Introduction to Graph Theory

Rian Neogi

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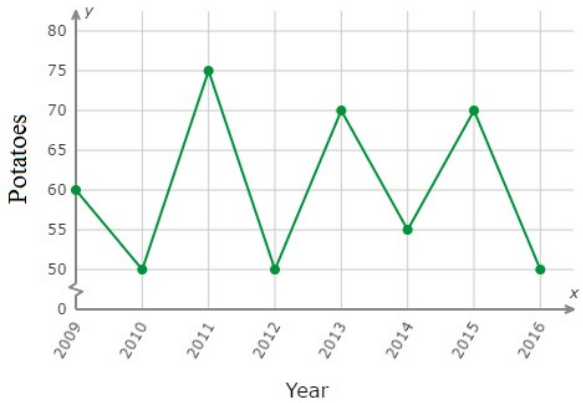

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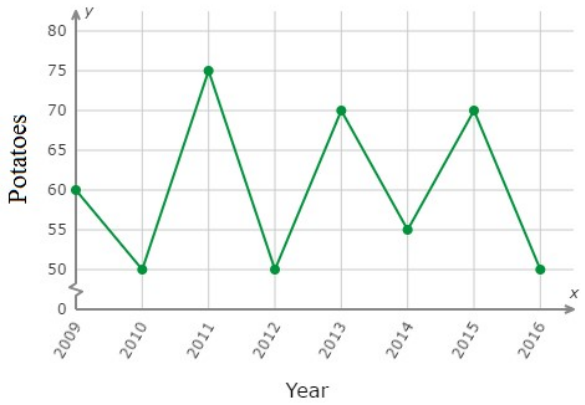

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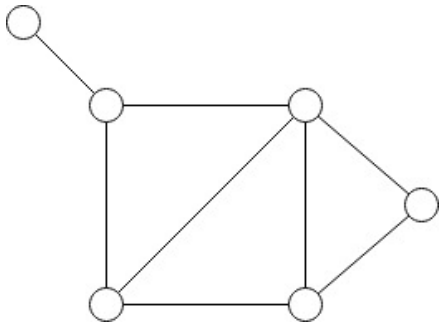

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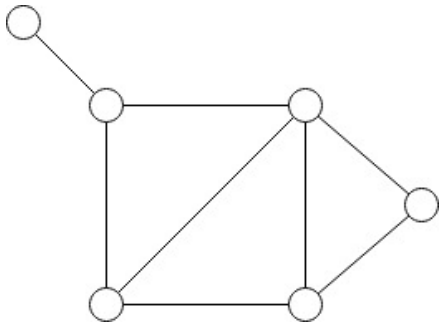

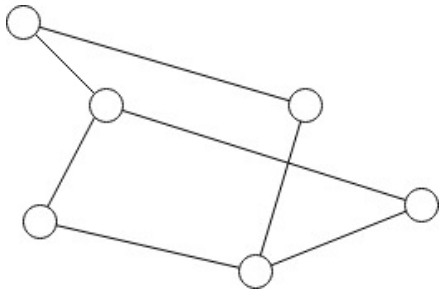

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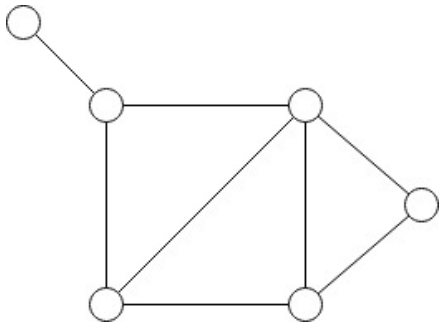

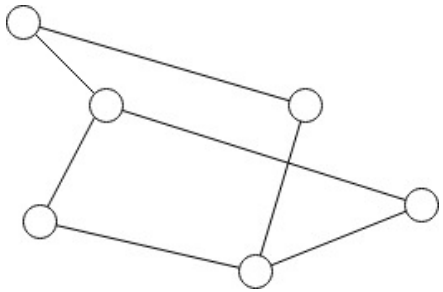

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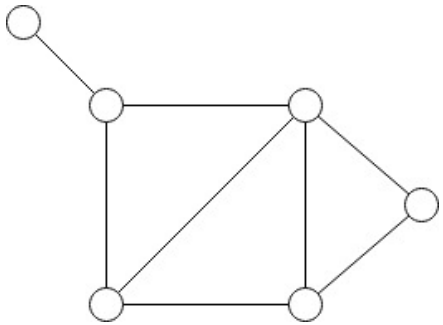

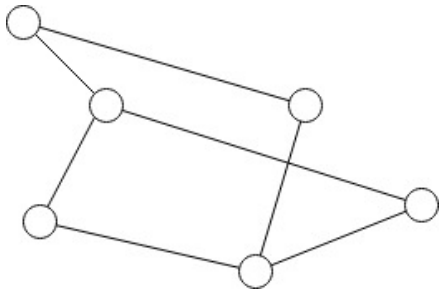

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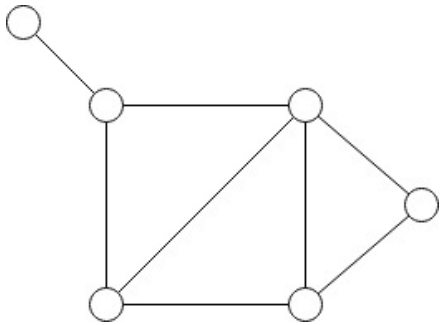

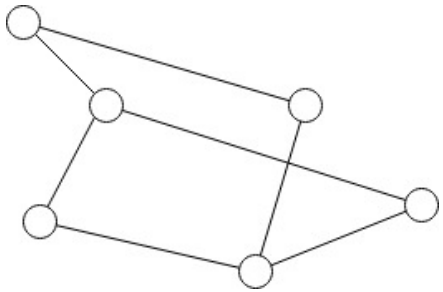

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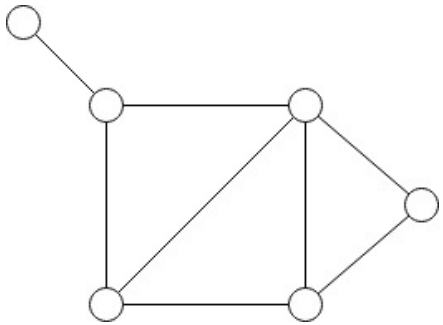

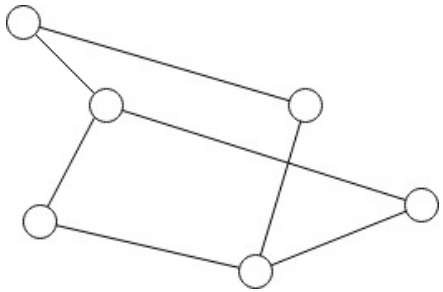

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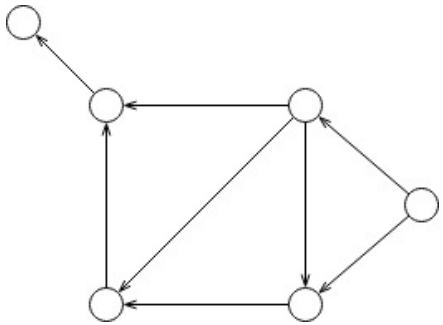

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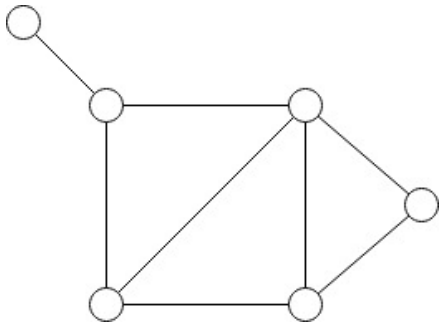

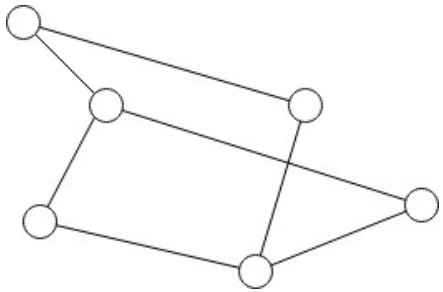

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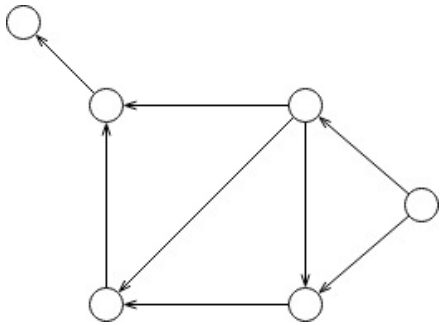

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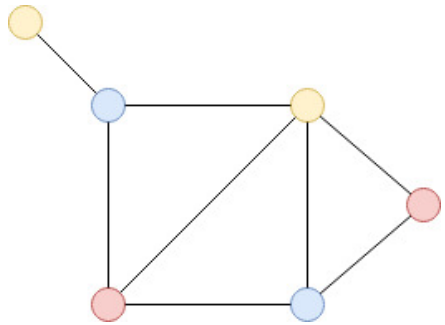

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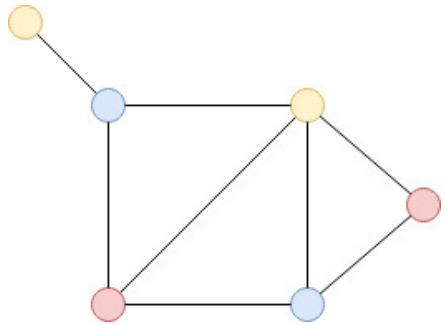

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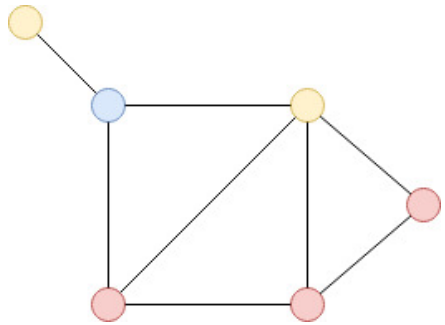

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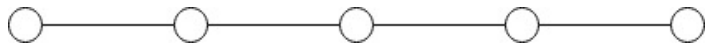

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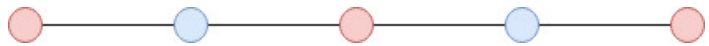

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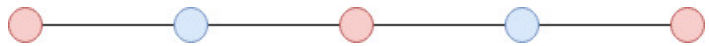

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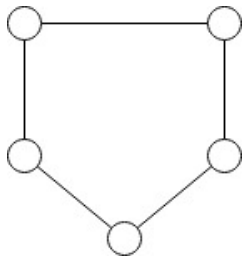

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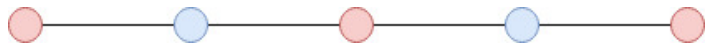

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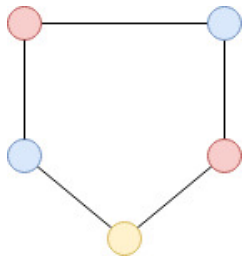

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This is only one instance of a scheduling problem. There are many: for example, aircraft scheduling.

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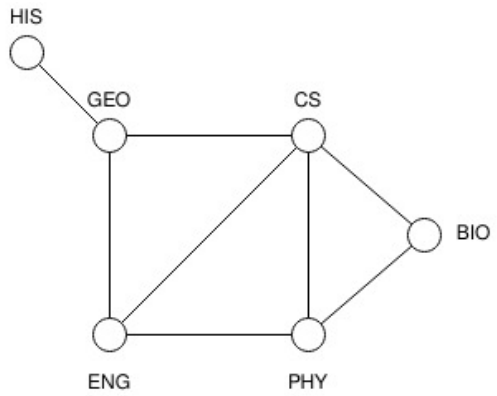

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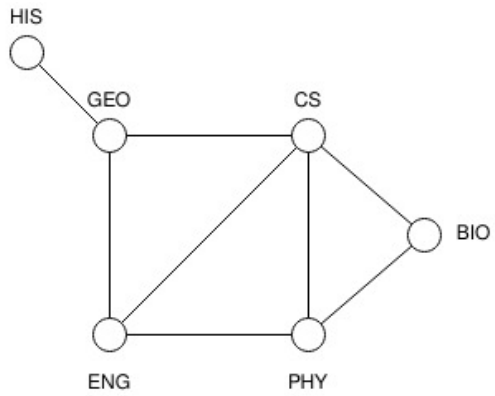

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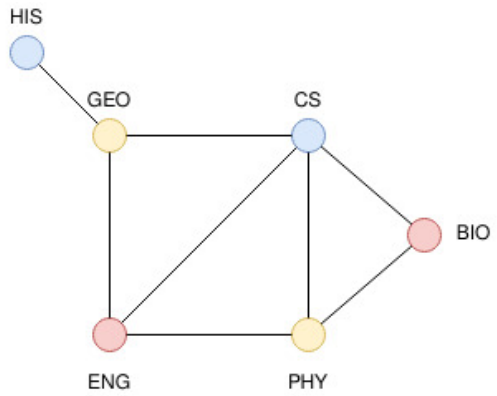

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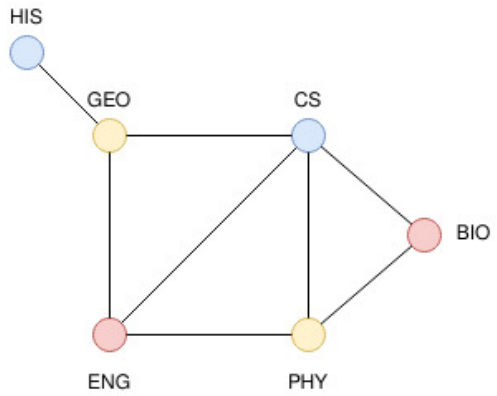

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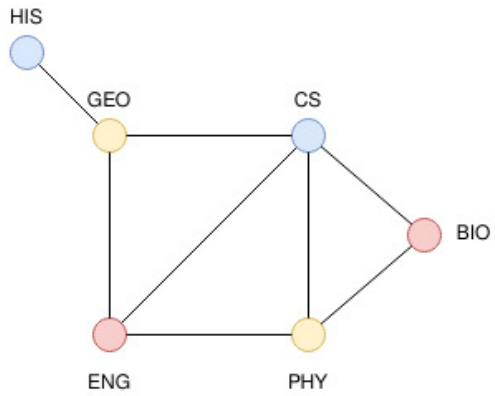

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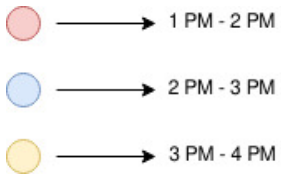

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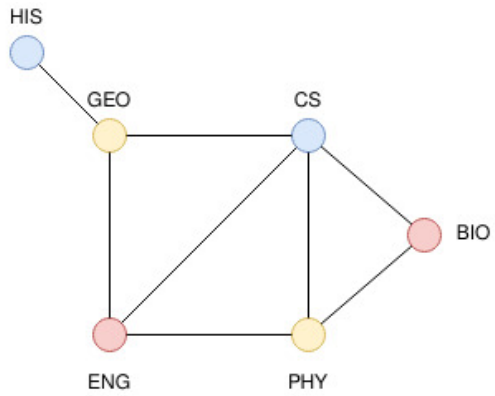

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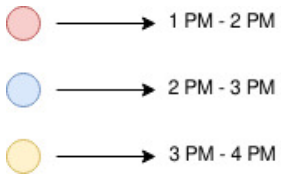

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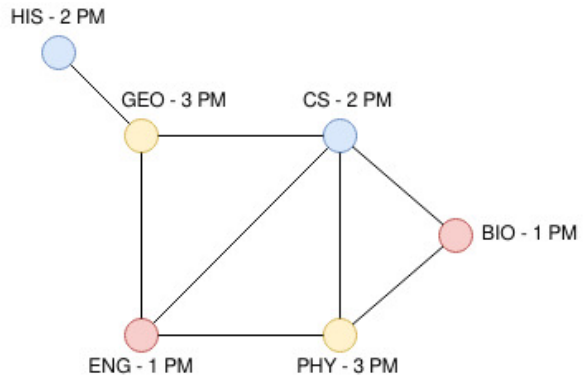

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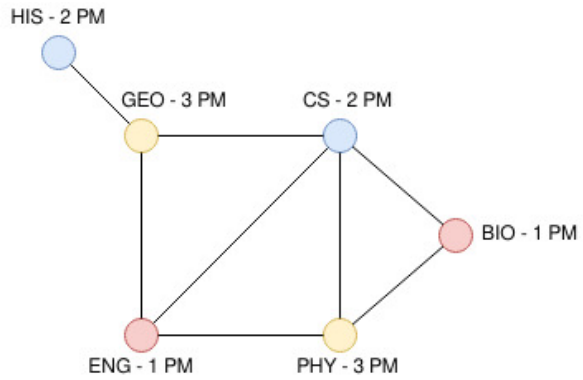

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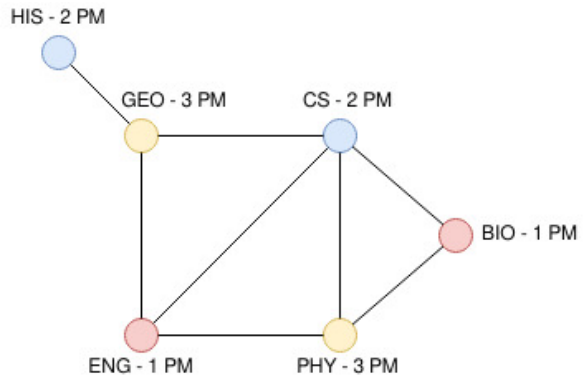

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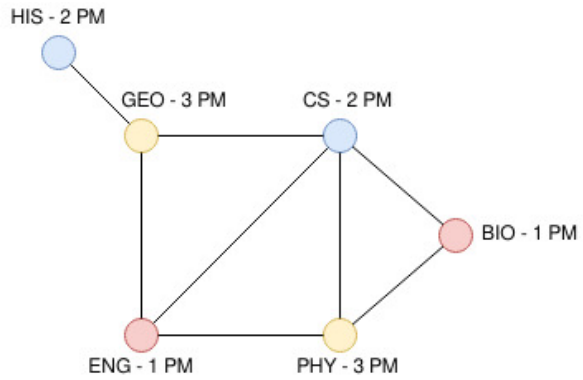

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This would imply that two nodes with an edge between them got the same color. However, then its not a valid coloring.

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- We saw what graph coloring is, and we saw some examples of it.
- We showed how scheduling problems can be solved using graph coloring.
- Computer scientists are interested in algorithms that color graphs optimal since it has applications in larger scale scheduling problems.

Thank You

Any Questions?