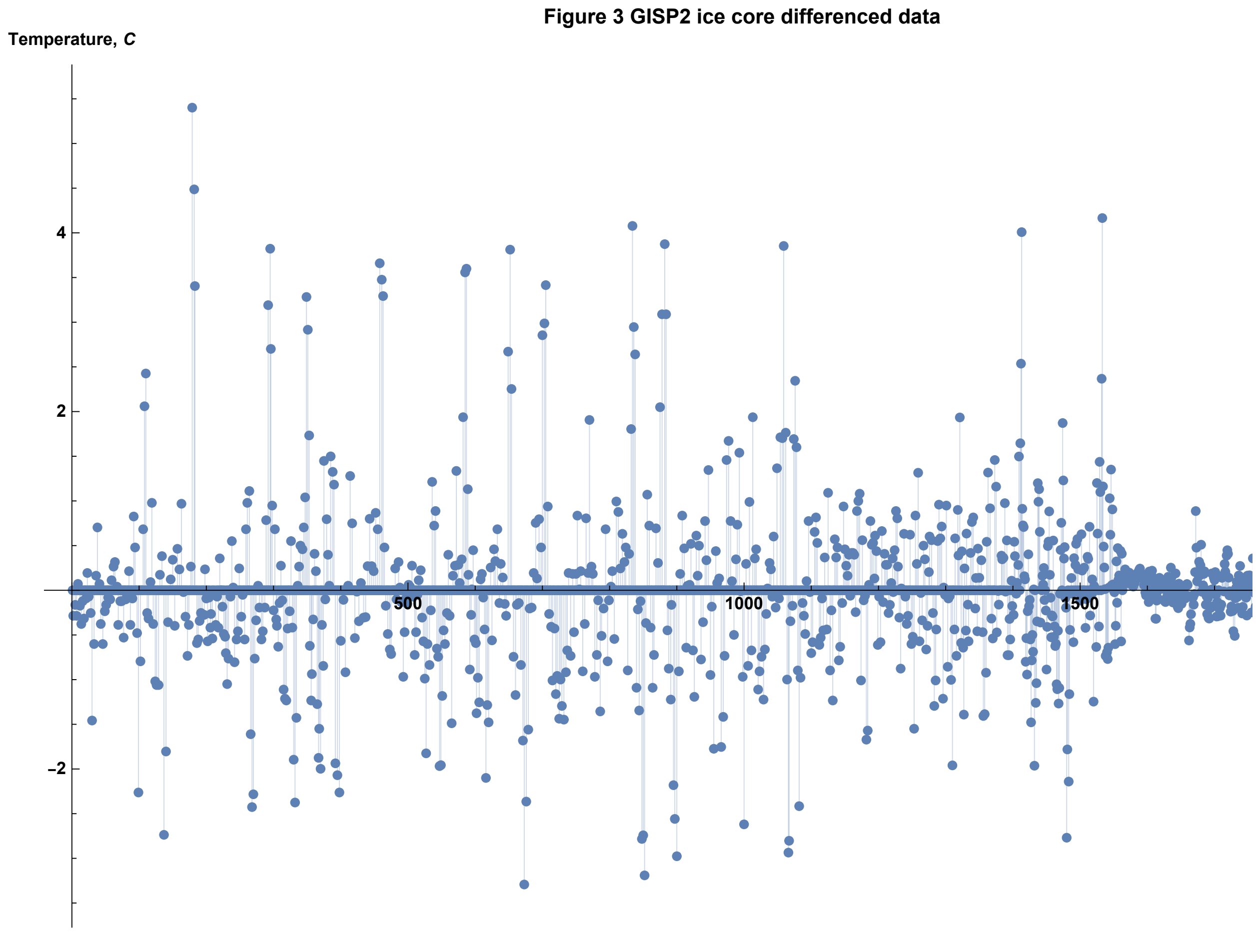
In[•]:=
```
diffdataAll = Differences[regulardataAll];
Length[diffdataAll]
ListLinePlot[diffdataAll, Joined → False, Filling → Axis, PlotRange → All,
 AxesLabel → {RawBoxes["datapoints"], RawBoxes["Temperature,C"]},
 PlotLabel → "Figure 3 GISP2 ice core differenced data",
 LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800]
```

Out[•]= 1995

Out[•]=


4. Time Series Analysis

In this section, recurrence plots are first plotted for the whole climate record to visualise the nature of the system, and a few measures of complexity based on the plot is carried out. Then, changes of state and complexity of the record are investigated using Hurst exponent, sample entropy and burstiness via a rolling window.

4.1 Recurrence plot and RQA

Recurrence of a system to its former states is the fundamental of many dynamical systems. [d] Recurrence plots (RPs) are graphical tools that depict the different occasions when dynamical systems visit the same region of phase space and hence revealing complex deterministic patterns in dynamical systems: [e]

$R_{i,j}(\epsilon) = \Theta(\epsilon - \| \vec{x}_i - \vec{x}_j \|)$, i,j = 1,2,3,...,N

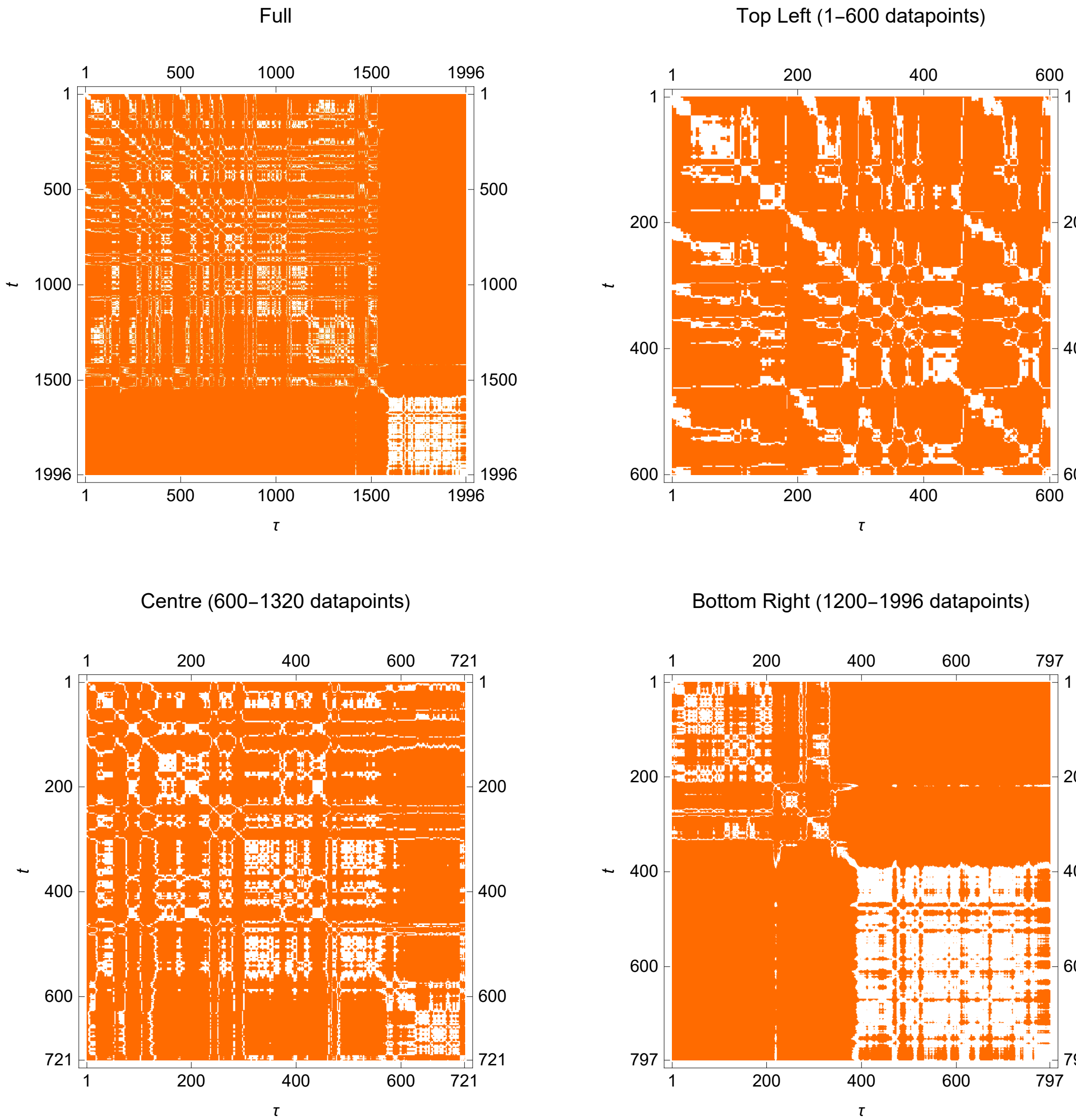
where N is the number of measured points $\vec{x}_i$, ϵ is a threshold distance, Θ (.) is a Heaviside function (i.e. Θ (x) = 0, if x <0, and Θ (x)=1 otherwise) and ||.|| is a norm. [d]
The length of diagonal lines which is parallel to the main diagonal line, Line of Identity reveals the nature of the system. One should expect long diagonal lines for predictable system, and short diagonal lines or even isolated points for stochastic system. [e] Selection of an appropriate ϵ has been a key question in Recurrence Quantification Analysis (RQA), and the fact that it strongly depends on the considered system [d], a value of ϵ = 0.15 standard deviation of climate time series is adopted in this study, based on the suggestion in [f]. The Recurrence Rate (RR) and Determinism (DET) are then calculated by using the Mathematica code adapted from [g]. The former estimates the probability that a certain state recurs while the latter provides an indication of determinism and predictability in the system. [e]

```
In[◦]:= 0.15 * StandardDeviation[regulardataAll]

Out[◦]= 1.11145
```

```
In[◦]:= ϵ = 1.1;

g1 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[regulardataAll[[t]] - regulardataAll[[τ]]]],
    {t, 1, Length[regulardataAll]}, {τ, 1, Length[regulardataAll]}],
   Mesh → False, FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Full"];

g2 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[regulardataAll[[t]] - regulardataAll[[τ]]]],
    {t, 1, 600}, {τ, 1, 600}], Mesh → False,
   FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Top Left (1-600 datapoints)"];

g3 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[regulardataAll[[t]] - regulardataAll[[τ]]]],
    {t, 600, 1320}, {τ, 600, 1320}], Mesh → False,
   FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Centre (600-1320 datapoints)"];

g4 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[regulardataAll[[t]] - regulardataAll[[τ]]]],
    {t, 1200, Length[regulardataAll]},
    {τ, 1200, Length[regulardataAll]}], Mesh → False, FrameLabel →
    TraditionalForm /@ {t, τ}, PlotLabel → "Bottom Right (1200-1996 datapoints)"];

Show[ GraphicsGrid[{{g1, g2}, {g3, g4}}],
 PlotLabel → Style["Figure 4 Recurrence Plot with distance = 1.1", "Subsubsection"]]
```

Figure 4 Recurrence Plot with distance = 1.1

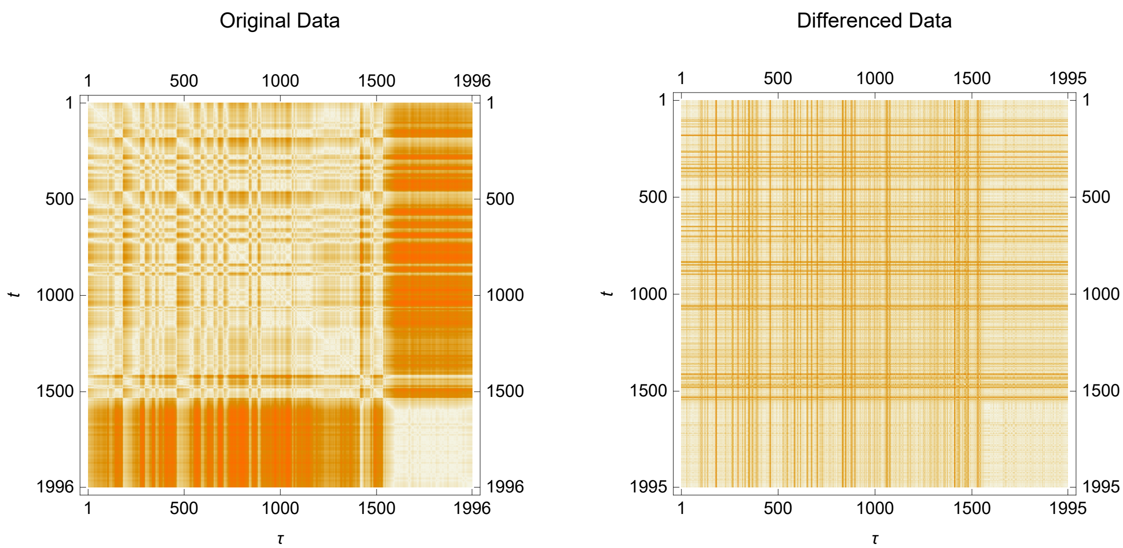
Out[]=



```
In[•]:= g5 = MatrixPlot[Table[Norm[regulardataAll[[t]] - regulardataAll[[τ]]],
      {t, 1, Length[regulardataAll]}, {τ, 1, Length[regulardataAll]}], Mesh → False,
    FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Original Data"];

g6 = MatrixPlot[Table[Norm[diffdataAll[[t]] - diffdataAll[[τ]]],
      {t, 1, Length[diffdataAll]}, {τ, 1, Length[diffdataAll]}], Mesh → False,
    FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Differenced Data"];

Show[GraphicsGrid[{{g5, g6}}],
 PlotLabel → Style["Figure 5 Recurrence Plot with all distances", "Subsubsection"]]
```

Figure 5 Recurrence Plot with all distances

Out[•]=



From Figure 4, it is observed that the temperature is characterised by larger distances in the upper left quadrant (the big square which covers most of the area) of the plot, and small distances in the lower right corner. This suggests that these indices underwent a notable evolution in the period covered by the data. The lower right quadrant in fact covers the Holocene period data points, where the temperature became warmer. According to [e], "butterfly" shaped structure on RP is unique and its position might be related to some sharp changes. From Figure 4 bottom right plot, there is a "butterfly" recurrence structure occurs at the middle upper left, and the position of this shape in fact is the Bølling Allerød warming period.

Looking at the RP with all distances (see Figure 5), two distinct squares of the plots again revealed that the transition of temperature between the squares, and the bands around point 1500 from original data plot could be related to both Bølling Allerød warming and the end of Younger Dryas (starting of Holocene).

```
In[ ]:= ϵ = 1.1;
rp = Table[UnitStep[ϵ - Norm[regulardataAll⟦t⟧ - regulardataAll⟦τ⟧]],
   {t, 1, Length[regulardataAll]}, {τ, 1, Length[regulardataAll]}];
n = Length[regulardataAll];
countlines = 0;
countpoints = 0;
For[j = 0, j < n, j++,
  linelength = 0;
  For[i = 1, i + j < n, i++,
   If[rp[[i, i + j + 1]] == 1, linelength++,
     If[linelength ≥ 2, countlines++,
      If[linelength == 1, countpoints++
       ];
     ];
     linelength = 0(*reset for a diagonal*)
    ];
  ];
  If[linelength ≥ 2, countlines++,
   If[linelength == 1, countpoints++
    ];
  ];
 ];
pr = N[(Total[Total[rp]]) / (n * n)];
pd = N[2 * countlines / (n * n)];
dr = N[pd / pr];
Print[StringJoin["Recurrence Rate (RR): ", ToString[pr]]];
Print[StringJoin["Determinisim(DET): ", ToString[dr]]];

Recurrence Rate (RR): 0.121143

Determinisim(DET): 0.164696
```

As reported from the code, the RR and DET of the system are 0.12 and 0.16 respectively. These measures are related to the length of diagonal lines in the recurrence matrix [d], hence, low values actually indicate that the low probability of a certain state recurs in this system and so the low determinism and predictability of the system, which is expected to see for climate records.

4.2 Hurst Exponent

Hurst Exponent is a measure of long-term memory of time series. If 1>H>0.5,there are long range time correlations, for 0.5>H>0, the series has long range anti-correlations, and if H=1.0, the process is deterministic while uncorrelated noise or random walk corresponds to H=0.5 [h]. The direct relationship of Hurst exponent and fractal dimension (which gives a measure of the roughness of a surface) is known by D = 2- H. The predictability index of temperature is then defined by $\mathrm{PI}_T = 2\,|\,D - 1.5\,|$. [h] Interpretation of this index is simple: a value close to 0 indicates the corresponding process approximates the usual

Brownian motion (as D = 1.5) and is therefore unpredictable while close to 1, the process is said to be very predictable. In this study, the single value of Hurst Exponent for whole climate record is first calculated. Then, a moving window of size 50 (which is approximately the length of Younger Dryas event) is applied for the record to detect the change of persistency in the data.

In[*]:=
```
windowPlot[data_, window_, f_, plegend_, color_, plabel_, xlabel_, ylabel_] :=
 Module[{movingmap, xAxis, windowedData, movingaverage, graph1, graph2, plot},
  movingmap = MovingMap[f, data, window];
  xAxis = Range[window, window + Length[movingmap]];
  windowedData = Table[{xAxis[[i]], movingmap [[i]]}, {i, Length[movingmap ]}];
  movingaverage = MovingAverage[windowedData, window];
  graph1 = ListLinePlot[windowedData, GridLines → Automatic, PlotStyle → {color},
    PlotRange → {{0, Length[data]}, Full}, PlotLegends → {plegend}];
  graph2 = ListLinePlot[movingaverage, PlotStyle → {Green},
    PlotLegends → {"Averange value"}];
  plot = Show[graph1, graph2, AxesLabel → {RawBoxes[xlabel], RawBoxes[ylabel]},
    PlotLabel → plabel, LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800];
  plot]
```

In[*]:=
```
hurstExp[data_] := h /. (FindProcessParameters[data, FractionalBrownianMotionProcess[h]])
```

In[*]:=
```
Hvalue = hurstExp[regulardataAll]
Dfractal = 2 - Hvalue
Pindex = 2 * Abs[Dfractal - 1.5]
```

Out[*]= 0.841255

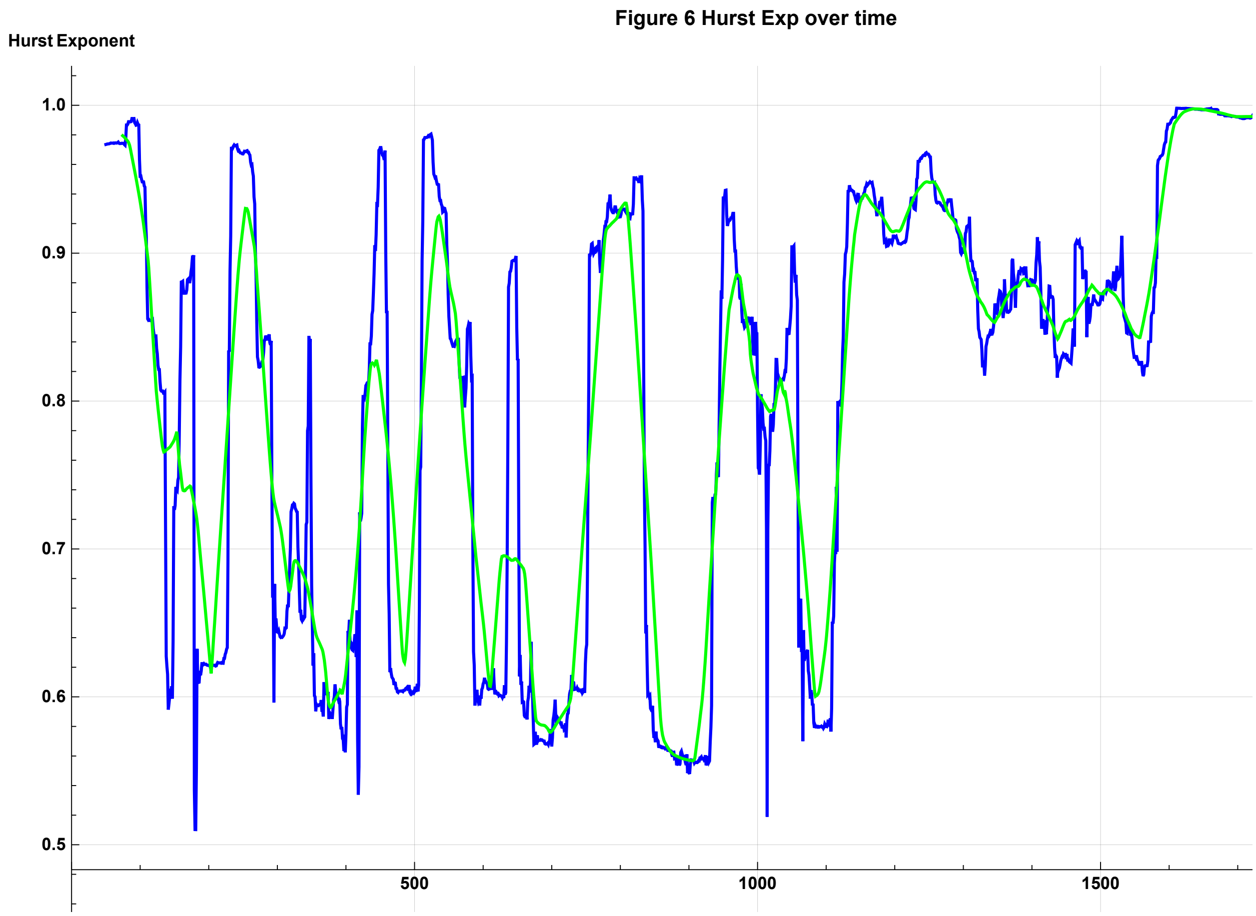
Out[*]= 1.15875

Out[*]= 0.682509

The Hurst for the whole record is 0.84, which tells the long-term memory and high persistence of the temperatures in Greenland before present. The Fractal Dimension is 1.16 and the Predictability Index is 0.68, suggests that the temperature data is far from random but also unpredictable. Figure 6 shows the moving window analysis of Hurst exponent over the data. From the figure, it is observed that in overall the time series exhibits reinforcing behaviour through time, with great fluctuations of persistency during the DO event and becomes relatively stable during the LGM. A final jump of H-value is seen onset of Holocene period, and the temperature depicts a very strong correlated behaviour entering the Holocene epoch.

In[•]:=
```
windowPlot[regulardataAll, 50, hurstExp, "Original data",
  Blue, "Figure 6 Hurst Exp over time", "datapoints", "Hurst Exponent"]
```

Out[•]=



4.3 Sample Entropy

Sample entropy (SampEn) is one of the regularity statistics been widely used to quantify randomness and complexity and is useful to classify systems and applicable to stochastic and deterministic processes [a]. Larger values of denote higher complexity and lower values imply organization and a higher degree of predictability. The Mathematica code applied is adapted from [i]. Similar to the previous part, a value of SampEn for whole data is first calculated, and followed by the rolling window time series. In order to avoid the cyclic behavior of the temperature influencing the obtained results, and the usefulness of SampEn to determine the complexity could be compromised, it is suggested by [j] to use the stationary version of data rather than the raw data. Two parameters, embedding dimension or the size of the template being compared, m and noise filter r where points within a distance of r are considered equal, are set to

2 and 0.2σ respectively, as suggested in [a,j].

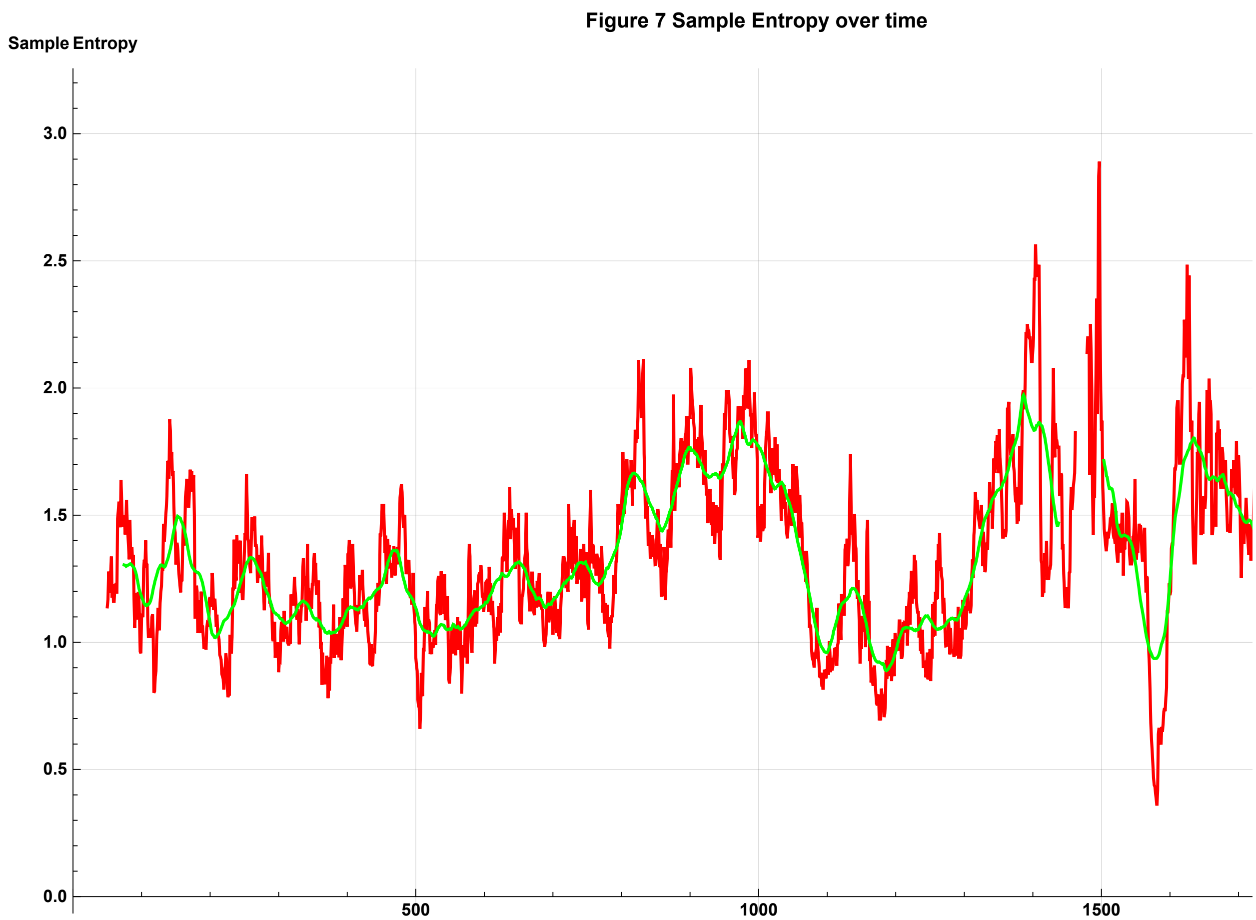
In[•]:=
```
SampEn[data_] := Module[{m, r, nF1, nF2, diff1, diff2, va1, va2},
  m = 2;
  r = 0.2 * StandardDeviation[data];
  va1 = Partition[data, m, 1];
  va2 = Partition[data, 1 + m, 1];
  nF1 = Nearest[va1 → Automatic, DistanceFunction → ChessboardDistance];
  nF2 = Nearest[va2 → Automatic, DistanceFunction → ChessboardDistance];
  diff1 = Total[(Length /@ nF1[va1, {All, r}])] - Length[va1];
  diff2 = Total[(Length /@ nF2[va2, {All, r}])] - Length[va2];
  -Log[N@(diff2 / diff1)]]
```

In[•]:=
```
SampEn[diffdataAll]
```

Out[•]= 1.17748

In[•]:=
```
windowPlot[diffdataAll, 50, SampEn, "Differenced data", Red,
 "Figure 7 Sample Entropy over time", "datapoints", "Sample Entropy"]
```

Out[•]=



Using a window of 1250 ages, the trend in Figure 7 shows that the complexity of the temperature in Greenland from 50 thousand ages before present fluctuates around 1.25 and increased slightly when entering LGM before a drop. The complexity then increases by time and eventually reaches a peak of 2.8 at the end of Bølling Allerød warming. This peak is then followed by a plummet to the lowest value at 0.35. Tracing back to the time series, this is actually the time after entering Holocene epoch, where the temperature raises linearly before stabilised at around -30°C.

4.4 Burstiness

Burstiness is a measure of the dispersion of a probability distribution where high values represent high volatility relative to the mean and therefore high complexity. [k] One of the measures of burstiness is the Fano Factor like the coefficient of variation, which is a ratio between the variance and the mean or absolute mean. [l,m] In this study, burstiness is measured using a modified coefficient of variation, $\bar{c}_v$ from [k]:

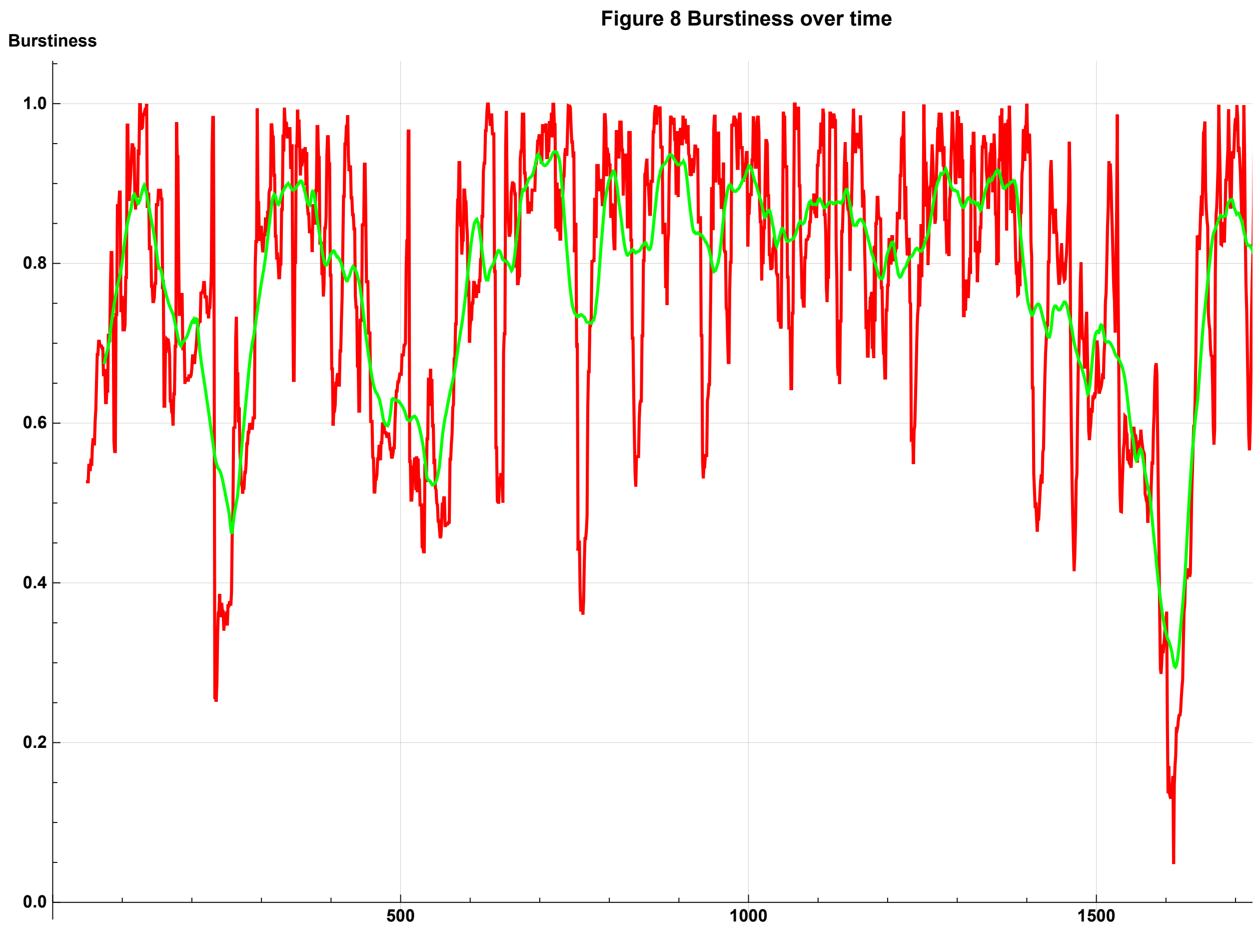
$\bar{c}_v = (c_v - 1) / (c_v + 1)$, where $c_v = \sigma / |\mu|$

```
In[]:= burstiness[data_] := Module[{std, mean, coeff, burst},
  std := StandardDeviation[data];
  mean := Abs[Mean[data]];
  coeff := std / mean;
  burst := (coeff - 1) / (coeff + 1);
  burst]
```

```
In[]:= burstiness[diffdataAll]
Out[]= 0.987912
```

In[•]:=
```
windowPlot[diffdataAll, 50, burstiness, "Differenced data",
 Red, "Figure 8 Burstiness over time", "datapoints", "Burstiness"]
```

Out[•]=



The burstiness of the data is 0.987. Figure 8 reveals the changes in burstiness of stationary data rolling over a window size of 50. The trend of complexity based on burstiness changes and fluctuates sharply before LGM, and becomes relatively stable and high after LGM. It is noticed that the burstiness begins to drop while approaching the Bølling Allerød warming, and continue to decrease until after the end of Younger Dryas. The complexity shoots up after reaching the trough, and relatively stable at value above 0.8 during the Holocene period.

5. Critical Slowing Down

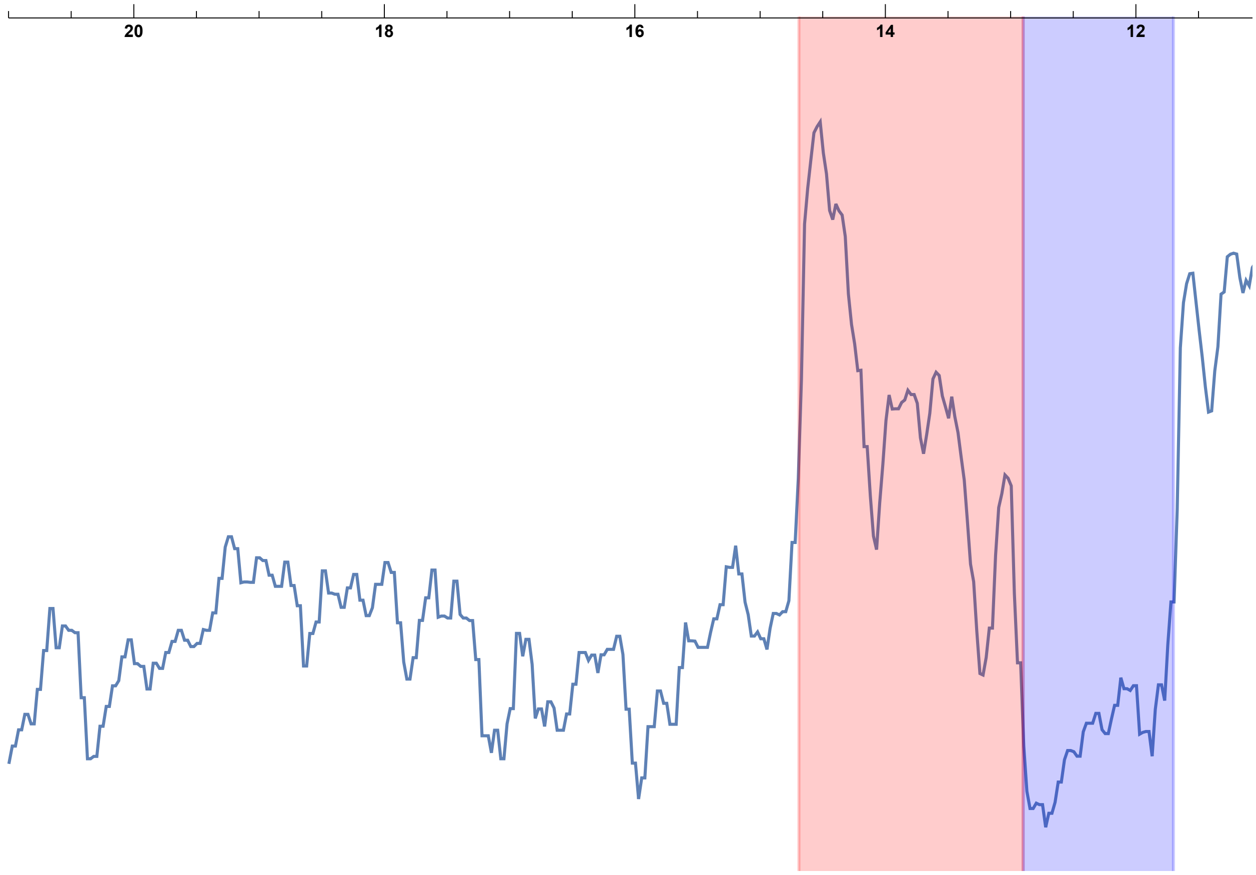
In this section, the climate record is cropped so that the dynamics and transitions of temperatures during Bolling Allerod Warming and end of Younger Dryas can be better captured. The aim is to use several indicators to detect early warning signals of those well-known abrupt climate changes during last deglaciation. The idea is based on the generic phenomenon called “critical slowing down”, that in the vicinity of many kinds of tipping points, the recovery rate of a system from small perturbations becomes very slow. [n] It can be inferred indirectly from rising “memory” in small fluctuations in the state of a system, as reflected, for instance, in a higher lag-1 autocorrelation, increased variance, change in skewness and kurtosis. In this study, the moving window size is taken as the half of the number of data points, as suggested in [b].

In[ ]:=
```
tsPart = TimeSeriesWindow[tsReg, {10, 21}]
Show[
 ListLinePlot[tsPart,
  ScalingFunctions → {"Reverse", Identity}, PlotRange → {{10, 21}, All},
  AxesLabel → {RawBoxes["time, x10³ yrs BP"], RawBoxes["Temperature,∘C"]},
  PlotLabel → "Figure 10 LGM to end of Younger Dryas (time series)",
  LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800],
 Plot[-60, {x, -12.9, -11.7}, PlotStyle → Blue, Filling → Axis],
 Plot[-60, {x, -14.69, -12.9}, PlotStyle → Red, Filling → Axis]
]
```

Out[ ]=
```
TimeSeries[ Time: 10. to 21.  Data points: 440 ]
```

Out[ ]=

Figure 10 LGM to end of Younger Dryas (time series)



In[ ]:=
```
AgePart = Reverse[tsPart["Times"]];
TemperaturePart = Reverse[tsPart["Values"]];
Length[AgePart]
Length[TemperaturePart]
regulardataPart = Table[TemperaturePart[[i]], {i, Length[TemperaturePart]}];
TableView[Table[{AgePart[[i]], TemperaturePart[[i]]}, {i, Length[TemperaturePart]}],
 AllowedDimensions → {2, Length[TemperaturePart]}]
```

Out[ ]= 440

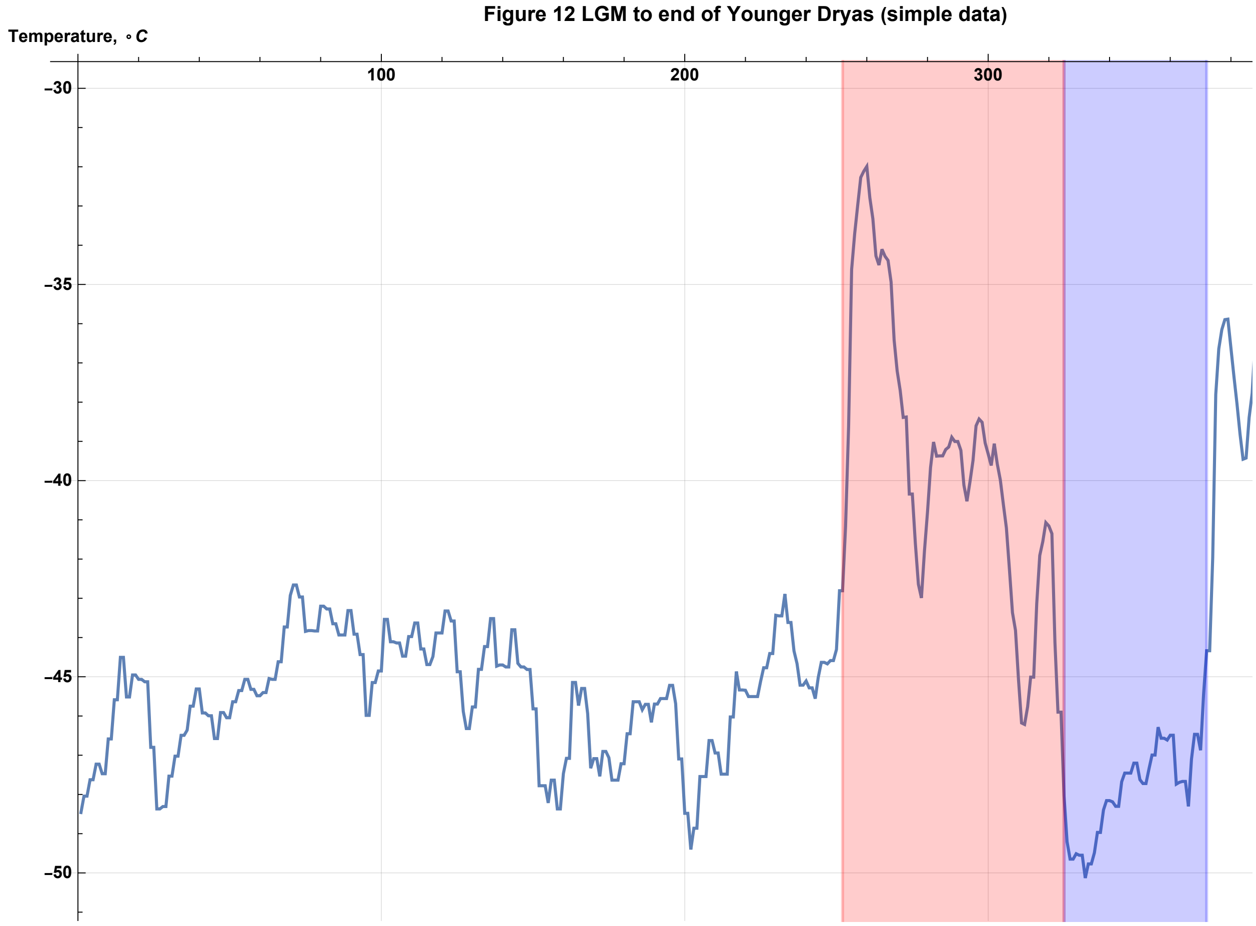
Out[ ]= 440

Out[ ]=

	1	2
1	20.9951	-48.4609
2	20.9701	-48.0428
3	20.9451	-48.0428
4	20.9201	-47.6248
5	20.8951	-47.6248
6	20.8701	-47.2272
7	20.8451	-47.2272
8	20.8201	-47.4719
9	20.7951	-47.4719
10	20.7701	-46.585
11	20.7451	-46.585
12	20.7201	-45.5862
13	20.6951	-45.5862
14	20.6701	-44.5055
15	20.6451	-44.5055
16	20.6201	-45.5147

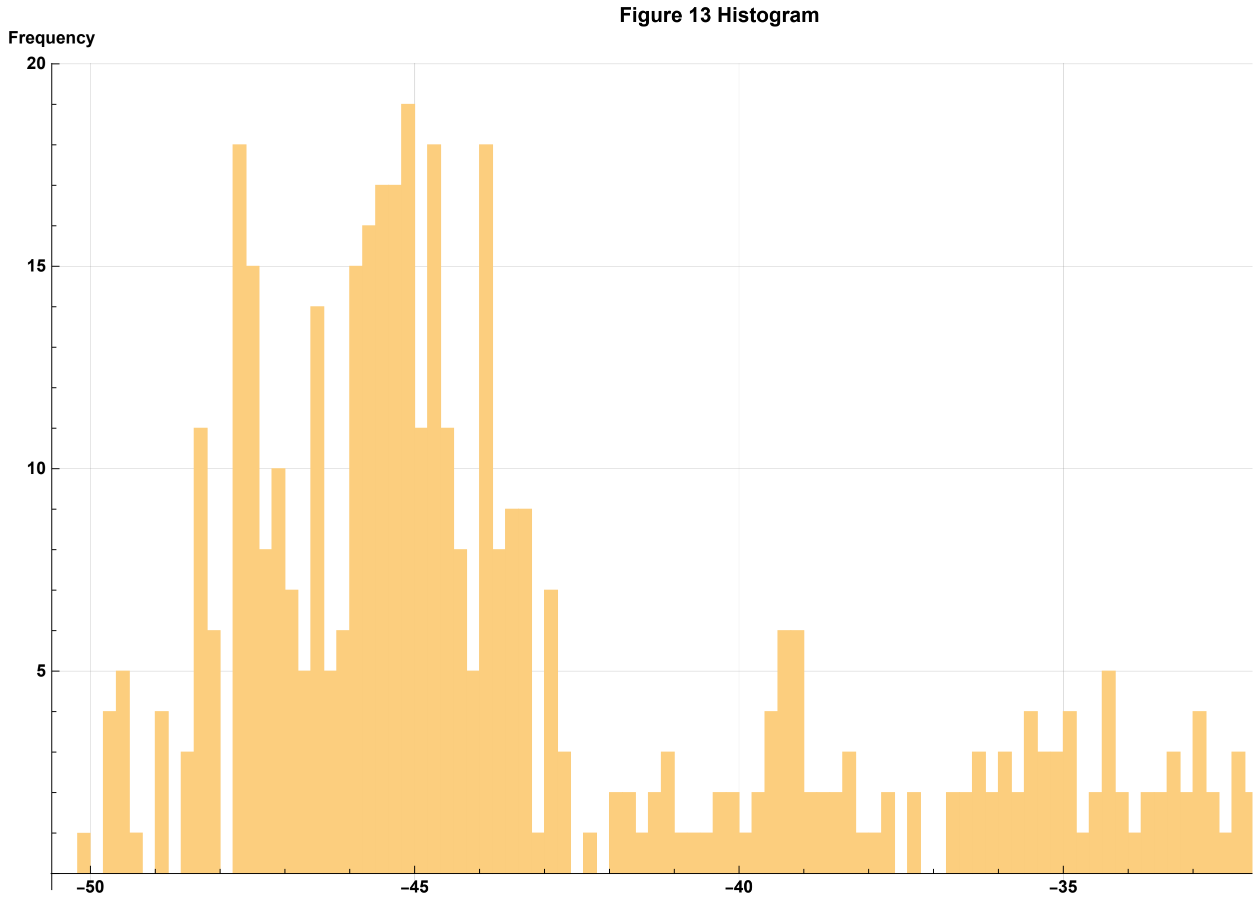
In[◦]:=
```
Show[
 ListPlot[regulardataPart , GridLines → Automatic, Joined → True, PlotRange → All,
  AxesLabel → {RawBoxes["datapoints"], RawBoxes["Temperature,∘C"]},
  PlotLabel → "Figure 12 LGM to end of Younger Dryas (simple data)",
  LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800],
 Plot[-60, {x, 325, 372}, PlotStyle → Blue, Filling → Axis],
 Plot[-60, {x, 252, 325}, PlotStyle → Red, Filling → Axis]
]
```

Out[◦]=



In[∘]:=
```
Histogram[regulardataPart, 100, PlotRange → All,
 AxesLabel → {RawBoxes["Temperature,∘C"], RawBoxes["Frequency"]}, GridLines → Automatic,
 PlotLabel → "Figure 13 Histogram", LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800]
```

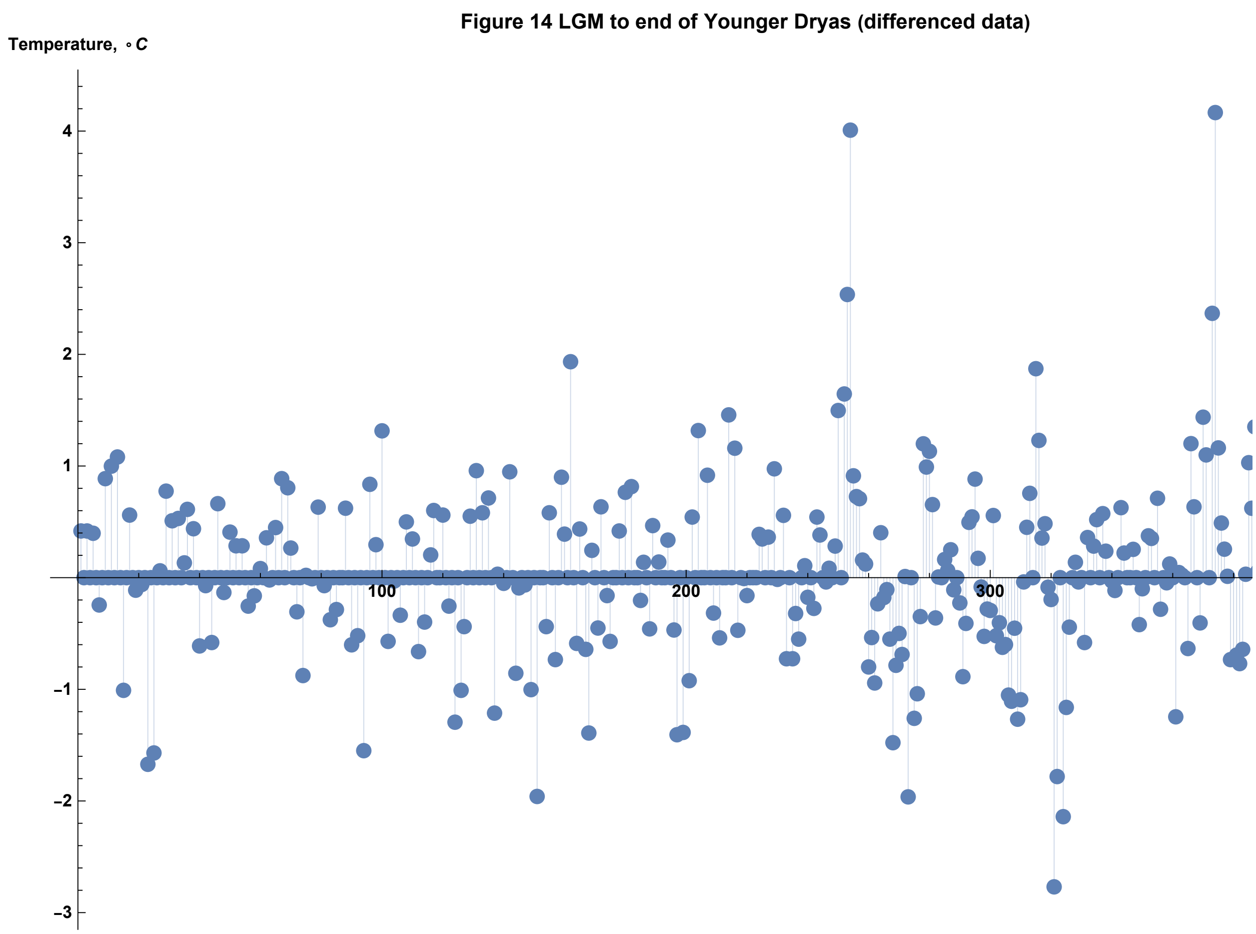
Out[∘]=



```
In[]:= diffdataPart = Differences[regulardataPart];
Length[diffdataPart]
ListLinePlot[diffdataPart, Joined → False, Filling → Axis, PlotRange → All,
 AxesLabel → {RawBoxes["datapoints"], RawBoxes["Temperature,∘C"]},
 PlotLabel → "Figure 14 LGM to end of Younger Dryas (differenced data)",
 LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800]
```

Out[]= 439

Out[]=



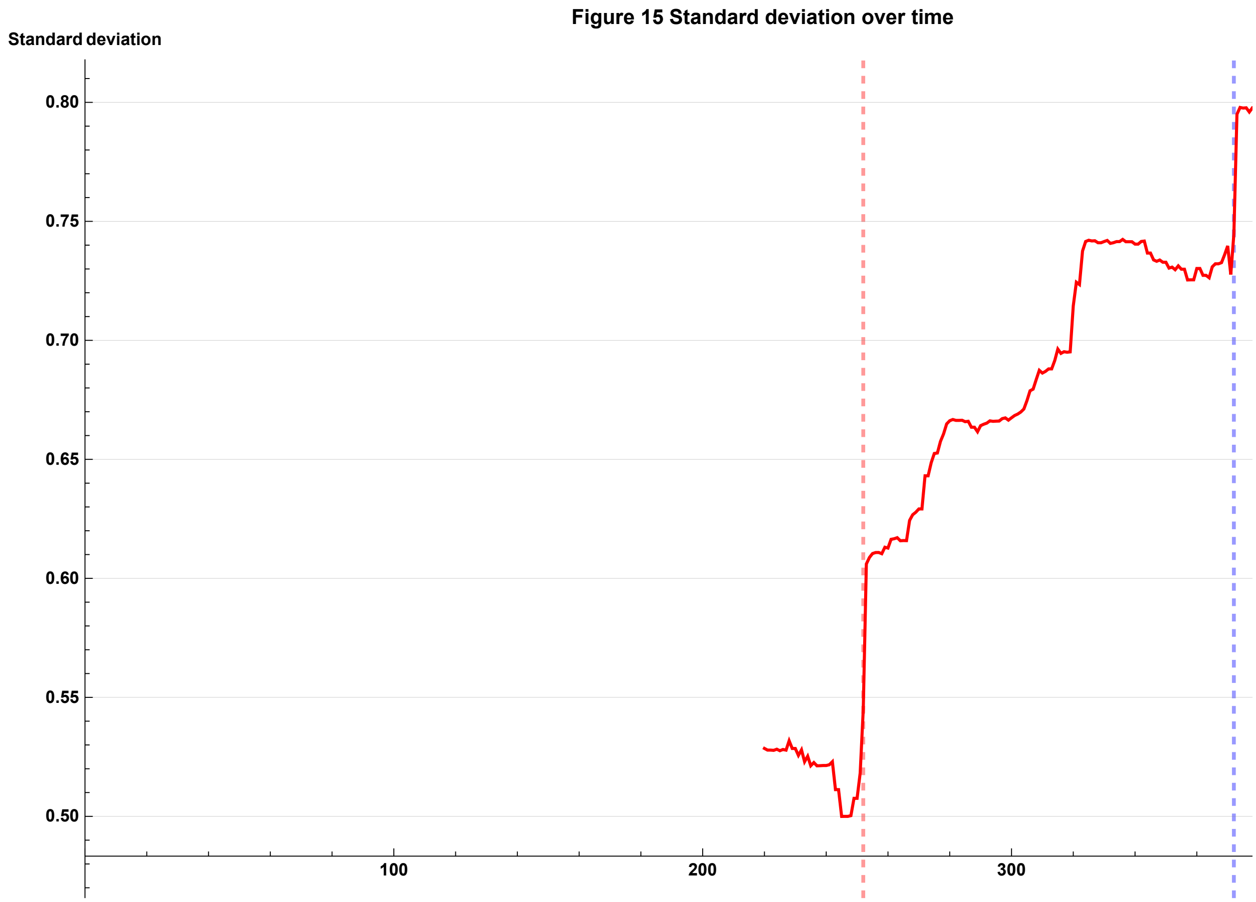
5.1 Variance

**Note that rolling window plot is modified such that it shows the events where abrupt changes occur, with red dashed line indicates the start of Bølling Allerød warming, and blue dashed line denotes the end of Younger Dryas. Last 25 data points are also been cropped so that the resultant graph focus more on the changes before transitions.

In[*]:=
```
windowPlot2[data_, window_, f_, plegend_, color_, plabel_, xlabel_, ylabel_] :=
 Module[{movingmap, xAxis, windowedData, graph, plot},
  movingmap = MovingMap[f, data[[1 ;; Length[data] - 25]], window];
  xAxis = Range[window, Length[data]];
  windowedData = Table[{xAxis[[i]], movingmap [[i]]}, {i, Length[movingmap ]}];
  graph = ListLinePlot[windowedData, GridLines → {{{372, Directive[Blue, Thick, Dashed]},
       {252, Directive[Red, Thick, Dashed]}}, Automatic}, PlotStyle → {color},
    PlotRange → {{0, Length[data]}, Full}, PlotLegends → {plegend}];
  plot = Show[graph, AxesLabel → {RawBoxes[xlabel], RawBoxes[ylabel]},
    PlotLabel → plabel, LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800];
  plot]
```

In[*]:=
```
windowPlot2[diffdataPart, 220, StandardDeviation, "Differenced data", Red,
 "Figure 15 Standard deviation over time", "datapoints", "Standard deviation"]
```

Out[*]=



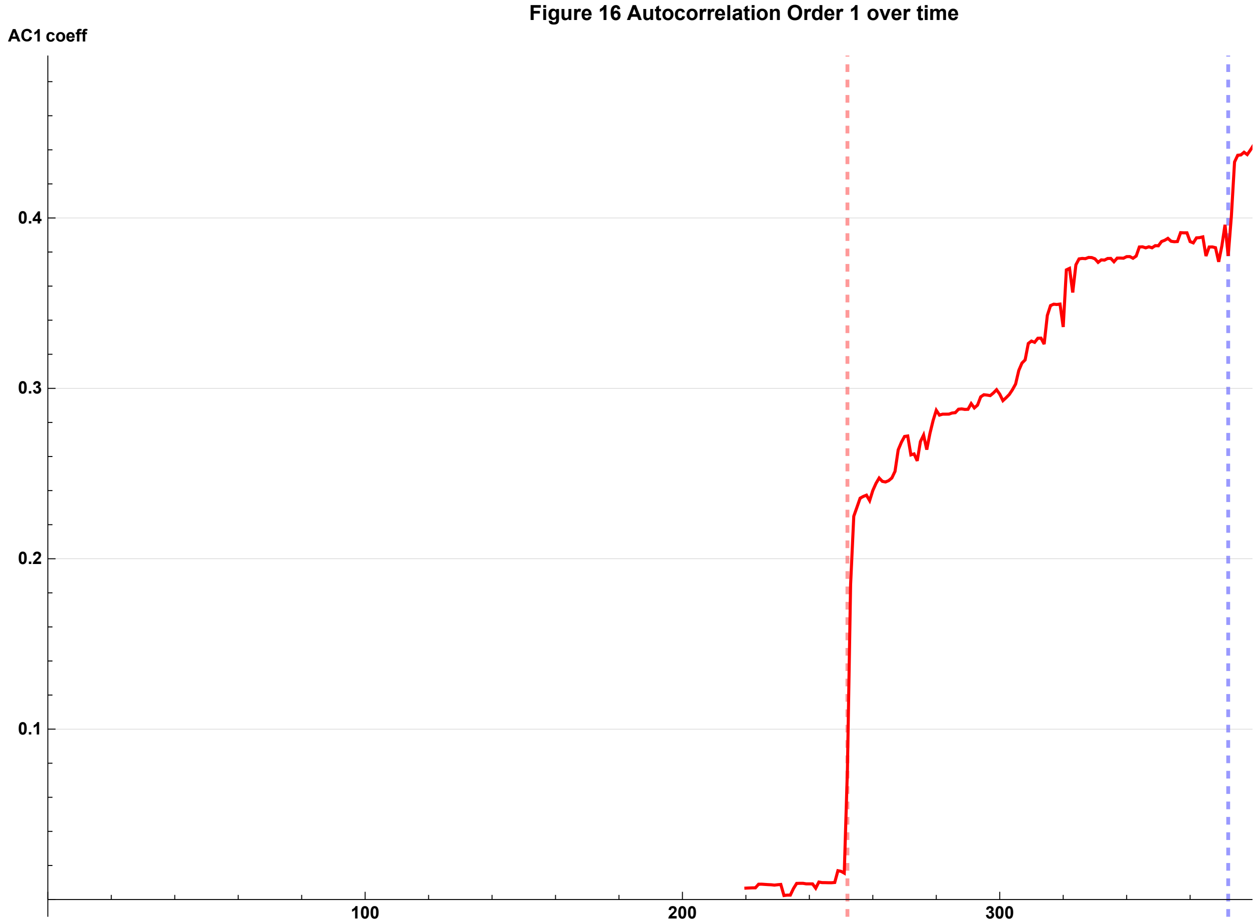
Discussion of the result in this part will be discussed together with next part (5.2).

5.2 Autocorrelation

In[*]:=
```
ACF1[data_] := CorrelationFunction[data, 1];
```

In[ ]:=
```
windowPlot2[diffdataPart, 220, ACF1, "Differenced data", Red,
  "Figure 16 Autocorrelation Order 1 over time", "datapoints", "AC1 coeff"]
```

Out[ ]=



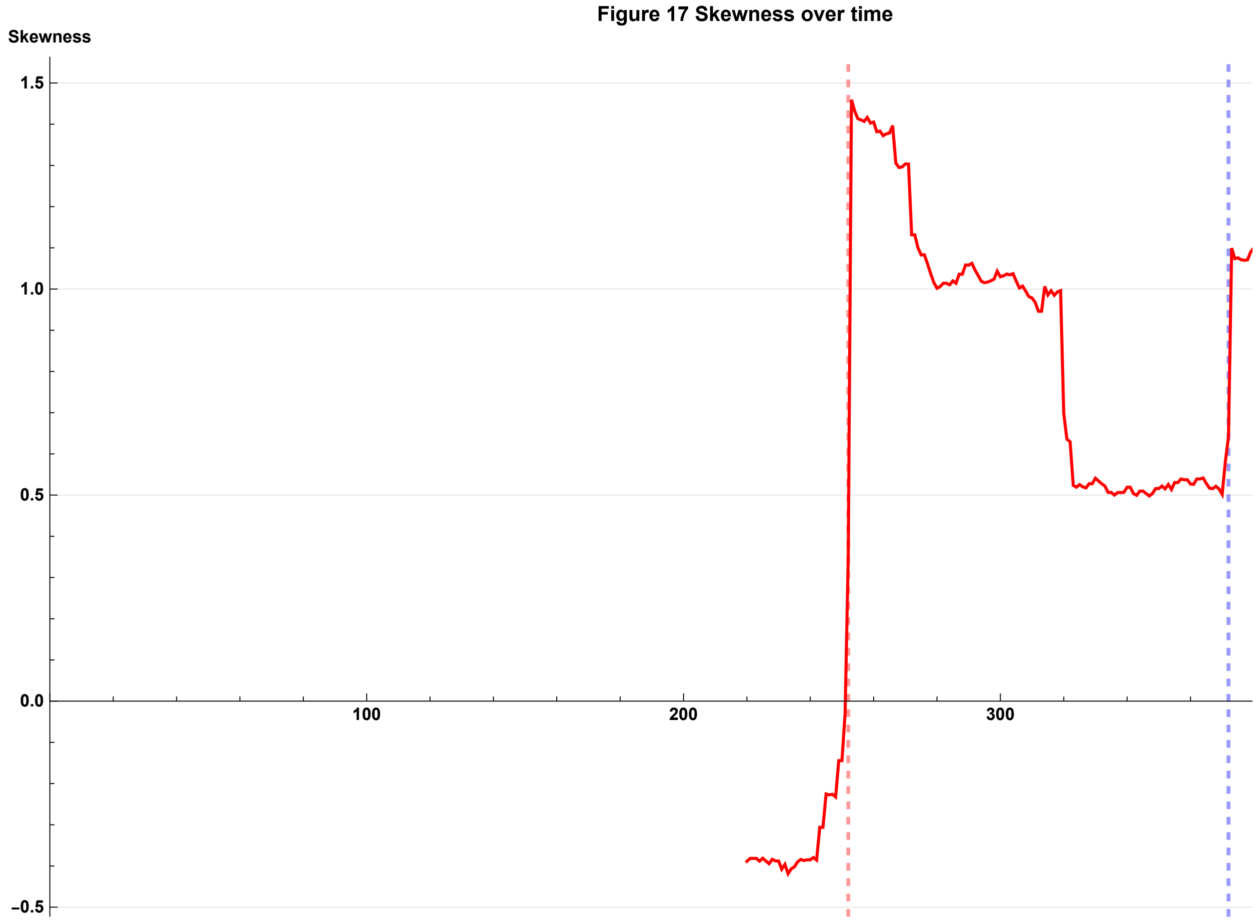
Both variance and autocorrelation graphs show a clear increase trend towards the end of of Younger Dryas (see Figure 15 and 16). As the window size is very large (half of the length of data), it is hard to tell, at least from these figures, if there are any signs of slowing down for Bølling Allerød warming.

5.3 Skewness

The skewness is a dimensionless measure of the degree of asymmetry of a probability distribution in which it vanishes for distribution symmetric about the mean and is positive or negative for an asymmetric distribution with a tail above or below the mean respectively. [o] The authors in [o] suggested that it is the change in skewness which acts as an early warning signal, and could be from zero skewness to either positive or negative values or from one sign of skewness to the other.

In[ ]:=
```
windowPlot2[diffdataPart, 220, Skewness, "Differenced data",
 Red, "Figure 17 Skewness over time", "datapoints", "Skewness"]
```

Out[ ]=



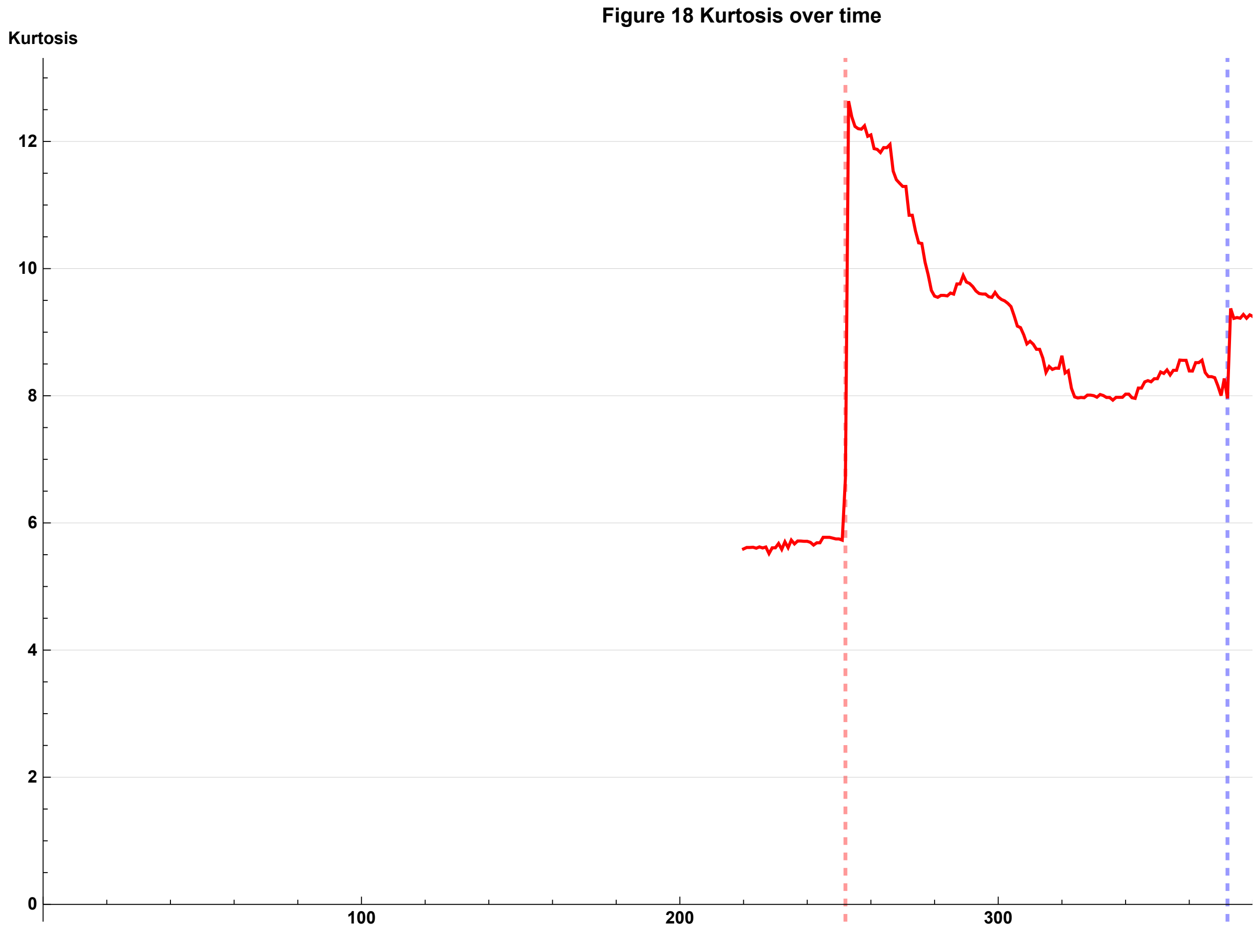
There is an increase in and hence a change of skewness from negative to positive values for Bølling Allerød warming (see Figure 17). The skewness of the system shows a decrease in general before the end of Younger Dryas, however, the values are still positive. Hence, based on the interpretation from [o], using skewness as an indicator shows an early warning signal for Bølling Allerød warming but not end of Younger Dryas.

5.4 Kurtosis

As mentioned, approaching tipping points, the variance of the system increases. Thus it enhances the tail of the distribution. Kurtosis is the measure of the tailedness of the distribution, so it is also used as an early warning signal for predicting tipping points. [p] It is expected to see a decrease in kurtosis near tipping points. It is observed that the kurtosis of system in fact shows a drop near the period of the end of Younger Dryas (see Figure 18).

In[ ]:=
```
windowPlot2[diffdataPart, 220, Kurtosis, "Differenced data",
  Red, "Figure 18 Kurtosis over time", "datapoints", "Kurtosis"]
```

Out[ ]=



6. Comparative Analysis of dataset

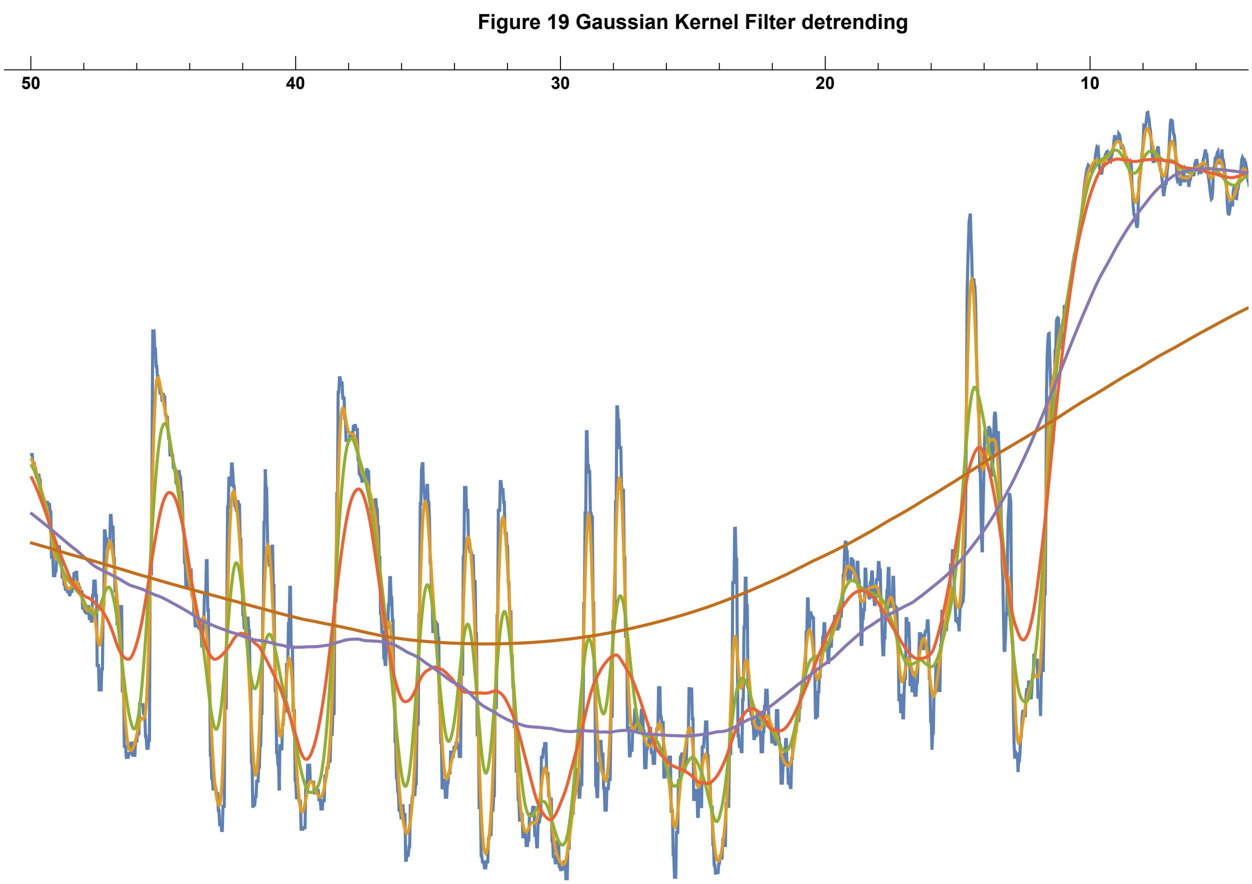
In this section, the real time series is detrended by using Gaussian kernel with a range of smoothing levels, r. The purpose is to apply the complexity measures in Section 4 such as SampEn and burstiness to the smoothed data, as a comparative analysis of complex time series with linear data. The comparative analyses of data are carried out by plotting measures of complexity value vs degree of smoothing. In the end of this section, a RP is constructed for the data with highest level of r, again, as a comparison with the original time series. Note that in Section 2, stationary version of data is used in application of SampEn and burstiness, hence, differenced data of linear data is also employed for the mentioned measures.

6.1 Detrend time series using Gaussian Kernel Filter

This part is to show the higher the smoothing factor, the more linear data is produced.

In[∘]:=
```
res = Table[GaussianFilter[tsReg, r], {r, {10, 25, 50, 200, 1000}}];
ListLinePlot[Join[{tsReg}, res], ScalingFunctions → {"Reverse", Identity},
 PlotLegends → {"data", "r = 10", "r = 25", "r = 50", "r = 200", "r = 1000"},
 AxesLabel → {RawBoxes["time, x10^3 yrs BP"], RawBoxes["Temperature,∘C"]},
 PlotLabel → "Figure 19 Gaussian Kernel Filter detrending",
 LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800]
```

Out[∘]=



6.2 Measures of complexity of linear data

6.2.1 Sample Entropy and burstiness

In[ ]:=
```
SampEncompare =
  Table[SampEn[Differences[GaussianFilter[tsReg, r]["Values"]]], {r, 1, 1001, 20}];
rValues = Range[1, 1001, 20];
dataSampEn = Transpose@{rValues, SampEncompare};
s1 = ListLinePlot[dataSampEn , PlotRange → All,
   GridLines → Automatic, PlotStyle → {Red}, PlotLegends → {"sample entropy"}];

burstcompare =
  Table[burstiness[Differences[GaussianFilter[tsReg, r]["Values"]]], {r, 1, 1001, 20}];
databurst = Transpose@{rValues, burstcompare};
b1 = ListLinePlot[databurst, PlotRange → All,
   GridLines → Automatic, PlotStyle → {Blue}, PlotLegends → {"burstiness"}];

Show[{s1, b1}, PlotRange → All,
 AxesLabel → {RawBoxes["smoothing factor r"], RawBoxes["Complexity measures"]},
 PlotLabel → "Figure 20 Measures of complexity vs smoothing factor",
 LabelStyle → {GrayLevel[0], Bold}, ImageSize → 800]
```

Out[ ]=

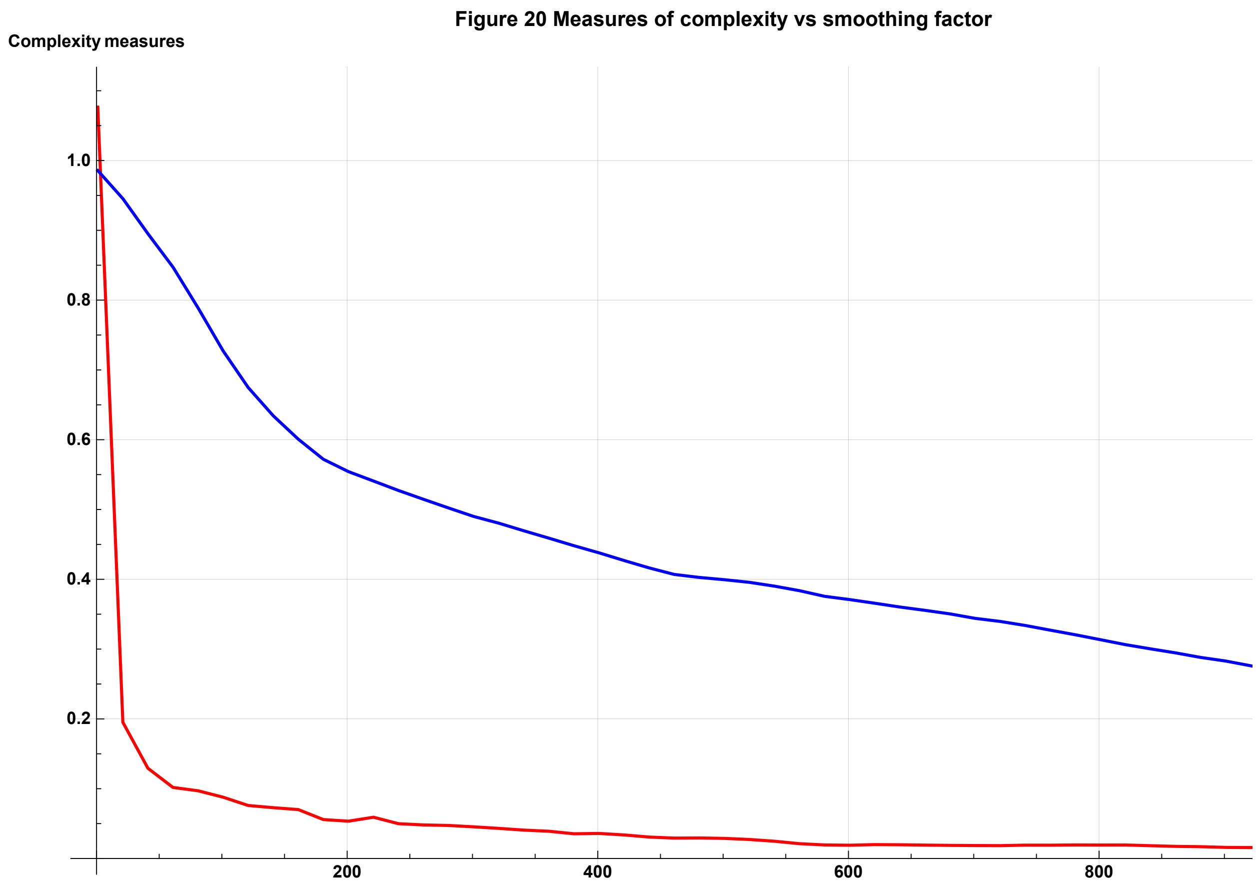


Figure 20 depicts that the higher linearity, the lower complexity of data in terms of sample entropy and burstiness. It also justifies that the original data (without any degree of detrending) is indeed complex and non-linear.

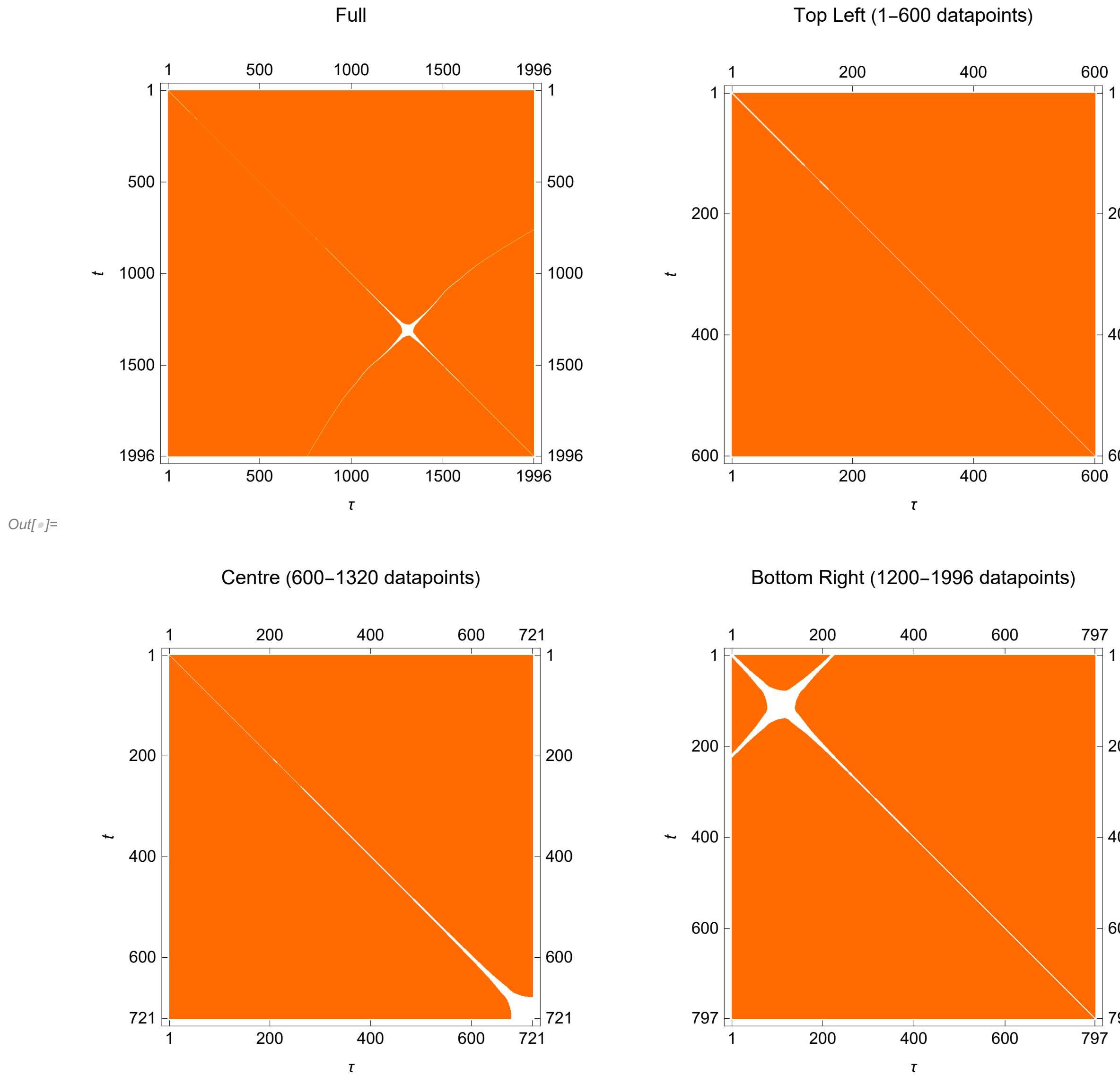
6.2.2 Recurrence plot

In[◦]:=
```
(*Extract one timeseries from Section 6.1*)
lineardata = res[[5]]["Values"]; (*when r = 1000*)
0.15 * StandardDeviation[lineardata]
```

Out[◦]= 0.573421

In[◦]:=
```
ϵ = 0.01;

g7 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[lineardata⟦t⟧ - lineardata⟦τ⟧]],
    {t, 1, Length[lineardata]}, {τ, 1, Length[lineardata]}],
   Mesh → False, FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Full"];

g8 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[lineardata⟦t⟧ - lineardata⟦τ⟧]],
    {t, 1, 600}, {τ, 1, 600}], Mesh → False,
   FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Top Left (1-600 datapoints)"];

g9 = MatrixPlot[Table[1 - UnitStep[ϵ - Norm[lineardata⟦t⟧ - lineardata⟦τ⟧]],
    {t, 600, 1320}, {τ, 600, 1320}], Mesh → False,
   FrameLabel → TraditionalForm /@ {t, τ}, PlotLabel → "Centre (600-1320 datapoints)"];

g10 = MatrixPlot[Table[
    1 - UnitStep[ϵ - Norm[lineardata⟦t⟧ - lineardata⟦τ⟧]], {t, 1200, Length[lineardata]},
    {τ, 1200, Length[lineardata]}], Mesh → False, FrameLabel → TraditionalForm /@ {t, τ},
   PlotLabel → "Bottom Right (1200-1996 datapoints)"];

Show[GraphicsGrid[{{g7, g8}, {g9, g10}}], PlotLabel →
  Style["Figure 21 Recurrence Plot with distance = 0.01 (linear data)", "Subsubsection"]]
```

Figure 21 Recurrence Plot with distance = 0.01 (linear data)



```
In[ ]:= ϵ = 0.01;
rp = Table[UnitStep[ϵ - Norm[lineardata⟦t⟧ - lineardata⟦τ⟧]],
    {t, 1, Length[lineardata]}, {τ, 1, Length[lineardata]}];
n = Length[lineardata];
countlines = 0;
countpoints = 0;
For[j = 0, j < n, j++,
  linelength = 0;
  For[i = 1, i + j < n, i++,
   If[rp[[i, i + j + 1]] == 1, linelength++,
     If[linelength ≥ 2, countlines++,
      If[linelength == 1, countpoints++
       ];
     ];
     linelength = 0(*reset for a diagonal*)
    ];
  ];
  If[linelength ≥ 2, countlines++,
   If[linelength == 1, countpoints++
    ];
  ];
 ];
pr = N[(Total[Total[rp]]) / (n * n)];
pd = N[2 * countlines / (n * n)];
dr = N[pd / pr];
Print[StringJoin["Recurrence Rate (RR): ", ToString[pr]]];
Print[StringJoin["Determinisim(DET): ", ToString[dr]]];

Recurrence Rate (RR): 0.0044794

Determinisim(DET): 0.088087
```

Figure 21 shows the recurrence plot of detrended time series with a factor of 1000, a big contrast with what is observed in Figure 4, where no patterns are seen. The RR and DET measured are 0.004 and 0.09 respectively, which are much lower than the values before (0.12 and 0.16).

7. Conclusion

In conclusion, in time series analysis, the dynamics and complexity of whole paleoclimate record from Greenland ice core are studied, and almost all of them show significant transition either in pattern (RP) or values (Hurst, SampEn, Burstiness) at the end of Younger Dryas. For detection of critical slowing down in period from Last Glacial Maximum to the end of Younger Dryas, strong signs of early warning are spotted for the end of Younger Dryas using indicator such as standard deviation, autocorrelation and kurtosis, while the abrupt change in warming of Bølling Allerød has only one positive result with indicator skewness. These results are somewhat parallel to previous studies, where [b,c] had found a positive trend in variability and autocorrelation for the end of Younger Dryas and a weak early sign in autocorrelation for the starting of Bølling Allerød**.** Then, the comparative analysis of the time series affirms its non-linearity nature. Finally, it is suggested to extend the complexity analysis by diving deeper into RQA. Comparison of Approximate Entropy and Sample Entropy on current data can also be carried out as in [a,j].

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