Answer to Q(2)

$$\max\ z = x_1 + 2x_2$$

s.t. $-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_1 + 3x_2 \ge 6$

$x_2, x_1 \geq 0$

① Let's convert the problem into standard LP problem,

$$\min z = -x_1 - 2x_2$$

$-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$-x_1 - 3x_2 \le -6$

Let's introduce some slack variable x_3, x_4, x_5 & x_6 to the inequalities

$-x_1 + 3x_2 + x_3 = 10$

$x_1 + x_2 + x_4 = 6$

$x_1 - x_2 + x_5 = 2$

$-x_1 - 3x_2 + x_6 = -6$

Here, non-basic variables are $= x_1, x_2$

basic variables are $= x_3, x_4, x_5, x_6$

1st Tableau:

Basic	x_1	x_2 (pivot column)	x_3	x_4	x_5	x_6	b
x_3	-1	3	1	0	0	0	$10 = 10/3$
x_4	$+1$	1	0	1	0	0	$6 = 6/1 = 6$
x_5	1	-1	0	0	1	0	$2 = 2/-1 = -2$
x_6 (Pivot row)	-1	-3	0	0	0	1	$-6 = -6/-3 = 2$
f	-1	-2	0	0	0	0	

Now, x_2 would be basic
& x_6 would be non basic

2nd Tableau

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_3	-1	3	1	0	0	0	10
x_4	1	1	0	1	0	0	6
x_5	1	-1	0	0	1	0	2
x_2	-1	-3	0	0	0	1	-6
f	-1	-2	0	0	0	0	f

row1 = row1 + row4
row2 = row4/3 + row2
row3 = row4/-3 + row3
row4 = row4 × $-\frac{1}{3}$
row5 = row4 × 3 + row5

pivot column

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
pivot row x_3	-2	0	1	0	0	1	$4 = 4/1 = 4$
x_4	2/3	0	0	1	0	1/3	$4 = 4 \times 3 = 12$
x_5	4/3	0	0	0	1	-1/3	$4 = 4 \times (-3) = -12$
x_2	1/3	1	0	0	0	-1/3	$2 = 2 \times (-3) = -6$
f	0	1	0	0	0	-1	f+6

Now, x_6 would be basic
and x_3 would be non basic

3rd Tableau

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_6	-2	0	1	0	0	1	4
x_4	2/3	0	0	1	0	1/3	4
x_5	4/3	0	0	0	1	-1/3	4
x_2	1/3	1	0	0	0	-1/3	2
f	0	1	0	0	0	-1	f+6

① row1 = row 1
② row2 = row1/-3 + row 2
③ row3 = row1/3 + row 3
④ row4 = row1/3 + row4
⑤ row5 = row1 + row5

pivot column

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_6	-2	0	1	0	0	1	$4 = 4/-2 = -2$
x_4	4/3	0	-1/3	1	0	0	$8/3 = 8/3 \times 3/4 = 2$
x_5	2/3	0	1/3	0	1	0	$16/3 = 16/3 \times 3/2 = 8$
x_2	-1/3	1	1/3	0	0	0	$10/3 = 10/3 \times (-3) = -10$
f	-2	1	1	0	0	0	f+10

pivot row

Now, x_1 is basic
x_4 is non basic

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_6	-2	0	1	0	0	1	4
x_1	4/3	0	-1/3	1	0	0	8/3
x_5	2/3	0	1/3	0	1	0	16/3
x_2	-1/3	1	1/3	0	0	0	10/3
f	-2	1	1	0	0	0	f+10

row1 = row2 × 3/2
row2 = row2 × 3/4
row3 = row2 × (-1/2) + row3
row4 = row2 × 1/4 + row4
row5 = row2 × 2 + row5

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_6	0	0	1/2	3/2	0	1	8
x_1	1	0	-1/4	3/4	0	0	2
x_5	0	0	1/2	-1/2	1	0	4
x_2	0	1	1/4	1/4	0	0	4
f	0	1	1/2	3/2	0	0	f+14

Now, x_3 & x_4 are non basic variable, therefore they are 0.

Now, $x_1^* = 2$
$x_5^* = 4$
$x_6^* = 8$
$x_2^* = 4$ } Ans.

Now, $x_2^* = f^* + 14$
$\Rightarrow 4 = f + 14$
$\Rightarrow f^* = -10$

Therefore, $\max z = +10$ (Ans.)

Validation: The LINPROG( ) code is attached at the appendix.

Answer to the Q(1)

(a) No.

First, we need to convert it to standard optimization problem,

$$\min F(x_1, x_2) = -4x_1^2 - 3x_2^2 + 5x_1x_2 + 8$$
$$\text{s.t. } x_1 + x_2 = 4$$

Secondly, we need to construct the Lagrange function.

$$L(x, v) = F(x_1, x_2) + v_1 h_J$$
$$= -4x_1^2 - 3x_2^2 + 5x_1x_2 + 8 + v_1 \times (x_1 + x_2)$$

Thirdly, we need to apply the KKT condition

$$\frac{dL}{dx_1} = v_1 - 8x_1 + 5x_2 = 0 \quad \text{--- (1)}$$

$$\frac{dL}{dx_2} = v_1 + 5x_1 - 6x_2 = 0 \quad \text{--- (2)}$$

$$\frac{dL}{dv_1} = x_1 + x_2 - 4 = 0 \quad \text{--- (3)}$$

By solving eqn (1), (2) & (3), we get;

$$x_1 = 11/6$$
$$x_2 = 13/6$$
$$v_1 = 23/6$$

$$[x_1^*, x_2^*] = [11/6, 13/6] \text{ (Ans)}$$

(b) No. First, we need to convert it to standard optimization problem,

$$\min f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$$
$$\text{s.t.} \; -x_1 - x_2 \le -4$$
$$x_1 - x_2 - 2 = 0$$

Second, we need to construct the lagrange function

$$\mathcal{L}(x, v_j, u_i) = f(x) + v_j h_j + u_i(g_i + s_i^2)$$

$$\mathcal{L}(x_1, x_2, v_1, u_1, s_1) = (x_1 - 2)^2 + (x_2 - 1)^2 - u_1 \times (-s_1^2 + x_1 + x_2 - 4) - v_1(x_2 - x_1 + 2)$$

Third, Apply KKT condition

$$\frac{dL}{dx_1} = v_1 - u_1 + 2x_1 - 4 = 0 \quad \text{--- (1)}$$

$$\frac{dL}{dx_2} = 2x_2 - v_1 - u_1 - 2 = 0 \quad \text{--- (2)}$$

$$\frac{dL}{dv_1} = x_1 - x_2 - 2 = 0 \quad \text{--- (3)}$$

$$\frac{dL}{du_1} = s_1^2 - x_1 - x_2 + 4 = 0 \quad \text{--- (4)}$$

$$u_1 s_1 = 0$$

Now, either $u_1 = 0$ or $s_1 = 0$

If $u_1 = 0$, then, ~~u_1~~
$u_1 = -1/2 < 0$

So, this solution is not feasible.

If $s_1 = 0$, then where $u_1 = 1 > 0$

So, $[x_1^*, x_2^*] = [3, 1]$ (Ans.)

(C) No.
$$\min. f(x_1, x_2) = 9x_1^2 - 18x_1x_2 + 13x_2^2 - 4$$
$$\text{s.t.} -x_1^2 - x_2^2 - 2x_1 \le -16$$

Now, the lagrange function would look like as follows:

$$\mathcal{L}(x_1, x_2, u_1, s_1) = 9x_1^2 - 18x_1x_2 + 13x_2^2 - u_1(-s_1^2 + x_1^2 + 2x_1 + x_2^2 - 16) - 4$$

$$\frac{dL}{dx_1} = 18x_1 - 18x_2 - u_1(2x_1 + 2) = 0 \quad (1)$$

$$\frac{dL}{dx_2} = 26x_2 - 18x_1 - 2u_1x_2 = 0 \quad (2)$$

$$\frac{dL}{du_1} = s_1^2 - x_1^2 - 2x_1 - x_2^2 + 16 = 0 \quad (3)$$

$$u_1s_1 = 0$$

Now, if we solve this system of equation, we obta

$x_1^* = 2.6$	$x_2^* = 2.02$	$u_1^* = 1.44 > 0$
$= 1.5$	$= -3.27$	$u_2^* = 17.15 > 0$
$= -3.73$	$= -3.09$	$u_3^* = 2.12 > 0$
$= -3.63$	$= 3.18$	$= 23.29 > 0$

Ans.

& $s = 0$ for all cases.

Therefore it satisfies KKT condition.

Appendix

```
%% Problem 1 (a)
syms x1 x2 v1

expr1 = -4*x1^2-3*x2^2+5*x1*x2+8;
eqConstr = x1+x2-4;
LM1 = expr1 + v1 * eqConstr;
LM_x1 = diff(LM1, x1)
LM_x2 = diff(LM1, x2)
LM_v1 = diff(LM1, v1)

sol1 = solve([LM_x1,LM_x2,LM_v1], [x1,x2,v1],'Real',true)

%% Problem 1 (b)
syms x1 x2 u1 v1 s1
expr2 = (x1 -2)^2+(x2 -1)^2;
ineqConstr2 = -x1-x2+4;
eqConstr2 = x1-x2-2;
LM2 = expr2+v1*eqConstr2+u1*(ineqConstr2+s1^2)
LM_x1 = diff(LM2, x1)
LM_x2 = diff(LM2, x2)
LM_v1 = diff(LM2, v1)
LM_u1 = diff(LM2, u1)

sol2 = solve([LM_x1,LM_x2,LM_v1,LM_u1], [x1,x2,v1,s1],"Real",true)

%% Problem 1 (c)

syms x1 x2 u1 s1
assume(u1 >= 0)
assume(s1 >= 0)
expr3 = 9*x1^2-18*x1*x2+13*x2^2-4;
ineqConstr3 = -x1^2-x2^2-2*x1+16;

LM3 = expr3+u1*(ineqConstr3+s1^2)
LM_x1 = diff(LM3, x1)
LM_x2 = diff(LM3, x2)
LM_u1 = diff(LM3, u1)
LM_s1 = u1*s1
sol3 = vpasolve([LM_x1,LM_x2,LM_u1,LM_s1], [x1,x2,s1,u1])
%% Problem 2

A = [-1 3
    1 1
    1 -1
    -1 -3];
b = [10 6 2 -6];
f = [-2 -1];
[x,fval] = linprog(f,A,b)
```