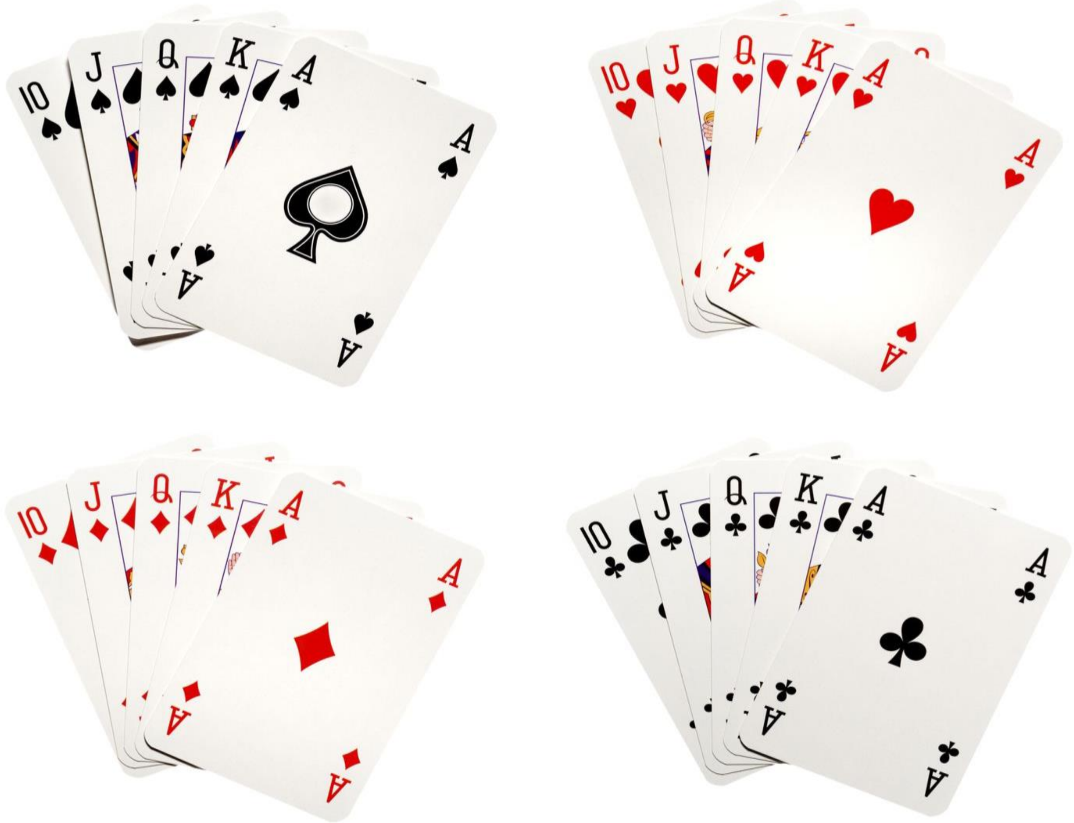
COMP30027 MACHINE LEARNING TUTORIAL

Workshop - 2

Head Tail

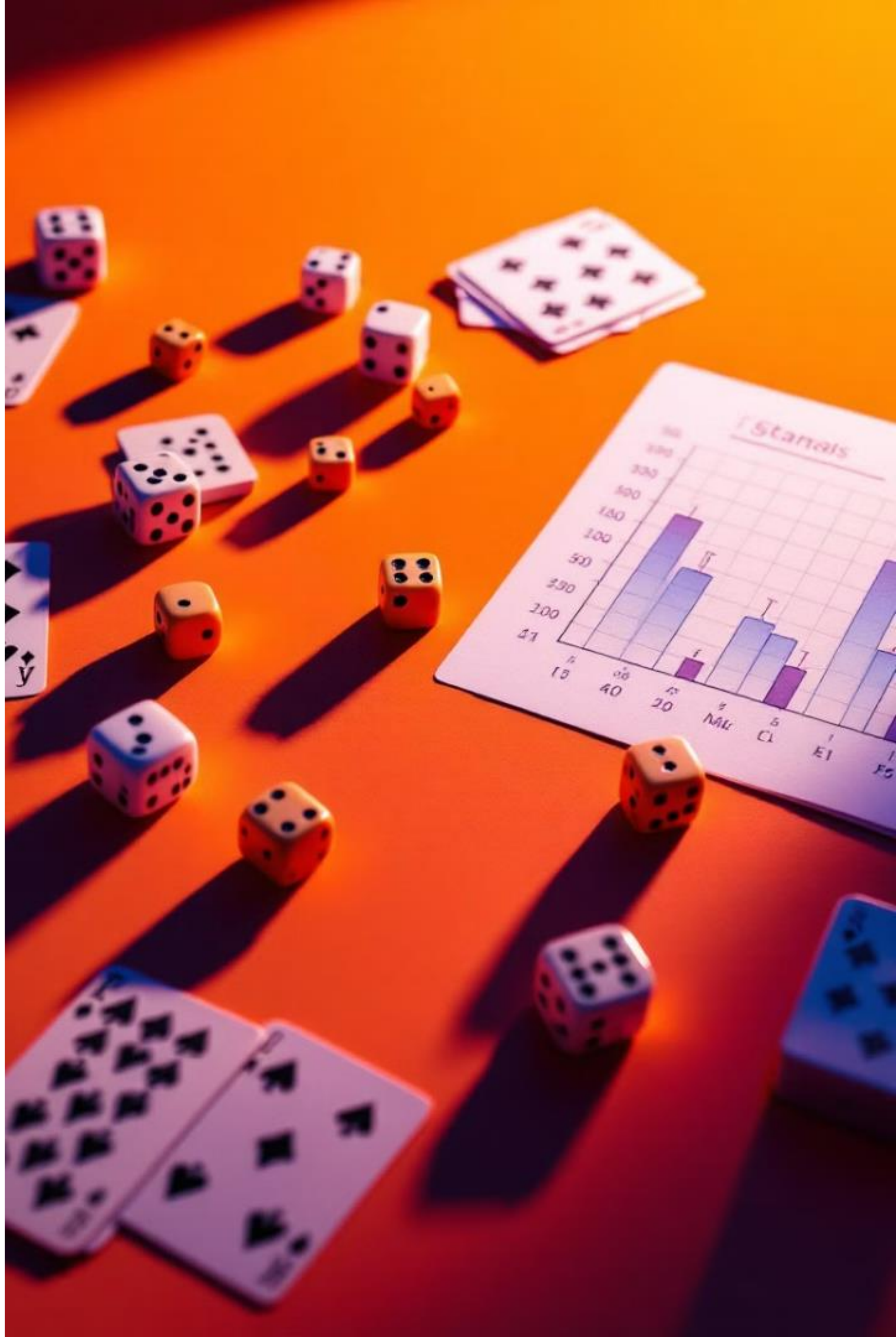


https://www.thoughtco.com/standard-deck-of-cards-3126599 https://www.toppr.com/guides/maths/probability/how-to-calculate-dice-probabilities/ https://www.amazon.com.au/Bocce-Ball-Premium-Set-Innovations/dp/B00EEQ09JS https://www.cuemath.com/data/tossing-a-coin/

Navigating the World of

Probability

Welcome to our exploration of probability. We'll journey from basic definitions to probability distributions



Understanding Basic Probability

Definition

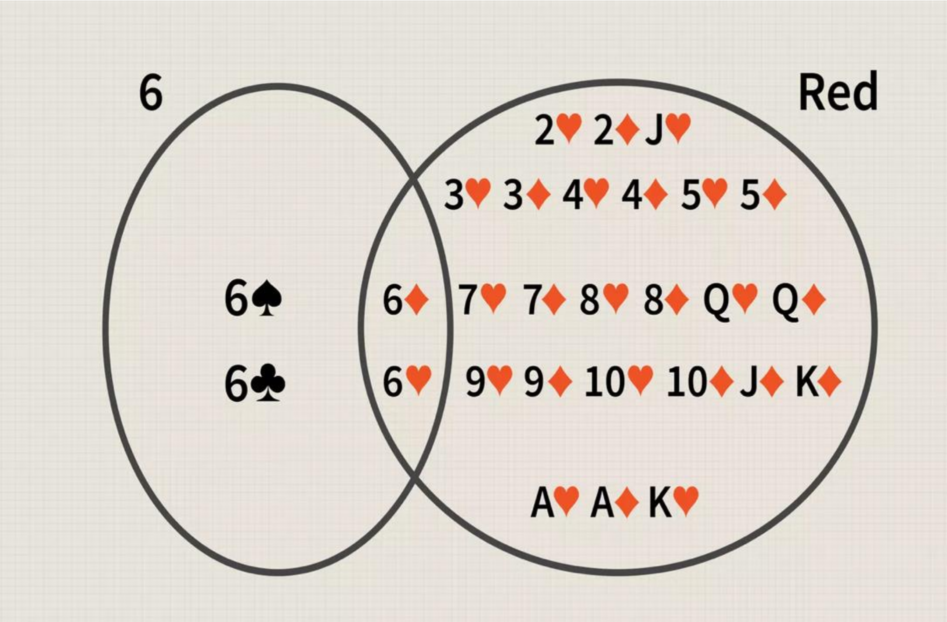
Probability measures the likelihood of an event occurring. It ranges from 0 (impossible) to 1 (certain).

Basic Rules

The sum of all probabilities in a sample space equals 1. The probability of impossible events is 0.

Deck of Card Example

P(red card) = P(black card) = 26/52=0.5



Joint Probability

Definition

Joint probability is the likelihood of two events occurring together. It's denoted as P(A and B) or P(A ∩ B).

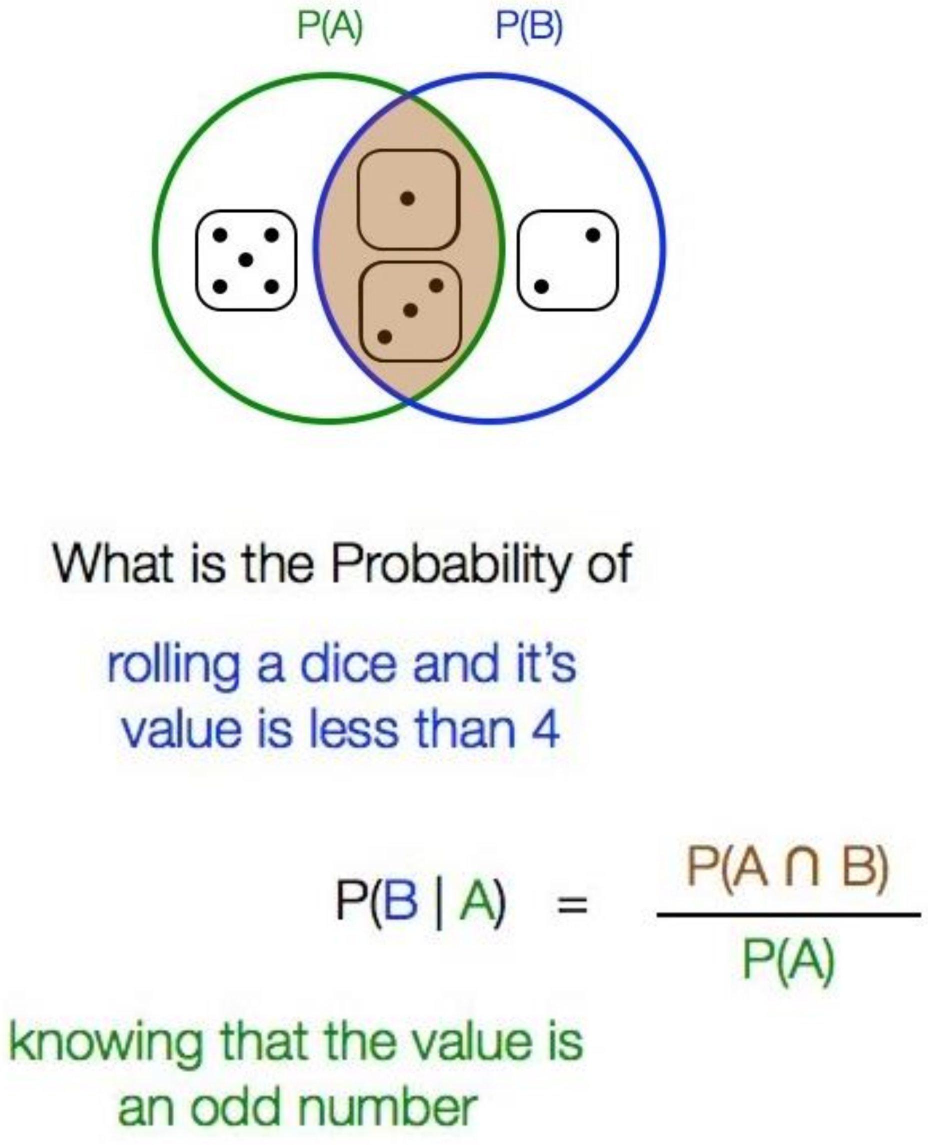
Formula

For independent events: P(A and B) = P(A) × P(B). For dependent events, we use the conditional probability formula.

Card Example

The probability of drawing a red `6′ card is

$P(6 \cap red) = P(6) \times P(red) = 4/52 \times 26/52 = 1/26$



Conditional Probability

1

Definition

The probability of event A occurring given that event B has occurred. It represents updated probabilities with new information.

2

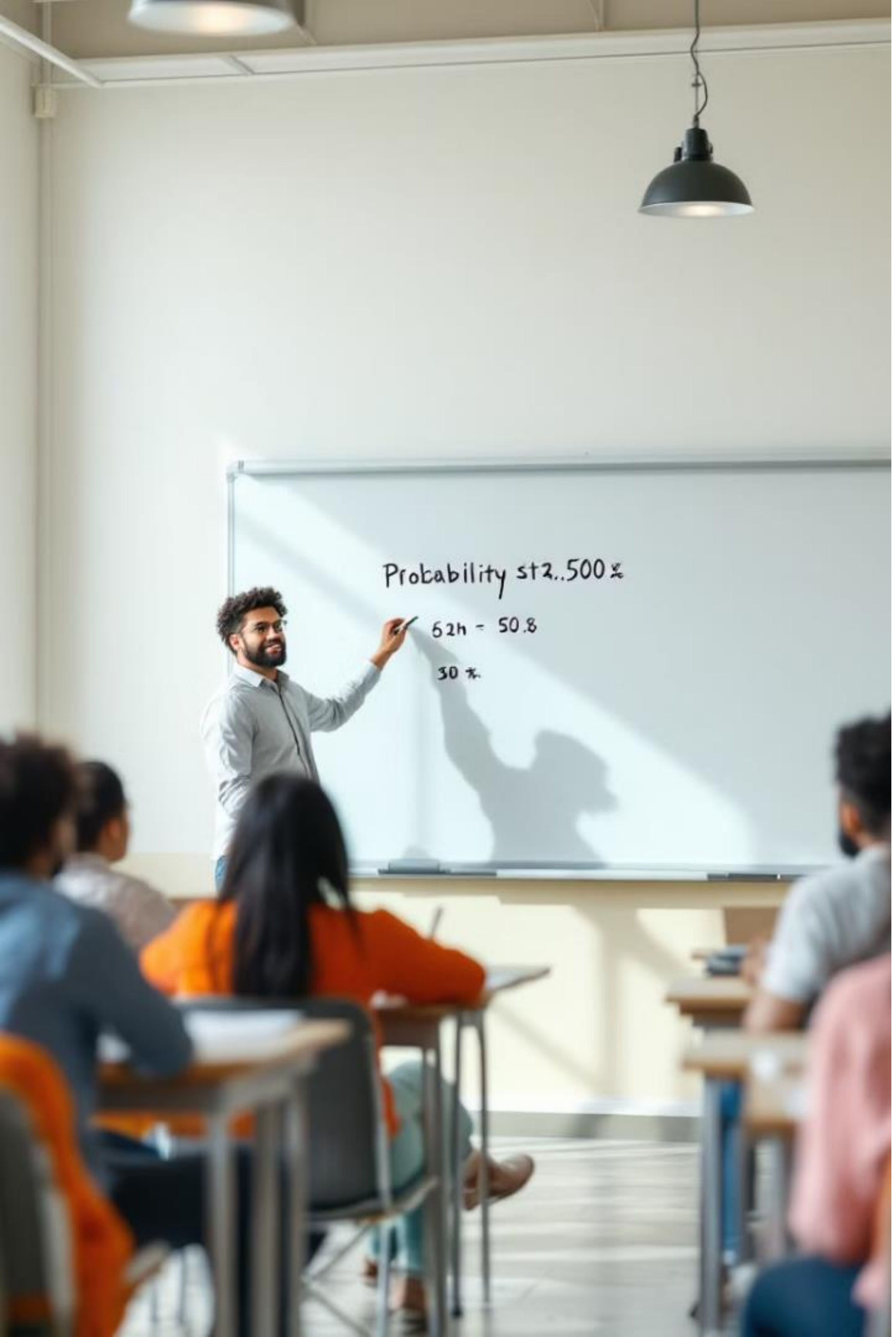
Formula

P(A|B) = P(A and B) / P(B), where P(B) > 0.

3

Example

https://www.youtube.com/watch?v=H02B3aMNKzE



Marginal Probability

Definition

Marginal probability is the probability of an event occurring regardless of the outcome of another variable.

Calculation

Found by summing joint probabilities across all values of the other variable.

Class Example

If 40% of students are male, P(Male) = 0.4 is a marginal probability, regardless of other student attributes.

Q1

Approximately 1% of women aged between 40 and 50 have breast cancer. 80% of mammogram screening tests detect breast cancer when it is there. 90% of mammograms DO NOT show breast cancer when it's NOT there. Use this information to complete the following table with:

•the **joint probabilities** P(Cancer,Test) for each possible pair of cancer status and test result

•the **conditional probabilities** P(Test|Cancer) for each test result given cancer status

Cancer	Test	Joint prob.	Conditional prob.
Yes	Positive		80%
Yes	Negative		
No	Positive		
No	Negative		90%

Cancer	Test	Joint prob.	Conditional prob.
Yes	Positive		80%
Yes	Negative		20%
No	Positive		10%
No	Negative		90%

Compute the Conditional Probabilities

- This method **ensures each row sums to 100%.**
- It's a simple subtraction method: **100% - Known Probability = Other Probability.**

1. **If a person has cancer:**
   - $P(Positive|Cancer) = 80\%$
   - $P(Negative|Cancer) = 100\% - 80\% = 20\%$

2. **If a person does NOT have cancer:**
   - $P(Negative|NoCancer) = 90\%$
   - $P(Positive|NoCancer) = 100\% - 90\% = 10\%$

Cancer	Test	Joint prob.	Conditional prob.
Yes	Positive	0.8%	80%
Yes	Negative	0.2%	20%
No	Positive	9.9%	10%
No	Negative	89.1%	90%

Formula for Joint Probability

$$P(A, B) = P(B|A) \times P(A)$$

where:

- A is the cancer status (Yes or No)
- B is the test result (Positive or Negative)

(1) Probability of having cancer and a positive test result

$$P(Cancer, Positive) = P(Positive|Cancer) \times P(Cancer)$$

$$= 0.80 \times 0.01 = 0.008$$

(2) Probability of having cancer and a negative test result

$$P(Cancer, Negative) = P(Negative|Cancer) \times P(Cancer)$$

$$= 0.20 \times 0.01 = 0.002$$

(3) Probability of not having cancer and a positive test result

$$P(NoCancer, Positive) = P(Positive|NoCancer) \times P(NoCancer)$$

$$= 0.10 \times 0.99 = 0.099$$

(4) Probability of not having cancer and a negative test result

$$P(NoCancer, Negative) = P(Negative|NoCancer) \times P(NoCancer)$$

$$= 0.90 \times 0.99 = 0.891$$

Q2

Given the table above, compute the **marginal probability** of a positive result in the mammogram screening test.

The **marginal probability** is the total probability of a positive result:

P(Test == Positive|Cancer == No)P(Cancer == No) + P(Test == Positive|Cancer == Yes)P(Cancer == Yes)

this is the sum of the two joint probabilities in the table above:
P(Test == Positive,Cancer == No) + P(Test == Positive,Cancer == Yes)

=> 0.8 + 9.9 = 10.7%

Cancer	Test	Joint prob.	Conditional prob.
Yes	Positive	0.8%	80%
Yes	Negative	0.2%	20%
No	Positive	9.9%	10%
No	Negative	89.1%	90%

Q3

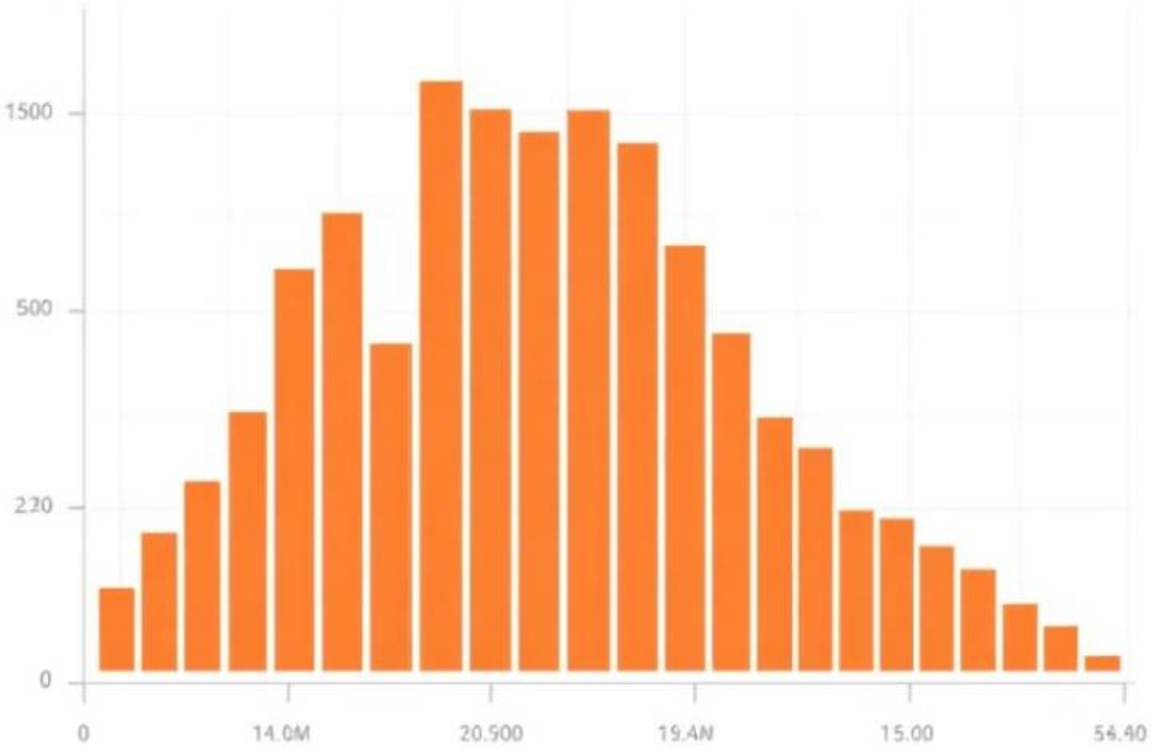
Suppose a woman in this age group receives a positive test result. Compute the **conditional probability** P(Cancer == Yes|Test == Postive).

This is equivalent to the joint probability P(Test == Positive, Cancer == Yes) over the total probability of a positive result:

P(Test == Positive,Cancer == Yes) / P(Test == Yes)

0.8 / 10.7 = 7.5%

This result shows that even if a screening test returns a positive result, the actual chance of cancer is only 7.5%.



Empirical Probability Distribution

Data Collection

Gather observations from real-world experiments or surveys. Record frequencies of each outcome.

Frequency Analysis

Count occurrences of each value. Divide by total observations to get relative frequencies.

Distribution Creation

Plot these relative frequencies. They approximate the true probability distribution as sample size increases.

Binomial Distribution

Definition

Models the number of successes in n independent trials, each with probability p of success.

Formula

P(X = k) = C(n,k) × p^k × (1-p)^(n-k) where C(n,k) is the binomial coefficient.

Coin Example

For 3 heads in 5 flips: P(X = 3) = C(5,3) × (0.5)^3 × (0.5)^2 = 10 × 0.03125 = 0.3125.

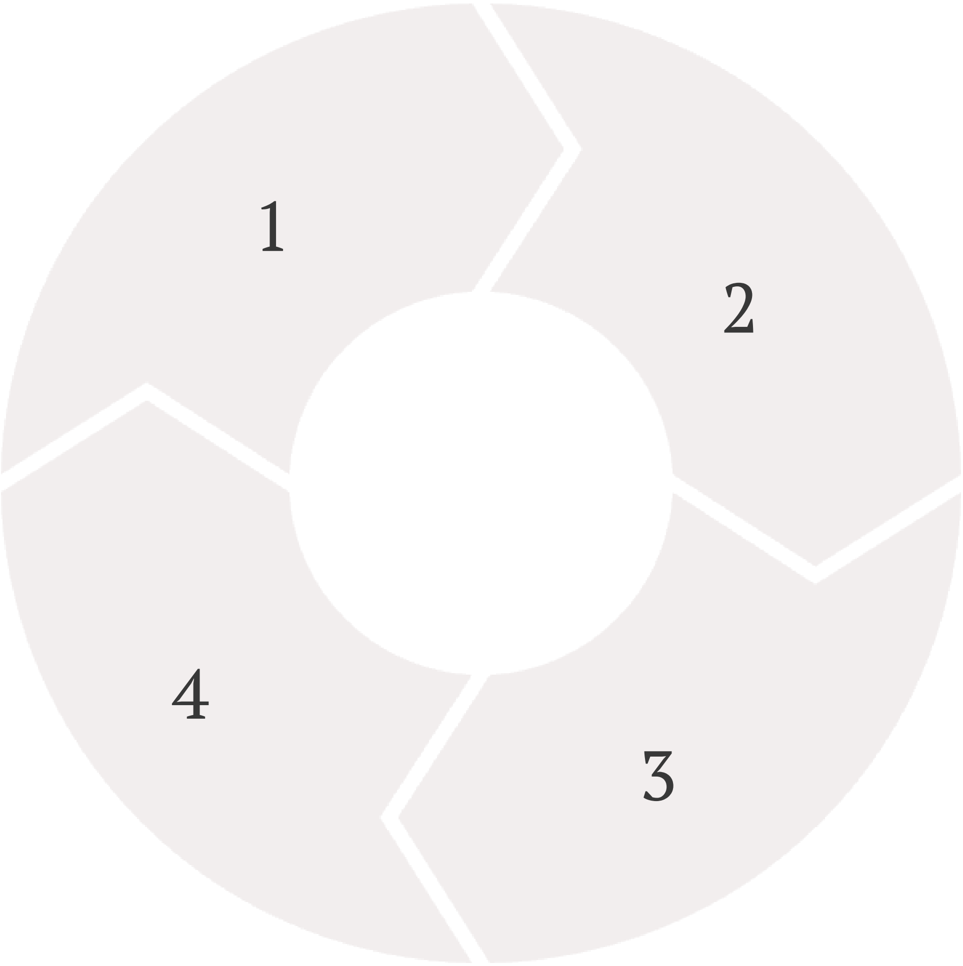
Q4

Suppose you flip a fair coin 10 times. What are the odds that you will get exactly K heads, for each possible value of K (0,1,...10)? Write a function to simulate this experiment and empirically estimate the probability distribution.

- Simulate flipping coin 10 time
- Count number of heads in each experiment
- Plot the probability distribution

https://in.pinterest.com/pin/coin-toss-probability-formula-solved-examples-in-2024--933652566490495490/

Probability Density Function



Definition

A function whose integral gives probabilities for continuous random variables.

Properties

PDFs must be non-negative and integrate to 1 over the entire domain.

Normal Distribution

The bell curve with PDF $f(x) = (1/\sigma\sqrt{2\pi}) \times e^{\wedge}(-(x-\mu)^2/2\sigma^2)$ is the most common example.

Interpretation

The PDF value itself isn't a probability. Probabilities come from areas under the curve.

Entropy in Probability

1

Definition

Entropy measures uncertainty or information content in a probability distribution.

2

Formula

$H(X) = -\Sigma P(xi) \times \log_2(P(xi))$ for all possible outcomes xi.

3

Coin Example

For a fair coin: $H(X) = -(0.5 \times \log_2(0.5) + 0.5 \times \log_2(0.5)) = 1$ bit.

4

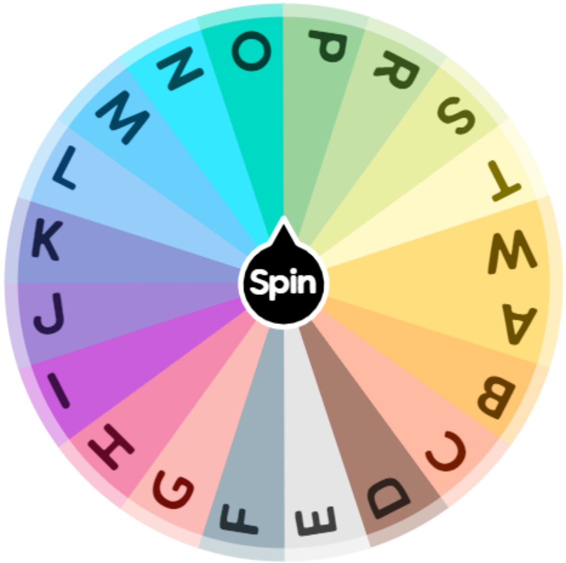
Properties

Entropy is maximized when all outcomes are equally likely. It quantifies unpredictability.

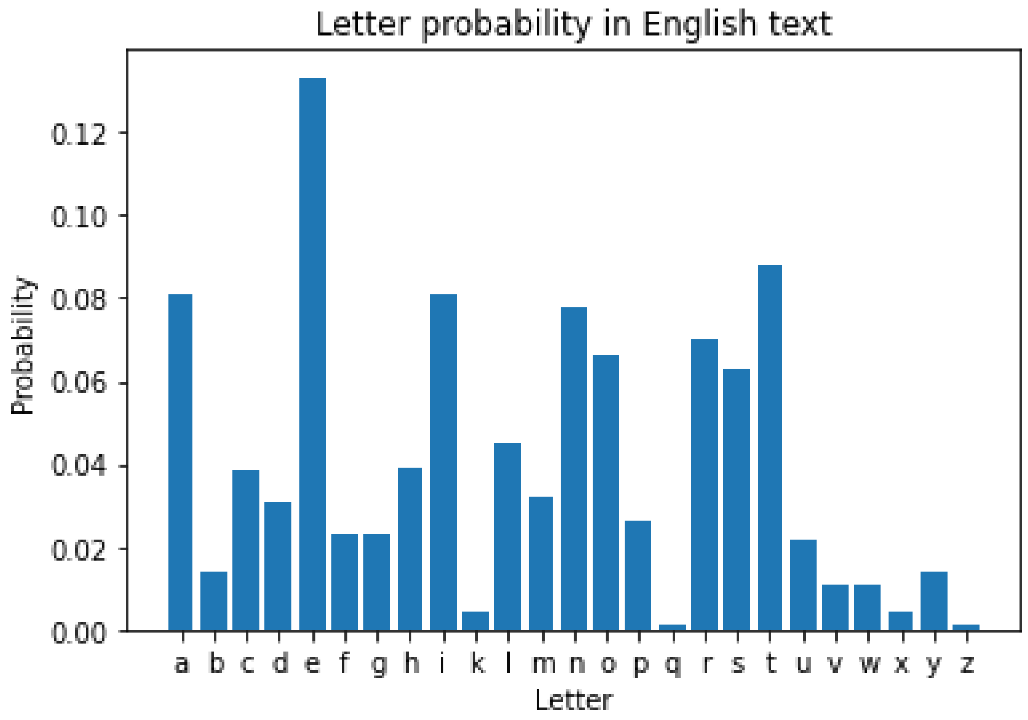
Q6

Compute the entropy of a random letter generator which can generate any of the 26 English letters (a-z), each with equal probability.

$$H(X) = -\sum_{x \in X} p(x) \log_2 p(x)$$



- All letters (A–Z) are **equally likely**
- Probability of each letter = 1/26
- Entropy is **maximum**
- High **uncertainty**: each letter is a total surprise



- Letters are **not equally likely** (e.g., E is common, Z is rare)
- More **predictable** due to patterns
- Entropy is **lower**
- Some information is **redundant** (due to structure and grammar)

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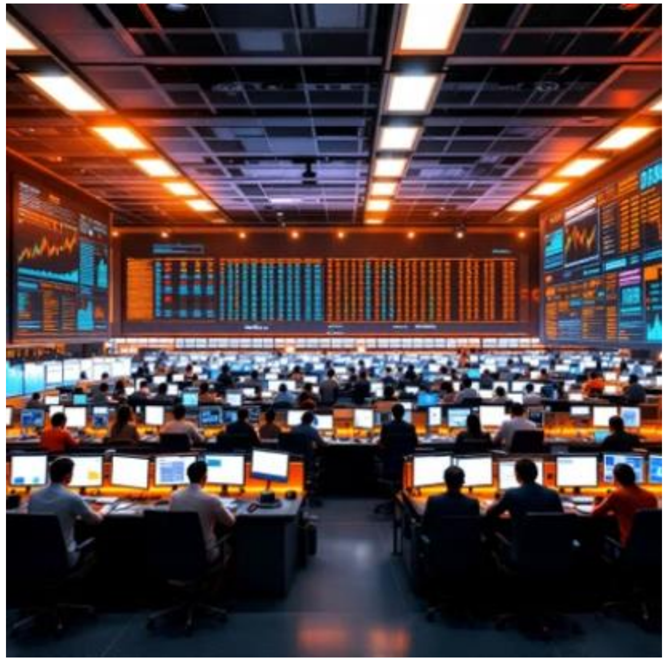
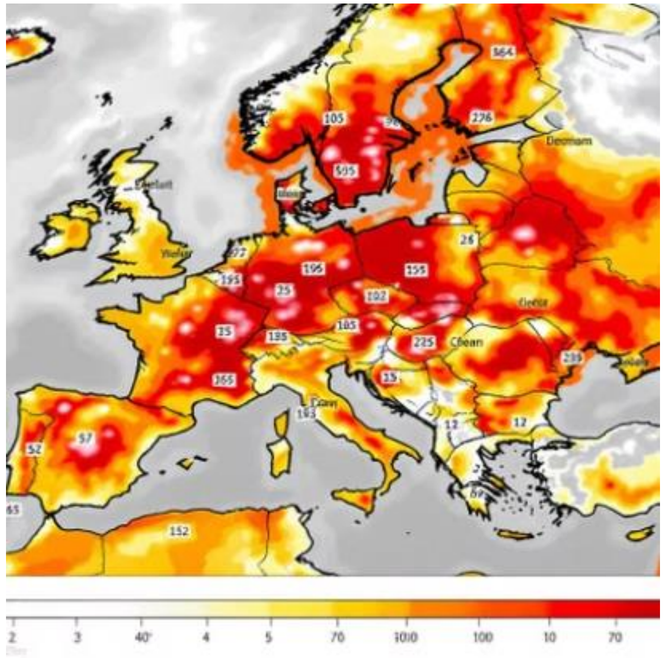
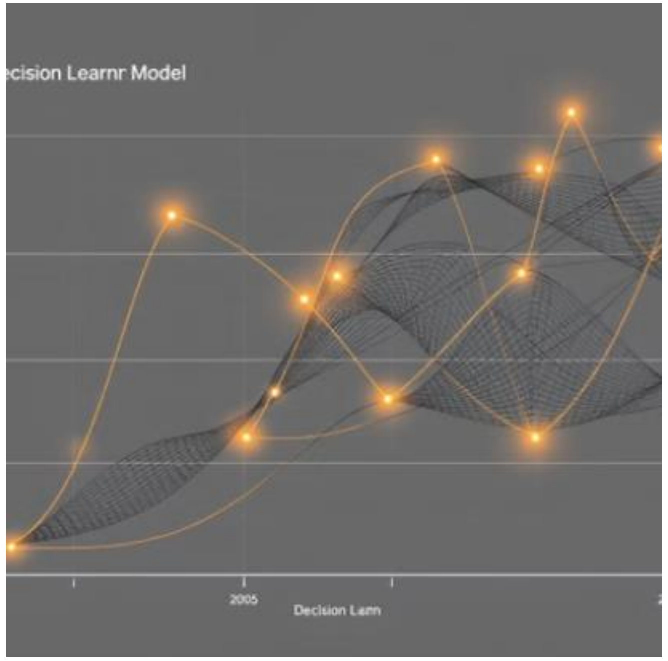
Real-world Applications of Probability







Probability concepts power financial risk assessment, weather forecasting, medical diagnostics, and AI systems. They help us navigate uncertainty in virtually every field.

THANK YOU!!

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