

605.621 - Foundations of Algorithms

Assignment 01

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Part 1

Please answer all ten (10) questions below.

1. [10 pts, insertion sort bug]

Find the bug in the insertion code given in module 1 lecture notes. Show with an example where the bug causes the sort happening incorrectly.

2. [10 pts, proof by induction]

Suppose we have the following sequence: $a_1 = 1, a_2 = 3, a_k = a_{k-2} + 2a_{k-1}$, for all integers $k \geq 3$. Show that for all integers $n \geq 1$, given the sequence a_1, a_2, \dots, a_k as defined above, a_n is always odd.

3. [10 pts, proof by contradiction]

Prove that $\log_2(3)$ is irrational.

4. [30 pts, time and space complexity]

For insertion sort algorithm,

- (a) Compute the best-case time complexity
- (b) Compute the worst-case time complexity
- (c) Compute the space complexity

5. [40 pts, algorithm analysis]

The **loop invariance** concept is explained on page 19 of the textbook. During our programming of the insertion sort algorithm in class, we implicitly used this concept while we design/program the algorithm loops in a Python script.

Read the relevant pages on the textbook and use loop invariance to solve problem 2-2 (page 40).