbf{C}_n. \quad (\mathbf{D12})$$

From (12) and (10b), the *net boundary flows* \mathbf{y}_s for the network are

$$\mathbf{y}_s \equiv \mathbf{y}_{rb} = \mathbf{C}_s\dot{\psi} + \mathbf{K}_s\mathbf{y}_h \quad (13)$$

where

$$\mathbf{C}_s = \mathbf{C}_b + \mathbf{K}_b\mathbf{N} \text{ and } \mathbf{K}_s = \mathbf{K}_b\mathbf{M}. \quad (\mathbf{D13})$$

From the equations of eco-cost for the j processes and the equations of interprocess eco-cost compatibility (not compiled here), the *enabling power* \mathbf{e}_s for the bounded network is

$$\mathbf{e}_s = \mathbf{e}_{os} + \dot{\psi}'\mathbf{G}_s + \mathbf{y}'_h\mathbf{F}_s \quad (14)$$

where

$$\mathbf{e}_{os} = \Sigma\mathbf{E}_o, \mathbf{G}_s = \mathbf{G} + \mathbf{N}'\mathbf{F}, \text{ and } \mathbf{F}_s = \mathbf{M}'\mathbf{F}. \quad (\mathbf{D14})$$

B. Ecosystem Economics

It has been shown that the vector of ecological boundary variables $\mathbf{v}' = [\mathbf{y}'_h\mathbf{y}'_s\mathbf{e}'_s\dot{\psi}']$ provides a mathematical basis for uniquely characterizing the ecology of a bounded network of processes, which is referred to here as an ecosystem. Let the vector of ecological boundary prices $\mathbf{p}'_v = [\mathbf{p}'_h\mathbf{p}'_r\mathbf{p}'_e\mathbf{p}'_\psi]$ represent the monetary values assigned to the boundary variables \mathbf{v}' by economic markets or other economic pricing mechanisms. Numerically, the vectors \mathbf{v} and \mathbf{p}_v represent the ecological and economic dimensions of the network as an industrial ecosystem. The scalar product $\mathbf{p}'_v\mathbf{v}$ uniquely defines the *ecosystem economics* with four scalar components

$$\mathbf{p}'_v\mathbf{v} = \mathbf{p}'_h\mathbf{y}_h + \mathbf{p}'_r\mathbf{y}_s + \mathbf{p}'_e\mathbf{e}'_s + \mathbf{p}'_\psi\dot{\psi} \quad (15)$$

where $\mathbf{p}'_h\mathbf{y}_h$ represents the temporal rate of revenue for products harvested from the ecosystem, $\mathbf{p}'_r\mathbf{y}_s$ and $\mathbf{p}'_e\mathbf{e}'_s = \mathbf{e}_s\mathbf{p}_e$, respectively, represent temporal rates of expenditures for ecosystem resources and residuals and for enabling factors of resource acquisition, conversion and transfer, and $\mathbf{p}'_\psi\dot{\psi}$ represents the capital investment rate in the ecological state of the ecosystem.

The components of ecosystem economics in (15) can be expressed as an explicit function of the ecosystem structure as follows.

- 1) Premultiply (12) by \mathbf{p}'_o , representing the unit prices of the output products \mathbf{y}_o of the constituent processes of the ecosystem. The resulting column vectors $\mathbf{p}'_o\mathbf{N} = \mathbf{p}'_\psi$ and $\mathbf{p}'_o\mathbf{M} = \mathbf{p}'_h$, respectively, represent prices imputed to the accumulating objects of ecological state ψ and the products \mathbf{y}_h harvested from the ecosystem.
- 2) Premultiply (13) by \mathbf{p}'_r , representing the unit prices of the material resources and residuals \mathbf{y}_s at the boundary of the ecosystem. The resulting vectors $\mathbf{p}'_r\mathbf{C}_s$ and $\mathbf{p}'_r\mathbf{K}_s$ represent monetized coefficients of resource conversion for ecological growth and for production, respectively.
- 3) Post-multiply (14) by \mathbf{p}_e , representing the unit prices of the enabling power \mathbf{e}_s for the ecosystem. The resulting scalar $\mathbf{e}_{os}\mathbf{p}_e$ represents the expenditure rate for sustaining power, and column vectors $\mathbf{G}_s\mathbf{p}_e$, and, $\mathbf{F}_s\mathbf{p}_e$ represent the expenditure rates for the marginal power for ecological growth and for production, respectively.
- 4) Premultiply (10s) by \mathbf{p}'_c , representing the unit prices of imported objects of ecological state at the boundary of the ecosystem. The resulting scalar $\mathbf{p}'_c\mathbf{y}_c$ represents the total expenditure rate and $\mathbf{p}'_c\mathbf{B}\psi$ represents the expenditure rate for replacements.

Ecosystem cash flow is defined by (15) for $\dot{\psi} = 0$. From 1)–3), it follows that cash flow (Cf) is mathematically related to the ecosystem structure and its boundary prices by

$$\text{Cf} = \mathbf{p}'_h\mathbf{y}_h + \mathbf{e}_{so}\mathbf{p}_e \quad (16)$$

where the vector $\mathbf{p}_h = \mathbf{p}_h + \mathbf{K}'_s\mathbf{p}_r + \mathbf{F}_s\mathbf{p}_e$ represents the marginal ecosystem cash flow per unit of harvested product. The scalar $\mathbf{p}'_h\mathbf{y}_h$, called the marginal ecosystem cash flow,

may be positive, negative, or zero, depending upon the numerical values of the boundary prices and the structure of the ecosystem. Marginal ecosystem cash flow vanishes identically when $\mathbf{p}_h = -\mathbf{K}'_s \mathbf{p}_r - \mathbf{F}_s \mathbf{p}_e$. In a similar manner, value added and other monetary measures of ecosystem performance can be expressed as explicit mathematical functions of ecosystem structure, the boundary prices, and extra-ecological factors of the political economy in which the industrial ecosystem functions.

Since the instruments for quantifying the ecological dimensions \mathbf{v} of industrial ecosystems are calibrated in terms of standardized units of mass, energy, space, and time, any changes in the ecological dimensions of industrial processes are conclusively a result of modifications in ecosystem structure, such as its processing technologies and/or network organization. Such modifications are usually characterized as changes in ecological “productivity,” as measured by the ratio of one or more components in $\mathbf{p}'_h \mathbf{y}_h$ to one or more components in $\mathbf{e}_s \mathbf{p}_e$. However, since monetary prices are not ecologically unique, revenues and expenditures may change over time as a result of price “inflation or deflation.” The change in the length $(\mathbf{p}'_v \mathbf{p}_v)^{1/2}$ of the price vector is perhaps a useful measure of price inflation for a given ecosystem. The structural relations in 1)–4) provide the information required to assess the impacts of changes in both ecological structure and boundary prices on ecosystem revenues, as required in economically based ecological design. But, modifications in network structure and boundary prices, clearly, cannot be computed from changes in ecosystem revenues.

Bounded networks of ecological processes, as ecological substructures of economic enterprises, are designed and managed by individual or corporate organizations. In open-market political economies, monetary prices for all interenterprise exchanges of material resources and enabling factors are established by interenterprise price negotiations, called interenterprise market processes. Exchanges between the ecological boundaries of enterprises and the boundaries of natural processes and ecosystems are generally referred to as “externalities” because they are external to interenterprise market processes. Pure interenterprise market economies can be ecologically sustainable only if the externalities of the interenterprise markets are numerically insignificant. If, for example, technological processes threaten the future availability of natural resources through excessive harvest rates or through the excessive dispersal of products and residuals, the externalities must be economically priced [3], [15]. The mathematics of risk-control pricing of the externalities of interenterprise market processes is discussed later in this paper.

IV. THE NATURE OF ECOLOGICAL DESIGN

Equations (D12), (D13), and (D14) are here identified as ecological design equations. These equations are based on the premise that the ecology of the network processes and the equations of network organization can be compiled in the given standard formats. These formats are both necessary and sufficient to assure that there are no mathematical accumulations of resources, residuals, or products within the

boundary of the ecosystem other than the objects of ecological state. The coefficients of resource conversion \mathbf{C}_n and \mathbf{K}_n and the network organization matrix \mathbf{A} , identified here as the *structural parameters* of the ecosystem, uniquely define the ecological design parameters \mathbf{N} and \mathbf{M} . The characteristic matrix \mathbf{B} for an ecosystem of j processes is always equal to the direct sum of the characteristic matrices \mathbf{B}_i , $i = 1, 2, \dots, j$, of the constituent processes, which typically can be aggregated into hierarchical objects of ecological state. In healthy environments, biotic objects of ecological states are generally self-reproducing from network resources \mathbf{y}_{rn} and boundary resources \mathbf{y}_{rb} according to the reproduction parameters \mathbf{B} . Abiotic objects of ecological state are typically produced from boundary resources \mathbf{y}_{rb} only; in which case the coefficient matrix \mathbf{C}_n in (7n) is an empty set.

Mathematically, an ecological process is defined as a *consumption* process when, and only when, \mathbf{y}_o in (9)–(11) is an empty set. Otherwise, it is identified as a *production* process. The distinction between networks of production processes and networks of consumption processes greatly simplifies ecological design procedures over what might otherwise be expected. For example, all natural landscapes and waters harvested by humans, such as commercial forests and fisheries, are characterized as industrial production processes, while those that are not harvested are characterized as natural consumption processes. Likewise, all human enterprises devoid of ecologically significant material products, as in education, performing-arts, and sports, for example, are characterized as consumption processes. Product research and development, product promotion, sales, and delivery, product insurance, accounting, and management, and all other resource acquisition, conversion, and transport activities are identified as enabling factors of production or of consumption, depending upon where the analytical boundaries are drawn between the processes.

It is easy to show that the ecology of a bounded network of consumption processes is mathematically derived as the direct sum of the ecologies of the network processes. Thus, networks of consuming processes can be easily aggregated and formulated as determinants of the product harvest rates from natural and engineered production networks, as well as determinants of the residuals that potentially can be cycled from them as resources. The ecological structures of production networks are the primary focus of ecological design, as discussed in this paper. The presumed objective of ecological design is a) to structure efficacious production networks for sustainable, efficient, and economic operation over the foreseeable future and b) within prevailing economic, social, and cultural constraints, contain the risks imposed by production and consumption technologies on significant natural processes and ecosystems of the biosphere.

Production networks evolve incrementally as temporal modifications or additions of product-specific production networks to existing production networks. The modifications and additions may be in response to changes in market competition, product or process innovation, and/or perceived new-market opportunities, for example. Each such incremental change is organized and engineered by one or more economic enter-

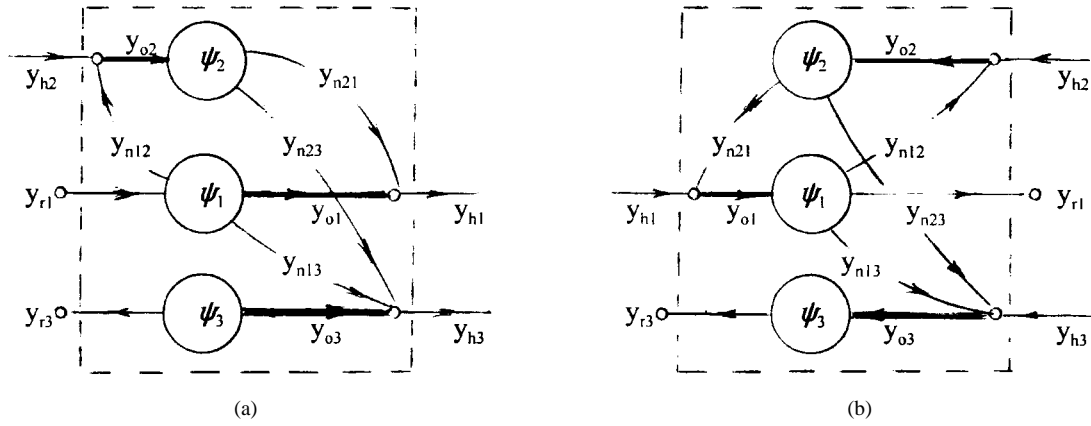


Fig. 1. Network organization for production and reduction modules in Example 1: (a) production module and (b) reduction module.

prises. The ecological design equations for typical product-specific production networks are illustrated next, starting with observable natural processes and engineered technologies at the lowest levels of ecological organization, and progressing sequentially to enterprise levels and beyond.

Example 1) Abiotic Production Modules: Consider the bounded network of first-order abiotic production and reduction processes with material linkages illustrated in Fig. 1, with no significant changes in ecological state. Referring to (9) and (10n), the structural parameters for the network when $\dot{\psi} = 0$ are

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{K}_n = \begin{bmatrix} 0 & k_{21} & 0 \\ k_{12} & 0 & 0 \\ k_{13} & 0 & 0 \\ 0 & k_{23} & 0 \end{bmatrix} \quad (17)$$

where the rows of \mathbf{A} and the columns of \mathbf{K}_n correspond to product flows $[y_{o1}y_{o2}y_{o3}]$, and where the columns of \mathbf{A} and the rows of \mathbf{K}_n correspond to resource flows $[y_{n21}y_{n12}y_{n13}y_{n23}]$.

From (D12), the production parameters for the network are

$$\mathbf{M} = 1/d \begin{bmatrix} 1 & -k_{21} & 0 \\ k_{12} & 1 & 0 \\ \kappa_1 & \kappa_2 & d \end{bmatrix} \quad (18)$$

where $\kappa_1 = k_{13} + k_{12}k_{23}$, $\kappa_2 = k_{23} - k_{13}k_{21}$, and $d = 1 + k_{12}k_{21}$.

The boundary flows y_{rb} for the network are given by (11) with $\mathbf{K}_s = \mathbf{K}_b\mathbf{M}$, as in (D13). The boundary flows specifically include resources y_{r1}, y_{r2} , and y_{r3} illustrated in Fig. 1. The columns of $\mathbf{K}_s = \mathbf{K}_b\mathbf{M}$ correspond to the products y_{h1} , harvested from the network and illustrated in Fig. 1 as y_{h1} , y_{h2} , and y_{h3} . The marginal time costs for resource conversion for the network \mathbf{F}_s are given by (14) with $\mathbf{F}_s = \mathbf{M}'\mathbf{F}$. The rows of \mathbf{F}_s represent the marginal time costs of the harvestable products y_{h1} , y_{h2} , and y_{h3} .

The network modules in Example 1 are illustrative of broad classes of production modules having many specific interpretations. In manufacturing, for example, substandard products are sometimes reprocessed, as illustrated by feed-

forward loop ψ_2 in Fig. 1(b). In many reduction processes, a fraction of the refined output is returned and combined with the raw feedstock, as illustrated by feedback loop ψ_2 in Fig. 1(b). Residuals from both ψ_1 and ψ_2 are collected and processed for transfer to unspecified processes as indicated by ψ_3 . Thus, the matrix of conversion coefficient \mathbf{K}_n can be viewed as parameters of “quality control” and recycling.

Whatever the specific application, the design equations in Example 1 provide the mathematics for exploiting network relationships among specific processing technologies to achieve design objectives at modular levels of network organization. At the same time, it provides a boundary representation of the network as a modular building block in the design of higher levels of network organization to achieve higher-level network objectives. For example, when y_{h1} is the only harvested product, only the first column \mathbf{k}_{s1} of \mathbf{K}_s and only the first row \mathbf{f}_{s1} of \mathbf{F}_s are included in the boundary representation, thereby providing a single-product equivalent technological representation of the modules in Fig. 1. This equivalent can be used to characterize the modular networks as a single process in higher-level network organizations.

Example 2) Food Chains and Sequential Production Processes: Consider the chain of j first-order processes with linkages illustrated in Fig. 2. Referring to (9) and (10n), the structural parameters for the network are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{C}_n = \begin{bmatrix} 0 & c_{22} & 0 & \cdots & 0 \\ 0 & 0 & c_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c_{jj} \end{bmatrix}$$

$$\mathbf{K}_n = \begin{bmatrix} 0 & k_{22} & 0 & \cdots & 0 \\ 0 & 0 & k_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k_{jj} \end{bmatrix} \quad (19)$$

where the rows of \mathbf{A} and the columns of \mathbf{K}_n correspond to product flows $[y_{o1}y_{o2} \cdots y_{oj}]$, the columns of \mathbf{A} and the rows of \mathbf{C}_n and \mathbf{K}_n correspond to network resources $[y_{n2}y_{n3} \cdots y_{nj}]$, and the columns of \mathbf{C}_n correspond to the accumulation rates $[\dot{\psi}_1\dot{\psi}_2\dot{\psi}_3 \cdots \dot{\psi}_j]$.

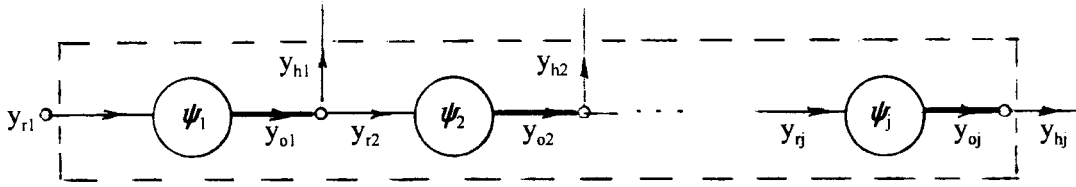


Fig. 2. Network organization for production chain in Example 2.

From (D12), the growth and production parameters for the network are, respectively

$$\mathbf{N} = \begin{bmatrix} 0 & c_{22} & k_{22}c_{33} & \kappa_{23}c_{44} & \cdots & \kappa_{2j}c_{jj} \\ 0 & 0 & c_{33} & \kappa_{33}c_{44} & \cdots & \kappa_{3j}c_{jj} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & c_{jj} \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 1 & k_{22} & \kappa_{23} & \cdots & \kappa_{2j} \\ 0 & 1 & k_{33} & \cdots & \kappa_{3j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k_{jj} \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \quad (20)$$

where $\kappa_{23} = k_{22}k_{33}$, $\kappa_{2j} = k_{22}k_{33} \cdots k_{jj}$, $\kappa_{3j} = k_{33}k_{44} \cdots k_{jj}$.

The boundary flows \mathbf{y}_{rb} for the network are given by (13), with resource conversion coefficients \mathbf{C}_s and \mathbf{K}_s , given by (D13). The boundary flows specifically include the resource input y_{r1} , illustrated in Fig. 2. The columns of \mathbf{C}_s correspond to accumulation rates $[\dot{\psi}_1 \dot{\psi}_2 \cdots \dot{\psi}_j]$ and the columns of \mathbf{K}_s correspond to product harvest rates $[y_{h1} y_{h2} \cdots y_{hj}]$. If only the end product y_{hj} is harvested from the network, then only the last column \mathbf{k}_{sj} of \mathbf{K}_s is retained in the boundary representation.

The marginal time costs of resource conversion for the network \mathbf{G}_s and \mathbf{F}_s are given by (D14) with the rows corresponding to accumulation rates $[y_{c1} y_{c2} \cdots y_{cj}]$ and product harvest rates, $[y_{h1} y_{h2} \cdots y_{hj}]$, respectively. The last row, \mathbf{f}_{sj} , of \mathbf{F}_s represents the marginal time costs of the end product, y_{hj} , of the production chain. If these are the only time costs of interest, then only the last row, \mathbf{f}_{hj} , is retained in the boundary representation.

The output-oriented production chain illustrated in Fig. 2 and its input-oriented counterpart (not illustrated here) are used extensively in manufacturing and refining. Mathematically, the design equations are the same for both input- and output-oriented chains. In the production operations where $\dot{\psi}_1 = 0$, this example provides a relatively simple design algorithm. From this algorithm it can be concluded, for example, that inefficiencies and error rates in resource conversion accumulate geometrically, rather than arithmetically, with the number of processes in the chain. Further, these inefficiencies and errors must be controlled by modular networks of the form illustrated in Example 1. They cannot be “designed out” of the system at chain levels of organization.

Chains of self-reproducing biotic processes, called food chains, are interlinked through intraspecies relationships, e.g., primary production, grazing, and predation. Such chains are commonplace and often appear as branches in very complex

food webs in natural ecosystems. If the objects of production are identical to the objects of ecological state, as in animal predation, then \mathbf{K} is an empty set and $\mathbf{y}_{ci} \equiv \mathbf{y}_{oi}$. In this instance, (13) and (D13) identify those biochemical components of interprocess flow that are synthesized into new objects of ecological state, i.e., new individuals in the predator population. Equations (14) and (D14) identify the biochemical components of interprocess flow that serve as the source of biotic power for the successive processes in the chain, each of which has a maintenance component e_{oi} and a marginal time cost \mathbf{f}_i , $i = 1, 2, \dots, j$. The enabling power of the resource conversion for the chain is computed as the direct sum of the powers of conversions for all of the network processes. The changes in populations $\dot{\psi}_i$, for the processes in the food chain are given by the respective differential equations of ecological state

$$\dot{\psi}_i = \mathbf{B}_i \psi_i + \mathbf{y}_{ci}, \quad i = 1, 2, \dots, j \quad (21)$$

where \mathbf{B}_i , with eigenvalues greater than zero, represents the reproduction parameters for species population ψ_i . The vector $\mathbf{y}_{ci} \equiv \mathbf{y}_{oi}$ represents the flow of objects of ecological state, as in grazing or predation, for example. The flow of resources and residuals at the boundaries of each process are those required to sustain the populations represented by the state vectors.

Example 3) Species Communities and Product-Specific Production Networks: Consider a network of production processes with interprocess material exchanges illustrated in Fig. 3. Referring to (9) and (10), it is easy to show that the growth and production parameters for the network are, respectively

$$\mathbf{N} = \begin{bmatrix} 0 & \mathbf{N}_{12} \\ \mathbf{C}_{n21} & 0 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{I} & \mathbf{M}_{12} \\ \mathbf{K}_{n21} & \mathbf{I} \end{bmatrix} \quad (22)$$

where \mathbf{C}_{n21} and \mathbf{K}_{n21} are diagonal matrices with entries c_{ii} and k_{ii} , respectively, $i = 3, 4, \dots, j$, and where

$$\mathbf{N}_{12} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ c_{33} & c_{44} & \cdots & c_{jj} \end{bmatrix}$$

and

$$\mathbf{M}_{12} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ k_{22} & k_{33} & \cdots & k_{jj} \end{bmatrix}. \quad (23)$$

The boundary flows \mathbf{y}_{rb} , as given by (13), include the resource input y_{r2} illustrated in Fig. 3. The first column, \mathbf{k}_{s1} in \mathbf{K}_s corresponds to harvested product y_{h1} . If all products are subject to harvesting, then all columns of \mathbf{K}_s are retained in the boundary representation.

The marginal time costs of resource conversion for the network are given by (D14), with the rows of \mathbf{G}_s and \mathbf{F}_s

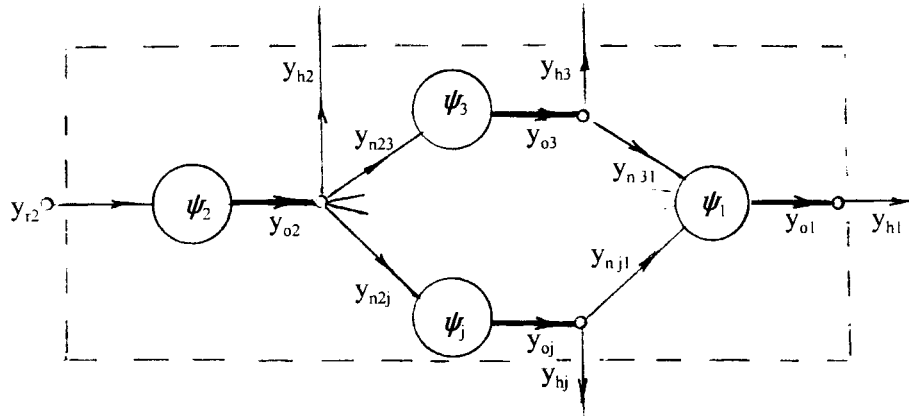


Fig. 3. Network organization for product-specific production network in Example 3.

corresponding to objects of ecological states and the objects of harvest, respectively. The first row, f_{s1} , of F_s represents the marginal time costs of the end product, y_{h1} . If all products of the network are subject to harvesting, then all rows of F_s are retained in the boundary representation.

In the realm of biotic ecosystems, Fig. 3 is characteristic of a community of different species populations in which ψ_1 may represent, for example, a population of carnivores feeding on populations $\psi_3 \dots \psi_j$ of animals grazing on a common grassland ψ_2 . The grassland is sustained by, among other things, a vector of nutrients, y_{r2} , extracted from terrestrial reservoirs or recycled from populations of microbes and invertebrates decomposing organic matter. The population ψ_1 is harvested at a temporal rate $y_{o1} = y_{h1}$. As in the food chains of Example 2, the interprocess material transfers y_{n2i} and y_{ni1} , $i = 3, 4, \dots, j$ have both biochemical components that are synthesized into new objects of ecological state, and biochemical components that serve as the source of biotic power for the respective processes. Also, the ecological state ψ_i for each constituent population is related to its accumulation, as represented by the differential equation of ecological state.

In the realm of abiotic ecosystems, Fig. 3 is characteristic of product-specific manufacturing operations in which a product y_{o1} or a vector of products y_{o1} are assembled from parts or subassemblies y_{oi} , $i = 3, 4, \dots, j$, produced by supplier processes 3— j according to engineering specifications for the final product(s). The suppliers typically utilize the same commodity y_{o2} or vector of commodities y_{o2} as feedstocks refined from ores, fossil hydrocarbons, or biotic products, y_{r2} that are extracted from the geosphere or biosphere, or drawn from stockpiles of recycled materials. Each process in the network may represent a chain of processes as already illustrated in Example 2. The entire network may be managed as a single enterprise or each process may be under separate management. In the latter instance, the assembly process is identified as the original equipment manufacturer (OEM) with suppliers.

Note that boundary representations for all enterprises in the network are formulated in terms of their outputs, rather than their inputs. Enterprise management based on such represen-

tations is referred to in the industry as output oriented and the managed network as a “just-in-time” manufacturing system. It is easy to show that structural information for the network of Fig. 3 can be compiled in the standard format only when the boundary representations for all enterprises in the network are output oriented. For all other orientations, products may accumulate as inventories within the network boundary. Any such inventories must be mathematically accounted for in the design equations. In practice, inventories must be periodically monitored so that production schedules can be adjusted accordingly, with all the attendant implications of economic inefficiencies.

The vector of end products y_{o1} illustrated in Fig. 3 typically accumulate as inventories of ecologically active objects in human cultures—items of food, clothing, shelter, communications, transportation, household appliances, automobiles, dwellings, etc. Inventories of ecologically active products that return no ecologically significant products to their host network have already been defined as consuming processes in industrial ecosystems. The following example illustrates how consuming processes can be mathematically characterized as ecological loads on production networks, thereby establishing product life cycles.

Example 4) Product Lifecycles: Consider the network of ecological processes illustrated in Fig. 4, for which the network organization is given by three vector equations

$$y_{o1} = y_{c1}, y_{o2} = y_{c2}, \text{ and } y_{o3} = y_{r2}. \quad (24)$$

Let the ecology of the product-specific production network illustrated in Example 3 be represented by ψ_1 , with $\dot{\psi}_1 = 0$

$$y_{b1} = K_{b1}y_{o1} + K_{b2}y_{o2} \quad (25)$$

$$e_1 = e_{o1} + y'_{o1}F_1 + y'_{o2}F_2 \quad (26)$$

where the vector y_{o2} represents products harvested from the network of suppliers 3, \dots, j , and y_{b1} represents the vector of all boundary flows for the network, including y_{r2} .

Let ψ_2 represent the product-specific inventory of functional products in a given spatial region. The ecology of the inventory is taken as

$$y_{c1} = B_1\psi_2, y_{c2} = B_2\psi_2, \text{ and } y_{r2} = B_3\psi_2 \quad (27)$$

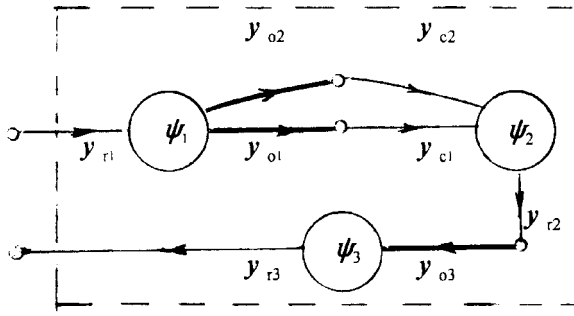


Fig. 4. Network organization for product-specific lifecycle in Example 4, featuring the product-specific production process ψ_1 , inventory of accumulated functional products ψ_2 , and a resource recovery process ψ_3 .

$$y_{b2} = C_1 y_{c1} + C_2 y_{c2} + C_3 y_{r2} \quad (28)$$

$$e_2 = e_{o2}(\psi_2) + y'_{c1} G_1 + y'_{c2} G_2 + y'_{r2} G_3 \quad (29)$$

where y_{c1} and y_{c2} represent inflows of products and repair parts, respectively, y_{r2} represents the outflow of discarded products, and y_{b2} represents the flow of all other boundary resources and residuals. The square matrices B_1 , B_2 , and B_3 identify the product accumulation rates, the product repair rates, and the product retirement rates, respectively. In general, the sustaining power $e_{o2}(\psi_2)$ is a function of the inventory of products, and it identifies both abiotic and biotic enabling power, such as kilowatts of energy, skill-specific persons per day, and computing and communication network time per day required to sustain the inventory of products as functional elements of the biosphere.

Let the ecology of process ψ_3 , with $\dot{\psi}_3 = 0$, be characterized as a reduction process

$$y_{b3} = K_{b3} y_{o3} \quad (30)$$

$$e_3 = e_{o3} + y'_{o3} F_3 \quad (31)$$

where y_{o3} represents the inflow of objects, and y_{b3} represents the flow of all boundary resources and residuals, including the outflow of recovered resources y_{r3} . If there are no recovered resources, as when products are dispersed, then $y_{r3} = 0$.

The ecology of the network is established by substituting (24) into (25) and (26), and then into (30) and (31), with subsequent substitution of (27) into the results. The resulting design equations show boundary flows and enabling power of the network as explicit functions of the network structure, the inventory of functioning products ψ_2 and the technical measures B_1 , B_2 , and B_3 of product durability, reliability, and repair ability.

The product-specific life cycle illustrated in this example is characteristic of virtually all products, ranging from processed food and clothing, with relatively short temporal life-cycle characteristics to computers, household appliances, automobiles, trucks, industrial equipment, housing facilities, etc., with relatively longer life-cycle characteristics. Examples 1–4 illustrate how the ecology of a given enterprise can be developed as a hierarchical sequence of design equations, starting with the technological parameters of engineered, first-order resource conversion processes (technologies) on the

factory floor, and culminating in the ecology of product lifecycles.

At subenterprise levels of ecological organization, monetary measures of ecosystem performance are known only implicitly because there generally are no markets within the analytical boundaries of enterprises to establish monetary prices for the boundary variables of constituent network processes. Application of ecosystem design concepts at subenterprise levels of organization are illustrated in [18]. It is at enterprise levels of economic organization that monetary measures of ecosystem performance are expressed as explicit mathematical functions of network structure and boundary prices, and where they become explicit criteria of both ecological design and interenterprise price negotiations. Applications at enterprise levels are illustrated in [16]. The ecology and the economics of alternative product-recycling strategies are illustrated in [11].

The baseline ecological and pricing information required to evaluate the sensitivities of enterprise-level ecosystems to changes in their structure and their boundary prices can, for the most part, be extracted from the business accounts of most industrial enterprises. However, for this information to be useful in the context of comprehensive ecological design and operations management, it should be recorded and processed in a tabular format consistent with the mathematics of the design equations. Input–output tables of the Leontief type developed more than 50 years ago [12], for example, can be modified and extended as data formats for ecological design, operations management, and ecologically based management accounting, as defined here.

V. PRINCIPLES OF SUSTAINABLE INDUSTRIALIZATION

In open-market political economies, populations of enterprises compete for niches in product-specific production networks, such as that illustrated in Fig. 3. The ecological substructures of political economies can be characterized as networks of ecologically interdependent enterprises and/or aggregates of competing enterprise, called industries. And, the biosphere can be characterized as networks of ecologically interdependent political economies, materially linked to the atmosphere, hydrosphere, geosphere, and the natural ecosystems of the planet. In principle, these characterizations can be developed into explicit mathematical representations. However, the goal at these levels of economic organization is not ecological design, as illustrated in the previous section. Rather, it is to provide reliable, systemic accounting of the ecological dimensions of evolving industrial ecosystems as required to a) assess the ecological risks political economies impose to the natural ecosystems of the biosphere and their abiotic reservoirs and b) to contain these risks through direct regulation or by pricing the externalities of interenterprise markets so as to assure ecologically sustainable industrialization.

When the unit prices on material dispersions from production and consumption processes are taken as zero, by default or otherwise, the ecological substructure and the price relationships in political economies become seriously flawed and beyond computation. Consequently, prescriptive regulations that are both ecologically and economically rational are, at best,

very difficult to synthesize [3]. The same is true when natural resources are priced below their value to political economies in stabilizing significant natural processes or ecosystems of the biosphere [15]. Further, ecological risks can be managed only at enterprise levels of economic organization, where changes in the prices of the externalities of interenterprise markets can motivate innovative changes in production and consumption technologies, natural resource requirements, and specific processing networks, such as those illustrated in the previous section. On the other hand, the ecological scope and scale of technologically generated risks generally depend on the aggregate of the material dispersion and resource harvest rates for all the engineered processes in a given political economy. An economic approach to containing the ecological risks of technologies is to price the externalities of interenterprise markets uniformly for all economic constituents in temporal relationship to the perceived ecological risks imposed by the political economy in which they function. The material dispersion and harvest rates for the constituent enterprises, for example, are specifically included as ecological variables in their respective boundary representations. The corresponding rates for the political economy are computed as aggregates of the material dispersion and harvest rates for the constituent enterprises.

A number of on-line pricing mechanisms can be devised to rationally and quantitatively link the unit prices of material dispersions and natural resource flows at enterprise boundaries to the ecological risks imposed by the political economy in which they function. For example, let the vector \mathbf{y}_d represent the temporal rate of material dispersions at the ecological boundary of a given political economy. Let the vector \mathbf{p}_d represent the unit prices assigned to corresponding residuals at enterprise boundaries within the economy, with

$$\mathbf{p}_d = \mathbf{D}_d(\mathbf{t}, \mathbf{y}_d) \quad (32)$$

where the entries of the time varying matrix $\mathbf{D}_d(\mathbf{t}, \mathbf{y}_d)$ are mathematically increasing functions of the perceived ecological risks the components of \mathbf{y}_d impose on the biosphere, including its supporting elements of the atmosphere, hydrosphere, and geosphere.

In principle, the flows \mathbf{y}_d can be computed as ecological boundary variables for the political economy, with subsequent assessments of the scope and scale of the perceived ecological risk. The perceived ecological risks can be assessed periodically and the unit prices \mathbf{p}_d increased incrementally over time and without bound, as required to contain the perceived risks; due consideration being given to safety factors, intrinsic time delays, and price schedules commensurate with the risk. Price schedules for high-risk residuals, for example, would increase more rapidly with dispersion rate than for low-risk residuals. Faced with unit price schedules on residuals, the constituent enterprises of the political economy are economically motivated to change the product and/or the processing technology, select alternative feedstocks, or modify product life cycles to reduce the risk. The same on-line pricing principles apply to both production and consumption enterprises. Also, similar on-line, risk-control pricing mechanisms can be devised to supplement interenterprise market processes, perhaps as an

ecologically based tax, to contain the ecological risks of natural resource harvests. However, issues concerning the means for generating such pricing mechanisms in political economies are outside the scope of this paper.

VI. CONCLUSIONS

A unique and central feature of the mathematical theory of industrial ecology presented here is the three generic design equations for mapping the ecology of a network of resource conversion processes into the ecology of the network at its boundary. The first of the three design equation is intermediate to the second and third. A different, ecologically incomplete form of the first design equation was developed by Loentief some 50 years ago as a theoretical aspect of production economics [12]. Collectively, the three design equations represent the ecology of bounded networks as explicit mathematical functions of the network structure. And, it is the vector of boundary prices assigned to the ecological variables at network boundaries that present the ecosystem economics as an explicit mathematical function of the same network structure, as required for quantitative economically based ecological design.

Most of the baseline information required for quantitative ecological design is already obtainable from the invoices, payrolls, and financial accounts of most enterprises. This information can be made directly accessible for economically based ecological design and for ecologically based operations management and management accounting by adaptations in recording formats and procedures. Counterpart information on natural ecological processes is perhaps less generally available, in part, because appropriate quantitative design paradigms are not normally available for characterizing natural processes as operating components of industrialized ecosystems.

In open-market political economies, unit prices on ecological variables are established at the ecological boundaries of enterprises, but pricing is generally incomplete because materials dispersed from engineered processes as residuals and products, as well as the ecological impacts of excessive harvest rates from natural ecosystems, are outside the scope of interenterprise market processes [3]. It has been shown that, in principle, the ecological risks imposed by the externalities of interenterprise markets can be contained by on-line, risk-control, feedback pricing mechanisms, thereby assuring ecologically sustainable industrialization. Issues concerning the various means for deploying such pricing mechanisms in political economies are outside the scope of this paper. However, it can be concluded that a) as human populations and industrialization continue to expand, engineered production and consumption processes must become increasingly compatible with the natural biotic and abiotic processes of the biosphere and b) choices among technological alternatives must be made at enterprise levels of economic organization. This is the only level of economic organization where changes in one or more boundary prices or market mechanisms can motivate innovations in resource materials, products, production networks, product lifecycles, and residuals processing through rational engineering design. The more widely economically

based ecological design is practiced, the greater the scope and scale of engineering innovation that can be drawn into the process of assuring that the earth's evolving industrialized biosphere is efficacious, sustainable, efficient, and economic.

With access to ever-improving information technologies, comprehensive economically based ecological design and ecologically based operations management can, and probably will, become an important aspect of interenterprise competition in open-market economies, including international trade and aid. The process can perhaps be hastened by a) developing and commercializing user-friendly software packages in support of economically based ecological design and ecologically based operations management and management accounting and b) begin implementing on-line, risk-control pricing of the externalities of interenterprise market processes as economic incentives for addressing regional- and global-scale ecological risks for which prescriptive regulations are scientifically formidable and politically frustrating.

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