Nonlinear Anisotropic Filtering

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Abstract. Image reslicing is an essential tool for mapping images into different coordinate systems, when overlaying images or registering to calibrate tracking. Nonlinear image filtering is an important tool for processing of medical images, and is particularly necessary when the resliced plane is nonorthogonal to the imaged plane. In this coursework, I have explored 2D anisotropic filtering after reslicing and 3D anisotropic filtering before reslicing, with different K values, controlling the strength of the filtering. Comparing the two statistically is challenging, but it is easily visible that 3D filtering is more robust against increasing diffusion.

1 Introduction

Medical imaging is becoming increasingly important in standard patient care. Our current imaging modalities are well suited for visual interpretation. Object visibility and the clarity of details have historically been favoured parameters during acquisition, as humans are highly effective at recognising structures in images even in the presence of a considerable amount of noise Gerig et al. [1992]. For image processing and computational methods of analysis, the signal to noise ratio and signal to contrast ratio become increasingly important. Acquisition based approaches to noise reduction can be costly and impractical, thus post-processing of raw data is an active field of research. Image filtering techniques have evolved as our computational capabilities have increased. Simple spatial averaging, reduced the amplitude of noise at the expense of a reduction in feature sharpness. The resulting blurring and diffusion can be suppressed by accounting for nonlinearity when filtering. Nonlinear filtering is particularly important when visualising non orthogonal planes.

2 Methods

2.1 Preparing Data

Reslicing of image in non-orthogonal plane, is implemented by applying a three-dimensional Euclidean distance transform the image. This allows for rotation about an axis, viewed from another axis, the image exists on a non-orthogonal plane.

2.2 Nonlinear filtering

Partial differential equations (PDE) are the basis of many image processing methods for smoothing. PDE methods model the original image as an initial state of a parabolic process, which develops into the filtered images through its time evolution. The Perona-Malik algorithm, models smoothing as a nonlinear diffusive process according to Equations (1,3) Nchama et al. [2020].

$$\frac{\delta}{\delta t}u(x,y,t) - div[g(|\nabla u(x,y,t)|)\nabla u(x,y,t) = 0, \Omega * (0,T] \tag{1}$$

$$\frac{\delta}{\delta n}u(x,y,t) = 0, \delta\Omega * (o,T] \tag{2}$$

$$u(x,y,0) = u_0(x,y), \Omega \tag{3}$$

Where $u(x,y,0), u(x,y,t)$ are the original and smoothed image respectively, as a function of time t. $\delta\delta n$ is the derivative normal to the boundary, and the diffusion coefficient function, is $g(|\nabla u(x,y,t)|)$ given by Equations (4,5), where K controls the strength of the diffusion Nchama et al. [2020].

$$g_1(U) = e^{(-U/K)^2} \tag{4}$$

$$g_2(U) = \frac{1}{1 + (U/K)^2} \tag{5}$$

The diffusion coefficient function stops the diffusion from effecting edges. Thus, by detecting the edges within an image, the algorithm can preserve them, favouring smoothing in regions with less detail.

3 Experiments

3.1 Positioning

Figure 1 shows reslicing of the 3D image to obtain an image on a non-orthogonal plane. Slices in each plane are sampled. Different rotations are applied to the image, until a suitable resampled image obtain. Care is taken to translate the centre point appropriately.

3.2 Comparison between 2D and 3D Anisotropic Filtering

2D filtering is performed after reslicing. The image is re-sampled, and filtering is simply performed on the single slice. 3D Filtering is performed on the entire image, before reslicing.

Diffusion strength is controlled by K according to Equations 4 and 5. It thus depends on the magnitude of the gradient of image intensity. When K is small, small intensity gradients block diffusion from crossing edges, shown in figure 2. The 3D implementation is more robust against increases in K. A reasonable value must be selected for the number of iterations used to solve for the gradient in a finite difference implementation. Then a slice can be visualised. The comparison of results in shown in Figure 3.

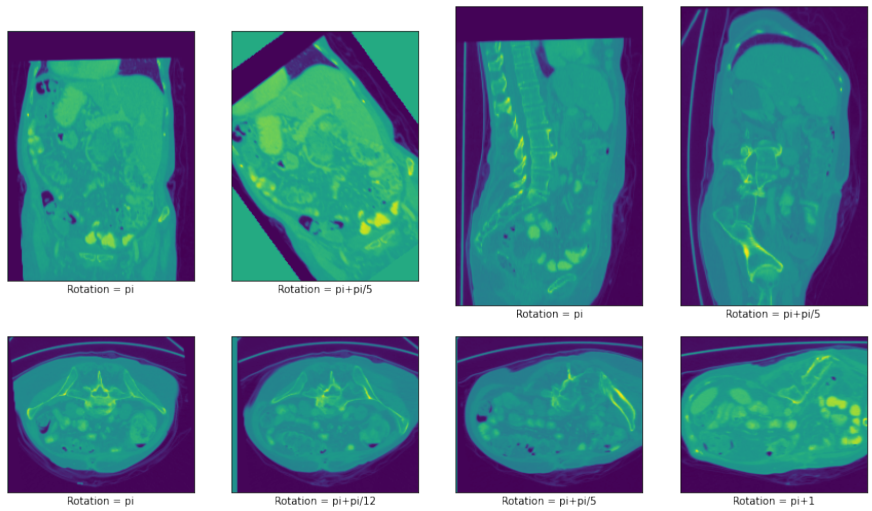

Fig. 1. Reslicing with different positioning parameters to visualise different non-orthogonal planes.

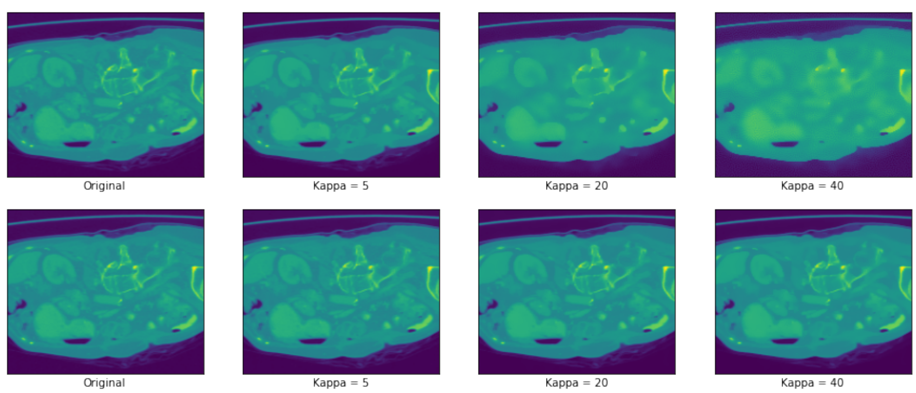

Fig. 2. Visualising the effect of different kappas on anisotropic filtering, on slice 140, using 2D and 3D methods

4 Results

Figure 3 shows that non-linear anisotropic filtering is much more robust when applied in 3D before reslicing. When applied after reslicing, diffusion travels more easily in the surface and thus, the organ boundaries are more easily deformed. Table 1 shows the mean and standard deviation are similar for both 3D and 2D method. The mean difference, is also small. This makes two images harder to differentiate statistically, but visually it is easy to see 3D filter images have maintain their details for larger values of K than 2D filtered images.

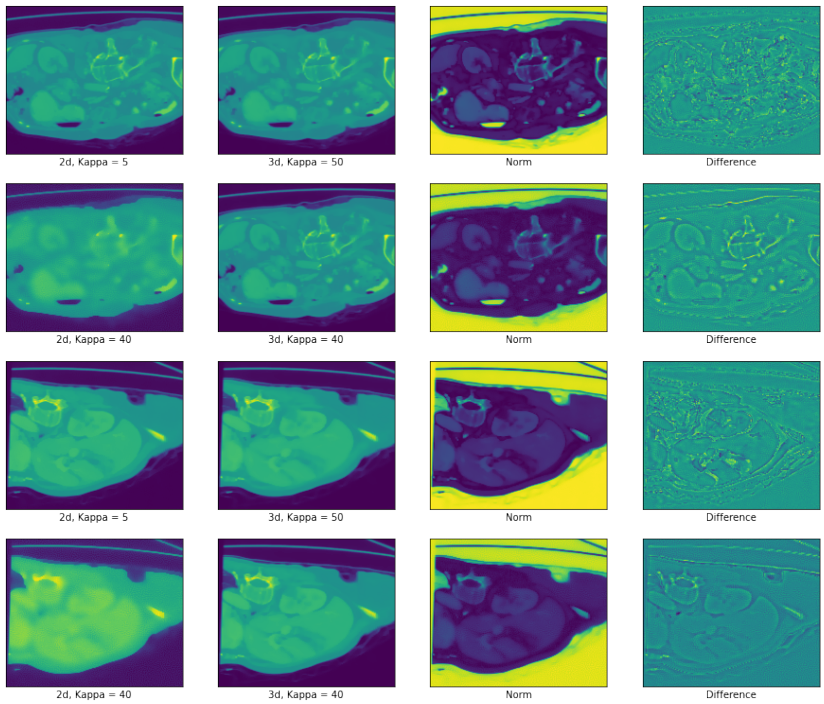

Fig. 3. Comparison of 3D and 2D filtering,Row(1,2); S=140, Row(3,4);S=90

Table 1. Comparison of 2D and 3D filtering for slice 140

Metric Slice 140 :	3D K = 50	2D K = 5	3D K = 40	2D K =40
Mean difference	0.0024		0.0059	
Max	995.1	836.1	1021.7	996.
Min	-1004.5	-1087.7	-1011.8	-1013.9
Mean	-235.4	-235.5	-235.4	-235.5
Std	452.2	455.6	455.6	454.4

5 Discussion

Calculating how much a filtered image overlaps the with the organ segmentation would enable a robust comparison, to see if boundaries have been preserved.

6 Conclusion

Nonlinear anisotropic filtering can correct for the loss of resolution when viewing an image at a high angle or reslicing images on a plane nonorthogonal to the original orientation. This is suitable method than linear filtering for dealing with diffusion like behaviour.

Bibliography

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