605.611 - Foundations of Computer Architecture

Assignment 02

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1. Convert the following fixed point numbers to binary fixed point. Give both the actual values, and normalize the values so that they have a binary 1 as the value for the left of the decimal point.

   (a) 7.25

   Answer: Integral part: $7 = (0111)_2$

   Repeatedly multiplying the fractional part by 2:

   $$0.25$$

   $$
   \begin{array}{rcccc}
    & 0 & . & 2 & 5 \\
   \times & & & & 2 \\
   \hline
   = & 0 & . & 5 & 0 \\
   \times & & & & 2 \\
   \hline
    & 1 & . & 0 & 0 \\
   \end{array}
   $$

   $$= (10)_2$$

   $$\Rightarrow 7.25 = (111.01)_2, \text{(normalized to } (1.1101)_2 \times 2^2)$$

   (b) 13.5

   Answer: Integral part: $13 = (1101)_2$

Repeatedly multiplying the fractional part by 2:

0.50

$$= \begin{array}{r} 0 \quad . \quad 5 \quad 0 \\ \times \quad\qquad\quad 2 \\ \hline 1 \quad . \quad 0 \quad 0 \end{array}$$

$$= (1)_2$$

$$\Rightarrow 13.5 = (1101.1)_2, (\text{normalized to } (1.1011)_2 \times 2^3)$$

(c) 0.5625

Answer: Integral part: 0

Repeatedly multiplying the fractional part by 2:

0.5625

$$= \begin{array}{r} 0 \quad . \quad 5 \quad 6 \quad 2 \quad 5 \\ \times \qquad\qquad\qquad\qquad 2 \\ \hline 1 \quad . \quad 1 \quad 2 \quad 5 \quad 0 \\ \times \qquad\qquad\qquad\qquad 2 \\ \hline 0 \quad . \quad 2 \quad 5 \quad 0 \quad 0 \\ \times \qquad\qquad\qquad\qquad 2 \\ \hline 0 \quad . \quad 5 \quad 0 \quad 0 \quad 0 \\ \times \qquad\qquad\qquad\qquad 2 \\ \hline 1 \quad . \quad 0 \quad 0 \quad 0 \quad 0 \end{array}$$

$$= (1001)_2$$

$$\Rightarrow 0.5625 = (0.1001)_2, (\text{normalized to } (1.001)_2 \times 2^{-1})$$

(d) 0.125

Answer: Integral part: 0

Repeatedly multiplying the fractional part by 2:

0.125

$$
\begin{array}{rcccc}
0 & . & 1 & 2 & 5 \\
\times & & & & 2 \\
\hline
0 & . & 2 & 5 & 0 \\
= \times & & & & 2 \\
\hline
0 & . & 5 & 0 & 0 \\
\times & & & & 2 \\
\hline
1 & . & 0 & 0 & 0
\end{array}
$$

$= (001)_2$

$\Rightarrow 0.5625 = (0.001)_2$, (normalized to $(1.0)_2 \times 2^{-3}$)

(e) 127.625

Answer: Integral part: $127 = (01111111)$

Repeatedly multiplying the fractional part by 2:

0.625

$$
\begin{array}{rcccc}
0 & . & 6 & 2 & 5 \\
\times & & & & 2 \\
\hline
1 & . & 2 & 5 & 0 \\
= \times & & & & 2 \\
\hline
0 & . & 5 & 0 & 0 \\
\times & & & & 2 \\
\hline
1 & . & 0 & 0 & 0
\end{array}
$$

$= (101)_2$

$\Rightarrow 127.625 = (1111111.101)_2$, (normalized to $(1.111111101)_2 \times 2^6$)

(f) 51,025.025

Answer: Integral part: $51025 =$

$\Rightarrow 51025$

$$
= \; \begin{array}{r} 25512 \\ 2\,\overline{)51025} \\ 4 \\ \hline 11 \end{array} \quad \begin{array}{r} 12756 \\ 2\,\overline{)25512} \\ 2 \\ \hline 05 \end{array} \quad \begin{array}{r} 6378 \\ 2\,\overline{)12756} \\ 12 \\ \hline 07 \end{array} \quad \begin{array}{r} 3189 \\ 2\,\overline{)6378} \\ 6 \\ \hline 03 \end{array} \quad \begin{array}{r} 1594 \\ 2\,\overline{)3189} \\ 2 \\ \hline 11 \end{array} \quad \begin{array}{r} 797 \\ 2\,\overline{)1594} \\ 14 \\ \hline 19 \end{array}
$$

$$
\begin{array}{r} 10 \\ \hline 10 \end{array} \quad \begin{array}{r} 4 \\ \hline 15 \end{array} \quad \begin{array}{r} 6 \\ \hline 15 \end{array} \quad \begin{array}{r} 2 \\ \hline 17 \end{array} \quad \begin{array}{r} 10 \\ \hline 18 \end{array} \quad \begin{array}{r} 18 \\ \hline 14 \end{array}
$$

$$
\begin{array}{r} 10 \\ \hline 02 \end{array} \quad \begin{array}{r} 14 \\ \hline 11 \end{array} \quad \begin{array}{r} 14 \\ \hline 16 \end{array} \quad \begin{array}{r} 16 \\ \hline 18 \end{array} \quad \begin{array}{r} 18 \\ \hline 09 \end{array} \quad \begin{array}{r} 14 \\ \hline 0 \end{array}
$$

$$
\begin{array}{r} 2 \\ \hline 05 \end{array} \quad \begin{array}{r} 10 \\ \hline 12 \end{array} \quad \begin{array}{r} 16 \\ \hline 0 \end{array} \quad \begin{array}{r} 18 \\ \hline 0 \end{array} \quad \begin{array}{r} 8 \\ \hline 1 \end{array}
$$

$$
\begin{array}{r} 4 \\ \hline 1 \end{array} \quad \begin{array}{r} 12 \\ \hline 0 \end{array}
$$

$$
\begin{array}{r} 398 \\ 2\,\overline{)797} \\ 6 \\ \hline 19 \end{array} \quad \begin{array}{r} 199 \\ 2\,\overline{)398} \\ 2 \\ \hline 19 \end{array} \quad \begin{array}{r} 99 \\ 2\,\overline{)199} \\ 18 \\ \hline 19 \end{array} \quad \begin{array}{r} 49 \\ 2\,\overline{)99} \\ 8 \\ \hline 19 \end{array} \quad \begin{array}{r} 24 \\ 2\,\overline{)49} \\ 4 \\ \hline 09 \end{array} \quad \begin{array}{r} 12 \\ 2\,\overline{)24} \\ 2 \\ \hline 04 \end{array} \quad \begin{array}{r} 6 \\ 2\,\overline{)12} \\ 12 \\ \hline 0 \end{array} \quad \begin{array}{r} 3 \\ 2\,\overline{)6} \\ 6 \\ \hline 0 \end{array}
$$

$$
\begin{array}{r} 18 \\ \hline 17 \end{array} \quad \begin{array}{r} 18 \\ \hline 18 \end{array} \quad \begin{array}{r} 18 \\ \hline 1 \end{array} \quad \begin{array}{r} 18 \\ \hline 1 \end{array} \quad \begin{array}{r} 8 \\ \hline 1 \end{array} \quad \begin{array}{r} 4 \\ \hline 0 \end{array}
$$

$$
\begin{array}{r} 16 \\ \hline 1 \end{array} \quad \begin{array}{r} 18 \\ \hline 0 \end{array}
$$

$$
\begin{array}{r} 1 \\ 2\,\overline{)3} \\ 2 \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ 2\,\overline{)1} \end{array}
$$

$\Rightarrow (1100\ 0111\ 0101\ 0001)_2$

4

Repeatedly multiplying the fractional part by 2:

0.025

$$
\begin{array}{r}
0\ .\ 0\ 2\ 5 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 0\ 5\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 1\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 2\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 4\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 8\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
1\ .\ 6\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
1\ .\ 2\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 4\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
0\ .\ 8\ 0\ 0 \\
\times \qquad\ 2 \\
\hline
1\ .\ 6\ 0\ 0 \\
\times \qquad\qquad\ 2 \\
\hline
1\ .\ 2\ 0\ 0
\end{array}
$$

(The line before $1\ .\ 6\ 0\ 0$ is marked with $=$.)

The pattern $(0000011)_2$ appears to keep repeating

$\Rightarrow 51,025.025 = (1100011101010001.0000011)_2,$

(normalized to $(1.1000111010100010000011)_2 \times 2^{15})$

5

(g) 7.1

Answer: Integral part: $7 = (111)_2$

Repeatedly multiplying the fractional part by 2:

0.1

$$
\begin{array}{r}
0 \;.\; 1 \\
\times \qquad 2 \\
\hline
0 \;.\; 2 \\
\times \qquad 2 \\
\hline
0 \;.\; 4 \\
\times \qquad 2 \\
\hline
0 \;.\; 8 \\
\times \qquad 2 \\
\hline
1 \;.\; 6 \\
= \quad \times \qquad 2 \\
\hline
1 \;.\; 2 \\
\times \qquad 2 \\
\hline
0 \;.\; 4 \\
\times \qquad 2 \\
\hline
0 \;.\; 8 \\
\times \qquad 2 \\
\hline
1 \;.\; 6 \\
\times \qquad 2 \\
\hline
1 \;.\; 2
\end{array}
$$

The pattern $(0011)_2$ appears to keep repeating

$\Rightarrow 7.1 = (111.0\overline{000110011})_2,$

(normalized to $(1.110\overline{00110011})_2 \times 2^2$)

(h) 5.2

Answer: Integral part: $5 = (101)_2$

Repeatedly multiplying the fractional part by 2:

0.2

$$
\begin{array}{r}
0 \quad . \quad 2 \\
\times \qquad 2 \\
\hline
0 \quad . \quad 4 \\
\times \qquad 2 \\
\hline
0 \quad . \quad 8 \\
\times \qquad 2 \\
\hline
1 \quad . \quad 6 \\
= \times \qquad 2 \\
\hline
1 \quad . \quad 2 \\
\times \qquad 2 \\
\hline
0 \quad . \quad 4 \\
\times \qquad 2 \\
\hline
0 \quad . \quad 8 \\
\times \qquad 2 \\
\hline
1 \quad . \quad 6 \\
\end{array}
$$

The pattern $(0011)_2$ appears to keep repeating

$\Rightarrow 7.1 = (101.\overline{00110011})_2,$

(normalized to $(1.01\overline{00110011})_2 \times 2^2$)

5. Convert the following from decimal to excess 127 format. Write your answers as hexadecimal digits.

   (a) -4

Answer:

$$-4 + 127$$
$$= 123 = 7B_{16}$$

(b) 4

Answer:

$$4 + 127$$
$$= 131 = 83_{16}$$

(d) 7

Answer:

$$7 + 127$$
$$= 134 = 86_{16}$$

(e) -7

Answer:

$$-7 + 127$$
$$= 120 = 78_{16}$$

8. Single precision floating point numbers have 7 digit decimal precision and double floating point numbers have 15 digit precision. Explain how these precision values are arrived at, and what they mean.

9. Convert the following numbers to IEEE 754 single precision numbers. Give your answers as hexadecimal numbers (do not give me binary, I cannot read it accurately. I WILL misread it and you WILL lose points).

(a) 7.25

Answer: Since the decimal is positive, the sign bit is 0

$$7.25 = (1.1101)_2 \times 2^2 \text{ from Part 1a}$$

Mantissa: $(1101)_2$

Exponent: $+2$

$$2 + 127$$
$$= (1000\ 0001)_2$$
$$\Rightarrow (0100\ 0000\ 1110\ 1000)_2 = (40E8\ 0000)_{16}$$

(b) 13.5

Answer: Since the decimal is positive, the sign bit is 0

$$13.5 = (1.1011)_2 \times 2^3 \text{ from Part 1b}$$

Mantissa: $(1011)_2$

Exponent: $+3$

$$3 + 127$$
$$= (1000\ 0010)_2$$
$$\Rightarrow (0100\ 0001\ 0101\ 1000)_2 = (4158\ 0000)_{16}$$

(c) 0.5625

Answer: Since the decimal is positive, the sign bit is 0

$$0.5625 = (1.001)_2 \times 2^{-1} \text{ from Part 1c}$$

Mantissa: $(001)_2$

Exponent: -1

$$-1 + 127$$
$$= (0111\ 1110)_2$$
$$\Rightarrow (0011\ 1111\ 0001\ 0000)_2 = (3F10\ 0000)_{16}$$

(d) 0.125

Answer: Since the decimal is positive, the sign bit is 0

$$0.125 = (1.0)_2 \times 2^{-3} \text{ from Part 1d}$$

Mantissa: 0

Exponent: -3

$$-3 + 127$$
$$= (0111\ 1100)_2$$
$$\Rightarrow (0011\ 1110\ 0000\ 0000)_2 = (3E00\ 0000)_{16}$$

(e) 127.625

Answer: Since the decimal is positive, the sign bit is 0

$$127.625 = (1.111111101)_2 \times 2^6 \text{ from Part 1e}$$

Mantissa: $(111111101)_2$

Exponent: 6

$$6 + 127$$
$$= (1000\ 0101)_2$$
$$\Rightarrow (0100\ 0010\ 1111\ 1111\ 0100)_2 = (42FF\ 4000)_{16}$$

(f) 51025.025

Answer: Since the decimal is positive, the sign bit is 0

$$51025.025 = (1.1000111010100010000011)_2 \times 2^{15} \text{ from Part 1f}$$
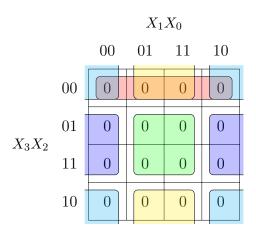
Mantissa: $(1000111010100010000011)_2$

Exponent: 15

$$15 + 127$$
$$= (1000\ 1110)_2$$
$$\Rightarrow (0100\ 0111\ 0100\ 0111\ 0101\ 0001\ 0000\ 0110)_2 = (4747\ 5106)_{16}$$

10. For each of the following truth tables:

(a) Give the DNF equation for the table.

(b) Give the minimal equation.

(c) Using Boolean algebra show the two Boolean equations are equivalent.

(d) Draw the circuit in Logisim. Be prepared to draw the circuit by hand.

A	B	C	f(A,B,C)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$X_1 X_0$$

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_3 X_2$$

A	B	C	f(A,B,C)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

A	B	C	f(A,B,C)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1