CSYS5040 Criticality in Dynamical Systems Assignment 2

Due Date: This is assignment is due in TurnItIn by Friday, October the 7th. This assignment is worth **25% of your final mark**.

You must do all of your working in a Mathematica notebook that I can run (no pdfs of Mathematica notebooks).

The * for some questions indicate the relative difficulty of the question. This is an individual assessment; your answers must reflect your own work. Marks will be based on the correctness of each answer, the effort put into exploring each question, and the originality of the examples you choose to look at. You are strongly encouraged o read beyond the class material to get a higher grade.

Question 1 (7.5%): The dynamics of a stochastic differential equation

a. Choose the constant valued parameters (i.e. μ, x_0 and ϵ) of a linear stochastic differential equation of the form: $\mathrm{d}x/\mathrm{d}t = \mu + \epsilon$ where ϵ is the stochastic (noise) term, x_0 is the initial starting point and then plot the time series of the solution, making sure μ is not equal to zero (it can be positive or negative though). Choose some numerical value b (for 'boundary') that has the same sign as μ, run some simulations of your solution, does your solution ever cross the boundary b and if so at what value of t does your solution cross it? Without simulating your solution, how can you know when to expect the solution to cross the boundary?
b. * Repeat part a. except that the stochastic differential equation is now non-linear, i.e. $\mathrm{d}x/\mathrm{d}t = \alpha x + \mu + \epsilon$ or uses even higher order terms in x, e.g. x^2or x^3such as $\mathrm{d}x/\mathrm{d}t = \beta x^2 + \alpha x + \mu + \epsilon$
c. Write a single paragraph on one application of the methods you used in parts a. and b. For example you can look up: "drift diffusion" and "neural network", "two alternative forced choice task" (e.g. here: https://tinyurl.com/yygku3oa), "Ornstein-Uhlenbeck process", or see here: https://en.wikipedia.org/wiki/Ornstein–Uhlenbeck_process

Question 2 (7.5%): Plotting non-linear functions for a non-linear map

a. Choose a discrete non-linear map (for example the Tent map, Logistic map etc.) and plot the return map of x(t) vs. x(t+1) for your example with parameters that generate a chaotic dynamic.
b. Plot the time series of data points generated by your map for both chaotic and non-chaotic parameter settings.
c. Show the bifurcation plot of your system for when one of the parameters in your system is changing. Include the Lyapunov values on the same plot for each of the parameter values, i.e. for each parameter value show whether the largest Lyapunov value is greater than zero or less than zero.

Question 3 (10%): Parameters in non-linear dynamical systems

a. **Based on your answer to Question 1 **for the non-linear system**, write a Mathematica function that shows what happens when a parameter value, e.g. β or α changes and the system switches from one equilibrium state to another. See the

Mathematica notebook from Week 1 where at the end there is a stochastic diffusion processes with more than one stationary state, note that the stationary state the system was attracted to (settled in) depended on the initial starting point x_0. For this question, using a system with more than one stationary state, I want you to let the system find its stationary state (i.e. it is in equilibrium) and then change the parameter values so that the system switches from one equilibrium point to another by going through a tipping point. The minimum you need to do to pass this question is to plot the time series of the system passing through this tipping point.

b. **Using the discrete map from your answer to Question 2 numerically estimate the Lyapunov exponent of your system using the technique shown in Week 4, i.e. where you calculate the gradient of the data plotted on a log-log plot (slide 39). Compare this with the exact value of the Lyapunov exponent using the analytical technique introduced in week 3. Then estimate the Fractal dimension of your system using the Kaplan-Yorke conjecture.