605.611 - Foundations of Computer Architecture

Assignment 02

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1. Convert the following fixed point numbers to binary fixed point. Give both the actual values, and normalize the values so that they have a binary 1 as the value for the left of the decimal point.

   (a) 7.25

   Answer: Integral part: $7 = (0111)_2$

   Repeatedly multiplying the fractional part by 2:

$$
\begin{array}{rcccc}
0.25 & & & & \\
 & 0 & . & 2 & 5 \\
 & \times & & & 2 \\
\hline
= & 0 & . & 5 & 0 \\
 & \times & & & 2 \\
\hline
 & 1 & . & 0 & 0
\end{array}
$$

$$= (10)_2$$

$$\Rightarrow 7.25 = (111.01)_2, (\text{normalized to } (1.1101)_2 \times 2^2)$$

   (b) 13.5

   Answer: Integral part: $13 = (1101)_2$

Repeatedly multiplying the fractional part by 2:

$$
\begin{aligned}
&0.50 \\
&= \begin{array}{r}
0\ .\ 5\ 0 \\
\times \qquad\quad 2 \\
\hline
1\ .\ 0\ 0
\end{array} \\
&= (1)_2 \\
&\Rightarrow 13.5 = (1101.1)_2, (\text{normalized to } (1.1011)_2 \times 2^3)
\end{aligned}
$$

(c) 0.5625

Answer: Integral part: 0

Repeatedly multiplying the fractional part by 2:

$$
\begin{aligned}
&0.5625 \\
&= \begin{array}{r}
0\ .\ 5\ 6\ 2\ 5 \\
\times \qquad\qquad 2 \\
\hline
1\ .\ 1\ 2\ 5\ 0 \\
\times \qquad\qquad 2 \\
\hline
0\ .\ 2\ 5\ 0\ 0 \\
\times \qquad\qquad 2 \\
\hline
0\ .\ 5\ 0\ 0\ 0 \\
\times \qquad\qquad 2 \\
\hline
1\ .\ 0\ 0\ 0\ 0
\end{array} \\
&= (1001)_2 \\
&\Rightarrow 0.5625 = (0.1001)_2, (\text{normalized to } (1.001)_2 \times 2^{-1})
\end{aligned}
$$

(d) 0.125

Answer: Integral part: 0

Repeatedly multiplying the fractional part by 2:

$$0.125$$

$$= \begin{array}{r} 0\ .\ 1\ 2\ 5 \\ \times \qquad\quad 2 \\ \hline 0\ .\ 2\ 5\ 0 \\ \times \qquad\quad 2 \\ \hline 0\ .\ 5\ 0\ 0 \\ \times \qquad\quad 2 \\ \hline 1\ .\ 0\ 0\ 0 \end{array}$$

$$= (001)_2$$

$$\Rightarrow 0.5625 = (0.001)_2, (\text{normalized to } (1.0)_2 \times 2^{-3})$$

(e) 127.625

Answer: Integral part: 127 = (01111111)

Repeatedly multiplying the fractional part by 2:

$$0.625$$

$$= \begin{array}{r} 0\ .\ 6\ 2\ 5 \\ \times \qquad\quad 2 \\ \hline 1\ .\ 2\ 5\ 0 \\ \times \qquad\quad 2 \\ \hline 0\ .\ 5\ 0\ 0 \\ \times \qquad\quad 2 \\ \hline 1\ .\ 0\ 0\ 0 \end{array}$$

$$= (101)_2$$

$$\Rightarrow 127.625 = (1111111.101)_2, (\text{normalized to } (1.111111101)_2 \times 2^{6})$$

(f) 51,025.025

Answer: Integral part: 51025 =

$\Rightarrow$ 51025

$$= \begin{array}{r} 25512 \\ 2\,\overline{)\,51025} \\ \underline{4} \\ 11 \\ \underline{10} \\ 10 \\ \underline{10} \\ 02 \\ \underline{2} \\ 05 \\ \underline{4} \\ 1 \end{array} \quad \begin{array}{r} 12756 \\ 2\,\overline{)\,25512} \\ \underline{2} \\ 05 \\ \underline{4} \\ 15 \\ \underline{14} \\ 11 \\ \underline{10} \\ 12 \\ \underline{12} \\ 0 \end{array} \quad \begin{array}{r} 6378 \\ 2\,\overline{)\,12756} \\ \underline{12} \\ 07 \\ \underline{6} \\ 15 \\ \underline{14} \\ 16 \\ \underline{16} \\ 0 \end{array} \quad \begin{array}{r} 3189 \\ 2\,\overline{)\,6378} \\ \underline{6} \\ 03 \\ \underline{2} \\ 17 \\ \underline{16} \\ 18 \\ \underline{18} \\ 0 \end{array} \quad \begin{array}{r} 1594 \\ 2\,\overline{)\,3189} \\ \underline{2} \\ 11 \\ \underline{10} \\ 18 \\ \underline{18} \\ 09 \\ \underline{8} \\ 1 \end{array} \quad \begin{array}{r} 797 \\ 2\,\overline{)\,1594} \\ \underline{14} \\ 19 \\ \underline{18} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

$$\begin{array}{r} 398 \\ 2\,\overline{)\,797} \\ \underline{6} \\ 19 \\ \underline{18} \\ 17 \\ \underline{16} \\ 1 \end{array} \quad \begin{array}{r} 199 \\ 2\,\overline{)\,398} \\ \underline{2} \\ 19 \\ \underline{18} \\ 18 \\ \underline{18} \\ 0 \end{array} \quad \begin{array}{r} 99 \\ 2\,\overline{)\,199} \\ \underline{18} \\ 19 \\ \underline{18} \\ 1 \end{array} \quad \begin{array}{r} 49 \\ 2\,\overline{)\,99} \\ \underline{8} \\ 19 \\ \underline{18} \\ 1 \end{array} \quad \begin{array}{r} 24 \\ 2\,\overline{)\,49} \\ \underline{4} \\ 09 \\ \underline{8} \\ 1 \end{array} \quad \begin{array}{r} 12 \\ 2\,\overline{)\,24} \\ \underline{2} \\ 04 \\ \underline{4} \\ 0 \end{array} \quad \begin{array}{r} 6 \\ 2\,\overline{)\,12} \\ \underline{12} \\ 0 \end{array} \quad \begin{array}{r} 3 \\ 2\,\overline{)\,6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 2\,\overline{)\,3} \\ \underline{2} \\ 1 \end{array} \quad \begin{array}{r} 0 \\ 2\,\overline{)\,1} \end{array}$$

$\Rightarrow (1100\ 0111\ 0101\ 0001)_2$

Repeatedly multiplying the fractional part by 2:

0.025

$$
= \begin{array}{cccccc}
 & 0 & . & 0 & 2 & 5 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 0 & 5 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 1 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 2 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 4 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 8 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 1 & . & 6 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 1 & . & 2 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 4 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 0 & . & 8 & 0 & 0 \\
\times & & & 2 & & \\ \hline
 & 1 & . & 6 & 0 & 0 \\
\times & & & & & 2 \\ \hline
 & 1 & . & 2 & 0 & 0
\end{array}
$$

The pattern $(0000011)_2$ appears to keep repeating

$\Rightarrow 51,025.025 = (1100011101010001.0000011)_2$,

(normalized to $(1.1000111010100010000011)_2 \times 2^{15}$)

(g) 7.1

Answer: Integral part: $7 = (111)_2$

Repeatedly multiplying the fractional part by 2:

$$0.1 = \begin{array}{cccc}
 & 0 & . & 1 \\
\times & & & 2 \\ \hline
 & 0 & . & 2 \\
\times & & & 2 \\ \hline
 & 0 & . & 4 \\
\times & & & 2 \\ \hline
 & 0 & . & 8 \\
\times & & & 2 \\ \hline
 & 1 & . & 6 \\
\times & & & 2 \\ \hline
 & 1 & . & 2 \\
\times & & & 2 \\ \hline
 & 0 & . & 4 \\
\times & & & 2 \\ \hline
 & 0 & . & 8 \\
\times & & & 2 \\ \hline
 & 1 & . & 6 \\
\times & & & 2 \\ \hline
 & 1 & . & 2
\end{array}$$

The pattern $(0011)_2$ appears to keep repeating

$\Rightarrow 7.1 = (111.0\overline{00110011})_2$,

(normalized to $(1.110\overline{00110011})_2 \times 2^2$)

(h) 5.2

Answer: Integral part: $5 = (101)_2$

Repeatedly multiplying the fractional part by 2:

$$
\begin{array}{l}
0.2 \\
= \begin{array}{cccc}
 & 0 & . & 2 \\
\times & & & 2 \\
\hline
 & 0 & . & 4 \\
\times & & & 2 \\
\hline
 & 0 & . & 8 \\
\times & & & 2 \\
\hline
 & 1 & . & 6 \\
\times & & & 2 \\
\hline
 & 1 & . & 2 \\
\times & & & 2 \\
\hline
 & 0 & . & 4 \\
\times & & & 2 \\
\hline
 & 0 & . & 8 \\
\times & & & 2 \\
\hline
 & 1 & . & 6
\end{array}
\end{array}
$$

The pattern $(0011)_2$ appears to keep repeating

$\Rightarrow 7.1 = (101.\overline{00110011})_2$,

(normalized to $(1.01\overline{00110011})_2 \times 2^2$)

5. Convert the following from decimal to excess 127 format. Write your answers as hexadecimal digits.

   (a) -4

Answer:

$$
\begin{aligned}
&-4+127\\
&=123=7B_{16}
\end{aligned}
$$

(b) 4

Answer:

$$
\begin{aligned}
&4+127\\
&=131=83_{16}
\end{aligned}
$$

(d) 7

Answer:

$$
\begin{aligned}
&7+127\\
&=134=86_{16}
\end{aligned}
$$

(e) -7

Answer:

$$
\begin{aligned}
&-7+127\\
&=120=78_{16}
\end{aligned}
$$

8. Single precision floating point numbers have 7 digit decimal precision and double floating point numbers have 15 digit precision. Explain how these precision values are arrived at, and what they mean.

9. Convert the following numbers to IEEE 754 single precision numbers. Give your answers as hexadecimal numbers (do not give me binary, I cannot read it accurately. I WILL misread it and you WILL lose points).

(a) 7.25

Answer: Since the decimal is positive, the sign bit is 0

$$7.25 = (1.1101)_2 \times 2^2 \text{ from Part 1a}$$

Mantissa: $(1101)_2$

Exponent: $+2$

$$\begin{aligned}
&2 + 127 \\
&= (1000\ 0001)_2 \\
&\Rightarrow (0100\ 0000\ 1110\ 1000)_2 = (40E8\ 0000)_{16}
\end{aligned}$$

(b) 13.5

Answer: Since the decimal is positive, the sign bit is 0

$$13.5 = (1.1011)_2 \times 2^3 \text{ from Part 1b}$$

Mantissa: $(1011)_2$

Exponent: $+3$

$$\begin{aligned}
&3 + 127 \\
&= (1000\ 0010)_2 \\
&\Rightarrow (0100\ 0001\ 0101\ 1000)_2 = (4158\ 0000)_{16}
\end{aligned}$$

(c) 0.5625

Answer: Since the decimal is positive, the sign bit is 0

$$0.5625 = (1.001)_2 \times 2^{-1} \text{ from Part 1c}$$

Mantissa: $(001)_2$

Exponent: -1

$$\begin{aligned} &-1 + 127 \\ &= (0111\ 1110)_2 \\ &\Rightarrow (0011\ 1111\ 0001\ 0000)_2 = (3F10\ 0000)_{16} \end{aligned}$$

(d) 0.125

Answer: Since the decimal is positive, the sign bit is 0

$$0.125 = (1.0)_2 \times 2^{-3} \text{ from Part 1d}$$

Mantissa: 0

Exponent: -3

$$\begin{aligned} &-3 + 127 \\ &= (0111\ 1100)_2 \\ &\Rightarrow (0011\ 1110\ 0000\ 0000)_2 = (3E00\ 0000)_{16} \end{aligned}$$

(e) 127.625

Answer: Since the decimal is positive, the sign bit is 0

$$127.625 = (1.111111101)_2 \times 2^6 \text{ from Part 1e}$$

Mantissa: $(111111101)_2$

Exponent: 6

$$
\begin{aligned}
&6 + 127 \\
&= (1000\ 0101)_2 \\
&\Rightarrow (0100\ 0010\ 1111\ 1111\ 0100)_2 = (42FF\ 4000)_{16}
\end{aligned}
$$

(f) 51025.025

Answer: Since the decimal is positive, the sign bit is 0

$$51025.025 = (1.1000111010100010000011)_2 \times 2^{15} \text{ from Part 1f}$$

Mantissa: $(1000111010100010000011)_2$

Exponent: 15

$$
\begin{aligned}
&15 + 127 \\
&= (1000\ 1110)_2 \\
&\Rightarrow (0100\ 0111\ 0100\ 0111\ 0101\ 0001\ 0000\ 0110)_2 = (4747\ 5106)_{16}
\end{aligned}
$$

10. For each of the following truth tables:

- Give the DNF equation for the table.
- Give the minimal equation.
- Using Boolean algebra show the two Boolean equations are equivalent.
- Draw the circuit in Logisim. Be prepared to draw the circuit by hand.

(a)

A	**B**	**C**	**f(A,B,C)**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Answer:

A \ BC	00	01	11	10
0	0	1	0	1
1	0	1	0	1

DNF: $\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}C + AB\overline{C}$

Minimal equation: $\overline{B}C + B\overline{C}$

Show $\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}C + AB\overline{C} = \overline{B}C + B\overline{C}$

$$\begin{aligned}
\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}C + AB\overline{C} &= \overline{A}(\overline{B}C + B\overline{C}) + A(\overline{B}C + B\overline{C}) \\
&= (\overline{A} + A)(\overline{B}C + B\overline{C}) \\
&= (1)(\overline{B}C + B\overline{C}) \\
&= \overline{B}C + B\overline{C}
\end{aligned}$$

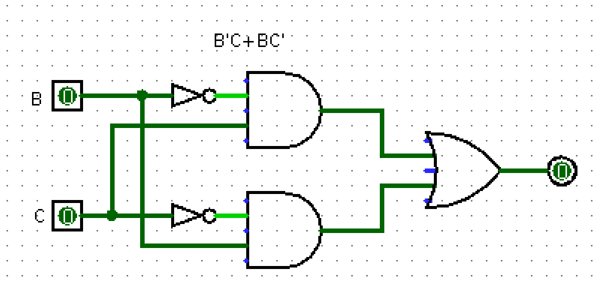

Figure 1: Circuit Diagram of the Truth Table 10a in Logisim

(b)

A	**B**	**C**	**f(A,B,C)**
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Answer:

A \ BC	00	01	11	10
0	1	1	1	1
1	0	1	1	0

DNF: $\overline{ABC} + \overline{AB}C + \overline{A}BC + \overline{A}B\overline{C} + \overline{AB}C + A\overline{B}C + \overline{A}BC + ABC$

Minimal equation: $\overline{A} + C$

Show $\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + \overline{A}B\overline{C} + \overline{A}\overline{B}C + A\overline{B}C + \overline{A}BC + ABC = \overline{A} + C$

$$
\begin{aligned}
&\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + \overline{A}B\overline{C} + \overline{A}\overline{B}C + A\overline{B}C + \overline{A}BC + ABC \\
&= \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C + ABC \\
&= \overline{A}(\overline{B}\overline{C} + \overline{B}C + BC + B\overline{C}) + AC(\overline{B} + B) \\
&= \overline{A}(\overline{B} + \overline{C} + BC + \overline{B}C + B\overline{C}) + AC(1) \\
&= \overline{A}(\overline{B} + \overline{B}C + \overline{C} + B\overline{C} + BC) + AC \\
&= \overline{A}(\overline{B} + \overline{C} + BC) + AC \\
&= \overline{A}(\overline{B} + \overline{C} + C) + AC \\
&= \overline{A}(\overline{B} + 1) + AC \\
&= \overline{A}(1) + AC \\
&= \overline{A} + AC \\
&= \overline{A} + C
\end{aligned}
$$

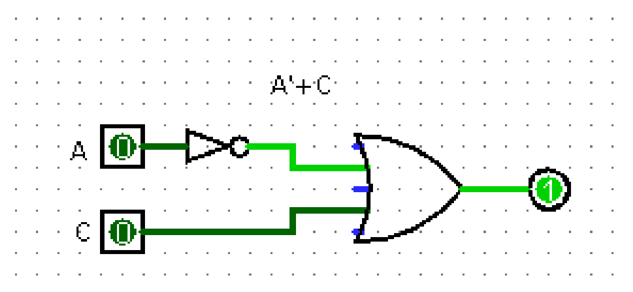

Figure 2: Circuit Diagram of the Truth Table 10b in Logisim

(c)

A	**B**	**C**	**f(A,B,C)**
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Answer:

A \ BC	00	01	11	10
0	1	0	1	0
1	1	1	1	1

DNF: $\overline{ABC} + A\overline{BC} + \overline{A}BC + ABC + A\overline{BC} + A\overline{B}C + ABC + AB\overline{C}$

Minimal equation: $A + \overline{BC} + BC$

Show $\overline{ABC}+A\overline{BC}+\overline{A}BC+ABC+A\overline{BC}+A\overline{B}C+ABC+AB\overline{C} = A+\overline{BC}+BC$

$$
\begin{aligned}
&\overline{ABC} + A\overline{BC} + \overline{A}BC + ABC + A\overline{BC} + A\overline{B}C + ABC + AB\overline{C} \\
&= \overline{ABC} + A\overline{BC} + \overline{A}BC + ABC + A\overline{B}C + AB\overline{C} \\
&= \overline{ABC} + \overline{A}BC + A(\overline{BC} + BC + \overline{B}C + B\overline{C}) \\
&= \overline{ABC} + \overline{A}BC + A(\overline{B} + \overline{C} + BC + \overline{B}C + B\overline{C}) \\
&= \overline{ABC} + \overline{A}BC + A(\overline{B} + \overline{C} + BC) \\
&= \overline{ABC} + \overline{A}BC + A(\overline{B} + \overline{C} + C) \\
&= \overline{ABC} + \overline{A}BC + A(\overline{B} + 1) \\
&= \overline{ABC} + \overline{A}BC + A(1) \\
&= \overline{A}(\overline{BC} + BC) + A \\
&= \overline{BC} + BC + A \\
&= A + \overline{BC} + BC
\end{aligned}
$$

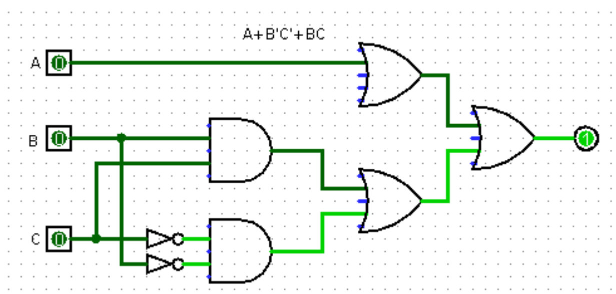

Figure 3: Circuit Diagram of the Truth Table 10c in Logisim