605.744: Information Retrieval
Problem Set (Module 9)

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1. (30%) Give a short definition or explanation of the following concepts:

   - web spam

     Answer: Content on the web that is designed to be favorable in relevance even though they may be completely irrelevant.
   - Broders' taxonomy

     Answer: Classification of search queries by users into 3 categories: informational, navigational, and transactional.
   - out-degree

     Answer: In a directed graph, out-degree is the number of edges going out of a vertex.
   - robots exclusion protocol

     Answer: Also known as robots.txt, it's used by web pages to inform crawlers on which portions to avoid indexing.
   - priority queue (in the context of web crawling)

     Answer:

2. (20%) Describe in your own words the process described in the course text to efficiently identify near duplicate documents in a large collection.

   Answer:

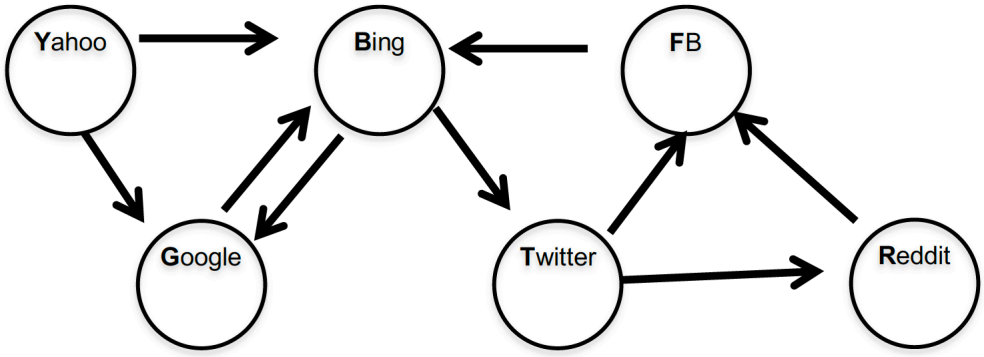
3. For this problem work with the directed web graph shown below. In the graph there are six nodes: Y, B, F, G, T, R (for the websites Yahoo, Bing, Facebook, Google, Twitter, and Reddit). Use a teleport probability of 0.20. Assume no other pages or links exist beside those shown in the figure.

   (a) (15%) Provide (i.e., write) the six recurrence equations that indicate how to iteratively calculate the PageRank score of each page at time t given scores from time t-1.

   Answer: Using the recurrence equation:

$$PR(a) = \frac{q}{N} + (1 - q) \sum_{i=1}^{n} \frac{PR(p_i)}{C(p_i)} \tag{1}$$

   With the values of the sites being initialized to equal probabilities:



Time	Y	B	F	G	T	R
t = 0	0.167	0.167	0.167	0.167	0.167	0.167

$$PR(Y,t_i) = \frac{0.20}{6} + 0$$
$$PR(B,t_i) = \frac{0.20}{6} + (0.80)\left(\frac{PR(Y,t_{i-1})}{C(Y)} + \frac{PR(F,t_{i-1})}{C(F)} + \frac{PR(G,t_{i-1})}{C(G)}\right)$$
$$PR(F,t_i) = \frac{0.20}{6} + (0.80)\left(\frac{PR(R,t_{i-1})}{C(R)} + \frac{PR(T,t_{i-1})}{C(T)}\right)$$
$$PR(G,t_i) = \frac{0.20}{6} + (0.80)\left(\frac{PR(B,t_{i-1})}{C(B)} + \frac{PR(Y,t_{i-1})}{C(Y)}\right)$$
$$PR(T,t_i) = \frac{0.20}{6} + (0.80)\left(\frac{PR(B,t_{i-1})}{C(B)}\right)$$
$$PR(R,t_i) = \frac{0.20}{6} + (0.80)\left(\frac{PR(T,t_{i-1})}{C(T)}\right)$$

(b) (25%) Using the brute-force iterative method of calculation shown in the video lecture calculate two iterations of PageRank scores for each page in the graph. Be sure to show scores at times t=0, t=1, and finally at t=2. Report scores using three digits of precision (e.g., 0.247, not 0.2 or 0.24696485932). Show work and do not merely provide a table of values.

Answer:

$$\begin{aligned} PR(Y,t_1) &= \frac{0.20}{6} + 0 \\ &= \frac{1}{30} \\ &= 0.033 \end{aligned}$$

$$
\begin{aligned}
PR(B,t_1) &= \frac{0.20}{6} + (0.80)\left(\frac{PR(Y,t_0)}{C(Y)} + \frac{PR(F,t_0)}{C(F)} + \frac{PR(G,t_0)}{C(G)}\right) \\
&= \frac{1}{30} + (0.80)\frac{1}{6}\left(\frac{1}{2} + \frac{1}{1} + \frac{1}{1}\right) \\
&= \frac{1}{30} + \frac{1}{3} \\
&= 0.367 \\
PR(B,t_2) &= \frac{1}{30} + (0.80)\left(\frac{0.033}{2} + \frac{0.233}{1} + \frac{0.167}{1}\right) \\
&= 0.367
\end{aligned}
$$

$$
\begin{aligned}
PR(F,t_1) &= \frac{0.20}{6} + (0.80)\left(\frac{PR(R,t_0)}{C(R)} + \frac{PR(T,t_0)}{C(T)}\right) \\
&= \frac{1}{30} + (0.80)\frac{1}{6}\left(\frac{1}{1} + \frac{1}{2}\right) \\
&= \frac{1}{30} + \frac{1}{5} \\
&= 0.233 \\
PR(F,t_2) &= \frac{1}{30} + (0.80)\left(\frac{0.100}{1} + \frac{0.100}{2}\right) \\
&= 0.153
\end{aligned}
$$

$$
\begin{aligned}
PR(G,t_1) &= \frac{0.20}{6} + (0.80)\left(\frac{PR(B,t_0)}{C(B)} + \frac{PR(Y,t_0)}{C(Y)}\right) \\
&= \frac{1}{30} + (0.80)\frac{1}{6}\left(\frac{1}{2} + \frac{1}{2}\right) \\
&= 0.167 \\
PR(G,t_2) &= \frac{1}{30} + (0.80)\left(\frac{0.367}{2} + \frac{0.033}{2}\right) \\
&= 0.193
\end{aligned}
$$

$$
\begin{aligned}
PR(T,t_1) &= \frac{0.20}{6} + (0.80)\left(\frac{PR(B,t_0)}{C(B)}\right) \\
&= \frac{1}{30} + (0.80)\frac{1}{6}\left(\frac{1}{2}\right) \\
&= \frac{1}{30} + \frac{1}{15} \\
&= 0.100 \\
PR(T,t_2) &= \frac{1}{30} + (0.80)\left(\frac{0.367}{2}\right) \\
&= 0.180
\end{aligned}
$$

$$
\begin{aligned}
PR(R, t_1) &= \frac{0.20}{6} + (0.80)\left(\frac{PR(T, t_0)}{C(T)}\right) \\
&= \frac{1}{30} + (0.80)\frac{1}{6}\left(\frac{1}{2}\right) \\
&= \frac{1}{30} + \frac{1}{15} \\
&= 0.100 \\
PR(R, t_2) &= \frac{1}{30} + (0.80)\left(\frac{0.100}{2}\right) \\
&= 0.073
\end{aligned}
$$

	Y	**B**	**F**	**G**	**T**	**R**
t = 0	0.167	0.167	0.167	0.167	0.167	0.167
t = 1	0.033	0.367	0.233	0.167	0.100	0.100
t = 2	0.033	0.367	0.153	0.193	0.180	0.073

(c) (5%) Which page (or pages) has/have the lowest PageRank score after two iterations?
Answer: Yahoo has the lowest PageRank score after two iterations with a value of 0.033.

(d) (5%) Which page (or pages) has/have the highest PageRank score after two iterations?
Answer: Bing has the highest PageRank score after two iterations with a value of 0.367.