Peirce's Modal Shift: From Set Theory to Pragmaticism

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IN THE FIRST OF A SERIES OF TWO ARTICLES published in *The Monist* in October 1896 and January 1897, Charles Peirce asserted that every type of possibility can be defined in terms of states of information: "*possibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false" (3.442).[1] But by the second article in this series, Peirce had changed his mind:

> I formerly defined the possible as that which in a given state of information (real or feigned) we do not know not to be true. But this definition today seems to me only a twisted phrase which, by means of two negatives, conceals an anacoluthon. We know in advance of experience that certain things are not true, because we see they are impossible. (3.527)[2]

Peirce himself took this change to be of the first importance. Around 1905, he characterized it as a return "to the Aristotelian doctrine of a *real possibility* . . . the

[1] References in this familiar decimal notation are to *Collected Papers of Charles Sanders Peirce*, ed. C. Hartshorne, P. Weiss, and A. Burks, 8 vols. (Cambridge: Belknap Press of Harvard University Press, 1931–60), by volume and paragraph number. Other references to Peirce's works are as follows. *'EP'* refers to *The Essential Peirce: Selected Philosophical Writings*, ed. N. Houser, C. Kloesel, and the Peirce Edition Project, 2 vols. (Bloomington: Indiana University Press, 1992–98); references are by volume and page number. *'W'* refers to *Writings of Charles S. Peirce: A Chronological Edition*, ed. M. Fisch, C. Kloesel, E. Moore, et al., 6 vols. (Bloomington: Indiana University Press, 1982–); references are by volume and page number. *'R'* refers to the Harvard manuscripts cataloged in *Annotated Catalogue of the Papers of Charles S. Peirce*, ed. R. Robin (Amherst: University of Massachusetts Press, 1967) and "The Peirce Papers: A Supplementary Catalogue," *Transactions of the Charles S. Peirce Society* 7 (1971): 37–57; these manuscripts are available in a microfilm edition, *The Charles S. Peirce Papers*, produced by Harvard University Library; references are by Robin's manuscript number and, when available, page number. *'NEM'* refers to *The New Elements of Mathematics*, ed. C. Eisele, 4 vols. (Atlantic Highlands, NJ: Humanities Press, 1976); references are by volume and page number. *'RLT'* refers to *Reasoning and the Logic of Things: The Cambridge Conferences Lectures of* 1898, ed. K. Ketner (Cambridge: Harvard University Press, 1992). *'PPM'* refers to *Pragmatism as a Principle and Method of Right Thinking: The 1903 Harvard Lectures on Pragmatism*, ed. P. Turrisi (Albany: State University of New York Press, 1997).

[2] An anacoluthon is a change within a sentence to a second grammatical construction inconsistent with the first. It is not clear what anacoluthon Peirce held to be concealed by the two negatives in that statement, which is grammatically correct.

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great step that was needed to render pragmaticism an intelligible doctrine" (*R* 288, 129). I will refer to this change as Peirce's *modal shift*.[3]

A casual reading of these passages might suggest that Peirce's modal shift was simply a change from anti-realism to realism about possibility. But this interpretation is too simple to be correct. First, Peirce seems to have been a realist about modality prior to the modal shift. His tychism—which he adopted no later than in 1883–84's "Design and Chance"[4]—is the view that there is *real chance*, i.e., real contingency, in the world; and this tychism supplemented Peirce's so-called "extreme scholastic realism," according to which there are real natural laws and thus *real necessity*. Furthermore, Peirce continued to rely on the "states of information" account of possibility after his 1896 criticism of it. In 1902, for example, he defined the physically possible as "that which a knowledge of the laws of nature would not enable a person to be sure was not true" (6.371), and beginning in 1903, he allowed the "states of information" account to play a central role in his interpretation of the modal part of his system of existential graphs. So the interpretation of the modal shift as a simple move from anti-realism to realism about modality will not do. We are in need of a more accurate interpretation, one that will explain exactly what Peirce changed his mind about and why he did so. The first goal of this essay is to provide such an interpretation. I will first provide an account of Peirce's thoughts on modality in the years preceding the modal shift. I will then consider the modal shift itself and attempt to explain exactly what Peirce did, and what he did not, change his mind about.

Understanding the modal shift itself will help us better understand the evolution of Peirce's general philosophical system during the last two decades of his life. His theory of modality was interwoven with his views on a number of other subjects, and as his position on possibility changed, his views on these other subjects changed along with it. An exhaustive account of Peirce's views on modality is far beyond the scope of a single essay. But it is possible, in a work of this length, to explain why Peirce took his criticism of the "states of information" account of modality to have been so important in the development of his "pragmaticism"—his later pragmatism[5]—and this is my second goal. After I explain the modal shift itself, I will "connect the dots," so to speak, between his modal shift and his pragmaticism. As we will see, those dots mark, not a straight line, but a winding path, one that begins in set theory and proceeds through Peirce's thoughts on continuity and his "extreme scholastic realism" about natural laws and kinds. In following this path, I will provide an account of Peirce's thinking about modality that is more

[3] The article in which Peirce's modal shift occurred was published in January 1897 but was written in 1896. I will cite 1897 when quoting from this article, but the actual year of Peirce's modal shift was 1896. 'Modal shift' usually refers to a fallacy involving modal terms, such as moving from the premise "It is necessary that either *p* or not-*p*," to the conclusion, "Either it is necessary that *p* or it is necessary that not-*p*." By calling Peirce's change of mind about modality his "modal shift," I do not mean to imply that Peirce was susceptible to this fallacy.

[4] *EP* 1:215–24, *W* 4:544–54.

[5] In 1905, Peirce renamed his version of pragmatism, calling it 'pragmaticism', a name he hoped was "ugly enough to be safe from kidnappers" (5.414, *EP* 2:335).

synoptic than its predecessors, one which I hope will settle the question of what Peirce was really up to in making his modal shift.[6]

1. BEFORE THE MODAL SHIFT

To provide a basis from which we can begin to consider the modal shift itself, I first need to describe some of Peirce's earlier thoughts about modality. I will not attempt an exhaustive account of his pre-1896 comments on the subject.[7] It will be sufficient to consider three ideas from Peirce's writings of the 1880s and 1890s: his interpretation of "ordinary" conditional propositions as quantifications, his "states of information" account of modality, and his tychism.

1.1 *"Ordinary" Conditionals as Quantifications*

Prior to the modal shift, Peirce gave an account of conditional propositions as covert quantifications, the quantifiers of which range over "states of things" or "possibilities." On its face, this account seems to anticipate contemporary possible world semantics. So, as a preliminary to examining Peirce's metaphysical views regarding modality prior to the modal shift, it is important to see that Peirce did not take this account of the conditional to imply any metaphysical doctrines about the reality of possibility or possible worlds.

An early appearance of this account was in 1880's "On the Algebra of Logic":

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[6] It is widely held that Peirce eventually adopted a more realist view of modality, but only a few commentators have taken note of Peirce's 1896 criticism of the "states of information" account and his later comments about it. Max Fisch mentions that criticism, noting that Peirce took it to be a significant step towards realism, but he does not mention Peirce's later reliance on that account. See Fisch, *Peirce, Semeiotic, and Pragmatism*, ed. K. Ketner and C. Kloesel (Bloomington: Indiana University Press, 1986), 193–94. Christopher Hookway (*Peirce* [New York: Routledge, 1985], 178–79 and 243) refers to Peirce's first criticism of the "states of information" account, mentions "the development in Peirce's views of modality during the 1890s," and connects that development with Peirce's thoughts on continuity, scholastic realism and pragmatism, but he does not mention Peirce's continued use of the "states of information" account after the 1890s. Charles Morgan acknowledges Peirce's criticism and subsequent use of the "states of information" account without attempting to explain the apparent inconsistency. See Morgan, "Peirce-Semantics for Modal Logics" ["Peirce-Semantics"], in *Proceedings of the C. S. Peirce Bicentennial International Congress*, ed. K. Ketner, J. Ransdell, C. Eisele, M. Fisch and C. Hardwick (Lubbock: Texas Tech Press, 1981), 207–15. Brian Noble correctly identifies one consequence of the modal shift, viz. a revision in Peirce's thinking about continuity, but he does not mention Peirce's continued reliance on the "states of information" account. See Noble, "Peirce's Definitions of Continuity and the Concept of Possibility" ["Definitions of Continuity"], *Transactions of the Charles S. Peirce Society* 25 (1989): 149–74. Manley Thompson (*The Pragmatic Philosophy of C. S. Peirce* [Chicago: University of Chicago Press, 1953], 289 n. 22) quotes Peirce's statement that he "formerly defined the possible" in terms of states of information and mentions his 1905 distinction between "subjective" and "objective" modality, but does not note his continued use of the "states of information" account. None of these commentators note the tension between Peirce's shift towards an apparently new realism about modality and his earlier realism about necessity and contingency.

[7] Peirce took a seemingly realist view of modality in an essay of August, 1859 (*W* 1:38). His comments in 1865's "An Unpsychological View of Logic . . . " (*W* 1:312), which strongly suggest an anti-realist view, are not necessarily incompatible with realism about modality; for an account of similar comments Peirce made around the same time regarding generals, and how those comments are compatible with realism about generals, see my "On Peirce's Early Realism," *Transactions of the Charles S. Peirce Society* 40 (2004): 575–605.

> [W]e not only require the form P ∴ C to express an argument, but also a form, $P_i \prec C_i$, to express the truth of its leading principle. Here P_i denotes any one of the class of premises, and C_i the corresponding conclusion. The symbol ≺ is the copula, and signifies primarily that every state of things in which a proposition of the class P_i is true is a state of things in which the corresponding propositions of the class C_i are true. (3.165, *W* 4:166)[8]

The "leading principle" of the inference from P to C is a conditional proposition, "If P, then C," which Peirce here interpreted as something like "Every state of things in which P is true is a state of things in which C is true." But Peirce did not yet have access to the quantifier, and so his ability to deal formally with such propositions was limited.[9] Peirce later criticized this early treatment of conditionals:

> Properly to express an ordinary conditional proposition the quantifier Π is required. In 1880, three years before I developed [my] general algebra [of logic], I published a paper containing a chapter on the algebra of the copula . . . I there noticed the necessity of such quantifiers properly to express conditional propositions; but the algebra of quantifiers not being at hand, I contented myself with considering consequences *de inesse*. (3.448, 1896)[10]

Here Peirce was alluding to the distinction between *de inesse* conditionals, which concern only what happens here and now, and what he called "ordinary" conditionals, which concern what happens, not just in the actual world, but across some "range of possibilities." (2.348, *R* 787, 1896; see also 3.442, 1896) "If P, then C," understood as a *de inesse* conditional, means "If P is true of the actual world, then C is true of the actual world," and is false if and only if P is actually true and C is actually false. But understood as an ordinary conditional, "If P, then C" means "In any possible state of things, *i*, if P is true in *i*, then C is true in *i*" and is false if and only if it is possible that (i.e., if and only if there is a possible state of things in which) P is true but C false. In modern terms, *de inesse* conditionals express material implication and ordinary conditionals express strict implication. The problem with Peirce's 1880 treatment of conditionals was that, without quantifiers, he had been unable to treat conditionals within formal logic as quantifying over multiple objects (in this case, states of things), and thus was limited to treating them as if they covered only a single state of things, viz., the way things actually are. He was thus limited to an interpretation of the conditional as *de inesse*, i.e., as expressing material implication. Once he could represent quantification formally, he was able to represent a conditional as quantifying over multiple states of things and thus able to represent ordinary conditionals, and thus strict implication. By 1885, he was working with an explicit distinction between material and strict implication, and he took the latter to be

> the sense which is most usefully attached to the hypothetical proposition in logic . . . [T]he peculiarity of the hypothetical [i.e., conditional] proposition is that it goes out beyond the actual state of things and declares what *would* happen were things other than they are or may be. (3.374, *W* 5:169)

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[8] See also 2.710, *W* 4:421, 1883. On Peirce's symbol '≺', see Hookway, *Peirce*, 135–36, and R. Dipert, "Peirce's Propositional Logic," *Review of Metaphysics* 34 (1981): 569–95.

[9] As Peirce acknowledged (3.393, *W* 5:178, 1885), his student O. H. Mitchell discovered the quantifier in 1883.

[10] Peirce makes the same criticism at 2.349, *R* 787, 1896.

Nonetheless, he was willing to interpret conditionals within his propositional logic as material conditionals (3.373 and 3.375, *W* 5:169–70, 1885).

Peirce seems to have put very little metaphysical weight on the "states of things" and "ranges of possibilities" in terms of which he understood strict implication. In one unpublished manuscript, he even indicated that reference to "states of things" is eliminable from the interpretation of a strict conditional:

> Suppose we say, "If Enoch and Elijah died, the Bible errs." This is the same as to say that every conceivable state of things in which Enoch and Elijah should die would be a state of things in which the Bible would be in error. To this the word-mongers sometimes retort, that a "state of things" is something whose meaning can only be elucidated by means of another hypothetical proposition. *But it is not necessary to use that phrase. We may speak of a pebble or a tree equally well.* Thus, we may express ourselves as follows: "Every pebble coexistent with Enoch and Elijah as dying men is coexistent with an error in the Bible." (*R* 410, 9–10, 1893; emphasis added)

There is no indication in any of his statements about ordinary conditionals that he took the states of things over which their quantifiers range to be fully real possible worlds, such as those required by David Lewis's modal realism.[11]

1.2 *The Information-Relative Account*

During this same period, Peirce held that all senses of the modal terms could be analyzed in terms of states of information (3.374, *W* 5:169–70, 1885; *W* 5:330 and 372, 1886; 4.65, 1893; 3.442, 1896). I will follow Morgan in calling this the *Information-Relative* account (IR).[12] On this account, the claim that "it is possible that *p*" means that it is not known to be false that *p*. My claim that it is possible that it will rain today means that, in my present state of information, I do not know that it will not rain. Further, the claim that it is necessary that *p* means that it is known to be true that *p*. Suppose that I know that my keys are either in my office or in my car. I check my office and discover that they are not there. So I say, "My keys must be in my car." This means that in my present state of information, I know that they are in my car.

On the IR account, different varieties of modality are to be defined by reference to different states of information. I will call the state of information that defines a given type of modality the *designated state of information* (DSI) for that type of modality. The DSIs involved in the above examples are states in which an *actual* person might find herself. Following Peirce, I will refer to the sort of modality captured by these senses of the modal terms as *subjective* modality.[13] But there are other types of modality, types that go beyond what an actual subject knows or fails to know. For example, the DSI in terms of which the IR account defines *physical* possibility and necessity is that of a hypothetical person who is "thoroughly acquainted with all the laws of nature and their consequences, but . . . ignorant of

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[11] D. Lewis, *On the Plurality of Worlds* (New York: Blackwell, 1986). On whether Peirce was ever a modal realist along the lines of Lewis, see note 24.

[12] C. Morgan, "Peirce-Semantics," and "Modality, Analogy, and Ideal Experiments According to C. S. Peirce," *Synthese* 41 (1979): 65–83.

[13] Peirce wrote that these senses of the modal terms express the "most subjective meaning" of those terms (5.454, *EP* 2:354, 1905).

all particular facts" (4.66, 1893). The physically possible is that which someone in that DSI would not know to be false, and the physically necessary is that which someone in that DSI would know to be true. Similarly, the *practical* modalities are to be defined by "imagin[ing] ourselves to know what the resources of men are, but not what their dispositions and desires are," and the DSI of the *mathematical* and *metaphysical* modalities are, respectively, the states of information of "the most perfect mathematician or metaphysician" (ibid.).

Peirce described two types of modality that are "of special interest to the logician more than to other men" (4.67, 1893). He defined *essential* (or *logical*) modality in terms of the information possessed by someone who knows "*nothing*, except the meanings of words, and their consequences" (ibid.). And he defined s*ubstantial* modality in terms of the knowledge possessed by someone who knows "everything now existing, whether particular fact or law, together with all their consequences" (ibid.). The DSI of substantial modality is not the omniscience of a trans-temporal God. Rather, it is the state of information of a hypothetical subject who knows all natural laws and all *present* facts; it is "supposed information of the present in the present" (ibid.).

In 1896, but still before the modal shift, Peirce again asserted that the IR account was sufficient to explain all senses of the modal terms, and he again distinguished between essential and substantial (or as he had come to say, "substantive") modality (3.442, 1896)[14]—this time in the context of his explanation of a historical disagreement about the interpretation of conditionals:

> [T]he Philonian logicians have always insisted upon beginning the study of conditional propositions by considering what such a proposition means in a state of omniscience; and the Diodorans have, perhaps not very adroitly, commonly assented to this order of procedure. Duns Scotus terms such a conditional proposition a "*consequentia simplex de inesse*." According to the Philonians, "If it is now lightening it will thunder," understood as a consequence *de inesse*, means "It is either not now lightening or it will soon thunder." According to Diodorus, and most of his followers (who seem here to fall into a logical trap), it means "It is now lightening and it will soon thunder." (Ibid.; see also 2.348, *R* 787, 1896)

The disagreement with which Peirce was here concerned was over whether conditionals are logically equivalent to disjunctions.[15] For present purposes, however,

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[14] Here Peirce wrote 'substantive' rather than 'substantial', but this appears to be a mere change in terminology that does not reflect a change in his ideas. I will continue to use his original term, 'substantial', when discussing this sort of modality.

[15] The distinction Peirce described between Philonian and Diodoran interpretations of the conditional is not the distinction between material and strict conditionals. Although Peirce sometimes used 'Philonian' to refer to material conditionals (e.g., *R* 410, 14, 1893), in the article in question he clearly meant by 'Philonian' a conditional that is equivalent to a disjunction, whether or not it is quantified. Had he not meant this, his description of the strict conditional as "an *ordinary* Philonian conditional" would make little sense (3.444, 1896). The *de inesse* (i.e., material) conditional is, on his view, equivalent to an unquantified disjunction, while the "ordinary" Philonian conditional is equivalent to a universally quantified disjunction. So Peirce's statement that he "prefer[ed] to build the algebra of relatives upon" the Philonian interpretation of the conditional (3.443) does not mean that he was in favor of the material conditional rather than the strict conditional, but rather that he interpreted conditionals, both *de inesse* and ordinary, as being logically equivalent to disjunctions. For more on Peirce and the "Philonian" conditional, see J. Zeman, "Peirce and Philo," in *Studies in the Logic of Charles Sanders Peirce* [*Studies*], ed. N. Houser, D. Roberts and J. Van Evra (Bloomington and Indianapolis: Indiana University Press, 1997), 402–17.

what is more important is that Peirce relied on the notion of substantial possibility in his explanation of *de inesse* conditionals and thereby indicated how the IR account of modality might be used in interpreting both kinds of conditional. Ordinary (i.e., strict) conditionals can be understood as quantifications over possible states of things, which in turn can be understood as cases which a subject in some state of information cannot rule out, i.e., which such a subject does not know are non-actual. An ordinary conditional is a quantification over multiple possible states of things; for any *limited* DSI, there will be multiple cases which a subject in that DSI cannot rule out. On the other hand, *de inesse* (i.e., material) conditionals are not quantifications, as they "range" over only a single state of things, viz. the way things actually are. And the way things actually are would exhaust the knowledge of someone in the virtually unlimited DSI of substantial modality. A subject in this DSI would know all there is to know about the world as it actually is at present, including all present facts and laws. So in the substantial sense, there are no *contingent* states of things. That is, there are no states of things that are substantially possible (states of things that the subject does not know *do not* obtain) but not substantially necessary (states of things that the subject knows *do* obtain). The only state of things that a subject of the DSI of substantial modality would not know does not obtain would be the one state of things that he knows *does* obtain: the *actual* present state of things. Peirce's IR account of substantial modality implies that, at least with regard to the present, there are no substantial contingents: everything substantially possible is substantially necessary.

1.3 *Tychism*

Further, Peirce's IR account of substantial modality implies that, if determinism is true, there are no future substantial contingents. Again, a subject in the DSI of substantial modality would know all present facts and laws and their consequences. If determinism is true, those present facts and laws will together determine *all* future facts. So, assuming determinism, there would be no present *or future* truths that the subject of the DSI of substantial modality would not know. Every proposition about the future that such a subject would not know to be false would be a proposition that that subject would know to be true. All substantially possible truths about the future will be substantially necessary. But Peirce did not have to accept this consequence of the IR account of substantial modality, since he denied determinism. According to Peirce's *tychism*, not all events are completely law-governed.[16]

Ordinary chance, said Peirce, "is merely relative to the causes that are taken into account" (*EP* 1:219, *W* 4:549, 1883–84). The factors determining how a pair of dice will land are simply too numerous and complicated for us to take them all into account. "Chance" is involved here, but only in the sense that how the dice will land is not completely determined by the limited set of factors of which we do take account. But taking into account *all* of the influences on the dice, how they

[16] Peirce's most elaborate defense of tychism is in "The Doctrine of Necessity Examined" (6.35–65, *EP* 1:298–311, 1892). He used the term 'tychism' not in that article, but in "The Law of Mind" (6.102–63, *EP* 1:312–33, 1892).

will land is not a matter of chance at all. It is completely determined by those influences; at least, this is the case *if* determinism is true. *Absolute chance*, on the other hand, is real indetermination. For the most part, the universe is law-governed, and so, for the most part, what actually does happen happens necessarily, given previous conditions and governing laws. But at least some events are not completely determined to happen just as they do. Absolute chance is the *real violation of natural law*. Such violations are neither obvious nor common. Laws are violated only "in some infinitesimal degree" and on "rare sporadic occasions" (*EP* 1:219, *W* 4:549, 1883–84). Nonetheless, there is a real element of chance in the world, such that, given previous conditions and governing laws, it is possible both that an event *e* happen and that *e* not happen. In such a case, *e*'s occurrence is contingent. Peirce's tychism is thus the view that, in addition to real necessity, there is real contingency in the world.[17] The contingency the reality of which is asserted by tychism is not, or not only, *physical* contingency; it is *substantial* contingency. In the substantial sense of the modal terms, "everything in the present which is possible is also necessary, and there is no present contingent" (4.67, c. 1893). However,

> we may suppose there are "future contingents." Many men are so cocksure that necessity governs everything that they deny that there is anything substantially contingent. But . . . they are unwarrantably confident, . . . wanting omniscience we ought to presume there may be things substantially contingent, and further that there is overwhelming evidence that such things are. (Ibid.)

So Peirce's tychism asserted, not (or not simply) the reality of physical contingency, but rather (or additionally) the reality of substantial contingency.

If the foregoing is correct, then we can explain tychism in terms of Peirce's IR account of modality. The contingency of a future event is not a matter of anyone's *actual* ignorance as to whether that event will take place. Such actual ignorance, the ignorance of someone in the DSI of subjective modality, reflects mere ordinary chance. When I roll a pair of dice, it is subjectively possible (for me) both that they land double-sixes and that they land some other way. In contrast, absolute chance transcends anyone's actual ignorance. The DSI in terms of which absolute chance should be defined is a hypothetical one: the state of information of a hypothetical being who knows all present facts and laws, plus all of their consequences. In this state of information, anything about the present not known to be false *is*

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[17] Peirce was hired in 1883 to write definitions for the *Century Dictionary*, and the *Century*'s entry for "physical necessity," which appeared in 1889 in volume 14 of the *Dictionary*, is worth noting. It reads: "**Physical necessity**, the necessity which arises from the laws of the material universe. This necessity is conditional, not absolute." On one plausible reading of this definition, it is consistent with the view that, although there is real physical necessity in the world, the laws of nature nonetheless have exceptions, such that there is also real physical contingency. So this entry seems to be in harmony with Peirce's tychism. However, according to Cornelis de Waal, Associate Editor with the Peirce Edition Project, this entry did not originate with Peirce. Rather, it first appeared in the *Imperial Dictionary*, to which the *Century*'s publishers had purchased the rights. De Waal reports that the only differences between the entries in the two dictionaries are that the *Imperial* entry has 'that which arises' instead of 'the necessity which arises' and has 'conditional' and 'absolute' in italics. But De Waal also informs me that this entry is marked in Peirce's personal interleaved copy of the *Century* (held at Harvard), indicating that he took responsibility for it. So even though he did not author the entry, it seems that he saw no reason to change it—not surprising, given that it could be read as being in harmony with his tychism, as described above.

known to be true, so there is no present substantial contingency. But there is *future* substantial contingency, i.e., possible truths about the future that are not necessarily true. According to tychism, even a quasi-omniscient subject with complete knowledge of all present laws and facts and their consequences would not know everything about the future.

1.4 *Two Questions About Modality*

So during the 1880s and early 1890s, Peirce defined all senses of 'possible', 'contingent', and 'necessary' in terms of *states of information*, i.e., in terms of the knowledge of some subject, either real or hypothetical. During this same period, he held that there is both real necessity and real contingency in the world. But from relatively early on, Peirce defined the real as that which is independent of what anyone thinks about it.[18] How then could Peirce consistently maintain that some events are *really* necessary or contingent while at the same time defining 'necessary' and 'contingent' in terms of states of information? Peirce anticipated this objection and replied as follows:

> To conclude from the above definitions [i.e., the IR account] that there is nothing analogous to possibility and necessity in the real world, but that these modes appertain only to the particular limited information which we possess, would be even less defensible than to draw precisely the opposite conclusion from the same premises. It is a style of reasoning most absurd. (4.68, 1893)

Clearly, Peirce thought that there was no conflict between the IR account and his realism about necessity and contingency. We can clarify Peirce's position here by distinguishing two sorts of question:

(1) *What modalities are exemplified?* Assuming that every actual event is possible, are there actual events that are contingent? Or are all actual events necessary?

(2) *What is the metaphysical status of modality?* Is it real (i.e., independent of what anyone thinks about it)? Can all modal terms be adequately defined in terms of states of information?

The distinction between these sorts of question is analogous to that between normative ethical questions regarding certain types of action (e.g., are there any supererogatory actions, or are all morally permissible actions either obligatory or morally neutral?) and meta-ethical questions regarding the nature of morality itself (e.g., is morality objective or subjective? do moral judgments convey truths or merely express emotion?). The first sort of question asks whether there are any events that fall under concepts such as *necessary* and *contingent*, while the second sort of question asks about the nature of necessity, contingency and possibility themselves. Peirce's tychism addresses a type-(1) question about what sorts of modality are exemplified: it asserts that some future events are substantially con-

[18] This is Peirce's definition of 'real', which he used as early as his 1871 review of Fraser's *Berkeley* (8.12, *EP* 1:88, *W* 2:467).

tingent. On the other hand, the IR account addresses type-(2) questions about the metaphysical status of modality.

Prior to the modal shift, Peirce held that all types of possibility, necessity and contingency ought to be understood in terms of (actual or hypothetical) states of information and that this was compatible with there being something "analogous" to modality "in the real world." This suggests a range of responses to type (2) questions, with Peirce's 1893 view occupying a moderate ground between two extremes:

(a) *strong realism*: modality is real, and at least some types of modality cannot be defined in terms of states of information, even hypothetical states of information;

(b) *weak realism*: modality is real and all types of modality can be defined in terms of states of information; and

(c) *anti-realism*: modality is not real.

Prior to the modal shift, Peirce was a weak realist. He took modality, including future substantial contingency, to be real, yet he took the IR account to be sufficient to define all types of modality. As I will show, the modal shift of 1896 was a move from weak to strong realism. What Peirce changed his mind about was not whether certain modal concepts were exemplified. Instead, he changed his mind about whether one sort of modality, viz. *substantial possibility*, could be defined in terms of states of information.

2. THE MODAL SHIFT

2.1 *The Rejection of the IR Account of Substantial Possibility*

Peirce's modal shift occurred in the second of a two-part series of review articles on Ernst Schröder's *Vorlesungen über die Algebra der Logik*, published in *The Monist* in 1896 and 1897. In the first article, "The Regenerated Logic," he again committed himself to the IR account, claiming that

> *possibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false. By varying the state of information all the varieties of possibility are obtained. (3.442, 1896)

But Peirce's view changed shortly after writing this article. In a manuscript dating from around May 1896, Peirce—after once again describing conditionals as quantifying over possible states of things—characterized the IR account as providing an analysis of only "negative" possibility and asserted that there was, in addition, "positive" possibility:

> In its primitive sense, that which is *possible* is a hypothesis which in a given state of information is not known, and cannot certainly be inferred, to be false. The assumed state of information may be the actual state of the speaker, or it may be a state of greater or less information. Thus arise various kinds of possibility. All these varieties of possibility are *ignorantial*, or *negative*. *Positive* possibility arises when our knowledge is such as is represented by a disjunctive proposition, that either A, or B, or C, or D, etc., is true. A, B, C, D, etc., are then the positively possible cases. Thus,

> in playing backgammon, there are twenty-one possible throws of the dice, at each play. The aggregate of the positively possible cases is the *range* or *universe* of possibility. (2.346, *R* 787)[19]

But the "positive" possibility that Peirce described in this passage is still a matter of what someone knows or would know. It seems that Peirce wanted to go beyond the IR account, but that he nevertheless continued to assume that all senses of 'possible' could be defined in terms of knowledge. It was in "The Logic of Relatives," the second *Monist* article about Schröder, that Peirce first articulated a concept of possibility that, he held, could not be defined in terms of the knowledge of an actual or hypothetical subject.

In this article, Peirce asked: "Is it, or is it not, logically possible for two collections to be so multitudinous that neither can be put into a one-to-one correspondence with a part or the whole of the other?" (3.526; see also *R* 14, *NEM* 3:49, 1896). In effect, Peirce was asking whether the following theorem of set theory is true:

> *The Cardinal Comparability Theorem* (CCT): "for any two sets, A and B, either A can be put into a one-to-one correspondence with a subset of B, or vice versa."[20]

Peirce's answer was that CCT is true: it is *not* logically possible for there to be two sets so large that neither can be mapped one-to-one onto even a part of the other.[21] For present purposes, what is important is that he believed that, in order to arrive at this conclusion, he needed a significant revision in his views about modality. As he saw it, the IR account did not capture the sense of "logically possible" relevant to questions of set theory,[22] and answering the question whether CCT is true required "not a mere *application* of logic, but a further *development* of the conception of logical possibility" (3.526, 1897; see also *R* 14, *NEM* 3:50, 1896).

Peirce's further development of the conception of logical possibility began with his denial that the IR account was adequate to define substantial possibility. We have seen that, before the modal shift, Peirce claimed that two types of modality of special interest to the logician—essential (or logical) modality and substantial modality—are adequately defined by the IR account. On the IR account, it is substantially possible that *p* if and only if a subject who knows all present facts and laws and their consequences would not know that it is false that *p*. On this way of thinking, it is substantially possible that *p because* such a subject would not know that it is false that *p*. Further, on the IR account substantial impossibility is

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[19] According to Cornelis de Waal, Peirce probably wrote *R* 787 in May 1896. De Waal also reports that Max Fisch believed that Peirce wrote *R* 787 between "The Regenerated Logic" and "The Logic of Relatives." If this is correct, then the passage quoted above is Peirce's first step away from the IR account. Nonetheless, I still locate Peirce's modal shift as having occurred in "The Logic of Relatives." It was there that he first insisted on a sort of modality that cannot be defined in epistemic terms, and as we will see, Peirce himself eventually identified "The Logic of Relatives" as the work in which his thinking about modality took an important leap forward.

[20] W. Myrvold, "Peirce on Cantor's Paradox and the Continuum" ["Cantor's Paradox"], *Transactions of the Charles S. Peirce Society* 31 (1995): 508–41, at 530.

[21] Peirce argued in support of CCT at *R* 14, *NEM* 3:49–50, 1896; 3.548–50, 1897; 4.179, 1897; and *NEM* 3:958–62, 1903.

[22] He expressed this same concern a few years later in his Baldwin's *Dictionary* entries for "Modality" (2.383, 1902) and "Possibility, Impossibility and Possible" (6.367, 1902).

a matter of knowledge. It is substantially impossible that *p* if and only if a subject who knows all present facts and laws and their consequences *would* know that it is false that *p*. So on this view, it is substantially impossible that *p because* such a subject would know that it is false that *p*.

Peirce's objection in "The Logic of Relatives" was that *this gets things backwards*: "We know in advance of experience that certain things are not true, because we see they are impossible" (3.527). That is, a person will know that it is false that *p because he sees that it is substantially impossible that p*, rather than *vice versa*. He gave the following example. Suppose that a chemist knows the following two claims to be true:

(A) Fluorine is present in the majority of a set of bottles.

(B) Oxygen is present in the majority of that same set of bottles.

Given this, no further experience is needed for her to "see" that the following claim is *not* true:

(C) Fluorine and oxygen cannot be present in the same bottle.

It is not that it is impossible that (C) is true *because* someone who knows (A) and (B) would know, or judge, that (C) is false. Rather, someone who knows (A) and (B) will judge that (C) is false because she will see that, given (A) and (B), it is substantially impossible that (C). In general, whether a claim is substantially possible does not depend on what someone in a hypothetical state of information would not know to be false. As Peirce later put it, "[i]t is not that certain things are possible because they are not known not to be true, but that they are not known not to be true because they are, more or less clearly, seen to be possible" (6.367, 1902).

2.2 *The Ideal World Account*

Peirce proceeded to give a new, non-IR account of substantial modality. In describing the fluorine and oxygen example, he said of claim (C) that he "know[s] it is not true, because I satisfy myself that there is no room for it even in that ideal world of which the real world is but a fragment" (3.527, 1897). On this new *Ideal World* account (IW), something that occurs in the ideal world is substantially possible, and something that does not occur in the ideal world is substantially impossible. We can "see" what does and does not occur in the ideal world, and such

> ideal experimentation establishes a much broader answer to the question than sensible experimentation could give. . . . In respect to the ideal world we are virtually omniscient; that is to say, there is nothing but lack of time, of perseverance, and of activity of mind to prevent our making the requisite experiments to ascertain positively whether a given combination occurs or not. Thus, every proposition about the ideal world can be ascertained to be either true or false. A description of thing [*sic*] which occurs in that world is *possible, in the substantive logical sense.* (Ibid.)

Peirce was careful to distinguish the ideal world from the real world, which he took to be merely a part of the ideal world:

> Of those [combinations] which occur in the ideal world some do and some do not occur in the real world; but all that occur in the real world occur also in the ideal world. For the real world is the world of sensible experience, and it is a part of the process of sensible experience to locate its facts in the world of ideas. This is what I mean by saying that the sensible world is but a fragment of the ideal world. (Ibid.)

The ideal world seems to be something like an abstract world into which we "see" when we examine logical relationships between propositions, when we consider relationships between sets, and so forth, and his description of the "ideal experimentation" by which an inquirer can explore the ideal world is suggestive of the thought experiments so frequently relied upon in philosophical inquiry. But Peirce's comments here are far from enough to convey exactly what he meant by "ideal world" and how we go about discovering what is and is not true of it. Still, the little that he does say is enough to suggest that, as with the possible states of things over which the implicit quantifier of an ordinary conditional ranges, Peirce's ideal world is nothing like the multiple possible worlds of the sort required by Lewis's modal realism. One certainly need not be committed to a modal realism like Lewis's—according to which non-actual but possible worlds are as fully real as the actual world—in order to maintain that thought experimentation can reveal truths about what would be the case in imagined circumstances, and Peirce's brief account of ideal experimentation seems no more likely than a commitment to the legitimacy of thought experiments to commit him to that variety of modal realism.

By the time of the modal shift, Peirce had come to see the difference between essential and substantial possibility as hinging, not on differing DSIs, but on whether the contradiction of a logically possible proposition is itself possible:

> Very many writers assert that everything is logically possible which involves no contradiction. Let us call that sort of logical possibility, *essential*, or *formal*, logical possibility. It is not the only logical possibility; for in this sense, two propositions contradictory of one another may both be severally possible, although their combination is not possible. (Ibid.)

Allow 'EM' to represent "It is essentially logically possible that." Peirce's point is that

$$\mathrm{EM}p$$

and

$$\mathrm{EM}{\sim}p$$

may each be true, although

$$\mathrm{EM}(p \bullet {\sim}p)$$

is always false. The passage continues: "But in the *substantive* sense, the contradictory of a possible proposition is impossible, because we are virtually omniscient in regard to the ideal world" (ibid.). If a combination occurs in the ideal world, then that combination is substantively (i.e., substantially) possible. That is, if 'p' describes something occurring in the ideal world, then it is substantially possible

that *p*, and it is substantially impossible that not-*p*. Allow 'SM' to represent "It is substantially possible that." Peirce took

$$SMp \supset \sim SM\sim p$$

to be true for all values of *p*. Thus, on his view, any proposition that is substantially possible is also substantially necessary:

$$SMp \supset SLp$$

is true for all values of *p* (where 'SL' means "It is substantially necessary that").

It seems, then, that for Peirce, there are no substantially contingent propositions of set theory. This is not necessarily a change from his earlier position, according to which there are future substantial contingents but no present substantial contingents. It is reasonable to take the propositions of set theory to be timeless, and so as not referring only to the future. Peirce could consistently maintain that there are substantially contingent propositions about the future of the actual world while still maintaining that there are no substantially contingent propositions of set theory.

He went on to illustrate his assumption that substantial possibility entails substantial necessity:

> [T]here is no contradiction in supposing that only four, or any other number, of independent atoms exist. But it is made clear to us by ideal experimentation, that five atoms are to be found in the ideal world. Whether all five are to be found in the sensible world or not, to say that there are only four in the ideal world is a proposition absolutely to be rejected, notwithstanding its involving no contradiction. (Ibid.)[23]

For any positive value of *n*, it is substantially possible that at least *n* atoms exist. However, it is not substantially possible that *only n* atoms exist; i.e., it is substantially impossible that only *n* atoms exist. Now, there is no contradiction in supposing that only *n* atoms exist. "There are only *n* atoms" is not contradictory, nor does it imply a contradiction. Nonetheless, it is still substantially impossible that there be only *n* atoms, since it is substantially possible that there be $n + 1$ atoms.

Peirce took the "ideal experimentation" in which we "see" whether or not things occur in the ideal world to be fallible. But he held that, with care and hard work, the risk of error can be reduced "indefinitely," unlike in sensible experimentation—thus his claim that we are "virtually omniscient in regard to the ideal world." No longer did Peirce think of substantial possibility in terms of the ignorance of a hypothetical, virtually omniscient subject. On his new, IW account of substantial modality, the substantially possible is that which occurs in the ideal world, with regard to which we actual subjects are virtually omniscient.

2.3 *Substantial Possibility and the Ideal World After the Modal Shift*

In "Multitude and Number" (1897), Peirce returned to the question whether CCT is true. He gave an argument for CCT, the "gist" of which was that

> it considers in what way contradiction can arise, and thus shows that the only circumstance which could render the one-to-one correspondence [between the members

[23] Cf. *R* 14, *NEM* 3:49–50, 1896.

> of two sets] impossible in one way, necessarily renders it possible in another way. (4.179)

For our purposes, what is interesting about this passage is not the argument itself, but that it was supposed to work by showing that the supposition that there are two sets neither of which can be mapped one-to-one to the other is *contradictory*. This is very different than "The Logic of Relatives," in which he had maintained that it is *not* contradictory to suppose that there are two such sets and that the absence of such a contradiction does not settle the question whether it is (substantially) possible for there to be two such sets. In "Multitude and Number," he did not rely on the concept of substantial possibility to argue for CCT. Rather, his view seems to have been that, to settle the question whether CCT is true, one need only see that the supposition that there are two sets each of which is greater than the other implies a contradiction. Peirce seems to have been assuming that the question whether CCT is true can be settled with recourse to essential logical possibility only.

This *seems* also to have been his view in the Baldwin's *Dictionary* entry for "Possibility, Impossibility, and Possible," in which he once again considered whether CCT is true. Interestingly, he asked whether we can suppose that there are two sets each larger than the other "without involving ourselves in contradiction," and responded that "[i]t is that positive supposition which will constitute the possibility, not the mere ignorance of whether such a supposition can be made or not" (6.367, 1902). He went on to deny that the relevant sort of possibility is "a question of ignorance" and to say that "[i]t is . . . desirable to state the logical principles of this general kind of possibility, which does not consist in ignorance, but, as it would seem, in hypothetic indetermination or disjunctive determination" (ibid.). His comments about contradiction, hypothetic indetermination and disjunctive determination might be taken to suggest that, as in "Multitude and Number," he was leaning toward the view that the sort of modality relevant to set theory was essential modality and that the question whether CCT is true could be settled by attending to whether particular suppositions regarding transfinite sets imply contradictions. But the rest of this entry shows that this interpretation is incorrect. Although he did not use either 'substantial' or 'substantive' in the above-quoted passage, he makes clear later in the entry that that is what he had in mind: "Substantive possibility: the admissibility of a pure hypothesis (as illustrated above)" (6.371). It is telling, however, that in this passage, he again raised the question of CCT without mentioning the IW account. Peirce seems to have continued to take substantial modality to be relevant to settling questions within set theory, but to have been dissatisfied with the IW account he had given in "The Logic of Relatives." In his discussion of CCT in "Multitude and Number," Peirce employed a notion reminiscent of his earlier IW account: "[a]s [a] collection enlarges and the individual distinctions [among its members] are little by little merged, it also passes out of the realm of brute force into the *realm of ideas* which is governed by rules" (4.178, emphasis added). And he referred to "the world of ideas, a Platonic world" in the last of his 1898 Cambridge Conferences lectures: "we must suppose that the existing universe . . . is an offshoot from, or an arbitrary determination of, a world of ideas, a Platonic world . . . " (6.191–92, *RLT* 258; see also 6.208,

RLT 263).[24] But in neither work did he say enough to make clear what he had in mind by that notion. Substantial possibility appeared again in the Lowell Lectures of 1903, as the mode of being which characterizes qualities and relations (*NEM* 3:350–51). But there Peirce did not mention the IW account (or the IR account), and he said too little about substantial possibility to reveal how his thinking on the subject had progressed by that point. In around 1905 he wrote of his IW treatment of substantive possibility in "The Logic of Relatives" that "it by no means affords a completely satisfactory account" of that sort of possibility. (*R* 288, 128) And so far as I have been able to discover, "The Logic of Relatives" was the only work in which Peirce defined substantial possibility in terms of "the ideal world."

Given all this, one might reasonably conclude that Peirce's modal shift was ultimately inconsequential. As I have shown, his move from weak to strong realism was motivated by the belief that substantial possibility cannot be adequately defined in terms of states of information, but he seems never to have articulated an alternative definition of substantial possibility with which he was satisfied. Had Peirce written nothing else about possibility, we might conclude that the modal shift was merely an unsuccessful attempt to articulate a conception of logical possibility not based on the IR account, an attempt which he ultimately found wanting and which he never significantly improved upon, and which had no real impact on the rest of his thinking.

3. AFTER THE MODAL SHIFT

But as we saw at the beginning, Peirce eventually came to view the modal shift as an extremely important development, describing it as "the great step that was needed to render pragmaticism an intelligible doctrine." As we will see, Peirce's pragmaticism implies a realism about possibility that was missing from his earliest formulation of that view. But it is not a realism about substantial possibility; the connection between Peirce's modal shift and his explicitly modal pragmaticism is more oblique than that. To understand the impact Peirce's modal shift eventually had on his pragmaticism, we first need to see how it affected his extreme scholastic realism. But before we can fully understand *that* consequence of the modal shift, we must consider what seems to have been a more immediate consequence of Peirce's move from weak to strong realism, viz. a change in his definition of continuity.[25]

3.1 *Continuity*

I say 'seems to have been' advisedly, because the evidence on this score is inconclusive. So far as I aware, Peirce himself never stated that his 1896 rejection of

[24] He went on to say: "We shall naturally suppose, of course, that existence is a stage of evolution. *This existence* is presumably but a *special* existence. We need not suppose that every [Platonic] form needs for its evolution to emerge into this world, but only that it needs to enter into *some* theatre of reactions, of which this is one" (6.195, *RLT* 258). Hilary Putnam takes this as evidence that Peirce *was* a modal realist along the lines of David Lewis. See Putnam, "Comments on the Lectures," in *RLT*, 55–102, at 97.

[25] In tracing Peirce's revision of his definition of continuity to his modal shift, I agree with Noble, "Definitions of Continuity."

the IR account of substantial modality influenced his thinking about continuity. Still, within a year of that rejection, he adopted a new account of continuity that, unlike those he had given previously, centrally involved the notion of possibility. So it is reasonable to suppose that there *was* a connection between Peirce's modal shift and this new, modal account of continuity. I believe that this connection is the key to understanding exactly how the modal shift, stemming from Peirce's work in set theory, came to impact pragmaticism.

Before the modal shift, Peirce had gone through at least three distinct stages in his thinking about continuity.[26] In the first stage, which lasted until no later than 1889, Peirce defined continuity as the property a thing has when all its parts have parts "in the same sense," i.e., when it has no "ultimate parts" (5.335, *EP* 1:68, *W* 2:256, 1869). According to his own later criticisms, during this period, he took this to be the same property as infinite divisibility (6.168, 1903). He called this property 'Kanticity'.[27] During the second, relatively brief, stage, Peirce wrote the entry for "Continuity" in the *Century Dictionary* (1889). There he rejected his earlier definition of continuity and accepted Cantor's definition, according to which "continuity is the *perfect concatenation* of a system of points." Still, Peirce was not completely satisfied, since he referred to Cantor's as "[t]he less unsatisfactory definition" (6.164). In the third stage, beginning no later than 1892, Peirce defined a continuum as that which has *both* Kanticity and "Aristotelicity," the latter being the property of any series that "contains the end point belonging to every endless series of points which it contains" (6.123, 1892).

By the time he wrote "The Logic of Relatives," he may have become dissatisfied with that definition. In that article, only a few lines before he rejected the IR account of substantial possibility, he wrote that "a perfectly satisfactory logical account of the conception of continuity is required" for the further development of topology (or as he preferred, "mathematical topics") (3.526). He offered no definition (old or new) of continuity in that article, but a new definition came soon afterwards, in 1897's "Multitude and Number." There he argued that the elements of a set of merely possible objects have no distinct identity:

> When we say that of all possible throws of a pair of dice one thirty-sixth part will show sixes, the collection of possible throws which have not been made is a collection of

[26] For more details on these stages, see my "Peirce's Triadic Logic Revisited," *Transactions of the Charles S. Peirce Society* 35 (1999): 284–311. For an alternative account of the evolution of Peirce's definitions of continuity, see V. Potter and P. Shields, "Peirce's Definitions of Continuity," *Transactions of the Charles S. Peirce Society* 13 (1977): 20–34; and V. Potter, "Peirce on Continuity," in *Peirce's Philosophical Perspectives*, ed. V. Colapietro (New York: Fordham University Press, 1996), 117–23. For an alternative account of Peirce's fourth definition of continuity, see J. Dauben, "Peirce on Continuity and His Critique of Cantor and Dedekind," in *Proceedings of the C. S. Peirce Bicentennial International Congress*, 93–98. My account of Peirce's modal conception of continuity having been brought about by his modal shift can be read as a supplement to the very detailed account given in Myrvold, "Cantor's Paradox." Potter, Shields, Dauben and Myrvold do not emphasize the modal nature of the conception of continuity Peirce adopted in 1897 or connect that conception to his modal shift.

[27] It is clear from his own later criticisms of this view that his definition during this first stage was Kant's; see *NEM* 3:780, 1899; 3.569, 1900; and 6.168, 1903. Peirce used the term 'Kanticity' to describe this property at 6.166, circa late 1891 to 1892; 6.122, 1892; and 4.121, 1893. (On the dating of 6.166 and 168, see my "Peirce's Triadic Logic Revisited," 307 n. 21.)

> which the individual units have no distinct identity. It is impossible so to designate a single one of those possible throws that have not been thrown that the designation shall be applicable to only one definite possible throw; and this impossibility does not spring from any incapacity of ours, but from the fact that in their own nature those throws are not individually distinct. (4.172, 1897)

When it comes to collections of "mere possibilities," such as the drops of water in the ocean, or the New Yorkers who will kill themselves the year after next, "the individuals merge together" (ibid.). A collection of mere possibilities is non-discrete, or *continuous*: "the units lose their individual identity because the collection exceeds every positive existence of the universe" (4.175).

Peirce conceived of generality as the opposite of individuality.[28] So it stands to reason that, having come to think of the merely possible as lacking individuality, Peirce would also begin to think of the merely possible as exemplifying generality:

> The possible is necessarily general; and no amount of general specification can reduce a general class of possibilities to an individual case. It is only actuality, the force of existence, which bursts the fluidity of the general and produces a discrete unit. (4.172, 1897)

Further, before the modal shift, Peirce took generality to be associated with continuity.[29] So for Peirce, his 1897 insight that possibility is a form of generality implied that there is an intimate connection between possibility and continuity: "the possible is general, and continuity and generality are two names for the same absence of distinction of individuals" (4.172). In an earlier piece, written at about the same time as "The Logic of Relatives," Peirce had described continuity as being incompatible with the presence of distinct individuals (*R* 14, *NEM* 3:62, 1896; see also *NEM* 3:799, 1897). But so far as I am aware, 1897's "Multitude and Number" contains Peirce's first description of this absence of distinct individuals in terms of mere possibility.

He went on to assume this connection between the merely possible and the continuous in an explanation of the continuity of a line:

> [A] line is nothing but a collection of points of a particular mode of multiplicity, yet in it the individual identities of the units are completely merged, so that not a single one of them can be identified, even approximately, unless it happen to be a topically singular point, that is, either an extremity or a point of branching, in which case there is a defect of continuity at that point. (4.219)

In other words, a continuous line is a collection of *merely possible* points having no distinct identities. It is like the collection of drops of water in an ocean, none of which has a distinct identity until it is made separate from the rest, thus interrupting the continuity. This new, modal conception of continuity appeared next in his 1898 Cambridge Conferences lectures:

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[28] Peirce contrasted individuals with universals as early as 1868 (*W* 2:175). For an explicit contrast between general facts (i.e., laws) and individual facts from about the same time as the modal shift, see 1.434, c. 1896.

[29] E.g., "in the light of the logic of relatives, the general is seen to be precisely the continuous" (*Collected Papers*, 8, 279, c. 1893; see also *NEM* 4:343, *RLT* 160, 1898).

> [A] continuum is a collection of so vast a multitude that in the whole universe of possibility there is not room for them to retain their distinct identities; but they become welded into one another. Thus, the continuum is all that is possible, in whatever dimension it be continuous. (*NEM* 4:343, *RLT* 160)[30]

So Peirce adopted a new, modal conception of continuity in 1897, the year after the modal shift. But he did not think of that new conception of continuity as *entirely* new. Before 1889, he held continuity to be Kanticity, and between 1893 and 1897, he took Kanticity to be a necessary (but not sufficient) condition of continuity. After his acceptance of the modal conception of continuity, he came to think that Kant's definition of continuity, and thus his own concept Kanticity, had been muddled. In 1903, Peirce wrote that his earlier definition of continuity as Kanticity plus Aristotelicity

> involves a misunderstanding of Kant's definition which [Kant] himself likewise fell into. Namely he defines a continuum as that all of whose parts have parts of the same kind. He himself, and I after him, understood that to mean infinite divisibility, which plainly is not what constitutes continuity . . . (6.168; see also *NEM* 2:482, c. 1904, and *NEM* 3:748, no date)

Having cleared up this confusion, Peirce came to think that his new, modal definition of continuity was roughly the same as Kant's (*NEM* 3:748, 1902; see also 3.569, 1900). So the modal shift did not result in a wholesale revision of his previous thinking about continuity. Rather, after the modal shift, Peirce reconceived Kanticity in modal terms, distinguishing it from infinite divisibility, and adopted this new, clarified concept of Kanticity as his definition of continuity.

3.2 *Extreme Scholastic Realism and Pragmaticism*

Nine years after the modal shift, Peirce wrote the following:

> In the *Monist* of Jan. 1897 [i.e., in "The Logic of Relatives"], quite assertorically, . . . and this same journal during 1892 [i.e., in "The Doctrine of Necessity Examined"] more tentatively, the author of the present article repudiated the nominalistic view of possibility, and explicitly returns to the Aristotelian doctrine of a *real possibility*. This was the great step that was needed to render pragmaticism an intelligible doctrine. The paper of Jan. 1878 [i.e., "How to Make Our Ideas Clear"] wavers palpably at this point, sensible of the advantages of a real possibility, yet wishing to save pragmaticism in case that doctrine should prove untenable. (*R* 288, 129, 1905)

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[30] For a detailed analysis of the modal account of geometrical continuity Peirce gave in these lectures, see H. Putnam, "Peirce's Continuum," in *Peirce and Contemporary Thought: Philosophical Inquiries*, ed. Ken Ketner (New York: Fordham University Press, 1995), 1–22. (A slightly different version of this was published as part of K. Ketner and H. Putnam's introduction to *RLT*.) For other statements of this modal conception of continuity, see, e.g., *NEM* 3:388, 1903; 3.568, 1900; and 6.168, 1903. Potter and Shields ("Peirce's Definitions of Continuity"), Potter ("Peirce on Continuity"), and Myrvold ("Cantor's Paradox") all maintain that Peirce adopted a fifth conception of continuity in 1908. While Peirce did claim to have made important progress in his thinking about continuity in 1908 (4.642), he seems nonetheless to have maintained the modal aspect of his 1897 conception of continuity: "there would be no actually existent points in an existent continuum, and . . . if a point were placed in a continuum it would constitute a breach of the continuity. Of course, there is a possible, or potential, point-place wherever a point might be placed" (6.182, c. 1911).

In another manuscript from the same period, Peirce wrote:

> In ["The Logic of Relatives"] the objectivity of possibility was asserted; and the hypothesis defended in ["The Architecture of Theories" (1891) and "The Doctrine of Necessity Examined," viz. the doctrine of tychism] supposes possibility to be real. . . . But ["How to Make Our Ideas Clear"] evidently endeavors to avoid asking the reader to admit a real possibility. The theory of modality is far too great a question to be treated incidentally to any other. But the distinct recognition of real possibility is certainly indispensable to pragmaticism. (5.527, c.1905)

So in hindsight, Peirce saw his criticism of the IR account of substantial modality, as well as his earlier defense of tychism, as "repudiations" of a nominalistic approach to the question of modality, and to see that move as having resulted in an improvement over his original treatment of pragmatism in "How to Make Our Ideas Clear." Although there seems to have been no immediate change in Peirce's pragmaticism following the modal shift and his adoption of a modal conception of continuity, there are unmistakable reflections of his new definition of continuity within his treatment of extreme scholastic realism in the early 1900s. And it is to that theory which we must now turn to understand, not only why Peirce came to think of the modal shift as having been decisive in the development of pragmaticism, but also why he came to view the IR account of possibility as nominalistic.

According to Peirce's *Extreme Scholastic Realism* (ESR), there are real generals, including real natural kinds and laws.[31] He gave the following well-known argument for the doctrine in his 1903 Harvard lectures on pragmatism. I hold a solid object, a rock for example, in my hand. I can predict with justification that if I release it, it will fall. According to Peirce, I know that the rock will fall because I know from past experience that *objects of this kind* always do fall**.** That is, I know that "all solid bodies fall in the absence of any upward forces or pressure," and "[i]f I *truly know* anything, that which I know must be *real*" (5.94–96, *EP* 2:181, *PPM* 191). So that which is represented by the general proposition "All solid bodies fall in the absence of any upward forces or pressure" is real. This law is an "active general principle" that is "really operative in nature" (5.100–01, *EP* 2:183, *PPM* 193).

That ESR requires the reality of necessity is not hard to see: since there are real natural laws, the necessity embodied by such laws is real. As we have already seen, Peirce's belief that there are real natural laws predates the modal shift. Peirce maintained the truth of ESR as early as 1868,[32] and in 1905 he wrote that "long before" writing "The Logic of Relatives," he had "argued for real generals, from which it follows pretty obviously that there are real necessities" (*R* 288, 129). It

[31] Peirce referred to his belief that there are real generals as "extreme scholastic realism" in a letter to Calderoni (8.208, c. 1905). Elsewhere he described himself as a "scholastic realist of a somewhat extreme stripe" (5.470, c. 1905). According to Fisch (*Peirce, Semeiotic, and Pragmatism*, 193), Peirce first called his realism about generals "extreme" in the "Grand Logic" of 1893. There he wanted to distinguish his realism both from nominalism and from "moderate realism," according to which there are "real" laws the being of which consists in nothing but the being of their individual cases (*R* 410, 15). In "On Peirce's Early Realism," I used the phrase 'extreme scholastic realism' to refer to the later version of Peirce's theory, according to which there is both real generality and real possibility (as explained below). However, I no longer think it historically accurate to restrict the phrase in that way, since Peirce considered himself an "extreme" scholastic realist *before* his scholastic realism became a form of realism about possibility.

[32] See my "On Peirce's Early Realism."

is no surprise, then, that by the early 1890s, Peirce had come to believe that the question of the reality of generals was, in essence, *the same* as the question of the reality of modality. In 1893's "Grand Logic," immediately after responding to the anticipated criticism that the IR account of modality implies that "there is nothing analogous to possibility and necessity in the real world," Peirce wrote:

> The question of realism and nominalism, which means the question how far real facts are analogous to logical relations, and why, is a very serious one, which has to be carefully and deliberately studied, and not decided offhand, and not decided on the ground that one or another answer to it is "inconceivable." Nothing is "inconceivable" to a man who sets seriously about the conceiving of it. (4.68)

So even before the modal shift, Peirce had come to associate the debate over whether there are real generals, including real natural laws, with the question of the nature of modality.

After the modal shift, Peirce's ESR took on a new dimension. It expanded to become the claim that there are not just real "generals" (and thus real necessity), but also real possibility, or as Peirce sometimes said, real "vagues" (5.453, *EP* 2:354, 1905). And the sort of possibility involved in this expanded ESR is the same as that posited in his new, modal conception of continuity: the possibility embodied by a collection that exceeds any definite multitude. In his 1903 Harvard lectures, he noted the "old" definition of a general: *Generale est quod natum aptum est dici de multis* ("A general is that whose expression naturally suits many things") (5.102, *EP* 2:183, *PPM* 193).[33] General terms like 'gold', 'lithium', and 'sun' are capable of applying truly to many different individual things. But to say that a general is that the expression of which suits merely many things is to describe "a very degenerate sort of generality" (5.103, *EP* 2:183, *PPM* 193). General terms are capable of applying truly, not just to many things, but to *any* number of objects whatsoever. On Peirce's view, there is no greatest multitude, so there is no limit to the number of objects to which a general term can truly apply. A general term is predicable, not only of a multitude greater than that of the individuals existing at any given time, but also greater than that of all individuals that will *ever* exist. So the reality of a general, like *sun*, requires the reality of possible but non-actual suns (ibid.). Merely possible suns are, in a sense, general, and therefore non-individual. They have no distinct identities of their own, no more so than the drops of water in the ocean or the possible points on a line.

So after the modal shift and his adoption of a modal conception of continuity, Peirce's ESR became the view that there are real generals, and thus real necessity, and real vagues, including real possibilities. It was also after the modal shift that Peirce came to think of pragmaticism as requiring the reality of unactualized possibilities, and as thus presupposing the truth of ESR. This, I believe, is ultimately why Peirce came to view the modal shift as having rendered pragmaticism "intelligible."

The classic statement of Peirce's *Pragmatic Maxim* (PM) is from 1878's "How to Make Our Ideas Clear": "Consider what effects, which might conceivably have

[33] I take this English translation from *EP* 2:523 n. 7.

practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object" (5.402, *EP* 1:132, *W* 3:266).[34] The PM identifies the pragmatic meaning of a concept with the concept of the phenomena that would result from some experimental interaction with the objects to which that concept applies. For example, the PM gives the meaning of the claim "*x* is a diamond" as a list of conditionals specifying phenomena that would result from interaction with *x*. The PM can be construed either as generating conditionals in the indicative mood (e.g., "If you apply pressure to *x*, it will resist") or in the subjunctive mood (e.g., "If you were to apply pressure to *x*, it would resist").

It is important which sort of conditional is generated by the PM. In "How to Make Our Ideas Clear," Peirce imagined a diamond that is never used to cut anything and which no one ever attempts to cut. It has been wrapped in cotton for its entire existence and, setting aside the pressure which is needed to create the diamond, the only pressure to which it has ever been exposed is that of the atmosphere and of its own weight. The diamond is destroyed without ever having any further pressure applied to it. Peirce asked, was the diamond hard or soft? The answer depends on how we construe the PM.

Indicative conditionals cover only actual events, so if the pragmatic meaning of "*x* is hard" is given by a list of indicative conditionals, the claim that the diamond was hard is analyzed in terms of actual events only and so refers only to interactions someone actually undertakes with *x*. Peirce originally construed the PM as generating indicative conditionals, and his initial answer to the question about the diamond was that *it is merely a matter of language* whether we say that it is "hard" or "soft" (5.403, *EP* 1:132, *W* 3:267, 1878). The difference between these descriptions will be a difference only in how we use the word 'hard' and will not reflect a difference in our idea of *hardness*, at least in so far as the pragmatic meaning of that idea is concerned.

On the other hand, subjunctive conditionals cover not only actual cases, but also merely possible cases. If the pragmatic meaning of 'hard' is given by subjunctive conditionals, then the diamond *was* hard, even if it was never subjected to pressure. It was hard in that, *if* pressure *had been* applied to it, it *would not have been* scratched. On this understanding of the PM, the pragmatic meaning of 'hard' includes the idea that a hard object is one which *would* resist pressure, even if pressure is never actually applied to it. After the modal shift, Peirce continued to maintain that whether the diamond is hard is a matter of language, but he had come to hold that it is not a *mere* matter of language. In 1905's "Issues of Pragmaticism," Peirce wrote that, in "How to Make Our Ideas Clear," he had

> said that if a diamond were to be formed in a bed of cotton wool, and were to be consumed there without ever having been pressed upon by any hard edge or point, it would be merely a question of nomenclature whether that diamond should be said to have been hard or not. No doubt, this is true, except for the abominable falsehood in the word MERELY, implying that symbols are unreal. Nomenclature involves clas-

[34] Later statements of the PM are at 5.412, *EP* 2:332, 1905; 5.438, *EP* 2:346, 1905; 5.9, c. 1905; and *EP* 2:402, 1907, a slightly different draft of which is at 5.467.

> sification; and classification is true or false, and the generals to which it refers are either reals in the one case, or figments in the other. (5.453, *EP* 2:354)

Peirce had come to hold that it is a mistake to construe the "experiments" described in the conditionals generated by the PM as isolated, individual actions. Rather, they should be understood as "*general kinds* of experimental phenomena," and thus, in order for any of the conditionals generated by the PM to be true, there must be real generals (5.425–26, *EP* 2:340; see also 5.3, 1902).

But there must also be real "vagues," real possibilities. In order for it to be true that "If you were to apply pressure to *x*, it would not be scratched," it must be really possible for pressure to be applied to *x*, whether or not it ever actually happens:

> Pragmaticism makes the ultimate intellectual purport of what you please to consist in conceived conditional resolutions, or their substance; and therefore, the conditional propositions, with their hypothetical antecedents, in which such resolutions consist, being of the ultimate nature of meaning, *must be capable of being true*, that is, of expressing whatever there be which is such as the proposition expresses, independently of being thought to be so in any judgment, or being represented to be so in any other symbol of any man or men. *But that amounts to saying that possibility is sometimes of a real kind.* (5.453, *EP* 2:354; emphases added)

Possibility is *not just a matter of ignorance*, even the ignorance of a hypothetical being. The truth of the conditionals generated by the PM requires a type of possibility that is not adequately captured by the IR account, the "real possibility" recognized by Peirce's later, modal ESR:

> It is plain that pragmaticism involves scholastic realism, since it makes all intellectual purport, and therefore, the meaning of reality itself, to consist in what *would be*, under conceivable conditions most of which can never be actualized. It thus involves making real being to include more than *existence*. Now that is precisely the point in dispute between Realists and Nominalists. 'A real possibility,' says the nominalist, 'is nonsense. For that is *possible* which we do not know is not true.' The realist says that there is, besides, a real possibility and real necessity (not mere compulsion, but rational necessity, as in the laws of nature). (*R* 845, 29–30, 1905)[35]

Just like the points in a continuous line or the drops of water in the ocean, the merely possible future events to which a subjunctive conditional refers do not constitute a collection of distinct individuals. To say that *x* is hard is not to say something about a collection of individual events or actions in the past, or even in the future. Rather, it is to make a general claim about what would happen in the future were specific conditions met. The 'would' of the subjunctive conditionals generated by the PM is inexhaustible by any multitude of actual events (8.208, c. 1905), just as the continuous line is inexhaustible by any multitude of individual points.

Peirce went on to say that in his original statement of pragmatism he had perhaps "wavered in his own mind" about the metaphysical presuppositions of the

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[35] Quoted in Fisch, *Peirce, Semeiotic, and Pragmatism*, 199, n. 24. According to Fisch, this comes from "Answers to Questions Concerning My Belief in God," most of which was published as 6.494–521. The passage in question is, Fisch says, from "a long passage omitted at the end of 6.501"; it is not included in the Harvard University Library microfilm edition of the Peirce manuscripts, which omits 21–31 of *R* 845. I take the date of this manuscript from Fisch.

PM (5.453, *EP* 2:354, 1905). A careful examination of that article reveals that it was not only with regard to the hardness of diamonds that Peirce wavered. When he first presented his pragmatic account of truth, he did so in terms of statements in the indicative mood:

> [A]ll the followers of science are fully persuaded that the processes of investigation, if only pushed far enough, *will* give one certain solution to every question to which they can be applied. . . . The opinion which *is* fated to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real. (5.407, *EP* 1:139, *W* 3:273; emphasis added)

But immediately after giving this account, Peirce considered a number of objections, and in responding to them he slipped into using the subjunctive mood with no acknowledgement of having done so. In responding to the objection that his pragmatic account of truth and reality is incompatible with his own definition of the real as that which is independent of what anyone thinks about it, Peirce wrote that

> if, after the extinction of our race, another should arise with faculties and disposition for investigation, that true opinion must be the one which they *would* ultimately come to. . . . the opinion which *would* finally result from investigation does not depend on how anybody may actually think. (5.408, *EP* 1:139, *W* 3:274; emphases added)

He then considered the well-known "Buried Secrets" objection to the pragmatic account of truth, viz. that it does not take account of truths that are forever beyond the reach of inquiry. His response was that "it is unphilosophical to suppose that, with regard to any given question (which has any clear meaning), investigation *would* not bring forth a solution of it, if it *were* carried far enough" (5.409, *EP* 1:140, *W* 3:274; emphases added). So even before the modal shift, Peirce seems to have been half-conscious of the need to modify the indicative-mood account of truth and reality and replace it with an account in the subjunctive mood.[36]

So Peirce first explicitly committed to the use of subjunctive conditionals in the application of the PM after the modal shift. But his interpretation of strict conditionals as covert quantifications over possible states of things did not change after the modal shift.[37] In 1898, he once again distinguished between *de inesse* and ordinary conditionals, and described each as he had previously: the former "relates to no range of possibilities at all, but merely to what happens, or is true, *hic et nunc*," while the latter "asserts . . . that in each possible state of things throughout a well-understood range of possibility either the antecedent is false or the consequent true" (*RLT* 125; see also 2.556, 1902; 4.410, 1903; and 4.564, 1906). And he once again committed to the use of the strict conditional in logic: " . . . in logic we are to understand the form 'If A, then B' to mean 'Either A is impossible or in every

[36] This earlier "wavering" between indicative and subjunctive conditionals has been overlooked by other commentators.

[37] Peirce did not take the distinction between material (*de inesse*) and strict (ordinary) conditionals to correspond exactly with the distinction between indicative mood and subjunctive mood conditionals; e.g., his position in 1896's "The Regenerated Logic" was that a given conditional, whether indicative or subjunctive, can be construed as either a *de inesse* or an ordinary conditional (3.442–43).

possible case in which it is true, B is true likewise,' or in other words it means 'In each possible case, either A is false or B is true'" (*RLT* 126).[38]

We are now in a position to understand Peirce's 1902 description of the IR account of possibility as "nominalistic" (6.367), as well as his later reference to the "nominalism" of "How to Make Our Ideas Clear" (8.216, c. 1910). Nominalism denies that there is real generality, so any conception of possibility according to which possibility is non-general is a nominalistic conception, even if it nonetheless maintains the reality of the possible. Peirce was an avowed realist about modality before the modal shift, but in hindsight he came to believe that his earlier view of modality had been nominalistic, since during that time he had not maintained that possibility was a form of generality. He came to see the modal shift as the first point in the development of his system at which possibility *conceived as generality* could enter. Looking back from 1905, his tychism appeared to be only a "tentative" repudiation of the nominalistic view that possibility is not real, in comparison with his later "quite assertoric" insistence on the reality of possibility. And it was only after that "quite assertoric insistence" that he came to think of ESR as implying that there is, besides real generality, also real possibility—hence his claim that the objectivity of possibility was "implied" in his 1871 defense of scholastic realism (5.527, c. 1905).

But Peirce never rejected the IR account outright.[39] Even after the modal shift, he thought of it as "extremely helpful up to a certain point" (6.367, 1902). It was, in his view, adequate to define a number of different types of possibility. He continued to recognize that modal terms sometimes express subjective modality (5.454, *EP* 2:354–55, 1905); he continued to use the IR definition of physical possibility (6.371, 1902); and he allowed the IR account to play a role in the interpretation of his gamma graphs, the modal part of his system of existential graphs (4.573, 1906). But he nonetheless viewed it as "superficial" (6.367, 1902), because it was inadequate to define the "real possibility" relevant to continuity, ESR and pragmaticism.

4. CONCLUSION

Before his 1896 modal shift, Peirce held that all senses of the modal terms are correctly analyzed in terms of states of information. He advocated strict conditionals as better expressing "the sense which is most usefully attached to the hypothetical proposition in logic" and interpreted them as quantifying over possible states of things. He was fully conscious that his extreme scholastic realism implied that there is real necessity and that his tychism implied that there is real contingency, and there are indications that he was at least somewhat conscious that an adequate

[38] In noting that Peirce's thinking about conditionals did not change after the modal shift, I disagree with Fisch, *Peirce, Semeiotic, and Pragmatism*, 196, and P. Skagestad, *The Road of Inquiry: Charles Peirce's Pragmatic Realism* (New York: Columbia University Press, 1981), 108–09.

[39] E.g., 4.517, 1903; 4.573, 1906. To my knowledge, the only previous commentators to acknowledge Peirce's use of the IR account after the modal shift are Morgan, "Peirce-Semantics," and R. Müller, "Interpretations of Modality: Epistemic Logic and Peirce's Logic of Ignorance," in *Phenomenology on Kant, German Idealism, Hermeneutics and Logic*, ed. O. K. Wiegand, et al. (Dordrecht-Boston: Kluwer Academic Publishers, 2000), 189–99.

version of pragmatism would pull one towards realism about unactualized possibilities. His modal shift was not a *complete* about-face. It was, rather, a move from weak realism to strong realism about one sort of possibility in particular: substantial possibility. Because of his desire to settle the question whether CCT is true, he adopted a non-IR account of substantial possibility, an account in terms of the "ideal world." Shortly after the modal shift, he introduced a new account of continuity as involving real, unactualized possibility, and this same sort of possibility was soon integrated into his ESR, completing that theory and thus, in his eyes, rendering "pragmaticism an intelligible doctrine." Even though he eventually set aside the IW account of substantial possibility and never articulated an account of that type of possibility with which he was fully satisfied, he came to see his initial rejection of the IR account of substantial possibility as the acceptance of an "Aristotelian" realism about possibility. But his modal shift did not prompt him to revise his interpretation of conditional propositions, and never—even at the time of the modal shift—did he intend *completely* to reject the IR account, which he continued to view as an adequate account of some types of modality.

This is far from an exhaustive account of Peirce's views on modality. There are several other issues upon which a complete account would have to touch and about which I have said little or nothing. These include his association of the concepts of *possibility* and *quality*, and his integration of each into his system of universal categories;[40] improvements in his theory of signs that followed on the heels of his modal shift;[41] his changing views about the nature of probability; the gamma portion of his existential graphs; and his denials that the principle of excluded middle applies to assertions of necessity and that the principle of contradiction applies to assertions of possibility.[42] What I have provided in this essay is a map through some of the darker territories of Peirce's thinking about modality—a map which, I hope, although only roughly sketched in some areas, nonetheless depicts more regions, and depicts them in more detail and with greater clarity, than previous commentators have managed.[43]

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[40] M. Murphey (*The Development of Peirce's Philosophy* [Cambridge: Harvard University Press, 1961], 394) and Fisch (*Peirce, Semeiotic, and Pragmatism*, 194) trace Peirce's mid-1890s revision of his system of categories directly to the modal shift. Fisch refers to a manuscript entitled "The Logic of Mathematics: An Attempt to Develop My Categories from Within" (1.417–520, *R* 900), which the *Collected Papers* dates as c. 1896 and which Fisch takes as having been written after "The Logic of Relatives." On the other hand, T. L. Short cites this same work as an *anticipation* of Peirce's modal shift, although therein Peirce does not mention the IR account. See Short, "The Development of Peirce's Theory of Signs" ["Development"], in *The Cambridge Companion to Peirce*, ed. Cheryl Misak (New York: Cambridge University Press, 2004), 214–40, at 238, n. 6.

[41] According to Short ("Development," 225), after Peirce's modal shift, he "was able ... to identify significance not with actual but with potential interpretation, or interpretability."

[42] E.g., *R* 678, 34, late 1910. For more on these claims, see my "Peirce's 'Entanglement' with the Principles of Excluded Middle and Contradiction," *Transactions of the Charles S. Peirce Society* 33 (1997): 680–703. Peirce did deny bivalence of a specific group of propositions, but not modal propositions; see my "Peirce's Triadic Logic Revisited."

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